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OPTIMAL EMPLOYMENT OF TACTICAL AIR FORCES IN THEATER AIR TASKS: A GAME-THEORETIC ANALYSIS

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SUMMARY

The optimal allocation of tactical aircraft among three air tasks—counter air, air defense, and support of ground operations—on each strike of a multistrike campaign is described in terms of a two-sided war game. The mathematical model assumes that counter-air missions destroy enemy aircraft, air-defense missions reduce the enemy's counter-air operations, and support of ground operations contributes to the accomplishment of the theater mission, or payoff, of the tactical forces.

The following air tactics are optimal, in the game-theoretic sense, for this type of campaign:

1. During the early phase of the campaign, the side with the larger force should split his effort between counter-air and air-defense operations. However, if his force is much larger than his opponent's, then he should split his force among all three tasks. The size of the split at any time depends on the expected duration of the campaign and the relative strengths of the forces at that time.

2. During the early phase of the campaign, the side with the smaller force should mix his tactics: he should choose, at random, between concentrating on counter air or concentrating on air defense. However, if his force is much smaller than his opponent's, then he should concentrate on one of the three tasks, the choice of task being made at random. The probabilities associated with the random choices at any time depend

on the expected duration of the campaign and the relative strengths of forces at that time.

3. During the closing phase of the campaign both sides concentrate on support of ground operations.

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OPTIMAL EMPLOYMENT OF TACTICAL AIR FORCES IN THEATER AIR TASKS: A GAME-THEORETIC ANALYSIS

1. INTRODUCTION

The problem of optimal employment of tactical air forces in the various theater air tasks, like many other military questions, can be analyzed as a multimove game between two sides: the strong, which seeks the largest payoff possible in the form of some theater mission, and the weak, which tries to make this payoff as small as possible. Both sides must make a large number of decisions: they must choose type of aircraft, type and size of weapon, target for attack, altitude of flight, and type of logistic support. An important decision during combat, which the two sides make independently, is the allocation of tactical air forces among the various theater air tasks during each period of combat.

We shall present the results of the analysis of a simplified version of this game, in which we assume that during each time period each side can employ its air forces in three air tasks: counter air, air defense, and support of ground operations. The mathematical proofs of these results are given in [1]. In a previous report, [2], we analyzed the problem for two air tasks and derived the optimal employment of tactical air forces. We shall see that increasing the number of air tasks to three yields substantially different optimal tactics.

2. THEATER AIR OPERATIONS

The tactical air war is characterized by a series of air actions or tasks undertaken in order to accomplish some defined theater mission. Among the usual tasks are the following:

Counter Air. These operations are against the enemy's theater air-base complex and organization in order to destroy his aircraft, personnel, facilities, etc.

Air Defense. These represent air-defense operations against the enemy's counter-air operations.

Close Air Support. The targets for close-support operations are concentrations of enemy troops or fortified positions in order to help the ground forces in the battle area. This is accomplished by aerial delivery of fire power against the enemy ground targets.

Interdiction. These operations reduce the enemy's military potential by attacking the transportation facilities.

Reconnaissance. The most important function of these operations is to obtain information about the targets.

Airlift. In this operation the planes are used to transport troops and equipment.

We shall take close air support, interdiction, reconnaissance for the ground forces, and airlift tasks to constitute what is termed "support of ground operations," or ground support, in this paper. Reconnaissance directed against enemy air will be assumed as part of the counter-air task.

3. FORMULATION OF GAME

We view the tactical air war game as consisting of a series of strikes, or moves, each of which consists of simultaneous counter-air, air-defense, and close-support operations by each side undertaken to accomplish a given theater mission or payoff. Let us assume that at the start of the air operations the stronger side (the opponent with the larger air force), Blue, has p planes and the weaker side, Red, has q planes, where q is smaller than p. Let us look at a strike in the campaign, say the initial strike. Suppose that on this strike Blue dispatches x planes on counter-air operations and u planes on air-defense operations, and the remaining amount, $m = p - x - u$ planes, on ground-support operations. Similarly, suppose that for his first strike Red allocates y planes to counter air, w planes to air defense, and the remaining number, $n = q - y - w$ planes, to support his ground forces. For this initial strike and for any future strikes, the above decisions are made by each side in ignorance of the allocation of the opposing side. It is assumed, however, that each side knows the number of planes that he and his opponent have.

Since Red allocates w planes to air defense we can expect a reduction in the number of Blue's planes that get through to counter-air targets. The number of interceptions by Red will be proportional to w, say cw, unless Blue's attacking planes are saturated. The proportionality constant, or kill potential, depends on the planes' characteristics and flying altitudes, and on their weapons' characteristics. The number of Blue attacking planes that penetrate Red's defenses is x – cw as long as cw is not larger than x. If cw is larger than x, no Blue aircraft will penetrate. Hence the number of Blue attacking planes that penetrate Red's defenses is the larger of the two numbers x – cw and 0, or

$$\max(0, x - cw).$$

The objective of Blue's counter-air operations is to reduce the enemy's air force by dropping bombs on certain targets, and the number of aircraft destroyed will vary with the number of attacking planes that penetrate Red's defenses. Increasing the number of Blue's penetrating planes will diminish the enemy's air force, but cannot reduce it by more than q. If we assume that each of Blue's penetrating planes can destroy b planes of the enemy, and that all of Red's aircraft are at risk at the time of a strike, then Blue's initial counter-air strike will destroy

$$\min[q, b \max(0, x - cw)]$$

Red planes. The proportionality constant b depends on the target as well as the aircraft characteristics.

Red's air force is further reduced during the strike by such factors as accidents and antiaircraft fire. Let us assume that these losses are proportional to the number of planes used by Red during the strike, or aq. Finally, the planes used in air defense are assumed to survive, and the Red aircraft that fail to penetrate the Blue air defense are assumed to return to base.

If we sum the losses, we see that during the initial strike Red's air force is reduced to

$$\overline{q}_1 = \max\,[0,\ q - b \max\,(0,\ x - cw)] - aq,$$

or to zero if the quantity $\overline{q}_1$ is negative. That is, if q_1 denotes the Red aircraft inventory at the conclusion of the initial strike, then

$$q_1 = \max\,(0,\ \overline{q}_1).$$

In exactly the same manner we can analyze the effect of the initial strike on Blue's inventory. At the end of the initial strike, Blue's inventory of planes is

$$\overline{p}_1 = \max\,[0,\ p - e \max\,(0,\ y - fu)] - dp$$

if this quantity is not negative, or is zero if $\overline{p}_1$ is negative. Thus if p_1 denotes the Blue inventory at the end of the first strike, then

$$p_1 = \max (0, \bar{p}_1).$$

Blue now has p_1 planes to allocate among the three tasks for the second strike, and Red has q_1 planes to allocate for the second strike. This strike will result in new inventories, p_2 and q_2, for the third strike. The process is repeated for the duration of the campaign.

4. PAYOFF

Let us look at Blue's employment of theater air forces during the campaign. We assume that his objective is to assist the ground forces in the battle area, and the results will vary with the number of planes he allocates to ground-support operations. We assume that it is possible to construct for Blue a payoff function, giving the payoff for each strike of the campaign, in the form of the distance advanced by the ground forces as a function of the number m of planes allocated to ground support. This function depends heavily on the characteristics of the ground-support targets—i.e., on the degree of concentration of troops, vehicles, and materiel, and on the fortification of positions. We make no attempt to give the explicit form of this function, but merely assume that the payoff, $f(m)$, is a positive function that increases with increasing allocations.

If Blue's ground forces now must advance while being subjected to Red's ground-support sorties, Blue's yield in ground support is no longer equal to $f(m)$ as described above, but is reduced in accordance with the number n of planes allocated by Red to close-support missions. If $g(n)$ is the function that measures the distance gained by Red's ground forces, then the net advance of Blue's ground forces, if he allocates m planes to ground support and Red allocates n planes to ground support, can be written as

$$Y(m, n) = f(m) - g(n).$$

The foregoing expression represents the payoff to Blue for this one period or one strike. The payoff for the entire campaign of N strikes is the sum of these net yields for each of the N strikes, or

$$M = \sum^{N} [f(m) - g(n)].$$

The problem faced by each side is now apparent. For example, Blue would like to allocate a large number of planes to ground-support missions and thereby increase the value of f at a given move, yet he would like to destroy the Red air force by means of counter-air operations in order to ensure that g is small, or zero, for subsequent moves. Further, if he does not provide for air defense he may suffer severe losses to his own air force if Red elects to mount a large counter-air strike. Each player has to take the future and the possibilities open to his opponent into account.

For the game described here, it turns out that optimal procedures for play, or optimal strategies in the game-theoretic sense, do exist. For particular functions f and g we shall give a qualitative description of the optimal tactics and then shall exhibit some explicit solutions.

5. FURTHER SIMPLIFYING ASSUMPTIONS

We assume, to simplify the computations, that Blue and Red have the same air-defense potential: each plane allocated to defense can prevent one attacking plane from reaching target—that is, we assume that $c = f = 1$. We also assume that each attacking plane that penetrates the defense can destroy one plane in an airfield strike, or $b = e = 1$, and that losses due to aborts, accidents, and antiaircraft fire are negligible. Then the inventory of planes at the end of a strike will be, for Blue and Red, respectively,

$$p_1 = \max\,[0,\ p - \max\,(0,\ y - u)],$$

$$q_1 = \max\,[0,\ q - \max\,(0,\ x - w)].$$

We again emphasize the fact that these simplifying assumptions have no effect on the general form of the optimal strategies. They are introduced merely to simplify the calculations. In a forthcoming report we shall show how a change in the value of the air defense potential influences the specific form of the optimal strategy.

Assume, to further simplify the computations, that the yield functions $f(m)$ and $g(n)$ are linear, say $f(m) = m$, $g(n) = n$. The payoff in the campaign then is

$$M(x, u; y, w) = \sum^{N} [(p - x - u) - (q - y - w)].$$

Blue wishes to make this payoff as large as possible by properly choosing the x's and u's during each of the N strikes, and Red wishes to make the payoff as small as possible by properly choosing the y's and w's.

6. STRATEGIES

The strategies available to Blue and Red are characterized by the strengths of their forces during each strike of the multistrike air campaign. Blue's strategy can be specified by the number of planes he allocates to counter-air operations, the number of planes he allocates to ground support, and the number of planes he commits to engage Red's attack. Red's strategy can be similarly described. These specifications are to be given for the first strike, the second strike, ..., and the last strike of the campaign. Of course, the allocations during any strike will depend on the strengths of the forces of the two sides at the beginning of that strike, since we are assuming that each side knows the strength of the opponent's forces at that time.

7. OPTIMAL TACTICS

In order to give a complete description of the optimal employment of tactical air forces it is necessary to do so in terms of the number of strikes and the relative strengths of the two sides. However, there are certain general conclusions that apply to all campaigns.

<u>Campaign ends with ground support</u>. The campaign always ends with a series of strikes on ground support—i.e., during the closing period of the campaign both Red and Blue concentrate all their forces on ground-support missions. In this terminal period both sides have the same optimal tactics, regardless of their initial forces.

<u>Blue (stronger) splits his forces</u>. At all times other than the closing phase of the campaign, Red and Blue have very different optimal tactics. During any of these early strikes, the stronger side, Blue, has a pure strategy. That is, there exists a best allocation of Blue's air force among the three air tasks. In this connection, there is a critical value (about 2.7) of the ratio of the Blue force size to the Red force size that governs Blue's allocation during the early period in the following manner: If the force ratio is less than this critical value, then the optimal allocation in the early period consists of splitting the stronger air force between two air tasks, counter air and air defense, and neglecting the ground-support task. The size of split depends on the relative strengths of the two air forces and the number of strikes left in the campaign. However, if Blue's strength relative to Red's is greater than the critical value, then Blue should divide his force in a fixed way, regardless of his strength, among the three tasks, counter air, air defense, and ground support. The number of aircraft allocated to each mission, however, is still dependent on the number of strikes remaining.

Red (weaker) mixes his tactics and concentrates his forces. The weaker combatant cannot use a single strategy, but must bluff during all the strikes other than those of the terminal phase. Unlike his opponent, the weaker combatant does not have a single allocation that is best. He must use a mixed strategy and gamble for high payoffs. If he is not too weak—i.e., if the force ratio is less than the critical value—then he concentrates his entire force either on counter air or on air defense; but which of these tasks receives the full effort is decided by some chance device. However, if Red is very weak (force ratio larger than critical value), then he allocates his entire air force to any one of the three air tasks with the particular task again chosen at random. In other words, if a player is very weak relative to the opponent, then he takes a chance on an early payoff. Of course, to be most effective, he must bluff correctly—i.e., the random device should select the tasks with the proper relative frequencies.

Mix and split the same tasks. It is of interest to note that on each strike Red, the weaker side, bluffs with the same tasks that Blue uses in his allocation. Thus if Red is very weak he bluffs with each of the three tasks, and Blue splits his forces among each of the three tasks. However, if Red is moderately weak, he bluffs with two tasks—counter air or air defense—and Blue splits his forces between the same two tasks, counter air and air defense.

Blue's defense decreases during campaign. As was noted above, prior to the closing phase of the campaign, Blue splits his forces among his air tasks. The actual split is a function of the force sizes of Blue and Red and the number of strikes left in the campaign. However, as the campaign proceeds, the fraction of Blue's force allocated to air defense will decrease. At the same time, the fraction allocated by Blue to counter air will increase. During this time, the chance that Red will attack Blue also decreases, but the chance that Red will defend himself increases.

Blue's defense in a long campaign. In the early stages of a relatively long campaign, the stronger side defends itself against a concentrated attack by the weak side. During this period, Blue dispatches on air defense a force of planes approximately the size of Red's entire force. Recall that we assumed a particular value for the air defense effectiveness.

8. SUMMARY TABLE

The attached table summarizes the optimal tactics for campaigns consisting of at most eight strikes. An example that illustrates the table and the above discussion is given in Sec. 9. The tabulation gives the optimal allocation for each strike (where the strike number is defined by the number of strikes remaining in the campaign) as a function of the relative sizes of the forces at the time of that strike. However, the value of the game, which is given in the last column of the tabulation, is for the entire campaign.

OPTIMAL TACTICS IN MULTISTRIKE TACTICAL AIR CAMPAIGN

(Strong Side Having p Planes and Weak Side Having q Planes, $p > q$)

Campaign Period	Duration of Campaign (No. of Strikes Remaining in Campaign)	Relative Initial Strengths of Opponents (Ratio of Strong Side to Weak Side) p/q	Optimal Initial Allocation by Strong Side (No. of Planes): Counter Air	Optimal Initial Allocation by Strong Side (No. of Planes): Air Defense	Optimal Initial Allocation by Strong Side (No. of Planes): Ground Support	Optimal Initial Allocation by Weak Side (Probability of Allocating All Planes): Counter Air	Optimal Initial Allocation by Weak Side (Probability of Allocating All Planes): Air Defense	Optimal Initial Allocation by Weak Side (Probability of Allocating All Planes): Ground Support	Expected Value of Campaign (Movement of Front Line)
I	1	1.00 to ∞	0	0	p	0	0	1	p − q
II	2	1.00 to ∞	0	0	p	0	0	1	2(p − q)
III	3	1.00 to 2.00	q	0	p − q	0.50	0.50	0	3(p − q)
	3	2.00 to ∞	1.5q	0.5q	p − 2q	0.50	0.50	0	3(p − q)
IV	4	1.00 to 2.33	0.5p + 0.5q	0.5p − 0.5q	0	0.50	0.50	0	4.5(p − q)
	4	2.33 to ∞	1.67q	0.67q	p − 2.33q	0.33	0.33	0.33	4.00p − 3.33q
V	5	1.00 to 1.70	0.41p + 0.59q	0.59p − 0.59q	0	0.53	0.47	0	6.35p − 6.35q
	5	1.70 to 2.45	0.55p + 0.36q	0.45p − 0.36q	0	0.45	0.55	0	5.82p − 5.45q
	5	2.45 to ∞	1.70q	0.75q	p − 2.45q	0.25	0.30	0.45	5.00p − 3.45q
VI	6	1.00 to 1.44	0.32p + 0.68q	0.68p − 0.68q	0	0.56	0.44	0	8.39p − 8.39q
	6	1.44 to 1.78	0.40p + 0.56q	0.60p − 0.56q	0	0.52	0.48	0	8.00p − 7.83q
	6	1.78 to 2.51	0.59p + 0.22q	0.41p − 0.22q	0	0.41	0.59	0	7.04p − 6.12q
	6	2.51 to ∞	1.71q	0.80q	p − 2.51q	0.20	0.29	0.51	6.00p − 3.51q
VII	7	1.00 to 1.29	0.25p + 0.75q	0.75p − 0.75q	0	0.58	0.42	0	10.50p − 10.50q
	7	1.29 to 1.53	0.29p + 0.70q	0.71p − 0.70q	0	0.57	0.43	0	10.26p − 10.19q
	7	1.53 to 1.84	0.41p + 0.51q	0.59p − 0.51q	0	0.50	0.50	0	9.54p − 9.09q
	7	1.84 to 2.55	0.63p + 0.11q	0.37p − 0.11q	0	0.37	0.63	0	8.21p − 6.64q
	7	2.55 to ∞	1.72q	0.84q	p − 2.55q	0.17	0.28	0.55	7.00p − 3.55q
VIII	8	1.00 to 1.25	0.20p + 0.80q	0.80p − 0.80q	0	0.60	0.40	0	12.60p − 12.60q
	8	1.25 to 1.40	0.22p + 0.78q	0.78p − 0.78q	0	0.59	0.41	0	12.48p − 12.45q
	8	1.40 to 1.59	0.28p + 0.69q	0.72p − 0.69q	0	0.56	0.44	0	12.06p − 11.86q
	8	1.59 to 1.88	0.42p + 0.46q	0.58p − 0.46q	0	0.49	0.51	0	11.03p − 10.22q
	8	1.88 to 2.58	0.66p + 0.01q	0.34p − 0.01q	0	0.34	0.66	0	9.36p − 7.07q
	8	2.58 to ∞	1.72q	0.86q	p − 2.58q	0.14	0.28	0.58	8.00p − 3.58q

9. EXAMPLE

Suppose that at the start of a five-strike campaign Blue has $p = 85$ planes and Red has $q = 68$ planes. Since the ratio of Blue to Red strength prior to the initial strike is 1.25, it is optimal for Blue to split his force between two tasks by dispatching 75 planes on counter air and 10 planes on air defense. On his initial strike, Red should bluff by dispatching all 68 of his planes either on counter-air missions (this is to happen with probability 9/17) or on air-defense missions (with probability 8/17).

If Blue and Red use these optimal tactics then the forces available to Blue and Red for the second strike will be either

$$p_1 = 85 - (68 - 10) = 27,$$

$$q_1 = \max (0, 68 - 75) = 0,$$

or

$$p_1 = 85 - 0 = 85,$$

$$q_1 = 68 - (75 - 68) = 61.$$

The former pair will occur with probability 9/17, while the latter will occur with probability 8/17. From the above, we note that if on his initial strike Red sends his 68 planes on counter air, and hence none on air defense, his force is reduced to zero; therefore, on each of the four remaining strikes the optimal policy for Blue is to send all his planes on ground-support strikes.

If on the initial strike the outcome of Red's random decision is to concentrate on air defense, then for the second strike Blue has 85 planes and Red has 61 planes. In this case, the optimal allocation for Blue on the second strike is 73 to counter air and 12 to air defense. Now again it is optimal for Red to concentrate either on counter air or on air defense, but this time with equal probabilities.

If Blue and Red make their optimal allocations on the second strike, then at the end of the strike the force sizes of Blue and Red will be either

$$p_1 = 85 - (61 - 12) = 36,$$

$$q_1 = \max(0, 61 - 73) = 0,$$

or

$$p_1 = 85,$$

$$q_1 = 61 - (73 - 61) = 49,$$

depending on the outcome of Red's random decision.

The force sizes for the third strike will be either $p_1 = 36$, $q_1 = 0$, or $p_1 = 85$, $q_1 = 49$, each with equal probability.

If the force sizes are $p_1 = 36$, $q_1 = 0$, then the optimal allocation for Blue is to send all 36 of his planes on close support. If the force sizes are $p_1 = 85$, $q_1 = 49$, then an optimal allocation (not unique) for Blue is 49 to counter air and zero to air defense. Red dispatches his 49 planes on counter air or air defense, each with equal probability.

If Red and Blue play optimally, then at the end of the third strike—or the beginning of the fourth strike—their force sizes will be either $p_1 = 36$, $q_1 = 0$, or $p_1 = 85$, $q_1 = 49$, each with equal probability. In either case, Blue and Red will concentrate on ground support during the fourth and fifth strikes, yielding a payoff of 2(36) = 72 for the last two strikes.

Combining the payoffs for each of the five strikes, we obtain as the expected payoff, with optimal allocations, the value of the game:

$$V = \frac{9}{17}(4)(27) + \frac{8}{17}\left[\frac{1}{2}(3)(36) + \frac{1}{2}(3)(36)\right] = 108$$

$$= 6.35(85 - 68).$$

10. SENSITIVITY TO INITIAL ALLOCATION

The importance of making the proper allocation on the initial strike (and therefore on every strike) can be forcefully illustrated by means of the foregoing example. Suppose that on the initial strike Blue concentrates his force of 85 planes on counter air (which is not an optimal tactic) and then makes optimal allocations during the remaining four strikes. Then if Red dispatches his force of 68 planes on counter air during the initial strike, Blue's force will be reduced to 17 planes and Red's force to zero. Optimal allocation for the last four strikes would demand dispatching the 17 planes on ground support each time. In this case the payoff for the campaign of five strikes would be 68, or a reduction of almost 40 per cent from the payoff of 108 if Blue had made an optimal allocation on the initial strike.

Suppose that Red, instead of using a mixed strategy on his initial strike, uses the strategy of allocating his entire air force to counter air on the first strike and then makes optimal allocations for the remaining four strikes. In this case, an allocation by Blue of 17 planes on counter air and 68 on air defense will reduce Red's force to 51 planes and Blue will still have 85 planes. Optimal allocations by both sides for the remaining four strikes will yield a total payoff to Blue of $\frac{9}{2}(85 - 51) = 153$, or almost 50 per cent higher to Blue than the expected payoff of 108 that is possible with optimal choice of a mixed strategy by Red.

11. GENERALIZATIONS OF MODEL

As already indicated, we have analyzed a simplified model of the conflict between two tactical air forces having three air tasks each. No great difficulty arises in carrying out a similar analysis for four or more air tasks.

We have assumed that each plane can destroy as many planes in the air as it can on the ground. Actually, the air-kill potential is much less than the ground-kill potential. However, the general properties of the optimal tactics as described in Sec. 7 are applicable for arbitrary values of the kill potentials; that is, the game still ends with a series of moves in which both sides concentrate on ground support, while at each move prior to this the stronger side splits his force between counter air and air defense and the weaker side randomizes among the various tasks. The magnitudes of the kill potentials determine the actual split of forces for the stronger side, and the probabilities associated with the mixed tactics for the weaker side.

In addition, we have assumed that each side has been concerned only with the effect on excess ground support without regard for his own plane losses in the air battle. Since the losses in the air duel are small compared to the losses on the ground, this omission cannot appreciably affect the solution.

Although the model does not contain force replacements, no difficulty arises in the analysis of the game if we assume a replacement schedule that is independent of the tactics employed. In such cases the optimal tactics are the same, though the game value is changed.

Not all aspects of tactical air war can be handled by the model described. Essential modifications must be made in order to analyze tactical air war. We list briefly a few of the limitations of our model:

Each plane is assumed to be capable of performing each of the three air tasks and to be equally effective on each task. Actually, of course, there are different types of planes, not all of which can perform all three tasks.

We have assumed a continuous payoff function. The realistic case where a discontinuity in the payoff occurs if the front line reaches a particular point has been excluded.

We have assumed that the counter-air strike is equally effective at all times, regardless of the defense. It would be more realistic to assume that if a combatant employs no air defense then the attacker is very effective since, among other things, he may now fly at a lower altitude.

The design is static. We have assumed that the destruction of a target always has the same value to the attacker regardless of the status of the campaign.

The duration of the campaign is known at the start of the war. Actually, the duration of the campaign may depend on the tactics employed.

In order to overcome these limitations, we must design additional models, perhaps many models, each of which takes into account only certain features of the tactical air problem. The results of these analyses have then to be taken together and carefully compared in terms of the assumptions of the models.

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