TRACKING ERROR
PROPAGATION AND ORBIT
PREDICTION PROGRAM

R. L. Mobley, L. N. Rowell and M. C. Smith

prepared for
UNITED STATES AIR FORCE PROJECT RAND
TRACKING ERROR PROPAGATION AND ORBIT PREDICTION PROGRAM

R. L. Mobley, L. N. Rowell and M. C. Smith
PREFACE

The detection and tracking of space vehicles and the prediction of their trajectories continues to be of vital importance to the Air Force and other agencies concerned with the national defense.

With the proliferation of both ground-based and space-borne sensors that are capable of detecting and tracking earth orbiting objects, the effective deployment and management of these sensors in the total surveillance function becomes the pressing concern.

This Memorandum describes a computer program designed for these kinds of studies. The program was especially designed for error analysis of deep-space-interplanetary probes but it is equally useful for low-altitude partial-orbit reentry vehicles in investigating the capabilities of various combinations of sensors and their optimum deployment. Reentry errors resulting from uncertainties in the area-mass-density parameter can also be computed.

The program is a modified version of a program which was developed for the Special Projects Branch, Theoretical Division of Goddard Space Flight Center, NASA, by the Sperry Rand Systems Group of the Sperry Gyroscope Company.

This program was used to obtain results for the APSC and ADC Joint Mission Analysis on "Surveillance of Objects in Space in the 1970s." More recently, it was used in Rand studies of the influence of down-range trackers in the determination of initial conditions and CEP.

Dr. M. C. Smith is an Advisory Physicist with IBM's Center for Scientific Study (Federal Systems Division) at Westlake, California, and is a consultant to The Rand Corporation.
SUMMARY

The computer program described in this report was designed to simulate a space flight and its surveillance by ground- and/or satellite-based sensors and reveal the effects of sensor and certain modeling errors on the estimation of a space vehicle's path. The statistical estimation is based upon an extension of the Schmidt-Kalman minimum variance technique.

This Memorandum is in two parts. The first is a mathematical formulation of the model used and is complete enough to be helpful in following the code. The second part, found in the appendix, can serve as a guide in using the program. A format of the input is shown with a description and the output to be expected.

A listing and trial solution are available from the authors upon request.
CONTENTS

PREFACE ......................................................... iii

SUMMARY ....................................................... v

Section
I. INTRODUCTION ............................................. 1

II. GENERAL STATEMENT OF PROGRAM OPERATION ............. 2
   Synthetic Data Generation Mode ........................... 2
   Orbit Determination Mode ................................. 3
   Variance-Covariance Propagation Mode ................... 3

III. MODEL DESCRIPTION ...................................... 4
   Coordinate Systems ....................................... 4
   Equations of Motion ..................................... 5
   Definition of Observing Sensors ......................... 13
   Determination of Data Points ............................. 16

IV. DEFINITION OF PARAMETERS ............................... 18
   State Parameters ....................................... 18
   Observed Parameters ..................................... 31

V. STATISTICAL ESTIMATION .................................. 36
   The Initial Estimate of State ............................ 36
   Improving the Estimate with Data ....................... 37

Appendix
A. DETAILED PROGRAM INPUT ................................. 49
B. SUBROUTINE DIRECTORY .................................. 64

REFERENCES .................................................. 67
I. INTRODUCTION

The function of the Tracking Error Propagation and Orbit Prediction (TEPOP) program is to demonstrate statistically the effects of sensor errors and certain modeling errors in the estimation of a space vehicle's path. The statistical estimation is based upon an extension of the Schmidt-Kalman minimum variance technique. The equations of motion are three degrees of freedom, point mass, solved in the Encke form, with perturbations included for atmospheric drag, earth's oblateness, and gravitational attraction of the Sun, Moon, Venus, Mars, and Jupiter.

The program was originally developed by the Sperry Rand Systems Group of the Sperry Gyroscope Company for the Special Projects Branch, Theoretical Division, Goddard Space Flight Center, NASA;\(^{(1)}\) however, the program has subsequently been modified to better meet the needs of researchers involved in these kinds of studies. These modifications include:

- The ability of sensors to be satellite-based as well as ground-based.
- The ability of sensors to rotate in a local horizontal plane and be limited in azimuth, elevation, and range.
- A drag bias may be included in the state vector.
- Substitution of a planetary ephemeris developed by Jet Propulsion Laboratories.
- Conversion to double-precision computation.

TEPOP is written in FORTRAN IV language and is currently operating on an IBM 360/65 computer.

In this study emphasis is on the formulation and a general description of program operation. Details required for using the program are appended.
II. GENERAL STATEMENT OF PROGRAM OPERATION

Encke's form of the equations of motion is used to define the vehicle path. The local two-body problem is solved numerically using a method developed by Herrick\(^{(2)}\) for general motion in an inverse square force field. The perturbing accelerations which cause deviations from this motion are integrated numerically using either a Runge-Kutta or Adams-Moulton method. Solving the problem in this manner requires that the two-body position and velocity be updated, or rectified, periodically. The criterion for rectifying the two-body path is the size of the deviations from it.

The state transition matrix, relating the differentials of vehicle state at one time to those at another time, is found analytically using only the two-body equations of motion. This simplification requires that the matrix be re-initialized, or rectified, at the same point where the two-body position and velocity are rectified.

The sequential minimum-variance filter used in the program is based upon the Schmidt-Kalman technique.\(^{(3)}\) It uses a set of parameters termed the Goddard Variational Parameters instead of the conventional orbital elements or position and velocity vectors to define the state of the vehicle. This set of parameters may be shown\(^{(4)}\) to be better in the differential correction scheme, especially when long time periods are involved. A set of finite rotations is used to transform corrections in these parameters to corrections in the position and velocity vectors for improving the vehicle path.

There are essentially three different modes of operation of the program, as outlined below.

SYNTHETIC DATA GENERATION MODE

This mode may be used to generate tracking data from as many as 54 sensors. The sensors may be assigned to the Earth, Sun, Moon, Venus, Mars, or Jupiter, and may be either orbiting in an elliptical unperturbed orbit or fixed on the body. They may also be rotating in a local horizontal plane and have their scan space limited in azimuth, elevation, and range. The observed vehicle may be orbiting any of the
bodies or may be on an interplanetary course. A tape of the observation data is created for later use in the orbit prediction mode. This mode may also be used to generate a trajectory without observations.

**ORBIT DETERMINATION MODE**

This mode may be used to determine a best fit to a vehicle’s path using real data as input, or for studying the effects of random noise and/or various biases on determining a vehicle path from synthetic data which may have been generated by this program. The input observation data may have random noise added as they are brought in for processing.

**VARIANCE-COVARIANCE PROPAGATION MODE**

This mode may be used to determine confidence regions for position and velocity from an a priori estimate, random and systematic tracking errors, and certain dynamic modeling errors. External data are not used for this mode.
III. MODEL DESCRIPTION

The success of any parameter estimation depends not only upon the technique being used, but also upon the completeness and accuracy of the simulation model. Most models necessarily contain some simplifying assumptions, and this is true of the model described here. Therefore, it is necessary for the user to be aware of how the model is constructed to determine its applicability to a particular problem.

A. COORDINATE SYSTEMS

The reference coordinate system chosen for this simulation is the rectangular equatorial coordinate system of the mean equator and equinox of 1950.0. Its center may be the Earth, Sun, Moon, Venus, Mars, or Jupiter. The center is determined initially by input and thereafter by which sphere of influence the vehicle is within. The radii of the spheres of influence are assumed to be: Earth = 123.3 eru, Moon = 9.0 eru, Venus = 82.1 eru, Mars = 72.9 eru, Jupiter = 0.0 (actual sphere of influence is not considered), and Sun = ∞. The sun is used as center only when the vehicle is outside all other spheres of influence. The relative positions of the bodies are found from the planet ephemeris tape described in Appendix A.

The rectangular planetocentric coordinate system in which the sensor locations are defined need not be concentric with the reference coordinate system and may rotate about its z-axis, which is perpendicular to the earth's equatorial plane. This rotated system is related to the reference system by an initial rotation, nominally the Greenwich hour angle for the base date of the reference system, a daily rotation rate, and an hourly rotation rate.

The topocentric coordinate system in which the data (azimuth, elevation, range, and range rate) are computed is defined for an ellipsoidal (eccentricity $c = 0.0818135$) body with $z$ pointed north, $y$ east, and $x$ vertical.
B. EQUATIONS OF MOTION

The form of the equations of motion used in this model assumes the dominant acceleration is due to the gravitational attraction of the reference body on the spacecraft. This assumption allows the equations to be separated into two components, Keplerian motion and a perturbed motion. The general form is derived from the general vector equation of motion,

\[ \ddot{\mathbf{R}} = -\frac{\mu_k}{r^3} \mathbf{R} + \mathbf{F}_a + \mathbf{F}_o + \mathbf{F}_d \]

by substituting

\[ \mathbf{R} = \mathbf{R}_{tb} + \mathbf{E} \]

where \( \mathbf{R}_{tb} \) is the two-body position which satisfies the differential equation

\[ \ddot{\mathbf{R}}_{tb} = -\frac{\mu_k}{r_{tb}^3} \mathbf{R}_{tb} \tag{1} \]

Therefore, the differential equation for the Encke term \( \mathbf{E} \), which must be solved by a numerical procedure, is:

\[ \ddot{\mathbf{E}} = -\frac{\mu_k}{r_{tb}^3} \left[ \frac{r_{tb}^3}{r^3} - 1 \right] \mathbf{R} + \mathbf{E} + \mathbf{F}_a + \mathbf{F}_o + \mathbf{F}_d \tag{2} \]

The first term is the deviation in the dominant acceleration term due to the displacement of the true path from the two-body path. The gravitational constant \( \mu_k \) may refer to any of the bodies which may become the reference center; the values of these constants are defined relative to that given for Earth as

*Symbols are either defined when used, or appear in Table 2 or Appendix A.
Earth $\mu_1 = \mu_e = 19.9094165 \, \text{eru}^3/\text{hr}^2$ nominally

Sun $\mu_2 = 332957.29 \, \mu_e \, \text{eru}^3/\text{hr}^2$

Moon $\mu_3 = 0.012299896 \, \mu_e \, \text{eru}^3/\text{hr}^2$

Venus $\mu_4 = 0.81476890 \, \mu_e \, \text{eru}^3/\text{hr}^2$

Mars $\mu_5 = 0.1078210 \, \mu_e \, \text{eru}^3/\text{hr}^2$

Jupiter $\mu_6 = 317.887 \, \mu_e \, \text{eru}^3/\text{hr}^2$

In the estimation process required in the trajectory calculation, the actual trajectory and the conic section must be kept very near one another. The size of $|z|$ is a measure of the distance the two solutions have drifted apart. It is used to initiate a rectification, i.e., the selection of a new conic orbit for which the initial conditions are the same as those of the actual orbit at the time of rectification.

1. Perturbing Accelerations Due to Other Planets

The perturbing gravitational accelerations due to the other bodies, $F_a$, may be derived from the N body equations as

$$F_a = \sum_{i \neq k, i=1}^{6} -\frac{\mu_i}{r_{(cb)i}^3} \left[ \frac{r_{(cb)i}^3}{r_{(vb)i}^3} - 1 \right] r_{(vb)i} + R$$

where $r_{(vb)} = |R_{(vb)}|$ refers to the position of the vehicle relative to the perturbing body,

and $r_{(cb)} = |R_{(cb)}|$ refers to the position of the perturbing body relative to the reference center.

A further substitution, as suggested by Battin, is made for

$$\left( \frac{r_{(cb)i}^3}{r_{(vb)i}^3} - 1 \right) = \zeta \frac{3 + 3\zeta + \zeta^2}{1 + (1 + \zeta)^{3/2}}$$
where

\[ \zeta = \frac{R + 2R_{(cb)}}{r_{(cb)}^2} \]

A similar substitution is made in Eq. (2) except that

\[ \zeta = \frac{E + 2R_{tb}}{r^2} \]

2. Perturbing Accelerations Due to Earth's Oblateness

The perturbing acceleration due to the oblateness of the earth may be included when the earth is the reference center; its form is

\[ F_0 = \sigma \frac{\sigma}{r} R + \nu \begin{bmatrix} 0^* \\ 0 \\ z \end{bmatrix} \]

where

\[ \sigma = \frac{C J}{r^4} \left[ 2.5 \left( \frac{z}{r} \right)^2 - 0.5 \right] + \frac{C A}{r^5} \left[ 7.5 - 17.5 \left( \frac{z}{r} \right)^2 \right] \frac{z}{r} + \frac{C K}{r^6} \left[ -63 \left( \frac{z}{r} \right)^4 + 42 \left( \frac{z}{r} \right)^2 - 3 \right] \]

\[ \nu = - \frac{C J}{r^4} + \frac{C A}{r^5} \left[ 7.5 \left( \frac{z}{r} \right) - 1.5 \left( \frac{z}{r} \right)^{-1} \right] + \frac{C K}{r^6} \left[ -12 + 28 \left( \frac{z}{r} \right)^2 \right] \]

and \( C_J = 0.064643882 \) (eru\(^5/\)hr\(^2\))
\( C_A = 0.45791657 \cdot 10^{-4} \) (eru\(^6/\)hr\(^2\))
\( C_K = 0.1343886 \cdot 10^{-3} \) (eru\(^7/\)hr\(^2\))

*These brackets denote a vector or column matrix.
3. Perturbing Accelerations Due to Air Drag

The perturbing accelerations due to the presence of the atmosphere may be included when the earth is the reference center; its form is

\[ F_d = -\frac{1}{2} (1 + \Delta)C_d \frac{A}{m} \rho v_r v_r \quad (5) \]

where

- \( |v_r| = v_r \): velocity relative to the rotating earth
- \( C_d \): constant drag coefficient
- \( \Delta \): constant drag bias
- \( \rho \): air density as described below

4. Atmosphere Model

The atmosphere model is one developed by G. Schilling.\(^{(6)}\) The pertinent equations are:

\[ h = r - \frac{(1 - \varepsilon)}{\sqrt{1 - (2\varepsilon - \varepsilon^2) \frac{x^2 + y^2}{r^2}}} = \text{geopotential altitude} \]

where \( r_e = 6378.165 \): equatorial radius,

- \( \varepsilon = 0.08181 \): eccentricity of the geoid,

\[ H = \frac{r h}{r_p + h} = \text{scale height} \]

where \( r_p = 6356.766 \): polar radius.

From Table I find \( i \) such that:

\[ H_i \leq H \leq H_{i+1} \]
The variation of molecular scale temperature is

\[ L = \frac{T_{i+1} - T_i}{H_{i+1} - H_i} \]

and the density is obtained from either:

\[
\rho = P_i \exp \left[ -\frac{mg \cdot 10^5}{RT_i} (H_i - H) \right] \quad \text{when } L = 0
\]

or

\[
\rho = P_i \left( \frac{T_i}{T_i + L(H - R_i)} \right) ^{m_0 g \cdot 10^5 / R L} \quad \text{when } L \neq 0
\]

where \( R^* \) (gas constant) = \( 8.31432 \cdot 10^7 \)

\( g \) (gravitational acceleration) = \( 980.665 \text{ cm/sec}^2 \)

\( m_0 \) (mean molecular mass) = \( 28.964 \text{ gm} \)

Table 1

<table>
<thead>
<tr>
<th>( i )</th>
<th>( H_i )</th>
<th>( P_i )</th>
<th>( T_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1.013 \cdot 10^6</td>
<td>288.160</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>2.263 \cdot 10^5</td>
<td>216.660</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>2.489 \cdot 10^4</td>
<td>216.660</td>
</tr>
<tr>
<td>4</td>
<td>47</td>
<td>1.250 \cdot 10^3</td>
<td>282.660</td>
</tr>
<tr>
<td>5</td>
<td>53</td>
<td>58.3</td>
<td>282.660</td>
</tr>
<tr>
<td>6</td>
<td>75</td>
<td>24.53</td>
<td>196.86</td>
</tr>
<tr>
<td>7</td>
<td>90</td>
<td>1.816</td>
<td>196.86</td>
</tr>
<tr>
<td>8</td>
<td>126</td>
<td>0.0142</td>
<td>322.86</td>
</tr>
<tr>
<td>9</td>
<td>175</td>
<td>19.727 \cdot 10^{-4}</td>
<td>1057.86</td>
</tr>
<tr>
<td>10</td>
<td>200</td>
<td>4.535 \cdot 10^{-4}</td>
<td>1182.86</td>
</tr>
<tr>
<td>11</td>
<td>500</td>
<td>7.221 \cdot 10^{-7}</td>
<td>2082.86</td>
</tr>
<tr>
<td>12</td>
<td>2036.828</td>
<td>5.896 \cdot 10^{-10}</td>
<td>18826.86</td>
</tr>
</tbody>
</table>
5. Solution of the Two-Body Problem

The approach to the solution of the two-body motion is that of Herrick. (2)* The equations of motion of a vehicle in a Keplerian orbit, written in terms of the initial position and velocity vectors \( \mathbf{R}_{\text{tb}_o} \) and \( \dot{\mathbf{R}}_{\text{tb}_o} \) are

\[
\mathbf{R}_{\text{tb}} = f \mathbf{R}_{\text{tb}_o} + g \dot{\mathbf{R}}_{\text{tb}_o} \tag{7}
\]

\[
\dot{\mathbf{R}}_{\text{tb}} = f \dot{\mathbf{R}}_{\text{tb}_o} + g \ddot{\mathbf{R}}_{\text{tb}_o} \tag{8}
\]

where \( f \) and \( g \) are explicit functions of the differential eccentric anomaly of the body in the Keplerian orbit. Using Herrick's approach, \( f, \dot{f}, g, \) and \( g \) are derived; then from Eqs. (7) and (8), \( \mathbf{R}_{\text{tb}} \) and \( \dot{\mathbf{R}}_{\text{tb}} \) are found.

More precisely, given the initial conditions at time \( t_o \)

\[
\mathbf{r}_{\text{tb}_o} = |\mathbf{R}_{\text{tb}_o}|, \quad \mathbf{v}_{\text{tb}_o} = |\dot{\mathbf{R}}_{\text{tb}_o}|, \quad d_{\text{tb}_o} = \mathbf{R}_{\text{tb}_o} \cdot \dot{\mathbf{R}}_{\text{tb}_o}, \quad \frac{1}{a} = \frac{2}{r_{\text{tb}_o}} - \frac{v_{\text{tb}_o}^2}{\mu}
\]

where \( a \) is the semimajor axis of the two-body ellipse, and the values of \( \dot{R}_{\text{tb}} \) and \( \ddot{R}_{\text{tb}} \) on the two-body trajectory are found at some later time \( t = t_o + \Delta t \) by first transforming the elapsed time \( \Delta t \) into a more easily handled variable, either \( \theta \) or \( \beta \). This is accomplished by solving Kepler's equation. The forms of Kepler's equation for elliptic, hyperbolic, and parabolic trajectories are, respectively (omitting the subscripts \( \text{tb} \))

\[
\sqrt{\mu} \Delta t = a^{3/2} (\theta - \sin \theta) + r_o \sqrt{a \sin \theta} + \frac{d_o}{\sqrt{\mu}} a (1 - \cos \theta) \quad \text{(elliptic)}
\]

\[
\sqrt{\mu} \Delta t = (-a)^{3/2} (\sinh \theta - \theta) + r_o \sqrt{-a \sinh \theta} + \frac{d_o}{\sqrt{\mu}} (-a)(\cosh \theta - 1) \quad \text{(hyperbolic)}
\]

*See also W. M. Boyce, "Analytic Derivation for the General Conic Over External Time-Arcs," NASA Manned Spacecraft Center (Internal Report).
\[
\sqrt{\mu} \Delta t = \frac{1}{6} a^{3/2} \theta^3 + r_o \sqrt{a} \theta + \frac{1}{2} \frac{d_o}{\sqrt{\mu}} a \theta^2
\]  
(parabolic)

where \( \theta \) is the differential eccentric anomaly, \( a \) is the orbit semimajor axis, and \( \mu \) is the gravitational attraction of the earth.

The general form of Kepler's equation for the three types of trajectories is

\[
\sqrt{\mu} \Delta t = \beta^2 \frac{f_1(\tau)}{2} + r_o \beta f_3(\tau) + \frac{d_o}{2} \beta^2 f_2(\tau)
\]  
(9)

where \( \beta = \sqrt{|a|} \theta \)

\[
\tau = \beta^2 \left( \frac{1}{a} \right)
\]

\[
f_1(\tau) = \sum_{i=0}^{\infty} \frac{(-\tau)^i}{(2i + 3)!}
\]

\[
f_2(\tau) = \sum_{i=0}^{\infty} \frac{(-\tau)^i}{(2i + 2)!}
\]

\[
f_3(\tau) = 1 - \tau f_1(\tau)
\]

\[
f_4(\tau) = 1 - \tau f_2(\tau)
\]

Once \( \beta \) or \( \theta \) is known, the functions \( f, g, \dot{f}, \dot{g} \) can be computed from the following:

**Elliptic Trajectories**

\[
\begin{align*}
  f &= 1 - \frac{a}{r_o} (1 - \cos \theta) \\
  g &= r_o \sqrt{a/\mu} \sin \theta + \frac{d_o}{\mu} \left( 1 - \cos \theta \right) \\
  \dot{f} &= -\frac{\sqrt{ma}}{rr_o} \sin \theta \\
  \dot{g} &= 1 - \frac{a}{r} (1 - \cos \theta)
\end{align*}
\]  
(10)
where \( r = a(1 - \cos \theta) + r_o \cos \theta + \frac{d_o}{\sqrt{\mu}} \sqrt{a} \sin \theta \)

**Hyperbolic Trajectories**

\[
\begin{align*}
  f &= 1 - \frac{|a|}{r_o} (\cos \theta - 1) \\
  g &= r_o \sqrt{\frac{|a|}{\mu}} \sinh \theta + \frac{|a| d_o}{\mu} (\cosh \theta - 1) \\
  \dot{f} &= -\frac{\mu |a|}{r r_o} \sinh \theta \\
  \dot{g} &= 1 - \frac{|a|}{r} (\cosh \theta - 1)
\end{align*}
\]

where \( r = |a| (\cosh \theta - 1) + r_o \cosh \theta + \frac{d_o}{\sqrt{\mu}} \sqrt{|a|} \sinh \theta \)

**Parabolic Trajectories**

\[
\begin{align*}
  f &= 1 - \frac{1}{2} \frac{\beta^2}{r_o} \\
  g &= \frac{r_o}{\sqrt{\mu}} \beta + \frac{d_o}{2\mu} \beta^2 \\
  \dot{f} &= -\frac{\sqrt{\mu}}{r r_o} \beta \\
  \dot{g} &= 1 - \frac{1}{2r} \beta^2
\end{align*}
\]

where \( r = \frac{1}{2} \beta^2 + r_o + \frac{d_o}{\sqrt{\mu}} \beta \)

The general form of each of the preceding equations is
\[ f = 1 - \frac{1}{r_0} \beta^2 f_2(\tau) \]

\[ g = \frac{r_0}{\sqrt{\mu}} \beta f_3(\tau) + \frac{d_0}{\mu} \beta^2 f_2(\tau) \]

\[ \dot{f} = -\frac{\sqrt{\mu}}{r} \frac{r_0}{\beta} f_3(\tau) \]

\[ \dot{g} = 1 - \frac{1}{r} \beta^2 f_2(\tau) \]

where \( r = \beta^2 f_2(\tau) + r_0 f_4(\tau) + \frac{d_0}{\sqrt{\mu}} \beta f_3(\tau) \)

After \( f, g, \dot{f}, \dot{g} \) have been evaluated, the position and velocity follow from Eqs. (7) and (8).

C. DEFINITION OF OBSERVING SENSORS

The sensors may be placed on or be orbiting any planet in an unperturbed elliptic orbit. All sensors, however, must be on or orbiting the same planet. The sensors may rotate at a constant rate in a horizontal plane and the sensor field of view may be limited in azimuth, elevation, and range.

The space being scanned by a sensor at any time is defined by:

\[ Y_r = \text{maximum range} \]

\[ Y_E = \text{maximum elevation angle} \]

\[ Y_e = \text{minimum elevation angle} \]

\[ 2\Delta Y_A = \text{azimuth window} \]

\[ \omega_r = \text{sensor rotation rate (in the local horizontal plane)} \]

\[ \phi(t) = \text{sensor heading azimuth} \]

\[ \lambda(t) = \text{sensor latitude (geodetic)} \]

\[ \mathbf{R}_s(t) = \text{sensor position vector in equatorial coordinates} \]
The first five of these \( (\bar{Y}_E, \bar{Y}_E, \bar{Y}_E, \Delta Y_A, \omega_r) \) are input; however, the last three depend upon whether the sensor is fixed on the observing planet or orbiting it.

**Fixed Sensor**

Fixed sensor positions are defined by the input:

\[ b = \text{sensor longitude (measured positive eastward)} \]

\[ \lambda = \text{latitude (geodetic)} \]

\[ h_g = \text{altitude (above the geoid)} \]

For a fixed sensor \( \phi(t) = \pi/2 \) and \( \lambda(t) = \lambda \) and the sensor position in equatorial coordinates is

\[
R_S(t) = \begin{bmatrix}
(1 - e^2 \sin^2 \lambda)^{-1/2} + h_g & \cos \lambda \cos [b + \phi(t)] \\
(1 - e^2 \sin^2 \lambda)^{-1/2} + h_g & \cos \lambda \sin [b + \phi(t)] \\
(1 - e^2)(1 - e^2 \sin^2 \lambda)^{-1/2} + h_g & \sin \lambda
\end{bmatrix}
\]

where \( \phi(t) = \text{Greenwich hour angle} \) and \( e = \text{eccentricity of the planet} \).

**Orbiting Sensor**

The position of the orbiting sensor is determined using the input:

\[ a_s = \text{semimajor axis} \]

\[ e_s = \text{eccentricity} \]

\[ i_s = \text{inclination} \]

\[ \omega_s = \text{argument of perigee} \]

\[ \nu_s(t_s) = \text{true anomaly at the reference time} \]

\[ \Omega_s = \text{right ascension of the ascending node} \]

\[ t_s = \text{reference time} \]
The eccentric anomaly of the satellite at time $t$ is found by solving the equation

$$E_s(t) = \sqrt{\frac{\mu}{a_s^3}} (t - t_s) + E_s(t_s) - e_s \sin E_s(t_s) + e_s \sin E_s(t) \quad (15)$$

where the eccentric anomaly at the reference time $t_s$ is found from

$$E_s(t_s) = 2 \tan^{-1} \left[ \sqrt{\frac{1 - e_s}{1 + e_s}} \tan \left( \frac{v_s(t_s)}{2} \right) \right]$$

Now the sensor's true anomaly and distance from the planet's center are obtained from

$$v_s(t) = \tan^{-1} \left[ \frac{\sqrt{1 - e_s^2 \sin E_s(t)}}{\cos E_s(t) - e_s} \right]$$

and

$$r_s(t) = a_s [1 - e_s \cos E_s(t)]$$

The sensor heading azimuth is

$$\phi(t) = \tan^{-1} \left( \frac{\cos i_s}{\sin i_s \cos [\omega_s + v_s(t)]} \right)$$

The sensor position vector in equatorial coordinates is

$$R_s(t) = r_s(t) \begin{bmatrix} \cos \lambda_I(t) \cos [b(t) + \psi(t)] \\ \cos \lambda_I(t) \sin [b(t) + \psi(t)] \\ \sin \lambda_I(t) \end{bmatrix}$$
where \( \lambda_i(t) \) and \( b(t) \), the geocentric latitude and longitude of the sensor, are obtained from
\[
\lambda_i(t) = \sin^{-1}\left(\sin\left(\omega_s + v_s(t)\right) \sin i_s\right)
\]
and
\[
b(t) = \tan^{-1}\left[\frac{\sin\left(\omega_s + v(t)\right) \cos i_s}{\cos\left(\omega_s + v_s(t)\right)}\right] - \psi(t)
\]

Finally, the geodetic latitude of the sensor is
\[
\lambda(t) = \tan^{-1}\left[\frac{z_s(t)}{\sqrt{x_s^2(t) + y_s^2(t)(1 - \varepsilon^2)}}\right]
\]

D. DETERMINATION OF DATA POINTS

The time spacing between observations from any sensor need not necessarily be the computation interval and the sensors need not observe at the same time points. Each sensor must have a minimum time interval between observations, which then determines how often data may be taken. If a limited scan space is given, this too may be a factor in determining how often data are taken.

At each trajectory computation point, every sensor is checked to determine if the next observation lies within the normal computation interval. If none occurs the following interval is considered. To simplify the calculation, only one computation interval at a time is searched for a solution. The ordering of the observations is found by solving:
\[
|Y_{AL} - Y_A| \leq \Delta Y_A
\]
\[
R_r \leq \overline{Y}_r
\]
\[
Y_E < Y_E < \overline{Y}_E
\]
for the time of occurrence of the next measurement from each station. The least of these times is the time of the next observation. In this computation the observed satellite path is assumed to be an unperturbed ellipse if its reference center is that of the observing planet, or motionless if its reference center is not the observing planet.

The solution is obtained by a Newton iteration on the first equation.

\[ Y_{AL} = \phi + \omega_r(t - t_o) = \text{sensor look azimuth} \]

\[ Y_A = \tan^{-1}\left(\frac{y_{tp}}{z_{tp}}\right) = \text{azimuth of the observed vehicle} \]

where \( y_{tp} \) and \( z_{tp} \) are the coordinates of the observed vehicle in the sensor topocentric coordinates. Further reduction in computation time is achieved by checking the range rate to determine if the observed vehicle will be within the range during the interval.
IV. DEFINITION OF PARAMETERS

A. STATE PARAMETERS

Since the differential corrections are obtained by an iterative scheme that depends on the inverse of the differential correction matrix, any tendency toward singularity in this matrix is a problem of the utmost concern. The choice of elements used in the differential correction affects the rate at which the determinant approaches zero. The partial derivatives that make up the matrix usually are periodic terms and consequently bounded in time. But any of the partial derivatives that are affected by the mean motion, energy, period, or semimajor axis will contain, in addition to the periodic term, a finite secular term. In time this secular term will account for the major contributions to the partial derivatives. If two or more of these partial derivatives contain secular coefficients, the matrix will become singular, since as time passes these secular vectors will become more and more parallel even though, strictly speaking, they are not proportional. The matrix becomes asymptotically singular in time.

The conventional elements $a$, $e$, $i$, $\omega$, $\Omega$, and $t_p$ are a set of variables in which only one parameter, i.e., the semimajor axis $a$, affects the period. Such a set would not be subject to the difficulties just described; however, three of the variables—the argument of perigee, the time of perigee passage, and the ascending node—become poorly defined for a near-circular and low-inclination orbit. Variables such as position and velocity avoid this last difficulty, but all six variables are functions of energy and therefore the differential correction matrix will tend toward singularity.

A set of parameters that avoids the difficulties of circular low-inclination orbits, and with only one parameter dependent on energy, is as follows:

\( a_1 = \text{rotation of } R \text{ about } \dot{R}_o \text{ from the initial position } R_o. \)

\( a_2 = \text{rotation of } \dot{R} \text{ about } R_o \text{ from the initial velocity } \dot{R}_o. \)

\( a_3 = \text{rotation of } \dot{R} \text{ about the angular momentum vector } V \text{ from the initial velocity } \dot{R}_o. \)
\( \alpha_4 \) = rotation of \( \mathbf{R} \) about \( \mathbf{H} \) caused by the change in the angle between \( \mathbf{R} \) and \( \dot{\mathbf{R}} \), leaving the magnitude of \( \mathbf{R} \), \( \dot{\mathbf{R}} \), and the semimajor axis unchanged.

\( \alpha_5 \) = change in the reciprocal semimajor axis. The magnitudes of \( \mathbf{R} \) and \( \dot{\mathbf{R}} \) are changed. This is the only parameter affected by the energy.

\( \alpha_6 \) = change in the magnitude of \( \mathbf{R} \) such that \( \mathbf{a} \cdot \mathbf{R}_0 \), and the angle between \( \mathbf{R} \) and \( \dot{\mathbf{R}} \) remain unchanged.

The first three parameters do not affect the magnitudes of \( \mathbf{R} \), \( \dot{\mathbf{R}} \), or the angle between them. The \( \mathbf{R} \), \( \dot{\mathbf{R}} \) is a rigid rotation pair relative to \( \mathbf{R}_0 \) and \( \dot{\mathbf{R}}_0 \).

The statistical computations are defined in terms of the \( \alpha \) parameters above. However, the equations of motion are in terms of the position and velocity vectors. Therefore, it is necessary to convert the variations in the \( \alpha \) parameters to equivalent variations in position and velocity. Finite rotations are required for this and the results are as follows:

\[ \mathbf{R}' = \mathbf{R} \cos \Delta \alpha_1 + \frac{\mathbf{R} \cdot \dot{\mathbf{R}}}{v^2} (1 - \cos \Delta \alpha_1) \dot{\mathbf{R}} - \frac{\sin \Delta \alpha_1}{v} \mathbf{R} \times \dot{\mathbf{R}} \]  

(16)

\[ \dot{\mathbf{R}}' = \cos \Delta \alpha_2 \dot{\mathbf{R}} + \frac{\mathbf{R}' \cdot \dot{\mathbf{R}}}{r^2} (1 - \cos \Delta \alpha_2) \mathbf{R}' + \frac{\sin \Delta \alpha_2}{r'} \mathbf{R}' \times \dot{\mathbf{R}}' \]  

(17)

\[ \mathbf{R}'' = \mathbf{R}' \cos \Delta \alpha_3 + \frac{\mathbf{R}' \times \dot{\mathbf{R}}' \times \mathbf{R}'}{|\mathbf{R}' \times \dot{\mathbf{R}}'|} \sin \Delta \alpha_3 \]  

(18)

\[ \dot{\mathbf{R}}'' = \mathbf{R}' \cos \Delta \alpha_3 + \frac{\mathbf{R}' \times \dot{\mathbf{R}}' \times \dot{\mathbf{R}}'}{|\mathbf{R}' \times \dot{\mathbf{R}}'|} \sin \Delta \alpha_3 \]  

(19)

\[ \mathbf{R}''' = \frac{\mathbf{r}'' + \Delta \alpha_6}{\mathbf{r}''} \left[ \cos (\gamma - \gamma') \mathbf{R}'' + \sin (\gamma - \gamma') \frac{\mathbf{R}' \times \mathbf{r}'' \times \mathbf{r}''}{|\mathbf{R}' \times \mathbf{r}''|} \right] \]  

(20)

\[ \dot{\mathbf{R}}''' = \sqrt{\frac{2}{\mathbf{r}'' + \Delta \alpha_6} - \left( \frac{2}{\mathbf{r}''} - \frac{v^2}{\mu} \right) - \Delta \alpha_5 \frac{\mathbf{R}''}{v''}} \]  

(21)
where

\[
\gamma = \tan^{-1} \left[ \sqrt{1 - \left( \frac{r'' \cdot \dot{r}'}{r'' \cdot v''} \right)^2} \left[ \frac{r'' \cdot \dot{r}'}{r'' \cdot v''} \right]^2 \right]
\]

and

\[
\gamma' = \tan^{-1} \left[ \sqrt{1 - \left( \frac{r'' \cdot \dot{r}'' + \Delta \alpha_4}{(r'' + \Delta \alpha_5)^2 \sqrt{\mu \left( \frac{2}{r'' + \Delta \alpha_5} - \frac{2}{r'' - \frac{r''^2}{\mu} + \Delta \alpha_5} \right)}} \right)^2 \right]
\]

then \( \Delta R = R'' - R \) and \( \Delta \dot{R} = \dot{R}'' - \dot{R} \).

Since the first three state parameters have the character of rigid rotation of the \( R, \dot{R} \) pair, an infinitesimal form of the parameter follows from the finite rotation given in Eqs. (16), (17), and (18). Consider the scalar product of \( H \) with \( R' \). Then Eq. (16) gives

\[
H \cdot R' = -\frac{h^2}{\nu} \sin \Delta \alpha
\]

where \( R' \) is \( R \), after having been rotated through the finite angle \( \Delta \alpha_1 \) about \( \ddot{R} \). For small \( \alpha_1(t) \) and \( R' = R_0 \), \( \Delta \alpha_1 = -\alpha_1(t) \) and \( \sin \Delta \alpha_1 = -\alpha_1(t) \). Therefore

\[
\alpha_1(t) = \frac{\nu(H \cdot R_0)}{h^2}
\]

where \( \alpha_1(t) \) represents an infinitesimal rotation of \( R \) about \( \dot{R}_0 \) from the initial position \( R = R_0 \). The expressions for \( \alpha_2(t) \) and \( \alpha_3(t) \) are obtained
from Eqs. (17) and (18) in a similar way. The remaining three parameters completely determine the lengths of \( \mathbf{R} \) and \( \mathbf{R} \) and the angle between them or \( r, v, \) and \( R \cdot \dot{R} \).

Therefore the components of the vector of state parameters, \( \hat{a} \), are as follows:

\[
\begin{align*}
\alpha_1 &= \left( v/\mathbf{h}^2 \right) (\mathbf{H} \cdot \mathbf{R}_0) \\
\alpha_2 &= -\frac{r}{h^2} (\mathbf{H} \cdot \dot{\mathbf{R}}_0) \\
\alpha_3 &= -\frac{1}{v^2 h} \left[ (\mathbf{H} \times \dot{\mathbf{R}}) \cdot \dot{\mathbf{R}}_0 \right] \\
\alpha_4 &= R \cdot \dot{R} \\
\alpha_5 &= \frac{1}{a} \\
\alpha_6 &= |\mathbf{R}|
\end{align*}
\]

(22)

The features of the orbit prediction mode include the provision to update one dynamic bias in addition to updating the six state variables and to account for additional observational and dynamic biases by degrading the estimate of the state without actually solving for the biases. In the variance-covariance propagation mode, the updating is omitted and a reference trajectory is used.

The following biases are considered:

**Dynamic Biases**

- Drag bias \( \delta \), updated (see Eq. (5))
- Gravitational bias \( \overline{\mu} \) (accounted for)

**Observational Biases** \( \hat{n} \) (accounted for; these apply to every station). Speed of light bias

Geodetic net bias (three components of position in the planetocentric axes), which should not be used for orbiting sensors.
\[ S^{-1} = \frac{\partial \hat{a}}{\partial \mathbf{X}} = \begin{bmatrix} -\frac{\mathbf{vH}^T}{\mathbf{h}^2} & 0 \\ 0 & \frac{\mathbf{rH}^T}{\mathbf{h}^2} \\ 0 & \left(\frac{\mathbf{h} \times \mathbf{R}}{\mathbf{h}^2}\right)^T \\ \mathbf{R}^T & \mathbf{R}^T \\ -\frac{2\mathbf{R}^T}{\mathbf{r}^3} & -\frac{2\mathbf{R}^T}{\mathbf{\mu}} \\ \mathbf{R}^T & 0 \end{bmatrix} \]

\[ I = \text{an } 8 \times 8 \text{ matrix} \]
\[ S_T = \text{a } 16 \times 16 \text{ matrix} \]
\[ S = \text{a } 6 \times 6 \text{ matrix} \]

\[ V = \frac{\partial \hat{a}}{\partial \mathbf{\mu}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{\mathbf{v}^2}{\mathbf{\mu}^2} \\ 0 \end{bmatrix} \quad (6 \times 1) \quad (24) \]

It may be shown that

\[ S_T = \begin{bmatrix} S & 0 & T & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \quad (16 \times 16) \]
Azimuth bias
Elevation bias
Range normalization bias
Range-rate bias

The complete vector of state parameters is then

\[
\begin{bmatrix}
\hat{\alpha} \\
\hat{\delta} \\
\hat{\nu} \\
\hat{\eta}
\end{bmatrix}, \text{ where } \hat{\alpha} \text{ and } \hat{\eta} \text{ are vectors}
\]

The complete vector of state variables is

\[
\begin{bmatrix}
X \\
\delta
\end{bmatrix}, \text{ where } X = \begin{bmatrix} R \\ \dot{R} \end{bmatrix} \text{ (the conventional state vector)}
\]

The state variables which are actually estimated are R, \( \dot{R} \), \( \delta \).

1. The Point Transformation Matrix, \( S_T \)

Assuming they are small enough, variations in the vectors may be linearly related at a time point by:

\[
\begin{bmatrix}
\Delta \hat{\alpha} \\
\Delta \hat{\delta} \\
\Delta \hat{\nu} \\
\Delta \hat{\eta}
\end{bmatrix} = S_T^{-1} \begin{bmatrix}
\Delta X \\
\Delta \delta \\
\Delta \nu \\
\Delta \eta
\end{bmatrix} \quad (16 \times 1)^* \quad (23)
\]

where

\[
S_T^{-1} = \begin{bmatrix}
S^{-1} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

*(16 \times 1) denotes a matrix with 16 rows and 1 column.*
where (1)

\[
\mathbf{S} = \begin{bmatrix}
\frac{H}{v} & 0 & \frac{H \times R}{h} & \frac{H \times R}{h^2} & \left(\frac{\mu R \cdot \hat{R}}{2v^2 h^2}\right) & \frac{H \times R}{r} & \frac{\mu R \cdot \hat{R}(1 - r/a)H \times R}{h^2 r^2 v^2} \\
0 & \frac{H}{r} & \frac{H \times \hat{R}}{h} & 0 & -\frac{\mu R}{2v^2} & \frac{\mu R}{r^2 v^2}
\end{bmatrix}
\]

and

\[
\mathbf{S}^{-1} \mathbf{T} + \mathbf{V} = 0
\]

\[
\mathbf{T} = -\mathbf{SV}
\]

\[
\begin{bmatrix}
-\frac{R \cdot \hat{R}}{2\mu h^2} & \frac{H \times R}{2\mu h^2} \\
\frac{R}{2\mu}
\end{bmatrix}
\]

\[
(6 \times 1)
\]

(25)

(26)

2. The State Parameter Transition Matrix, \( \Lambda_T \)

Variations in the state parameters at one time may be linearly related to variations at another time by

\[
\begin{bmatrix}
\Delta \hat{\alpha}(t) \\
\Delta \delta(t) \\
\Delta \hat{\mu}(t) \\
\Delta \hat{n}(t)
\end{bmatrix}
= \Lambda_T(t, t_o)
\begin{bmatrix}
\Delta \hat{\alpha}(t_o) \\
\Delta \delta(t_o) \\
\Delta \hat{\mu}(t_o) \\
\Delta \hat{n}(t_o)
\end{bmatrix}
\]

(27)

where

\[
\Lambda_T(t, t_o) =
\begin{bmatrix}
\Lambda(t, t_o) & W(t, t_o) & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & I
\end{bmatrix}
\]

(28)

I = an 8 \times 8 matrix
\[
\Lambda(t, t_o) = \frac{\partial^2 \alpha(t)}{\partial \alpha(t_o)} (6 \times 6)
\]
\[
= S^{-1}(t) \left[ \frac{\partial X(t)}{\partial X(t_o)} \right] S(t_o)
\]
\[
\Psi D(\alpha_1, t_o) = \left[ \frac{\partial \alpha(t)}{\partial \alpha(t_o)} \right] (6 \times 1)
\]
\[
= S^{-1}(t) \left[ \frac{\partial X(t)}{\partial \alpha(t_o)} \right]
\]
\[
W(t, t_o) = \left[ \frac{\partial \alpha(t)}{\partial \mu(t_o)} \right] (6 \times 1)
\]
\[
= S^{-1}(t) \left[ \frac{\partial X(t)}{\partial \mu(t_o)} \right]
\]

The partials required for the parameter transition matrix are found as described below.

The Transition Matrix of NASA Parameters, \(\Lambda\). If the partials of the perturbing accelerations with respect to position and velocity are assumed to be small over the interval \((t_1, t_o)\), then this transition matrix may be found analytically.

\[
\Lambda(t, t_o) = \begin{bmatrix}
\frac{fv}{v_o} & -\frac{gv}{r_o} & 0 & 0 & 0 & 0 \\
-\frac{fr}{v_o} & \frac{gr}{r_o} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & \lambda_{3,4} & \lambda_{3,5} & \lambda_{3,6} \\
0 & 0 & 0 & \hat{g} & \lambda_{4,5} & \hat{r}_o \\
0 & 0 & 0 & 0 & 1 & \hat{r}_o \\
0 & 0 & 0 & g/r & \lambda_{6,5} & \hat{r}_o \\
\end{bmatrix} (6 \times 6) (29)
\]
\[ \lambda_{3,4} = \frac{\hbar}{v} \left\{ \frac{\delta - 1}{r^2} \left[ \frac{r(f_4 + 1)}{r_o} + 1 \right] + \frac{\bar{r}}{h^2} \left( d_o - \frac{\mu g}{r} \right) \right\} \]

\[ \lambda_{3,5} = \frac{\mu^* \bar{r}}{2h v^2 v_o} \left[ d_o \left( \delta d - gv^2 \right) - \delta h^2 \right] \]

\[ + \frac{h^2 \beta^2}{2v^2 v_o^2} \left[ \frac{r f_3}{v^3} + \frac{d_o \beta f_2 (f_3^2 + f_2)}{\sqrt{\mu}} \right. \]

\[ + \beta^2 \left( -3f_6 + f_5 - f_1 f_2 + f_2^2 + f_1 f_3^2 \right) \left] \right\} \]

\[ \lambda_{3,6} = \frac{h \bar{r}}{v^2 v_o} \left\{ \frac{r}{r} \left( f_4 + \frac{r_o}{r} \right) - \frac{\mu^* \bar{r}}{r_o v_o^2} \right\} \]

\[ - \frac{d_o}{h^2} \left[ 1 - \frac{\mu}{r_o v_o^2} \left( d_o - \frac{\mu g}{r} \right) \right] \}

\[ \lambda_{4,5} = \frac{\sqrt{\mu}}{r} \left( - \frac{r_o^2 \beta \Sigma_1}{\sqrt{\mu}} - \frac{d_o r_o \beta^2 \Sigma_2}{\sqrt{\mu}} \right. \]

\[ - \frac{d_o^2 \beta^3 \Sigma_3}{\mu} + \frac{r_o \beta^3 \Sigma_4}{\sqrt{\mu}} + \frac{d_o \beta^4 \Sigma_5}{\sqrt{\mu}} + \beta^5 \Sigma_6 \right) \]

\[ \lambda_{6,5} = \frac{\beta}{r} \left( - \frac{r_o^2}{2} S_1 - \frac{d_o r_o \beta S_2}{\sqrt{\mu}} + \frac{r_o \beta^2 S_3}{\mu} + \frac{d_o \beta^2 S_4}{\sqrt{\mu}} + \frac{d_o \beta^3 S_5}{\sqrt{\mu}} + \beta^4 S_6 \right) \]
where \( r = |R_{tb}| \)
\[ r_o = |R_{rb}| \]
\[ d_o = R_o \cdot \hat{R}_o \]
\[ d = R_{tb} \cdot \hat{R}_{tb} \]
\[ v = |\hat{R}_{tb}| \]
\[ v_o = |\hat{R}_o| \]
\[ h = R_{tb} \times \hat{R}_{tb} \]

\( \beta \) is found in solving the two-body problem (Section III.5)

\( f, g, \dot{f}, \dot{g} \) from Eq. (13)

\( f_1, f_2, f_3, f_4 \) from Eq. (9)

\[ f_6 = \sum_{i=0}^{\infty} \frac{(-\tau)^i}{(2i + 5)!} \]

\[ f_7 = \sum_{i=0}^{\infty} \frac{(-\tau)^i}{(2i + 6)!} \]

\[ f_5 = 1/24 - \tau f_7 \]

\[ \tau = \beta^2/a \]

\[ \Sigma_1 = f_4 f_4 \]

\[ \Sigma_2 = f_4 (1 + 2 f_4) \]

\[ \Sigma_3 = f_2 f_3 \]

\[ \Sigma_4 = 3 f_2/2 - 3 f_1/2 - 2 f_2 f_3 \]

\[ \Sigma_5 = 3 f_1/2 - 3 f_5 - 2 f_2^2 \]

\[ \Sigma_6 = f_5/2 - 3 f_6/2 - f_1 f_2 \]

\[ s_1 = f_3 \]

\[ s_2 = f_2 f_3 \]

\[ s_3 = f_1/2 - f_5 - f_1 f_3 \]

\[ s_4 = f_5 - f_1/2 - f_1 f_3/2 \]

\[ s_5 = f_5/2 - 3 f_6/2 - f_1 f_2 \]

\[ s_6 = f_6/2 - 2 f_7 - f_1^2/2 \]
As mentioned at the beginning of Section III.B, the two-body solution may be rectified if the Encke terms become too large. The discontinuity at the time of a rectification, \( t_r \), also affects this transition matrix; it is accounted for using the equation

\[
A(t, t_0) = A(t, t_{r+}) S^{-1}(t_{r+}) S(t_{r-}) A(t_{r-}, t_0)
\]

where \( S^{-1}(t_{r+}) \) is computed after \( R_{tb}, \dot{R}_{tb} \) have been corrected and \( S(t_{r-}) \) before.\(^{(7)}\)

The Transition Matrix of Drag Bias, \( \Psi_{\delta S} \). This part of the total transition matrix, \( \Lambda_T \), cannot be found analytically, due to the fact that the perturbing acceleration due to drag cannot be eliminated in taking the partials. The form of the equation is:

\[
\Psi_{\delta S}(t, t_0) = S^{-1}(t) \begin{bmatrix} \partial X(t) \\ \partial \delta(t_0) \end{bmatrix} (6 \times 1) \tag{30}
\]

where

\[
\begin{bmatrix} \partial X(t) \\ \partial \delta(t_0) \end{bmatrix} = \int_{t_0}^{t} \left\{ \begin{bmatrix} \partial \dot{X}(t) \\ \partial X(t) \\ \partial \delta(t_0) \end{bmatrix} + \begin{bmatrix} \partial \dot{X}(t) \\ \partial \delta(t) \end{bmatrix} \right\} dt
\]

since \( \dot{\delta} = 0 \) and

\[
\begin{bmatrix} \partial X(t_0) \\ \partial \delta(t_0) \end{bmatrix} = 0
\]

Assuming that

\[
\begin{bmatrix} \partial F \\ \partial X \end{bmatrix} = \begin{bmatrix} \partial F \\ \partial \delta \end{bmatrix} = \begin{bmatrix} \partial \delta \\ \partial X \end{bmatrix} = 0
\]
the terms of the integrand are written as follows:

\[
\begin{bmatrix}
\frac{\dot{a}X(t)}{\dot{a}X(t)} \\
\frac{\dot{a}R(t)}{\dot{a}R(t)} \\
\frac{\dot{a}\hat{R}(t)}{\dot{a}R(t)}
\end{bmatrix}
= \begin{bmatrix}
\frac{\dot{a}R(t)}{\dot{a}R(t)} & \frac{\dot{a}\hat{R}(t)}{\dot{a}R(t)} \\
\frac{\dot{a}R(t)}{\dot{a}R(t)} & \frac{\dot{a}\hat{R}(t)}{\dot{a}R(t)} \\
\frac{\dot{a}\hat{R}(t)}{\dot{a}R(t)} & \frac{\dot{a}\hat{R}(t)}{\dot{a}R(t)}
\end{bmatrix} (6 \times 6)
\]

where

\[
\begin{bmatrix}
\frac{\dot{a}R(t)}{\dot{a}R(t)} \\
\frac{\dot{a}\hat{R}(t)}{\dot{a}R(t)}
\end{bmatrix} = 0 (3 \times 3)
\]

\[
\begin{bmatrix}
\frac{\dot{a}R(t)}{\dot{a}R(t)} \\
\frac{\dot{a}\hat{R}(t)}{\dot{a}R(t)}
\end{bmatrix} = I
\]

\[
\begin{bmatrix}
\frac{\dot{a}\hat{R}(t)}{\dot{a}R(t)}
\end{bmatrix} = \frac{\mu}{r^3} \begin{bmatrix}
\frac{3RR_T}{r^2} - I
\end{bmatrix}
\]

\[-\frac{(1 + \delta)}{2} \rho C_D A m v_{rp} \begin{bmatrix}
v_{rp} & v_{rp}^T
\end{bmatrix} v_{rp}^{-1} + I \begin{bmatrix}
0 & \psi & 0
d - \psi & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} (3 \times 3)
\]

\[
\frac{\dot{a}\hat{R}(t)}{\dot{a}R(t)} = -\frac{(1 + \delta)}{2} \rho C_D A m v_{rp} \begin{bmatrix}
v_{rp} & v_{rp}^T
\end{bmatrix} v_{rp}^{-1} + I (3 \times 3)
\]

\[\dot{\psi} = \text{earth rotation rate}\]

\[
v_{rp} = \dot{R} - \begin{bmatrix}
0 \\
0 \\
\dot{\psi}
\end{bmatrix} \times R (3 \times 1)
\]

\[v_{rp} = |v_{rp}|\]

and

\[
\begin{bmatrix}
\frac{\dot{a}X(t)}{\dot{a}\delta(t)}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} (6 \times 1)
\]

\[-\frac{1}{2} \rho C_D A/m v_{rp} v_{rp}^{-1}\]
The Transition Matrix of Gravitation Bias, \( W \). This matrix may be found analytically by assuming the partials of the perturbing accelerations are negligible.

\[
W(t, t_o) = A(t, t_o)W(t_o, t_o) + W'(t, t_o)
\]

where

\[
W'(t, t_o) = \begin{bmatrix}
0 \\
0 \\
- \frac{h}{2v^2} \left( \frac{g}{\mu} - \frac{d_o \beta f_3}{\sqrt{\mu} v} \right) - \left( t - t_o + \frac{d_o f_2 \beta^2}{\mu} \right) \frac{1}{r^3} \\
\frac{f h}{2 \mu v^2} \left[ \dot{g} + \frac{d_o}{h^2} \left( \frac{\mu g}{r} - d_o \right) \right] \\
\frac{1}{2} \left[ \frac{\beta}{\sqrt{\mu}} + \left( t - t_o \right) \left( 1 - \frac{2r}{a} \right) + \frac{d_o \beta^2 f_2}{\mu r} \right] \\
0 \\
\frac{(t - r_o) d - d_o g}{2 \mu r}
\end{bmatrix}
\]

\[
W(t_o, t_o) = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
\dot{R}_o \cdot \dot{R}_o \\
\mu^2
\end{bmatrix}
\]
and \( v = \| \dot{R}_{tb}(t) \| \)

\[
d = R_{tb}(t) \cdot \dot{R}_{tb}(t)
\]

\[
d_o = R_o \cdot \dot{R}_o
\]

\[
r = |R_{tb}(t)|
\]

The discontinuity at the time of a rectification, \( t_r \), also affects the transition matrix of gravitation bias \( W \) and is accounted for using

\[
W(t_{r+}, t_o) = s^{-1}(t_{r+})S(t_{r-})[\Lambda(t_{r-}, t_o)W(t_o, t_o) + W'(t_{r-}, t_o)]
\]

\[
W(t, t_o) = W'(t, t_{r+}) + \Lambda(t, t_{r+})W(t_{r+}, t_o)
\]

B. OBSERVED PARAMETERS

A sensor may observe any or all of the parameters:

Azimuth \hspace{1cm} \text{Range}

Elevation \hspace{1cm} \text{Range rate}

The vector of estimates of these parameters is

\[
\mathbf{y}_c = \begin{bmatrix}
\tan^{-1} \frac{y_{tp}}{z_{tp}} \\
\tan^{-1} \frac{x_{tp}}{\sqrt{y_{tp}^2 + z_{tp}^2}} \\
r_{tp} \\
\frac{R_{tp}}{r_{tp}} \cdot \left[ \dot{R}_{tp} - \dot{R}_{stp} \right]
\end{bmatrix}
\]

(32)

where \( R_{tp} = C \left[ R_{(vb)} - R_s \right] \)

\( R_{(vb)} = R + R_{(cb)} \)
\( \mathbf{R}_{(cb)} \) = position of vehicle relative to the \( k \)th planet; the observing planet.

\[
\mathbf{C} = \begin{bmatrix}
\cos \lambda \cos (\psi + b) & \cos \lambda \sin (\psi + b) & \sin \lambda \\
- \sin (\psi + b) & \cos (\psi + b) & 0 \\
- \sin \lambda \cos (\psi + b) & - \sin \lambda \sin (\psi + b) & \cos \lambda
\end{bmatrix}
\]

\( \lambda \) = station geodetic latitude
\( \psi \) = Greenwich hour angle
\( b \) = station longitude
\( \mathbf{R}_s \) from Section III.C, Eq. (14)
\( \dot{\mathbf{R}}_{tp} = \mathbf{C} \dot{\mathbf{R}} \)

For a fixed sensor (see Section III.C.1):

\[
\dot{\mathbf{R}}_{stp} = \begin{bmatrix}
0 \\
\dot{\psi} [(1 - \epsilon^2 \sin^2 \lambda)^{-1/2} + h] \cos \lambda \\
0
\end{bmatrix}
\]

where \( \dot{\psi} \) = earth rotation rate
\( \epsilon \) = earth eccentricity
\( h \) = height above the geoid

For an orbiting sensor (see Section III.C.2):

\[
\dot{\mathbf{R}}_{stp} = \begin{bmatrix}
\frac{1}{r_s} \sqrt{\mu_a} \left[ e_s \left( 1 - \frac{r_s^2}{a_s^2} \right) \right] \\
\sqrt{\frac{\mu}{r_s}} \left( \frac{2}{r_s} - \frac{1}{a_s} \right) \cos \gamma \sin \phi \\
\sqrt{\frac{\mu}{r_s}} \left( \frac{2}{r_s} - \frac{1}{a_s} \right) \cos \gamma \cos \phi
\end{bmatrix}
\]
\[
\gamma = \sin^{-1}\left(\frac{1}{r_s} \sqrt{\mu a_s \left[e_s^2 - \left(1 - \frac{r_s^2}{a_s^2}\right)\right]}\right)
\]

where \(\gamma\) = flight path angle

\[
\phi = \tan^{-1}\left(\frac{\sin \left(\frac{\pi}{2} - i_s\right)}{\cos \left(\frac{\pi}{2} - i_s\right) \cos (\omega_s + \nu_s)}\right)
\]

\(\phi\) = heading angle

\[
r_s = a_s \left(1 - e_s \cos E_s\right)
\]

\(r_s\) = true anomaly

\[
v_s = \tan^{-1}\left(\frac{\sqrt{1 - e_s^2 \sin E_s}}{\cos E_s - e_s}\right)
\]

\(v_s\) = eccentric anomaly (see Section III.C.2), Eq. (15)

Small variations in the observed parameters may be linearly related to variations in the state parameters by

\[
\begin{align*}
\Delta Y &= N_T \\
\begin{bmatrix}
\Delta \alpha \\
\Delta \delta \\
\Delta \mu \\
\hat{\Delta} \hat{n}
\end{bmatrix}
\end{align*}
\]

where \(N_T = M_{ST}\)

\[
M_T = [M 0 0 F]
\]

\(S_T\) from Section IV.A
1. Matrix to Transform $\Delta X$ to $\Delta Y$, $M$

This matrix of partials may be written (see p. 32)

$$
M = \begin{bmatrix}
\frac{\partial Y}{\partial X}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial Y}{\partial R_{tp}}, \frac{\partial Y}{\partial R_{stp}}, \frac{\partial Y}{\partial R_{tp}}
\end{bmatrix}
$$

(n x 6) \hspace{1cm} (34)

where $\frac{\partial R_{tp}}{\partial R} = \frac{\partial R_{tp}}{\partial R} = C$

and

$$
\begin{bmatrix}
\frac{\partial Y}{\partial R_{tp}}
\end{bmatrix} = \begin{bmatrix}
\begin{array}{ccc}
\frac{R_{tp}}{\sqrt{y_{tp}^2 + z_{tp}^2}} & 0 & 0 \\
0 & 0 & -1 \\
0 & 1 & 0
\end{array}
\end{bmatrix}
$$

(35)

$$
\begin{bmatrix}
\frac{\partial Y}{\partial R_{tp}}
\end{bmatrix} = \begin{bmatrix}
\begin{array}{ccc}
\frac{R_{tp}}{\sqrt{z_{tp}^2 + y_{tp}^2}} & \frac{R_{tp}}{r_{tp}} \\
\frac{R_{tp}}{r_{tp}} & 0 & 0 \\
\langle R_{tp} - R_{stp} \rangle & \langle R_{tp} - R_{stp} \rangle & I
\end{array}
\end{bmatrix}
$$

(36)
2. Matrix to Transform $\Delta \hat{\mathbf{n}}$ to $\Delta \mathbf{Y}$, $\mathbf{F}$

This matrix of partials may be written

$$
\mathbf{F} = \begin{bmatrix} \frac{\partial \mathbf{Y}}{\partial \mathbf{C}} \end{bmatrix} (n \times 8)
$$

Assuming the geodetic net biases are not used when orbiting sensors are used, then

$$
\mathbf{F} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{c'} \\ \mathbf{r}_{tp} \end{bmatrix} - \begin{bmatrix} \frac{\partial \mathbf{Y}}{\partial \mathbf{C}} \\ \frac{\partial \mathbf{Y}}{\partial \mathbf{R}_{tp}} \end{bmatrix} C' - \begin{bmatrix} \frac{\partial \mathbf{Y}}{\partial \mathbf{C}} \\ \frac{\partial \mathbf{Y}}{\partial \mathbf{R}_{tp}} \end{bmatrix} \begin{bmatrix} 0 & \psi & 0 \\ -\psi & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, I
$$

where $\mathbf{r}_{tp} = \mathbf{R}_{tp} \cdot \left\{ \mathbf{R}_{tp} - \mathbf{R}_{stp} \right\}$

$c = \text{speed of light}$

$\psi = \text{earth rotation rate}$

$I = 4 \times 4 \text{ matrix}$

$$
\mathbf{C} = \begin{bmatrix} \cos \lambda \cos b & \cos \lambda \sin b & \sin \lambda \\ -\sin b & \cos b & 0 \\ -\sin \lambda \cos b & -\sin \lambda \cos b & \cos \lambda \end{bmatrix}
$$

and

$$
\begin{bmatrix} \frac{\partial \mathbf{Y}}{\partial \mathbf{C}} \\ \frac{\partial \mathbf{Y}}{\partial \mathbf{R}_{tp}} \end{bmatrix} \begin{bmatrix} \frac{\partial \mathbf{Y}}{\partial \mathbf{C}} \\ \frac{\partial \mathbf{Y}}{\partial \mathbf{R}_{tp}} \end{bmatrix}
$$

may be obtained from Section IV.B.1, Eqs. (35) and (36).
V. STATISTICAL ESTIMATION

THE INITIAL ESTIMATE OF STATE

Given an estimate of the vector of state variables at a time $t_o$, the best initial estimate at any other time $t$ is given by:

\[
\begin{bmatrix}
X(t) \\
\delta(t) \\
\bar{\mu}(t) \\
\hat{n}(t)
\end{bmatrix} = \int_{t_o}^{t} \begin{bmatrix}
\dot{X} \\
\dot{\delta} \\
\dot{\bar{\mu}} \\
\dot{\hat{n}}
\end{bmatrix} dt + \begin{bmatrix}
X(t_o) \\
\delta(t_o) \\
\bar{\mu}(t_o) \\
\hat{n}(t_o)
\end{bmatrix} \tag{16 \times 1}
\]

However, since $\dot{\delta} = \dot{\bar{\mu}} = \dot{\hat{n}} = 0$ and

\[
X(t_o) + \int_{t_o}^{t} \dot{X} dt = \begin{bmatrix}
R_{tb}(t) \\
\dot{R}_{tb}(t)
\end{bmatrix} + \int_{t_o}^{t} \begin{bmatrix}
\bar{E} \\
\dot{\bar{E}}
\end{bmatrix} dt = \begin{bmatrix}
R(t) \\
\dot{R}(t)
\end{bmatrix}
\]

(see Sections III.B and IV.A), the initial estimate at time $t$ may be rewritten:

\[
\begin{bmatrix}
\bar{X}(t) \\
\bar{\delta}(t) \\
\bar{\bar{\mu}}(t) \\
\bar{\hat{n}}(t)
\end{bmatrix} = \begin{bmatrix}
R(t) \\
\dot{R}(t)
\end{bmatrix}
\]

The initial estimate of the vector of state parameters at any time may then be found as shown in Section IV.A; however, it is not necessary to compute the state parameters as such.

The variance-covariance matrix corresponding to the initial estimate of the state parameters, $Q$, at $t_o$ is found from the variance-covariance matrix of state variables, $P$, by

\[
Q_T(t_o) = S^{-1}_T(t_o)p_T(t_o)[S^{-1}_T]_T(t_o) \tag{16 \times 16}
\]

(37)
Then the initial $Q_T$ matrix at any time $t$ may be found by

\[ Q_T(t) = \Lambda_T(t, t_0) Q_T(t_0) \Lambda_T^T(t, t_0) \]  \hspace{1cm} (38)

where $\Lambda_T$ is defined in Section IV.A, and $Q_T$ may be partitioned so that:

\[
Q_T = \begin{bmatrix}
Q & C_{\alpha\delta} & C_{\alpha\mu} & C_{\alpha\eta} \\
C_{\alpha\delta}^T & C_{\delta} & 0 & 0 \\
C_{\alpha\mu}^T & 0 & B & 0 \\
C_{\alpha\eta}^T & 0 & 0 & U \\
\end{bmatrix} \hspace{1cm} (16 \times 16)
\]

where $\beta = \mu^2$ and the other symbols are defined in Table 2 at the end of this Section.

**IMPROVING THE ESTIMATE WITH DATA**

At this point it is advantageous to extend or augment the six-element vector, $X$, to include $\delta$ so that

\[
X_A = \begin{bmatrix} X \\ \delta \end{bmatrix} \quad \text{and} \quad \hat{\alpha}_A = \begin{bmatrix} \hat{\alpha} \\ \delta \end{bmatrix} \hspace{1cm} (7 \times 1)
\]

then

\[
\Lambda_A = \begin{bmatrix} \Lambda & \psi_{D\delta} \\ 0 & 1 \end{bmatrix} \hspace{1cm} (7 \times 7) \quad S_A = \begin{bmatrix} S & 0 \\ 0 & 1 \end{bmatrix} \hspace{1cm} (7 \times 7) \quad S_A^{-1} = \begin{bmatrix} S^{-1} & 0 \\ 0 & 1 \end{bmatrix} \hspace{1cm} (7 \times 7)
\]

\[
W_A = \begin{bmatrix} W \\ 0 \end{bmatrix} \hspace{1cm} (7 \times 1) \quad V_A = \begin{bmatrix} V \\ 0 \end{bmatrix} \hspace{1cm} (7 \times 1) \quad T_A = \begin{bmatrix} T \\ 0 \end{bmatrix} \hspace{1cm} (7 \times 1)
\]
so that

\[
\Lambda_T = \begin{bmatrix} \Lambda_A & W_A & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I \end{bmatrix} \quad (16 \times 16)
\]

\[
S_T = \begin{bmatrix} S_A & T_A & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I \end{bmatrix} \quad (16 \times 16)
\]

\[
S_T^{-1} = \begin{bmatrix} S_A^{-1} & V_A & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I \end{bmatrix} \quad (16 \times 16)
\]

and

\[
M_A = [M, 0] \quad (n \times 7) \quad M_T = [M_A^T, 0, \mathbf{F}] \quad (n \times 16)
\]

where \([M, 0] = [M, \mu]\).

Also,

\[
N_T = [M_A S_A^T, M_A T_A^T, \mathbf{F}] \quad N_A = M_A S_A \quad (n \times 7)
\]

(where \(M_A T_A^T = -N_A V_A\) \((n \times 1)\)

and

\[
Q_T = \begin{bmatrix} Q_A & C_{\delta \alpha \mu} & C_{\delta \alpha \eta} \\ C_{\delta \alpha \mu}^T & B & 0 \\ C_{\delta \alpha \eta}^T & 0 & U \end{bmatrix}
\]

If data occur at the time point \(t_1\), the parts of the covariance matrix \(Q_T\) are found at the time point as follows:
\[ Q_A(t_1) = \Lambda_A(t_1, t_o) \left[ Q_A(t_o) \Lambda_T_A(t_1, t_o) + C_{\delta \alpha \mu}(t_o) \mu^T_A(t_1, t_o) \right] \\
+ \bar{W}_A(t_1, t_o) \left[ C_{\delta \alpha \mu}(t_o) \Lambda_T_A(t_1, t_o) = B(t_o) \bar{W}_A(t_1, t_o) \right] (7 \times 7) \]

\[
C_{\delta \alpha \mu}(t_1) = \Lambda_A(t_1, t_o) C_{\delta \alpha \mu}(t_o) + \bar{W}_A(t_1, t_o) B(t_o) (7 \times 1) 
\]

\[
C_{\delta \alpha \eta}(t_1) = \Lambda_A(t_1, t_o) C_{\delta \alpha \eta}(t_o) (7 \times 8) 
\]

then, using the Kalman filter, the corrections to the state parameters which are to be estimated (leaving off the time notation since all are the same) are

\[
\Delta \hat{a}_A = L \{ Y_{\text{obs}} - Y_c \} 
\]

where the weighting matrix, L, is

\[
L = \left[ Q_A N^T_A - C_{\delta \alpha \mu} V^T_A + C_{\delta \alpha \eta} F^T \right] \bar{Y}^{-1} (7 \times n) 
\]

\[
\bar{Y} = N A^T A C_{\delta \alpha \mu} F^T + \bar{F} C_{\delta \alpha \eta} N^T_A + \bar{F} F^T - N A^T C_{\delta \alpha \mu} V^T_A 
\]

\[
- N A^T C_{\delta \alpha \mu} N^T_A + N A^T B^T V^T_A + \bar{E} (n \times n) 
\]

\[
\bar{E} = \text{covariance matrix associated with the observed data} \\
Y_{\text{obs}} = \text{actual observed data} 
\]

The parts of the covariance matrix are then updated and
\[ Q_A(t/N) = Q_A(t/N-1) \]
\[ - L \left[ N_A Q_A(t/N-1) - N_A V_A C_{\delta \alpha \mu}^T(t/N-1) + F C_{\delta \alpha \eta}^T(t/N-1) \right] (7 \times 7) \]
\[ C_{\delta \alpha \mu}(t/N) = C_{\delta \alpha \mu}(t/N-1) - L \left[ N_A C_{\delta \alpha \mu}(t/N-1) - N_A V_A B \right] (7 \times 1) \]
\[ C_{\delta \alpha \eta}(t/N) = C_{\delta \alpha \eta}(t/N-1) - L \left[ N_A C_{\delta \alpha \eta}(t/N-1) + F U \right] (7 \times 8) \]
\[ U(t/N) = U(t/N-1) = U(t_o) (8 \times 8) \]
\[ B(t/N) = B(t/N-1) = B(t_o) (1 \times 1) \]

**Table 2**

**DEFINITION OF PROGRAM AND EQUATION SYMBOLS IN COMMON**

<table>
<thead>
<tr>
<th>Program</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>a</td>
<td>Semimajor axis of the orbit</td>
</tr>
<tr>
<td>AABS</td>
<td></td>
<td>Magnitude of a</td>
</tr>
<tr>
<td>ALAMDA</td>
<td>(\Lambda)</td>
<td>Parameter transition matrix, Eq. (29)</td>
</tr>
<tr>
<td>ALMAT</td>
<td>L</td>
<td>Weighting matrix of the minimum variance filter, Eq. (41)</td>
</tr>
<tr>
<td>AMASS</td>
<td>(A/m)</td>
<td>Area-to-mass ratio</td>
</tr>
<tr>
<td>AMMAT</td>
<td>M</td>
<td>Matrix of partials of observations with respect to state variables, Eq. (34)</td>
</tr>
<tr>
<td>AUERAD</td>
<td>A.U</td>
<td>Conversion constant, eru/astronomic unit</td>
</tr>
<tr>
<td>BIASMU</td>
<td>(\bar{\mu})</td>
<td>Gravitational bias uncertainty</td>
</tr>
<tr>
<td>BIAS</td>
<td>U</td>
<td>Diagonal matrix of observational bias uncertainties squared</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1. speed of light</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. (u, v, w) of station position</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. range</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. range rate</td>
</tr>
</tbody>
</table>
Table 2 (cont.)

<table>
<thead>
<tr>
<th>Program</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMU</td>
<td>$\mu_i$</td>
<td>Gravitational constants for the perturbing bodies</td>
</tr>
<tr>
<td>CBX</td>
<td>$C_{\delta \mu \eta}$</td>
<td>Matrix of covariance of satellite state parameters and the accounted-for observational bias, Eqs. (40) and (42)</td>
</tr>
<tr>
<td>CA</td>
<td>$C_a$</td>
<td>Parameters pertinent to a rectification</td>
</tr>
<tr>
<td>CB</td>
<td>$C_b$</td>
<td></td>
</tr>
<tr>
<td>CC</td>
<td>$C_c$</td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>$C_d$</td>
<td></td>
</tr>
<tr>
<td>CE</td>
<td>$C_e$</td>
<td></td>
</tr>
<tr>
<td>CF</td>
<td>$C_f$</td>
<td></td>
</tr>
<tr>
<td>CDRAG</td>
<td>$C_D$</td>
<td>Drag coefficient</td>
</tr>
<tr>
<td>CHMU</td>
<td>$C_{\delta \mu \eta}$</td>
<td>Matrix of covariance of satellite state parameters and the accounted-for dynamic bias, Eqs. (40) and (42)</td>
</tr>
<tr>
<td>CKMER</td>
<td></td>
<td>Conversion factor, eru to km</td>
</tr>
<tr>
<td>CKSERH</td>
<td></td>
<td>Conversion factor, eru/hr to km/sec</td>
</tr>
<tr>
<td>CONA</td>
<td>$C_A$</td>
<td>Second gravitational harmonic coefficient</td>
</tr>
<tr>
<td>CONJ</td>
<td>$C_J$</td>
<td>First gravitational harmonic coefficient</td>
</tr>
<tr>
<td>CONK</td>
<td>$C_K$</td>
<td>Third gravitational harmonic coefficient</td>
</tr>
<tr>
<td>CPOS</td>
<td>$R_{cb_1}$</td>
<td>Block of perturbing body-position vectors</td>
</tr>
<tr>
<td>CRAD</td>
<td></td>
<td>Conversion factor, deg to rad</td>
</tr>
<tr>
<td>CWLIN(1)</td>
<td>$\Xi$</td>
<td>Position vector in the Encke axis</td>
</tr>
<tr>
<td>CWLIN(4)</td>
<td>$R_6$</td>
<td>Partialials of position with respect to drag bias, Eq. (30)</td>
</tr>
<tr>
<td>CWLIN(7)</td>
<td>$\dot{\Xi}$</td>
<td>Velocity vector in the Encke axis</td>
</tr>
<tr>
<td>CWLIN(10)</td>
<td>$\dot{R}_6$</td>
<td>Partialials of velocity with respect to drag bias, Eq. (30)</td>
</tr>
<tr>
<td>Program</td>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>-------------</td>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>GWLIN(13)</td>
<td>( \dddot{z} )</td>
<td>Acceleration vector in the Encke axis, Eq. (2)</td>
</tr>
<tr>
<td>GWLIN(16)</td>
<td>( \dddot{R}_5 )</td>
<td>Partialials of acceleration with respect to drag bias, Eq. (30)</td>
</tr>
<tr>
<td>DAYK</td>
<td></td>
<td>Number of days since epoch</td>
</tr>
<tr>
<td>DAYS</td>
<td></td>
<td>Day of year of launch</td>
</tr>
<tr>
<td>DEL</td>
<td>( \delta )</td>
<td>Variation in drag</td>
</tr>
<tr>
<td>DELALP</td>
<td>( \Delta \alpha )</td>
<td>Variations in the NASA parameters: ( \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6 ), Eq. (22)</td>
</tr>
<tr>
<td>DELX</td>
<td>( \Delta X )</td>
<td>Variations in state variables: ( x, y, z, \dot{x}, \dot{y}, \dot{z} )</td>
</tr>
<tr>
<td>DELY</td>
<td>( \Delta Y )</td>
<td>Array of observation residuals, Eq. (33)</td>
</tr>
<tr>
<td>DFRHO</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>DTFE</td>
<td></td>
<td>Integration interval for far-earth trajectories ( r &gt; 4 \text{ eru} )</td>
</tr>
<tr>
<td>DTI</td>
<td></td>
<td>Current integration step size</td>
</tr>
<tr>
<td>DTNE</td>
<td></td>
<td>Integration interval for near-earth trajectories ( r &lt; 4 \text{ eru} )</td>
</tr>
<tr>
<td>EBAR</td>
<td>( \bar{E} )</td>
<td>Covariance matrix of observation noise. The square root of diagonal terms are the standard deviations used for noise generation</td>
</tr>
<tr>
<td>EN</td>
<td>( n )</td>
<td>Mean motion</td>
</tr>
<tr>
<td>EPSSQ</td>
<td>( e^2 )</td>
<td>Square of earth's eccentricity</td>
</tr>
<tr>
<td>ERAD</td>
<td>( r_e )</td>
<td>Earth's equatorial radius</td>
</tr>
<tr>
<td>FA</td>
<td>( f_1 )</td>
<td>Series expansion used in the Keplerian Model</td>
</tr>
<tr>
<td>FB</td>
<td>( f_2 )</td>
<td>Series expansion used in the Keplerian Model</td>
</tr>
<tr>
<td>FC</td>
<td>( f_3 )</td>
<td>Series expansion used in the Keplerian Model</td>
</tr>
<tr>
<td>FD</td>
<td>( f_4 )</td>
<td>Series expansion used in the Keplerian Model</td>
</tr>
<tr>
<td>FPTH</td>
<td>( F'(\theta) )</td>
<td>Partial of Kepler's equation used in the Newton iteration scheme</td>
</tr>
</tbody>
</table>
Table 2 (cont.)

<table>
<thead>
<tr>
<th>Program</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRHO</td>
<td>Not used</td>
<td></td>
</tr>
<tr>
<td>FTH</td>
<td>F(θ)</td>
<td>Iterate in solving Kepler's equation $F(θ) \approx \sqrt{μ} \ (t-t_0)$</td>
</tr>
<tr>
<td>HAFPI</td>
<td>$π/2$</td>
<td></td>
</tr>
<tr>
<td>HMIN</td>
<td>Minutes of launch hours</td>
<td></td>
</tr>
<tr>
<td>HMU</td>
<td>$μ_e$</td>
<td>Earth's gravitational constant</td>
</tr>
<tr>
<td>HRS</td>
<td>Hours of launch day</td>
<td></td>
</tr>
<tr>
<td>IFLAG</td>
<td>Indicator for frequency of printing</td>
<td></td>
</tr>
<tr>
<td>INTPD</td>
<td>Indicator for Kalman scheme</td>
<td></td>
</tr>
<tr>
<td>IOBS</td>
<td>Indicator for computation of observations</td>
<td></td>
</tr>
<tr>
<td>ITYPE</td>
<td>Indicates type of observation data</td>
<td></td>
</tr>
<tr>
<td>JDRAG</td>
<td>Not used</td>
<td></td>
</tr>
<tr>
<td>KDATA</td>
<td>Number of observation points</td>
<td></td>
</tr>
<tr>
<td>KLM</td>
<td>Indicates units for position and velocity vectors</td>
<td></td>
</tr>
<tr>
<td>KCOMP</td>
<td>Indicates criterion causing a rectification</td>
<td></td>
</tr>
<tr>
<td>KOND</td>
<td>Indicator for frequency of printing the trajectory time history</td>
<td></td>
</tr>
<tr>
<td>KPRTR</td>
<td>Not used</td>
<td></td>
</tr>
<tr>
<td>KRUNT</td>
<td>Indicates number of observation points at a time point</td>
<td></td>
</tr>
<tr>
<td>KSTA</td>
<td>Station number of station currently observing</td>
<td></td>
</tr>
<tr>
<td>KTAB</td>
<td>Number of observation points processed</td>
<td></td>
</tr>
<tr>
<td>KTAPE</td>
<td>Not used</td>
<td></td>
</tr>
<tr>
<td>KTHC</td>
<td>Number of iterations used in solving Kepler's equation</td>
<td></td>
</tr>
<tr>
<td>KWBMU</td>
<td>Working body numbers</td>
<td></td>
</tr>
<tr>
<td>KWDDXI</td>
<td>Not used</td>
<td></td>
</tr>
</tbody>
</table>
Table 2 (cont.)

<table>
<thead>
<tr>
<th>Program</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>KWDXI</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>KWLIN</td>
<td></td>
<td>Number of second-order equations to be solved</td>
</tr>
<tr>
<td>KWLIND</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>KWXI</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>LDDXIA</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>LDDXIZ</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>LDXIA</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>LDXIZ</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>LTBCD</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>LTBIN</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>LXIA</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>LXIT</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>MBMAX</td>
<td></td>
<td>Number of working bodies</td>
</tr>
<tr>
<td>MOSOL</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>MREF</td>
<td></td>
<td>Number of reference bodies</td>
</tr>
<tr>
<td>MUD</td>
<td></td>
<td>Error indicator</td>
</tr>
<tr>
<td>MWREF</td>
<td></td>
<td>Indicates position of reference planet $\mu$ in the array BMU</td>
</tr>
<tr>
<td>NBTAS</td>
<td></td>
<td>Number of observational biases</td>
</tr>
<tr>
<td>NOSOL</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>NSRT</td>
<td></td>
<td>Number of sampling rates</td>
</tr>
<tr>
<td>NSLTH</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>NTE3</td>
<td></td>
<td>Number of $\Sigma^2$ matrices</td>
</tr>
<tr>
<td>NTYPE</td>
<td></td>
<td>Array of indicators for observation types</td>
</tr>
<tr>
<td>Program</td>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>NUMDAT</td>
<td></td>
<td>Number of observation types</td>
</tr>
<tr>
<td>NUMSTA</td>
<td></td>
<td>Number of observation stations</td>
</tr>
<tr>
<td>NYEAR</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>NYEARF</td>
<td></td>
<td>Year of launch or beginning of trajectory</td>
</tr>
<tr>
<td>PDOT</td>
<td>( \dot{p} )</td>
<td>Earth rotation rate in radians per solar day</td>
</tr>
<tr>
<td>PERDRG</td>
<td>( F_d )</td>
<td>Drag perturbation, Eq. (5)</td>
</tr>
<tr>
<td>PEROBL</td>
<td>( F_o )</td>
<td>Perturbations due to oblateness of earth, Eq. (4)</td>
</tr>
<tr>
<td>PERRAP</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>PI</td>
<td>( \pi )</td>
<td></td>
</tr>
<tr>
<td>PMAT</td>
<td>( P )</td>
<td>Covariance matrix of state variables, see p. 37</td>
</tr>
<tr>
<td>PRNTFE</td>
<td></td>
<td>Print interval for far-earth trajectory</td>
</tr>
<tr>
<td>PRNTNE</td>
<td></td>
<td>Print interval for near-earth trajectory</td>
</tr>
<tr>
<td>PSI60</td>
<td>( \psi_{60} )</td>
<td>Greenwich hour angle of 0(^{h}) January 0, 1960</td>
</tr>
<tr>
<td>PSIB</td>
<td>( \psi_o )</td>
<td>Greenwich hour angle of epoch of trajectory</td>
</tr>
<tr>
<td>PSIDOT</td>
<td>( \dot{\psi} )</td>
<td>Earth rotation rate in radians per solar hour</td>
</tr>
<tr>
<td>QMAT</td>
<td>( Q )</td>
<td>Parameter covariance matrix, Eq. (38)</td>
</tr>
<tr>
<td>RC</td>
<td>( R )</td>
<td>Position vector</td>
</tr>
<tr>
<td>RCB</td>
<td>( R_{cb_i} )</td>
<td>Block of perturbing body position vectors (see CPOS above)</td>
</tr>
<tr>
<td>RCIN</td>
<td>( R_I )</td>
<td>Position vector at beginning of the trajectory</td>
</tr>
<tr>
<td>RDC</td>
<td>( \dot{R} )</td>
<td>Velocity vector</td>
</tr>
<tr>
<td>RDCIN</td>
<td>( \dot{R}_I )</td>
<td>Velocity vector at beginning of the trajectory</td>
</tr>
<tr>
<td>RDI</td>
<td>( \dot{R}_o )</td>
<td>Velocity vector at last rectification</td>
</tr>
<tr>
<td>RDTB</td>
<td>( \dot{R}_{tb} )</td>
<td>Two-body velocity vector</td>
</tr>
<tr>
<td>Program</td>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>RECTT</td>
<td>$t$</td>
<td>Time of last rectification</td>
</tr>
<tr>
<td>RHODOT</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>RI</td>
<td>$R_o$</td>
<td>Position vector at last rectification</td>
</tr>
<tr>
<td>RMIN</td>
<td>$r_{min}$</td>
<td>Minimum perigee distance</td>
</tr>
<tr>
<td>RTAB</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>RTB</td>
<td>$R_{tb}$</td>
<td>Two-body position vector</td>
</tr>
<tr>
<td>RVB</td>
<td>$R_{vb}$</td>
<td>Vectors from vehicle to the perturbing bodies</td>
</tr>
<tr>
<td>SAMPLE</td>
<td></td>
<td>Data sampling intervals and rates</td>
</tr>
<tr>
<td>SCALDS</td>
<td></td>
<td>Scale factors</td>
</tr>
<tr>
<td>SCALEA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCALED</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCALEV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCALVS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SDDXI</td>
<td>$F_a$</td>
<td>Acceleration vectors for perturbations due to attractions of bodies other than the reference body, and the differential acceleration in the Encke axis due to the reference body, Eq. (3)</td>
</tr>
<tr>
<td>SEC</td>
<td></td>
<td>Seconds of launch minute</td>
</tr>
<tr>
<td>SMAT</td>
<td>$S_A$</td>
<td>The point transformation matrix, Eqs. (25) and (39)</td>
</tr>
<tr>
<td>SQTMU</td>
<td>$\sqrt{\mu_e}$</td>
<td>Square root of HMU above</td>
</tr>
<tr>
<td>STAC</td>
<td>$R_{s_i}$</td>
<td>Array of station coordinates in the inertial axis</td>
</tr>
<tr>
<td>STAHT</td>
<td>$h_{g_i}$</td>
<td>Array of station altitudes (above the geoid)</td>
</tr>
<tr>
<td>STALN</td>
<td>$b_{i}$</td>
<td>Array of station longitudes</td>
</tr>
<tr>
<td>STALT</td>
<td>$\lambda_{i}$</td>
<td>Array of station latitudes</td>
</tr>
<tr>
<td>STANM</td>
<td></td>
<td>Array of station names</td>
</tr>
<tr>
<td>T</td>
<td>$t$</td>
<td>Current time</td>
</tr>
<tr>
<td>TADD</td>
<td></td>
<td>Starter for random noise generator</td>
</tr>
</tbody>
</table>
Table 2 (cont.)

<table>
<thead>
<tr>
<th>Program</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TARG</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>TBF</td>
<td>$\tilde{f}$</td>
<td></td>
</tr>
<tr>
<td>TBFD</td>
<td>$\tilde{f}$</td>
<td>Coefficients for the two-body solution, Eqs. (10)-(13)</td>
</tr>
<tr>
<td>TCG</td>
<td>$g$</td>
<td></td>
</tr>
<tr>
<td>TBCD</td>
<td>$\tilde{g}$</td>
<td></td>
</tr>
<tr>
<td>TEBAR</td>
<td>$E_i$</td>
<td>Array of variance-covariance matrices of observations, for the observing stations</td>
</tr>
<tr>
<td>TH</td>
<td>$\tilde{e}$</td>
<td>Differential eccentric anomaly estimate for Kepler's equation solution</td>
</tr>
<tr>
<td>THC</td>
<td>$\theta$</td>
<td>Differential eccentric anomaly</td>
</tr>
<tr>
<td>TI</td>
<td>$t_o$</td>
<td>Time of last rectification</td>
</tr>
<tr>
<td>TIMEB</td>
<td></td>
<td>Array of station numbers for the $E$ matrices</td>
</tr>
<tr>
<td>TIN</td>
<td>$t_i$</td>
<td>Initial time</td>
</tr>
<tr>
<td>TIRT</td>
<td></td>
<td>Time of last rectification</td>
</tr>
<tr>
<td>TK</td>
<td></td>
<td>Time of next observation</td>
</tr>
<tr>
<td>TKEP</td>
<td></td>
<td>Time of last Kepler solution</td>
</tr>
<tr>
<td>TMAX</td>
<td>$t_{\text{max}}$</td>
<td>Maximum time of trajectory</td>
</tr>
<tr>
<td>TTABLE</td>
<td></td>
<td>Time of ephemeris position block</td>
</tr>
<tr>
<td>TMOPI</td>
<td>$2\pi$</td>
<td></td>
</tr>
<tr>
<td>TZERO</td>
<td></td>
<td>Elapsed time from $0^h$ January 0 of year to the beginning of the trajectory</td>
</tr>
<tr>
<td>TZHERS</td>
<td>$T_0$</td>
<td>Elapsed time from $0^h$ of present day to the beginning of the trajectory</td>
</tr>
<tr>
<td>UPBIAS</td>
<td>$\hat{n}$</td>
<td>Vector of observation bias uncertainties</td>
</tr>
<tr>
<td>UREF</td>
<td></td>
<td>Not used</td>
</tr>
<tr>
<td>WBMU</td>
<td>$\nu_1$</td>
<td>Packed array of gravitational constants (BMU packed) (Section III.B)</td>
</tr>
</tbody>
</table>
### Table 2 (cont.)

<table>
<thead>
<tr>
<th>Program</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>WBNAME</td>
<td></td>
<td>Packed array of names of perturbing bodies</td>
</tr>
<tr>
<td>WVECT</td>
<td>$W_a$</td>
<td>Parameter transition matrix of the accounted-for dynamic bias, Eq. (39)</td>
</tr>
<tr>
<td>XI</td>
<td>$i$</td>
<td>Inclination of the orbit</td>
</tr>
<tr>
<td>YCOM</td>
<td>$Y_c$</td>
<td>Array of computed values for observations</td>
</tr>
<tr>
<td>YOBS</td>
<td>$Y_{obs}$</td>
<td>Array of actual observation values</td>
</tr>
</tbody>
</table>
Appendix A

DETAILED PROGRAM INPUT

This appendix gives a detailed description of the input format. The quantities in the description column are entered on the specified card in the appropriate columns. The names are those used for the quantities in the program.

Some constants which are subsequently defined are set initially, but may be changed by input. If these are to be changed in multiple-case runs, they must be changed in each case.

\[ \bar{c}_J = 0.032321941 \text{ eru}^7/\text{hr}^2 = \mu J_{20} \]
\[ \bar{c}_A = 0.000045791657 \text{ eru}^6/\text{hr}^2 = -\frac{2}{3} \mu J_{30} \]
\[ \bar{c}_K = 0.0001343886 \text{ eru}^7/\text{hr}^2 = -\mu J_{40} \]
\[ \psi_{60} = 1.72218633 \text{ rad} \]
\[ \psi = 0.2625164 \text{ rad/hr} \]
\[ \dot{\psi} = 0.0172027915 \text{ rad/day} \]

SIMULATION DATA

The input data are read by FORTRAN with FORMAT statements and are, therefore, rigidly fixed except that they are divided into groups preceded by identification cards; if the identification card for a group does not appear, then its data will not be read. A group ID number 000013 signals the end of the data and must always appear. The format is shown in the figure, and the individual format type is given below. The units used in the program are eru and hr; however, the input is slightly more flexible. An input parameter (KLM) may be used to change the units of some parameters for both input and output. In the following list the units are given where applicable and the symbol (u) is used for those which are to be defined by the input value of KLM.

The order of the variables in the following list is the same as that given in the figure, which describes the card format:
<table>
<thead>
<tr>
<th>Group</th>
<th>Name</th>
<th>Type</th>
<th>Units</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TITLE</td>
<td>12A4</td>
<td></td>
<td></td>
<td>Title of simulation run</td>
</tr>
<tr>
<td>2</td>
<td>TIN</td>
<td>E12.0 hr</td>
<td></td>
<td>$t_o$</td>
<td>Reference time for the beginning of simulation</td>
</tr>
<tr>
<td>2</td>
<td>TMAX</td>
<td>E12.0 hr</td>
<td></td>
<td>$t_{\text{max}}$</td>
<td>Final time for the simulation</td>
</tr>
<tr>
<td>2</td>
<td>DTNE</td>
<td>E12.0 hr</td>
<td></td>
<td>$\Delta t_{\text{ne}}$</td>
<td>Integration interval for near-earth (less than 4 eru from earth center) trajectories</td>
</tr>
<tr>
<td>2</td>
<td>DTFE</td>
<td>E12.0 hr</td>
<td></td>
<td>$\Delta t_{\text{fe}}$</td>
<td>Integration interval for far-earth (greater than 4 eru from earth center) trajectories</td>
</tr>
<tr>
<td>2</td>
<td>PRNTNE</td>
<td>E12.0 hr</td>
<td></td>
<td></td>
<td>Trajectory print interval for near-earth trajectories</td>
</tr>
<tr>
<td>2</td>
<td>PRNTFE</td>
<td>E12.0 hr</td>
<td></td>
<td></td>
<td>Trajectory print interval for far-earth trajectories</td>
</tr>
<tr>
<td>2</td>
<td>NYEARP</td>
<td>16 yr</td>
<td></td>
<td></td>
<td>Year of initial time of simulation</td>
</tr>
<tr>
<td>2</td>
<td>DAYS</td>
<td>F6.0 day</td>
<td></td>
<td></td>
<td>Day at initial time of simulation</td>
</tr>
<tr>
<td>2</td>
<td>HRS</td>
<td>F6.0 hr</td>
<td></td>
<td></td>
<td>Hour of initial time of simulation</td>
</tr>
<tr>
<td>2</td>
<td>HMIN</td>
<td>F6.0 min</td>
<td></td>
<td></td>
<td>Minute at initial time of simulation</td>
</tr>
<tr>
<td>2</td>
<td>SEC</td>
<td>F6.0 sec</td>
<td></td>
<td></td>
<td>Second at initial time of simulation</td>
</tr>
<tr>
<td>3</td>
<td>MREF</td>
<td>I6</td>
<td></td>
<td></td>
<td>Indicates the initial reference body</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1 - Earth</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2 - Sun</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3 - Moon</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4 - Venus</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5 - Mars</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6 - Jupiter</td>
</tr>
<tr>
<td>Group</td>
<td>Name</td>
<td>Type</td>
<td>Units</td>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
<td>------</td>
<td>-------</td>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>3</td>
<td>KLM</td>
<td>I6</td>
<td></td>
<td></td>
<td>Determines the units for those input parameters whose units are given as $u$ (undetermined)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>$\text{ft, ft/sec}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>$\text{km, km/sec}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>$\text{mi, m/sec}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>$\text{n mi, n mi/sec}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>$\text{eru, eru/hr}$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>ISTAT</td>
<td>I6</td>
<td></td>
<td></td>
<td>Number of state variables (must be 6 or 7)</td>
</tr>
<tr>
<td>3</td>
<td>IFLAG</td>
<td>I6</td>
<td></td>
<td></td>
<td>Print control for the Kalman Scheme. Any combination of the following sets of parameters may be printed by summing $1 - Q_A, C_{\delta \alpha \mu}, C_{\delta \alpha \mu}, \Delta Q$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>$F, N_A, L, Y, Y$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>100</td>
<td>$A_A^{-1}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1000</td>
<td>$S_A^{-1}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10,000</td>
<td>$E$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>When $IFLAG &gt; 0$ print will occur only once in each print interval. If $IFLAG &lt; 0$ print will occur at each observation</td>
</tr>
<tr>
<td>3</td>
<td>IOBS</td>
<td>I6</td>
<td></td>
<td></td>
<td>Observation computation indicator</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td>do not compute observations</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>compute observations</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>compute observations and store on FORTRAN Logical Unit 8 (only if observations not input)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>LTBIN</td>
<td>I6</td>
<td></td>
<td></td>
<td>Control for observation data input</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td>no observations input</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>observations on cards</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>observations are on FORTRAN Logical Unit 8</td>
<td></td>
</tr>
<tr>
<td>Group</td>
<td>Name</td>
<td>Type</td>
<td>Units</td>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
<td>------</td>
<td>-------</td>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>3</td>
<td>KOND</td>
<td>I6</td>
<td></td>
<td></td>
<td>Control for trajectory and state covariance print. Sums of the indicators will cause each to print. 0 - do not print 1 - print position and velocity 2 - print osculating elements 4 - print state covariance matrix -1, or -2, or -3, print the applicable parameters at each observation point also</td>
</tr>
<tr>
<td>3</td>
<td>INTFD</td>
<td>I6</td>
<td></td>
<td></td>
<td>Procedure control for the Kalman Scheme ±1 - process observation parameters at a data point separately and use only the one which gives the minimum trace on the covariance matrix ±2 - process observation parameters simultaneously at a data point &gt;0 - correct the orbit at observation points and conclude by computing an updated estimate of initial state parameters and covariance &lt;0 - do not correct the orbit and do not update the initial estimate =0 - do not process observations</td>
</tr>
<tr>
<td>3</td>
<td>IPLAN</td>
<td>I6</td>
<td></td>
<td></td>
<td>Indicates the observing planet 1 - Earth 2 - Sun 3 - Moon 4 - Venus 5 - Mars 6 - Jupiter</td>
</tr>
<tr>
<td>Group</td>
<td>Name</td>
<td>Type</td>
<td>Units</td>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
<td>------</td>
<td>-------</td>
<td>--------</td>
<td>-------------</td>
</tr>
</tbody>
</table>
| 3     | IBMU  | I6   |       |        | Control which bodies will be used in computing perturbations due to gravitational attraction. Sum the indicators for those to be used.  
100,000 - Earth  
10,000 - Sun  
1,000 - Moon  
100 - Venus  
10 - Mars  
1 - Jupiter |
| 4     | CDRAG | E12.0|       | CD     | Drag coefficient |
| 4     | AMASS | E12.0| cm²/gm| A/m    | Area-to-mass ratio |
| 4     | DELIN | E12.0|       | Δ      | Initial drag uncertainty |
| 5     | VNAME | A4   |       |        | A one-word vehicle name |
| 5     | RMIN  | E12.0| eru   |        | Minimum radius of perigee (used for terminating the run) |
| 5     | TADD  | E12.0|       |        | Control for noise generation and starter value for the random number generator  
>0 - generate random noise in observation  
=0 - do not generate noise |
<p>| 5     | BIASMU| E12.0| eru³/hr²| μ | Value of uncertainty in μ (gravitational constant) |
| 6     | PSI60 | E12.0| rad   | ψ¹950 | Greenwich hour angle of mean equinox 0th January 1, 1950 |
| 6     | PDOT  | E12.0| rad/day| ρ   | Daily rate of earth's rotation about the sun |
| 6     | PSIDOT| E12.0| rad/day| ψ   | Hourly rate of earth's rotation on its axis |
| 6     | CONJR | E12.0| eru⁵/hr²| C_J | First harmonic of the earth's potential |</p>
<table>
<thead>
<tr>
<th>Group</th>
<th>Name</th>
<th>Type</th>
<th>Units</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>CONAR</td>
<td>E12.0</td>
<td>$\text{er}^6/\text{hr}^2$</td>
<td>$\overline{C}_A$</td>
<td>Second harmonic of earth's potential</td>
</tr>
<tr>
<td>6</td>
<td>CONKR</td>
<td>E12.0</td>
<td>$\text{er}^7/\text{hr}^2$</td>
<td>$\overline{C}_K$</td>
<td>Third harmonic of earth's potential</td>
</tr>
<tr>
<td>7</td>
<td>SRCIN(1)</td>
<td>E12.0</td>
<td>u</td>
<td>x</td>
<td>Coordinate of position in equatorial plane, vernal equinox direction</td>
</tr>
<tr>
<td>7</td>
<td>SRCIN(2)</td>
<td>E12.0</td>
<td>u</td>
<td>y</td>
<td>Coordinate of position in equatorial plane</td>
</tr>
<tr>
<td>7</td>
<td>SRCIN(3)</td>
<td>E12.0</td>
<td>u</td>
<td>z</td>
<td>Coordinate of position perpendicular to equatorial plane</td>
</tr>
<tr>
<td>7</td>
<td>SRDCIN(1)</td>
<td>E12.0</td>
<td>u</td>
<td>$\dot{x}$</td>
<td>Coordinate of velocity in X direction</td>
</tr>
<tr>
<td>7</td>
<td>SRDCIN(2)</td>
<td>E12.0</td>
<td>u</td>
<td>$\dot{y}$</td>
<td>Coordinate of velocity in Y direction</td>
</tr>
<tr>
<td>7</td>
<td>SRDCIN(3)</td>
<td>E12.0</td>
<td>u</td>
<td>$\dot{z}$</td>
<td>Coordinate of velocity in Z direction</td>
</tr>
<tr>
<td>8</td>
<td>NUMSTA</td>
<td>I6</td>
<td></td>
<td></td>
<td>Number of observing stations</td>
</tr>
<tr>
<td>8</td>
<td>STANM(I)</td>
<td>2A4</td>
<td></td>
<td></td>
<td>Station names (two words each)</td>
</tr>
<tr>
<td>8</td>
<td>NSTA(I)</td>
<td>I6</td>
<td></td>
<td></td>
<td>Array of station identification numbers; negative numbers indicate orbiting</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>stations; positive numbers indicate ground stations</td>
</tr>
<tr>
<td>8</td>
<td>ITYPE(I)</td>
<td>I6</td>
<td></td>
<td></td>
<td>Indicates the observed parameters for each station. Sum indicators for</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>more than one parameter</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1 - azimuth</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10 - elevation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100 - range</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1000 - range rate</td>
</tr>
<tr>
<td>8</td>
<td>ELMAX(I)</td>
<td>E12.0</td>
<td>deg</td>
<td></td>
<td>Maximum elevation angle for each radar</td>
</tr>
<tr>
<td>Group</td>
<td>Name</td>
<td>Type</td>
<td>Units</td>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>----------</td>
<td>--------</td>
<td>-------</td>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>8</td>
<td>ELMIN(I)</td>
<td>E12.0</td>
<td>deg</td>
<td></td>
<td>Minimum elevation angle for each radar</td>
</tr>
<tr>
<td>8</td>
<td>OMEGS(I)</td>
<td>E12.0</td>
<td>deg/</td>
<td></td>
<td>Radar rotation rate for each radar</td>
</tr>
<tr>
<td>8</td>
<td>RANGS(I)</td>
<td>E12.0</td>
<td>u</td>
<td></td>
<td>Maximum range for each radar</td>
</tr>
<tr>
<td>8</td>
<td>DELAZ(I)</td>
<td>E12.0</td>
<td>deg</td>
<td></td>
<td>Azimuth bandwidth for each radar</td>
</tr>
<tr>
<td>8</td>
<td>DELT(I)</td>
<td>E12.0</td>
<td>sec</td>
<td></td>
<td>Minimum time increment between observations for each station</td>
</tr>
<tr>
<td>8</td>
<td>STALN(I)</td>
<td>E12.0</td>
<td>deg</td>
<td></td>
<td>Station longitudes (for ground stations)</td>
</tr>
<tr>
<td>8</td>
<td>STALT(I)</td>
<td>E12.0</td>
<td>deg</td>
<td></td>
<td>Station latitudes (for ground stations)</td>
</tr>
<tr>
<td>8</td>
<td>STAHT(I)</td>
<td>E12.0</td>
<td>u</td>
<td></td>
<td>Station altitudes (for ground stations)</td>
</tr>
<tr>
<td>8</td>
<td>ORBT(I)</td>
<td>E12.0</td>
<td>hr</td>
<td></td>
<td>Orbit reference times for which orbital elements are given (for orbiting stations)</td>
</tr>
<tr>
<td>8</td>
<td>ORBV(I)</td>
<td>E12.0</td>
<td>deg</td>
<td></td>
<td>True anomalies of station orbits at reference time (for orbiting stations)</td>
</tr>
<tr>
<td>8</td>
<td>ORBA(I)</td>
<td>E12.0</td>
<td>u</td>
<td></td>
<td>Semimajor axis of orbits (for orbiting stations)</td>
</tr>
<tr>
<td>8</td>
<td>ORBE(I)</td>
<td>E12.0</td>
<td></td>
<td></td>
<td>Eccentricities of orbit ellipses (for orbiting stations)</td>
</tr>
<tr>
<td>8</td>
<td>ORBI(I)</td>
<td>E12.0</td>
<td>deg</td>
<td></td>
<td>Inclination angles (for orbiting stations)</td>
</tr>
<tr>
<td>8</td>
<td>ORBCO(I)</td>
<td>E12.0</td>
<td>deg</td>
<td></td>
<td>Right ascension of ascending node of each orbit (for orbiting stations)</td>
</tr>
<tr>
<td>8</td>
<td>ORBSO</td>
<td>E12.0</td>
<td>deg</td>
<td></td>
<td>Argument of perigee of each orbit (for orbiting stations)</td>
</tr>
<tr>
<td>Group</td>
<td>Name</td>
<td>Type</td>
<td>Units</td>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>--------------</td>
<td>--------</td>
<td>-------</td>
<td>--------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>9</td>
<td>SAVEP(I,J)</td>
<td>E12.0</td>
<td>(u^2)</td>
<td></td>
<td>The initial covariances matrix ((P))</td>
</tr>
<tr>
<td>10</td>
<td>NTEB</td>
<td>I6</td>
<td></td>
<td></td>
<td>Number of (E) matrices input (maximum is 5)</td>
</tr>
<tr>
<td>10</td>
<td>ISTE(I)</td>
<td>I6</td>
<td></td>
<td></td>
<td>Largest station number to which each (E) matrix applies. Values must be in ascending order</td>
</tr>
<tr>
<td>10</td>
<td>SAVE(E)</td>
<td>E12.0</td>
<td>(u)</td>
<td></td>
<td>The (E) matrix (standard deviations in observed parameters). Units are:</td>
</tr>
<tr>
<td></td>
<td>(I,J,K)</td>
<td></td>
<td></td>
<td>(\sigma_a)</td>
<td>second of arc</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(\sigma_b)</td>
<td>seconds of arc</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(\sigma_r)</td>
<td>(=) (km) if (KLM=2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(\sigma_{r^*})</td>
<td>(u)</td>
</tr>
<tr>
<td>11</td>
<td>UPBIAS(I)</td>
<td>E12.0</td>
<td>(u)</td>
<td></td>
<td>Uncertainty in biases in:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I=1</td>
<td>speed of light-eru/hr</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I=2</td>
<td>(u) coordinate of station position-(u)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I=3</td>
<td>(v) coordinate of station position-(u)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I=4</td>
<td>(w) coordinate of station position-(u)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I=5</td>
<td>azimuth-seconds of arc</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I=6</td>
<td>elevation-seconds of arc</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I=7</td>
<td>range-(u)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I=8</td>
<td>range rate-(u)</td>
</tr>
<tr>
<td>12</td>
<td>A</td>
<td>E12.0</td>
<td>(u)</td>
<td></td>
<td>Semimajor axis of target satellite orbit</td>
</tr>
<tr>
<td>12</td>
<td>E</td>
<td>E12.0</td>
<td>(e)</td>
<td></td>
<td>Eccentricity of the observed satellite orbit</td>
</tr>
<tr>
<td>12</td>
<td>AIN</td>
<td>E12.0</td>
<td>deg</td>
<td>(i)</td>
<td>Inclination of the observed satellite orbit</td>
</tr>
<tr>
<td>12</td>
<td>SO</td>
<td>E12.0</td>
<td>deg</td>
<td>(\omega)</td>
<td>Argument of perigee of the observed satellite orbit</td>
</tr>
<tr>
<td>Group</td>
<td>Name</td>
<td>Type</td>
<td>Units</td>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
<td>-------</td>
<td>-------</td>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>12</td>
<td>CO</td>
<td>E12.0</td>
<td>deg</td>
<td>Ω</td>
<td>Right ascension of ascending node of the observed satellite orbit</td>
</tr>
<tr>
<td>12</td>
<td>V</td>
<td>E12.0</td>
<td>deg</td>
<td>ν</td>
<td>True anomaly of target satellite orbit at reference time</td>
</tr>
</tbody>
</table>

**OBSERVATION DATA**

The observation data, if any, may be input in either of two forms:

1. For cards immediately following the simulation data and preceded by a format definition card, the format definition card must be included. Although the format of the data is flexible, the order and number of parameters are not. The proper order of the input parameters is:

   a. Station number (fixed point value)
   b. Day from reference data (floating point value)
   c. Hour from reference time (floating point value)
   d. Minute from reference time (floating point value)
   e. Second from reference time (floating point value)
   f. Azimuth (in degrees)
   g. Elevation (in degrees)
   h. Range (units determined by KLM)
   i. Range rate (units determined by KLM)
   j. Type of observation; determines which observation parameter will be used (fixed point value)

   1 - azimuth
   10 - elevation
   100 - range
   1000 - range rate

   This is overridden by ITYPE(I) from simulation input unless ITYPE(I) = 0. When card input is used, an observation tape is created on FORTRAN Logical Unit 8.

2. A binary tape may be used for observation data input. It must be on FORTRAN Logical Unit 8. It may have been created by a previous run which either had card input of observations or generated observations itself. The format of each record is:
Item 1 - hour from reference time (double precision)
Item 2 - minute of hour (double precision)
Item 3 - second of minute (double precision)
Item 4 - station number (INTEGER*4)
Item 5 - azimuth (rad) (double precision)
Item 6 - elevation (rad) (double precision)
Item 7 - range (eru) (double precision)
Item 8 - range rate (eru/hr) (double precision)

The switch settings for creating the binary tape are:

a. When generating observations the following switches must be set:
   IOBS = 2
   LTBIN = 0

   The value of ITYPE for each station will be stored with its observation data; however, all four parameters will be computed and stored on tape regardless of ITYPE.

   Other switches should be set as follows:
   ISTAT = 6
   IFLAG = 0

b. When reading data from cards, a binary tape is created on FORTRAN Logical Unit 8 by:
   LTBIN = 1

**PLANETARY TYPE**

The ephemeris used in TEP0P is essentially the Jet Propulsion Laboratory Development Ephemeris Number 19.\(^9\) The data contained on the tape are integrated positions and velocities of the planets Mercury, Venus, Earth-Moon barycenter, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto. Similarly, positions and velocities of the moon with the nutations and nutation rates in longitude and obliquity are included. The positions and velocities are referred to the rectangular equatorial coordinate system of the mean equator and equinox of the Julian Date 1950 = Julian Ephemeris Date (JED) of 2433282.423. The JED that appears on the tapes is the actual Julian Date number 2433280.5. The intervals spanned by the tapes are as follows:
Tape Julian Date (calendar date) to Julian Date (calendar date)
1 2433280.5 December 30, 1949 2440584.5 December 29, 1969
2 2440544.5 November 19, 1969 2445752.5 February 22, 1984
3 2445712.5 January 13, 1984 2451544.5 January 1, 2000

For detailed description consult Ref. 9.

The planetary tape is required only when IBMU (group 3) indicates more than one planet.

SUMMARY OF INPUT

The following items provide a checklist for setting up data for a computer run.

1. Coordinate Systems and Time. The orientation of the rotating coordinate system with respect to the inertial reference coordinate system (see Section III.A) depends upon the angle, PSI60 (group 6); the angular rates, PDOT and PSIDOT (group 6); and time, TIN, NYEARP, DAYS, HRS, HMIN, SEC (group 2). The origin of one with respect to the other can also depend on time, since the two need not necessarily be referred to the same planet (IPLAN ≠ MREF, group 3). However, when the perturbing accelerations due to other bodies are not accounted for and the observing planet and reference planet are the same, then the user may simplify the problem of determining the initial position of the vehicle with respect to the observing stations by setting: PSI60 = DAYS = HRS = HMIN = SEC = 0, NYEARP = 1950, and TIN = 0. The two-axis systems are then initially aligned.

2. Equations of Motion of the Observed Vehicle. The vehicle may or may not be interplanetary. In either case, the origin of the inertial axis system in which the equations of motion are solved is given initially by MREF = k in group 3, where k is the planet number. When the path is interplanetary the reference center may change, during flight, to any of the other bodies; in order for this to happen the user must set IBMU = 111111 for interplanetary flights. When the path
is on orbit about one of the six bodies, the user must set IBMU = $10^{6-k}$ (group 3) as well as MREF. The gravitational attraction due to bodies other than the reference body may be accounted for by setting IBMU = $10^{6-k} + 10^{6-i}$ + ..., etc., where i refers to the body number (e.g., for an earth orbit with the sun and moon gravitational attraction accounted for, IBMU = 111000 and MREF = 1). When the reference center is earth, the perturbing accelerations due to oblateness and atmospheric drag may be accounted for by setting CDRAG ≠ 0 and AMASS ≠ 0 (group 4) for drag, and CONJR ≠ 0, CONAR ≠ 0, CONKR ≠ 0 (group 6) for oblateness. The initial position and velocity may be input in either group 7 or group 12. The simplest case is an earth orbit with no perturbing accelerations; the input for this case would be:

MREF = 1  
IBMU = 100,000  
CONJR = CONAR = CONKR = 0  
AMASS = 0  
CDRAG = 0

with initial position and velocity given by the orbital elements in group 12.

3. Observing Sensors. There may be 54 observing sensors. They all must be either orbiting or fixed on the same planet (IPLAN = k, where k is the body number; and IBMU = $10^{6-k} + 10^{6-i}$, where i is the reference planet for the vehicle when it is different than the observing planet). The sign given to the station number, NSTA (group 8), determines whether the sensor is fixed or orbiting. The scan space must be defined for each sensor, as well as a rotation rate and minimum interval between measurements. It is usually only necessary to limit the elevation angle. The data for a 10-deg minimum elevation angle limit with 1-sec data rate would be (in group 8):

ELMAX(I) = 90  
ELMIN(I) = 10  
OMEGS(I) = 0  
RANGS(I) = 1.E20  
DELZ(I) = 360  
DELT(I) = 1
When azimuth limits are required for a problem, care should be taken because an iterative scheme is required for finding the time of an observation. The solution can be difficult when (a) the azimuth window is too small, (b) the rotation rate is too large, or (c) the vehicle passes too near the sensor.

When sensor data are input, the above limit inputs are meaningless because the external measurement data dictate the time of the observations.

4. Statistical Computations. The number of state variables (ISTAT) must be determined first. There are seven which may be estimated—the position and velocity vectors and a drag bias. The initial estimate for the drag bias, \( \delta \), is input in DELIN (group 4). The uncertainty in other variables may be accounted for but they cannot be estimated; they are the gravitational bias, whose uncertainty is given in BIASMU (group 5), and the observational biases in speed of light, geodetic net bias (three components), and the biases in observations directly, whose uncertainties are given in UPBIAS (group 11). Noise in the data may be accounted for by the input standard deviations in SAVEB (group 10).

There are essentially three modes of operation insofar as the statistical computations are concerned. These are:

a. Synthetic Data Generation Mode. In this mode no statistical computations are necessary; therefore, groups 9, 10, and 11 need not be input. Data may be stored on tape for later use by setting IOBS = 2 (group 3). In group 3, INTPD = 0 and LTBIN = 0.

b. Orbit Determination Mode. This is the mode in which the state variables are actually estimated. An initial covariance must be given in group 9, and covariance of observation noise is given in group 10. Group 11 and BIASMU (group 5) need be input only if the uncertainty in these biases is to be accounted for. The indicators need to be:

- INTPD > 0
- IOBS = 1
- LTBIN > 0
- ISTAT = 6 or 7
The data must be input in either card form or on FORTRAN Logical Unit 8. If the data are on FORTRAN Logical Unit 8 (synthetic data), noise may be added by setting TADD = 41 (group 5).

c. Variance-Covariance Propagation Mode. This mode is similar to the previous mode except that the state variables are not actually estimated (INTPD < 0); therefore, no observation data need be input (LTBIN = 0). The initial estimate of covariance (group 9) does need to be input as well as uncertainties in the biases which are to be accounted for (group 11 and BIASMU, group 5) and standard deviations in observations (group 10).

5. Output. The amount and type of output is determined by the time intervals PRNTNE, PRNTFF, and possibly by the interval between observations and the indicators IFLAG, KOND, INTPD. The last indicator, INTPD, affects the output because it controls the computation of those parameters which may be printed due to IFLAG and it may cause the program to integrate backward to give a new estimate of state variables at the initial time and print these.
Appendix B

SUBROUTINE DIRECTORY

AIMS Computes atmospheric density as shown in Section III.B.4.

CROSS Computes the crossproduct of two vectors, \( C = A \times B \).

CWLR A dual-purpose routine which computes either the true position and velocity

\[
\mathbf{R} = \mathbf{R}_{tb} + \mathbf{E}, \quad \mathbf{\dot{R}} = \mathbf{R}_{tb} + \mathbf{\dot{E}}
\]

or the initial values for the Encke terms

\[
\mathbf{E} = \mathbf{R} - \mathbf{R}_{tb}, \quad \mathbf{\dot{E}} = \mathbf{\dot{R}} - \mathbf{\dot{R}}_{tb}
\]

DALFA Computes corrections to the state variables, \( \Delta X \), due to corrections in the state parameters, \( \Delta a \), by the finite rotations shown in Section IV.A.

DERIV Computes the perturbing accelerations, \( \mathbf{\ddot{E}} \) (Section III.B, Eq. (2)), and the variational equations defining the Parameter Transition Matrix of the Updated Dynamic Bias (Section IV.B, Eq. (33)) which are to be integrated by DRKAM.

DINT Converts a double precision number to an integer.

DOT Computes the dot product of two vectors, \( \mathbf{C} = \mathbf{A} \cdot \mathbf{B} \).

DRAG Computes the perturbing drag accelerations as shown in Section III.B.3.

DRIVE Sets initial values for some constants. It is the ENTRY point for the program.

DRKAM Numerically integrates the derivatives computed by DERIV.

DTEST Computes the time of the next data point when external data are not used. See Section III.D.

EIGEN Computes the eigenvalues and eigenvectors of a real symmetric matrix, namely, the covariance matrix for output.

FIX Separates a decimal integer into its vector of coefficients and counts the number of nonzero elements.
GAUSS  Normally distributed random number generator.
GETTAP  Reads the planetary tape described in Appendix A.
INPUT   Reads the input data described in Appendix A, and does some post-data initialization.
INTM    Is merely a link to the integration routine, DRKAM.
KEPLER  Solves the general form two-body equations as shown in Section III.B.5.
KQUIK   Solves the elliptical two-body equations as shown in Section III.B.5.
MAIN    Directs the flow of computations.
MATAVG  Computes the average of the sum of a matrix and its transpose.
MATINV  Computes the inverse of a matrix.
MATMP4  Computes the product of two matrices.
MATMP2  Computes the product of three matrices.
MINVAR  Updates the covariance matrix and computes the corrections to the vector of state parameters as shown in Section IV.
OBLATE  Computes perturbing accelerations due to the aspherical body.
OBSEB   Computes observation data.
OSCUUL  Computes the osculating elements: semimajor axis, eccentricity, inclination, node, argument of perigee, eccentric anomaly, mean anomaly, and the mean motion.
PART    Computes the parameter transition matrix A as shown in Section IV.A.2.
PLANET  Causes planet positions to be computed by READE and if required for a reference planet switch, computes the vehicle position in the new axes.
PRINT   Prints the history of computations.
RCTTST  Tests the Encke terms to determine if a rectification is necessary.
READE   Interpolates planetary positions and velocities, converts units, and translates reference centers if required.
RECORD  Reads external data as described in Appendix A.
RECT Updates the two-body equations at a rectification point.

RPERG Computes perigee radius.

SMATRX Computes the point transformation matrix or its inverse as defined in Section IV.A.

SPHERE Determines whether a change of reference body is necessary.

VECTOR Computes the magnitude of a vector, its square, and its cube. The vector of position, for instance, occupies a six-element array; the first three are the vector elements, the fourth is the cube of the magnitude, the fifth is the magnitude, and the sixth is the square of the magnitude. The last three elements of the array are supplied by this subroutine.
REFERENCES


