THE RATE OF ADVANCE OF THE FRONT
LINE IN SOME WORLD WAR II CAMPAIGNS

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Summary: This document presents some historical data that deal with the velocity of the front line and the strengths of the opposing forces in the European and Mediterranean theaters of World War II.
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In a model of air-ground warfare the position of the battle line, relative to its initial position, is of interest for several reasons. For example, the changing length of a supply line is an important logistic variable; the vulnerability or even availability of forward air fields, as well as the effectiveness of fields in the rear are influenced by their distance from the front; again, one may use the change in position of the front line as part of the payoff in the model war game. This is, in fact, the case in both the RAND study of tactical air systems and that of the symmetric air war where the motion of the front line decides the outcome of the war game. How accurately the front line position need be known depends on the particular model being investigated and on how sensitively the conclusions drawn therefrom vary with changes in the battle line. The purpose of this document is to present some historical data that deal with the velocity of the front line and the strengths of the opposing forces in the European and Mediterranean theaters of WW II.
SOME VARIABLES THAT AFFECT THE VELOCITY OF THE FRONT LINE

Even a superficial consideration of air-ground war leads to a lengthy list of factors, each of more or less influence on the progress of the war. Many of these factors do not lend themselves readily to numerical description, and hence are usually omitted in a mathematical model. They may, of course, be of equal or greater importance than some of those quantifiable terms included in the model. We mention some of the obvious variables.

(1) The strengths of the opposing forces. We leave both the definition and the discussion of this term "strength" to the next section.

(2) Tactical terrain. The terrain over which a battle is fought has many and diverse effects. It may, for one thing, multiply several-fold the effective force of the defender. In the Italian campaign, for example, a single German 88mm gun, backed up by a tank carrying a 75mm gun, was able to hold up an American division in its advance up the Sele valley.*

(3) Strategic terrain. Topographic features may influence the course of events even though no battles then rage over them. For example, the timing and execution of a withdrawal may rest on the presence of rivers or other barriers to the rear.

* Reference 1, p. 81.
in the mathematical model, a war game, or in an historical example) do not include those of the entire theater, then the course of events may be explicable only when the

for example, one is following the fortunes of the U.S. Fifth Army in Italy, it is at first difficult to understand the sudden advance beginning one week after the Salerno invasion. But the British Eighth Army made contact with the Fifth at this time, and the German XIV Panzer Corps is presumed to have been ordered to withdraw to the north in order to avoid being outflanked.

(5) Interdiction that prevents a division from reaching the front is easily taken into account in a mathematical model. Interdiction that prevents a division already in the line from moving to a more vital portion of the line requires a much more sophisticated model.

(6) Mobility of firepower. Rockets fired from a ground launcher or from an aircraft using a tube cluster, a 75mm gun arming an aircraft or a medium tank -- these are obviously very different animals, but this difference does not make for simplicity of mathematical models.

(7) Reconnaissance and intelligence. The qualitative importance of intelligence is unquestioned. However,
assigning a numerical measure for its worth is difficult.

(8) Logistics. Difficulties of supply can be taken into account, in a model, at least in part, if "strength" is measured in terms of the firepower actually applied. Rationing of ammunition would then enter the model. Shortages of POL, however, affect mobility of firepower, a factor less easily introduced into a mathematical model of ground war.

(9) Close support

(10) The special mobility of airborne troops.

(11) Weather. This is part of the tactical terrain; it also affects the amount and effectiveness of close support

(12) Homogeneity of forces. The heterogeneous mixture of Italians and Germans encountered in Sicily was a factor in that campaign.

(13) Quality of troops. The age, morale, training, and battle experience of the German troops, for instance, varied greatly from division to division. For example, the 3rd Parachute Division was young, Nazi-indoctrinated, and fought best of all the German units on the Normandy front; it was not to be compared with a division of raw recruits from the Hitler Jugend. In his classic nineteenth century work [Ref. 2] on war models, Major Livermore had a term in his equations for the percentage of troops that are
recruits. Most quantitative models, however, stop somewhat short of this attempt at realism.

The list given above can probably be expanded considerably; we stop at this point, however, and return to the first factor mentioned: the strengths of the opposing forces.

THE STRENGTHS OF THE OPPOSING FORCES

The simplest measure of the strength of an army is given by a count of the number of divisions in the line as shown by the Order of Battle.* This immediately raises the obvious question as to whether or not the divisions were up to their T/O strengths. U.S. divisions were normally maintained at or near combat strength in the field by a steady flow of replacements; as casualties occurred, the positions were filled by new men. There are instances, however, of a division short 1000 riflemen, of battalions down to 1 or 2 companies, and of tank battalions reduced to 50 tanks each. Nevertheless, we have assumed, when necessary, that Allied divisions were maintained at their T/O figure. German divisions, on the other hand, fluctuated widely in strength. Although some replacements were

* Even the official Order of Battle for the American forces in the ETO [Ref. (19)] is a document of somewhat limited availability,
received, reorganization of units was necessary in many cases in order to keep units intact. This was usually done by cannibalizing depleted units to provide replacements for less worn outfits. During some campaigns this was done so extensively as to preclude keeping an orderly account of the German divisions present.

Even a count of the number of divisions on the opposing side -- that is, the German order of battle -- can not be guaranteed as completely accurate. We have depended, in large part, on the G-2 estimate of the enemy situation. We recall, however, that Germany Seventh Army Intelligence magnified the three American divisions attacking von Schlieben's forces in Fortress Cherbourg into six and a half divisions. We have assumed that by the time the G-2 sections of the after-action reports were written all errors had been eliminated.

Given the number of divisions on each side, there still remains the problem of finding the exchange ratio of the Axis and Allied divisions -- the problem, that is, of determining the equivalent strength of the Axis divisions involved. Here we have had to depend on the occasional estimates given by intelligence officers and commanders of the opposing forces. Unfortunately, however, these estimates are not always consistent. Thus Genlt. Bodo Zimmerman, Operations Officer of OB WEST,
estimated* that the 63 German divisions on the Western front in early September 1944 were "at most comparable to about 27 enemy units." On the other hand, on September 8, 1944, Genfldm. von Rundstedt reported** to OKW that the effectiveness of his 63 divisions was that of "27 infantry divisions and 6 - 7 panzer divisions, at most."

Instead of making a count of divisions or a numerical count of manpower in the opposing forces, one might try to compare opposing forces in terms of the firepower each can bring to bear. From the T/O and E's giving the organic composition of each type of division, one can determine the combat manpower, the armament, and the vehicles nominally assigned to each division. Now numerical values have been assigned*** to each weapon, and for various ranges, which numbers purport to represent approximately the relative effect of the several weapons. This numerical scale of firepower has been extended by some military experts by dropping the dependence on range and by including terms for armor and for manpower. Thus, a weighted over-all measure for the combat effectiveness of a division can be obtained and can be used to compare two divisions (each assumed to be at its T/O strength); The physical meaning of this measure, however, is a bit obscure. Moreover, the difference in T/O and E strength,

* Ref. (3), MS B-308, p. 180
** Ibid, p. 188
*** Ref. (19), p. 15
as given by this measure, between an American infantry division, for example, and a German Volks Grenadier division is of the order of 10% and is unimportant relative to other uncertainties encountered. In the Appendix we give, purely for illustrative purposes, a table that purports to compare the firepower of a U.S. infantry division (T/O and E7, 15 July 1943) with that of a German infantry division, old type (before October 1943).

If strength is to be measured in terms of firepower, then presumably one should use not the armament assigned to each division in the T/O and E's but rather the firepower actually brought to bear on the opposing force. Some figures for Allied artillery expenditures are readily available; to find how the volume of small arms fire varied throughout a given campaign, however, is more difficult, and data on enemy artillery fire is very scanty. In view of these facts as well as of Col. S. L. A. Marshall's astonishing figures* for the probability that on a given day of battle an infantryman will fire a shot in anger, we take a dim view of estimating strength by measuring the firepower applied.

THE HISTORICAL DATA

We turn now to a report of the historical data on a number of World War II campaigns in the ETO and the MTO. We ignore terrain, climate, initiative, logistic support, air support, the

*Reference (23), Chapter 5.
tactical situation, quality of personnel and equipment, mobility, etc., and seek such gross relation as may exist merely between the Allied and Axis strengths and the rate of advance of the battle line. The strength of each side is measured by the number of equivalent divisions in the line; this number is corrected, by such numerical values for manpower and estimates of equivalent strength as we could find. Rates of advance are estimated from the situation maps of the campaigns. These data are taken from the After Action Reports and Histories of the various Allied Armies and from other documents listed in the Bibliography.

The historical data are presented in the figures below; in these we exhibit the strength of each side and the velocity of the front as they vary throughout the campaign; the velocity is

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DISCUSSION

In Fig. 8 we exhibit the values of relative strength and velocity of front for all the campaigns considered in Figures 1 through 6. We observe that these variables are related only in a very gross way. If, for purposes of a mathematical model, one wants a formula with numbers, then we suggest that two such formulas be used both in order to bracket much of the observed data and to find out how sensitive various conclusions from the model are with respect to the position of the front line. Our own nominations for these two formulas are as follows:

(Velocity-Strength Relation A) \[ V = \frac{M-M}{M+M} \]

(Velocity-Strength Relation B) \[
V = \begin{cases} 
30 \left( \frac{M-M}{M+M} + \frac{1}{3} \right) & \text{for } \frac{M}{M} \leq \frac{1}{2} \\
0 & \text{for } \frac{1}{2} \leq \frac{M}{M} \leq 2 \\
30 \left( \frac{M-M}{M+M} - \frac{1}{3} \right) & \text{for } 2 \leq \frac{M}{M}
\end{cases}
\]

*Note that in Fig. 8 we have not shown velocities greater than 5 miles/day or strength ratios greater than 5 to 1.
In these equations we denote by $V$ the velocity of advance (or retreat, if $V$ is negative) of the front line (measured in miles per day) and by $M$ and $\bar{M}$ the strengths of the opposing forces. In Relation B the battle line is held fixed unless one side is stronger by at least a two-to-one ratio; the unopposed rate of advance is 20 miles/day. The two velocity-strength relations are exhibited graphically in Fig. 8. Figures 9 through 14 show, for the campaigns considered above, the historical progress of the battle line as well as that predicted by using the velocity strength relations A and B.
Fig. 1—The campaign in Sicily
Fig. 2—The Fifth U.S. Army in Italy—Salerno to the Winter Line
Fig. 3 — First U.S. Army — Normandy to St. Lo
Fig. 4—The Battle of the Bulge
Fig. 5—First U.S. Army—From the breakout to the West Wall
Fig. 6—Seventh U.S. Army—October 1944—February 1945
Fig. 7 — The European Theater of Operations
Fig. 8

\[ V = \begin{cases} 
30 \left( \frac{M - \bar{M}}{M + \bar{M}} + \frac{1}{3} \right) & \text{for } \frac{M}{\bar{M}} \leq \frac{1}{2} \\
0 & \text{for } \frac{1}{2} < \frac{M}{\bar{M}} \leq 2 \\
30 \left( \frac{M - \bar{M}}{M + \bar{M}} - \frac{1}{3} \right) & \text{for } 2 \leq \frac{M}{\bar{M}} 
\end{cases} \]
Fig. 9—The campaign in Sicily

Fig. 10—The Fifth Army in Italy
Fig. 11 — First U.S. Army Normandy to St. Lo

Fig. 12 — The Battle of the Bulge
Fig. 13—First U.S. Army from the breakout to the West Wall

Fig. 14—Seventh U.S. Army October 1944—February 1945
APPENDIX

In the tables below, which are given for illustrative purposes only, a crude comparison is attempted between the firepower of a U.S. infantry division (T/O and E 7, 15 July 1943) and a German infantry division, old type (before October 1943). This comparison is made simply by weighting each weapon by the "point value" assigned to it by military judgment.

U.S. Infantry Division (T/O and E 7, 15 July 1943)

<table>
<thead>
<tr>
<th>Weapon</th>
<th>Number in Div.</th>
<th>Point Value</th>
<th>Total Points For Weapon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rifle, cal. 30</td>
<td>6,518</td>
<td>1</td>
<td>6,518</td>
</tr>
<tr>
<td>Auto. rifle, cal. 30</td>
<td>243</td>
<td>3</td>
<td>729</td>
</tr>
<tr>
<td>Machine gun, cal. 30</td>
<td>157</td>
<td>6</td>
<td>942</td>
</tr>
<tr>
<td>Machine gun, cal. 50</td>
<td>236</td>
<td>10</td>
<td>2,360</td>
</tr>
<tr>
<td>Mortar, 60 mm.</td>
<td>90</td>
<td>6</td>
<td>540</td>
</tr>
<tr>
<td>Mortar, 81 mm.</td>
<td>54</td>
<td>15</td>
<td>810</td>
</tr>
<tr>
<td>AT rocket launcher</td>
<td>557</td>
<td>6</td>
<td>3,342</td>
</tr>
<tr>
<td>AT gun, 57 mm.</td>
<td>57</td>
<td>8</td>
<td>456</td>
</tr>
<tr>
<td>Howitzer, 105 mm.</td>
<td>54</td>
<td>25</td>
<td>1,350</td>
</tr>
<tr>
<td>Howitzer, 105 mm.</td>
<td>12</td>
<td>50</td>
<td>480</td>
</tr>
<tr>
<td><strong>Total Number of Points</strong></td>
<td></td>
<td></td>
<td><strong>17,527</strong></td>
</tr>
</tbody>
</table>
German Infantry Division, Old Type (before October 1943)

<table>
<thead>
<tr>
<th>Weapon</th>
<th>Number in Div.</th>
<th>Point Value</th>
<th>Total Points For Weapon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rifle or Carbine*</td>
<td>9,750</td>
<td>0.7</td>
<td>6,825</td>
</tr>
<tr>
<td>Submachine gun</td>
<td>700</td>
<td>1</td>
<td>700</td>
</tr>
<tr>
<td>Light machine gun</td>
<td>527</td>
<td>6</td>
<td>3,162</td>
</tr>
<tr>
<td>Heavy machine gun</td>
<td>116</td>
<td>10</td>
<td>1,160</td>
</tr>
<tr>
<td>Mortar, 81 mm.</td>
<td>58</td>
<td>15</td>
<td>870</td>
</tr>
<tr>
<td>AT rocket</td>
<td>90</td>
<td>10</td>
<td>900</td>
</tr>
<tr>
<td>AT or Tk gun, 20 mm.</td>
<td>11</td>
<td>5</td>
<td>55</td>
</tr>
<tr>
<td>AT gun, 75 mm.</td>
<td>75</td>
<td>25</td>
<td>1,875</td>
</tr>
<tr>
<td>Inf. How., 75 mm.</td>
<td>20</td>
<td>25</td>
<td>500</td>
</tr>
<tr>
<td>Inf. How., 150 mm.</td>
<td>6</td>
<td>40</td>
<td>240</td>
</tr>
<tr>
<td>Gun/How, 105 mm.</td>
<td>36</td>
<td>30</td>
<td>1,080</td>
</tr>
<tr>
<td>Howitzer, 150 mm.</td>
<td>12</td>
<td>40</td>
<td>480</td>
</tr>
<tr>
<td><strong>Total Number of points</strong></td>
<td></td>
<td></td>
<td><strong>17,847</strong></td>
</tr>
</tbody>
</table>

* The German T/O and E actually assigned 15,500 rifles or carbines to the 17,200 personnel of this division. However, the number of personnel in the three infantry regiments was only 9,750 and we have arbitrarily taken this as the effective number of rifles or carbines.
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(11). "Final After Action Report, 12th Army Group, October 1943 - July 1945," Volumes I through XIV.


