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PROJECT RAND

RESEARCH MEMORANDUM

A PARAMETRIC STUDY OF THE PERFORMANCE OF AIR-LAUNCHED BALLISTIC MISSILES

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Assigned to ____________________

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SUMMARY

This memorandum is intended as an aid to rapid estimation of the effect of launch velocity, path angle, and altitude on the performance of air-launched ballistic missiles. Most of the data shown are for a representative missile configuration flown on a maximum-range trajectory. Sufficient information is given to permit extrapolating the basic data to other missile configurations and trajectories. One- and two-stage missiles are considered.

Data are shown for missile ranges of 200 to 1500 n mi, launch velocities from Mach 0.4 to 3.0, and launch altitudes from sea level to 60,000 ft.

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SYMBOLS

A = reference area, maximum missile frontal area, ft^2

C_D = drag coefficient = $\frac{D}{qA}$

C_{L_α} = lift-curve slope, per radian

D = aerodynamic drag, lb

g = acceleration of gravity, ft/sec^2

g_s = sea level value of g = 32.2 ft/sec^2

h = altitude, ft

I = motor specific impulse, sec

K_L = induced-drag factor

L = lift, lb

M = Mach number

MR = mass ratio, $\frac{W_G}{W_E}$

m = mass, $\frac{W}{g_s}$

n = maximum normal acceleration, g's

n_i = initial longitudinal acceleration during boost, ft/sec^2

p = atmospheric pressure, lb/ft^2

R = radius of the earth = 20,900,000 ft

S = missile range, ft

T = motor thrust, lb

t = time from launch, sec

t_b = time of motor burnout, sec

V = velocity, ft/sec

V_b = velocity at time t_b, ft/sec

$V_{pot} = gI \log MR + V_o$

W = missile weight, lb

W_E = missile empty weight = $(W_G - W_P)$, lb

W_G = missile launch weight, lb

W_L = payload weight, lb (includes all weight not specifically a part of the propulsion system)

W_P = propellant weight, lb

W_{PP} = weight of inert motor parts, lb

α = angle of attack, radians

θ = missile flight path angle, referred to local horizontal, radians

θ_b = missile flight path angle at burnout, radians

$$\Lambda = \frac{W_P}{W_P + W_{PP}}$$

$\nu = \frac{W_P}{W_G}$ per stage

ϕ = missile range angle = $\frac{S}{R}$, radians

SUBSCRIPTS

o = launch conditions

1 = first stage

2 = second stage

I. INTRODUCTION

The concept of launching relatively large ballistic missiles from aircraft has been receiving increasing attention in the past year. Air-launched ballistic missiles have already achieved ranges of several hundred miles in test programs, and the flight mechanics of air launch have proven entirely feasible. The many novel uses for air-launched ballistic missiles that have already been suggested indicate that this subject will receive continued attention.

The performance estimation of an air-launched ballistic missile is complicated by the fact that the launch path angle, velocity, and altitude are all variable to some extent. This memorandum is intended as an aid in estimating the effect of these three variables on missile performance. Because of the many possible variations in motor design and in missile configuration, emphasis is placed on the relative effect of the three launch variables on missile performance rather than on the absolute values of missile range and mass ratio shown.

Most of the data shown are for maximum-range trajectories with a horizontal launch path. Sufficient information is given to permit extrapolating the basic data to a variety of other trajectories.

II. METHOD AND ASSUMPTIONS

In order to properly interpret the performance data shown in the next section it is necessary to understand the general technique and assumptions used in the trajectory calculations. The detailed calculation procedure is described in Appendix B.

A family of missile trajectories was calculated with various values of missile mass ratio and of launch velocity, altitude, and path angle. One- and two-stage missiles were considered. It is assumed that the motor is immediately ignited when the missile is dropped from the airplane. A negligible error is introduced by ignoring the several seconds of free fall that would probably be allowed in actual practice for the safety of the aircraft.

When the missile is dropped, the aircraft flight path and hence the missile flight path will usually be nearly horizontal. During first-stage burning, the missile velocity vector is rotated upward so that a specified flight-path angle is achieved at burnout. Body aerodynamic lift is used for normal force. The exact program or procedure used to rotate the velocity vector upward to achieve a specified flight-path angle has a relatively small effect on missile range performance. The simple pull-up program used in this study is described on page 25.

After first-stage burnout the missile follows a zero-lift path for the remainder of the flight. If a two-stage motor is used, the second stage is ignited two seconds after first-stage burnout. This procedure was used to obtain maximum missile range, the primary objective in this study.

A generic missile configuration was used, with the characteristics shown in Table 1.

Table 1

CHARACTERISTICS OF GENERIC MISSILE CONFIGURATION

$\frac{A}{W_E}$	0.001 ft^2/lb
n	3 g's
C_{L_α}	4/radian
K_L	0.2
Λ_1	0.85[a]
Λ_2	0.87[b]
n_1	150 ft/sec^2

[a] For one-stage, and the first of two-stage, motors.
[b] For the second stage.

The effect of variations in the values of $\frac{A}{W_E}$ and K_L is discussed on page 8 . The value of 3 g's maximum normal acceleration was arbitrarily chosen, but the effect was insignificant because the missiles seldom achieved this value in the maximum-range trajectories. The effect of variations in other parameters is discussed on page 7.

The variation of drag coefficient with Mach number and the variation of specific impulse with altitude is shown in Appendix B.

In all two-stage missiles the mass ratio of the two stages is identical. Again this is a minor compromise in the interest of simplicity.

A typical boost trajectory for a one-stage missile is shown in Fig. 1 (Appendix A).

III. MISSILE PERFORMANCE

The missile performance data are divided into two groups: (a) maximum-range performance of the assumed missile configuration, and (b) the effect on missile range of perturbations in the configuration and the trajectory.

MAXIMUM-RANGE DATA

The data shown in Figs. 2 through 14 (Appendix A) are for the assumed missile configuration (see Table 1) launched on a horizontal flight path and flown on a maximum-range trajectory. To obtain these data a first-order optimization was made of initial angle of attack and of first-stage burnout path angle. As shown in Appendix B, the value of initial angle of attack was used to shape the boost trajectory.

The variation of missile range with fuel-weight/gross-weight ratio and launch altitude and velocity is shown in Fig. 2 (one-stage missile) and Fig. 3 (two-stage missile). To make the data shown in Figs. 2 and 3 more readily useful, the variation of missile range with gross-weight/payload-weight ratio (W_G/W_L) is shown in Fig. 4 (one-stage missile) and Fig. 5 (two-stage missile). In all figures showing W_G/W_L the motor weight ratios shown in Table 1 are used. As used in this report the payload weight (W_L) includes all weight not specifically a part of the propulsion system, so that $W_G/W_L = \frac{\Lambda}{\Lambda - \nu}$.

Figures 2 through 5 are most useful for observing the effect on missile range of launch velocity and altitude. Because of the many assumptions of missile configuration and motor performance characteristics used in the calculations, the absolute value of range or of fuel-weight/gross-weight ratio must be used with caution.

It should be noted that only nine combinations of launch Mach number and altitude were investigated, as shown in Table 2.

Table 2

LAUNCH CONDITIONS INVESTIGATED

Altitude, h_o (ft)	Mach Number, M_o			
	0.4	0.8	2.0	3.0
0	x	x		
20,000	x	x		
40,000	x	x	x	
60,000			x	x

Therefore, when Figs. 2 through 5 are used, the data for subsonic launch velocity are valid only at altitudes of 40,000 ft or less, and the data for supersonic launch velocity are valid only at altitudes of 40,000 ft or more.

The basic data shown in Figs. 2 through 5 can be cross-plotted in many different ways. Three examples are shown in Figs. 6 through 8 to illustrate the effect on missile performance of variations in one parameter, with the other parameters held constant.

A different presentation of maximum-range data is shown in Fig. 9, the variation of burnout velocity (V_b) and potential velocity (V_{pot}) with missile range. As used here, potential velocity is defined as $V_{pot} = gI \log MR + V_o$. For any given missile range the difference between potential and burnout velocity represents the total of all velocity losses encountered during boost. Ideally there should be a separate potential velocity curve for every combination of launch velocity and altitude because of the different amounts of velocity loss due to aerodynamic drag. However, the difference between the data for various launch conditions is small, and for the sake of simplicity only the two sets of curves are shown.

The approximate values of altitude at burnout and of altitude at apogee are shown in Figs. 10 and 11 for one-stage missiles, and in Figs. 12 and 13 for two-stage missiles. The missile time-of-flight is shown as a function of range in Fig. 14.

The missile flight-path angle at first-stage burnout was an independent variable in the calculation procedure, and an optimum value was determined for each of the launch conditions shown in Table 2. The value of burnout path angle giving maximum missile range varied from 35 deg for long-range, high-altitude launch to slightly over 45 deg for short-range, low-altitude launch.

PERTURBATION EFFECTS

All of the missile performance data previously presented have been based on the assumed missile configuration and motor performance characteristics and on a horizontal-launch flight path. In this section information will be shown to illustrate the effect of changes in some of the previously fixed parameters. It is beyond the scope of this report to describe completely the effect of perturbations in every parameter. Instead, several examples were arbitrarily chosen to illustrate the effect on missile performance of small variations in each of several parameters.

The effect of launching the missile on an elevated path angle varies with the launch velocity. If the missile is launched at a flight-path angle identical to the desired burnout path angle, the burnout velocity is increased by about 400 ft/sec for a Mach 0.4 launch, or by about 800 ft/sec for a Mach 3.0 launch. The variation in effect is probably attributable to the fact that the drag-due-to-lift is greater at higher launch speeds. This variation is nearly insensitive to changes in launch altitude.

The effect of changes in missile area/weight ratio and in the induced-drag factor is shown in Fig. 15 (Appendix A). Again this is a specific example, intended to show the approximate magnitude and direction of these effects. Obviously the effect of changes in drag and lift characteristics will be different at different launch conditions, but the magnitude of the effects is so small that for the purpose of this study they are relatively unimportant.

All of the performance calculations were based on an initial longitudinal acceleration of approximately 150 ft/sec^2, or about 5 g's. This value can be varied plus-or-minus about 2 g's without changing missile range more than 5 per cent.

One of the most important inputs to the performance calculations is the variation of rocket-motor specific impulse with altitude. The set of motor performance data used in this study, shown in Fig. 17 (Appendix B), does not represent any particular rocket motor. It was synthesized from theoretical rocket-nozzle performance considerations and typical values of motor chamber pressure and nozzle expansion ratio. The potential velocity of a missile is directly proportional to the time-averaged motor specific impulse. Therefore, if a motor with performance significantly different from that shown in Fig. 17 is used, it will be necessary to adjust the missile performance data contained herein accordingly. However, a change in rocket-motor performance characteristics would have only a secondary effect on the variation of missile range with changes in launch conditions, and illustration of the latter variation is the primary aim of this memorandum.

APPENDIX A

Figures 1 – 15

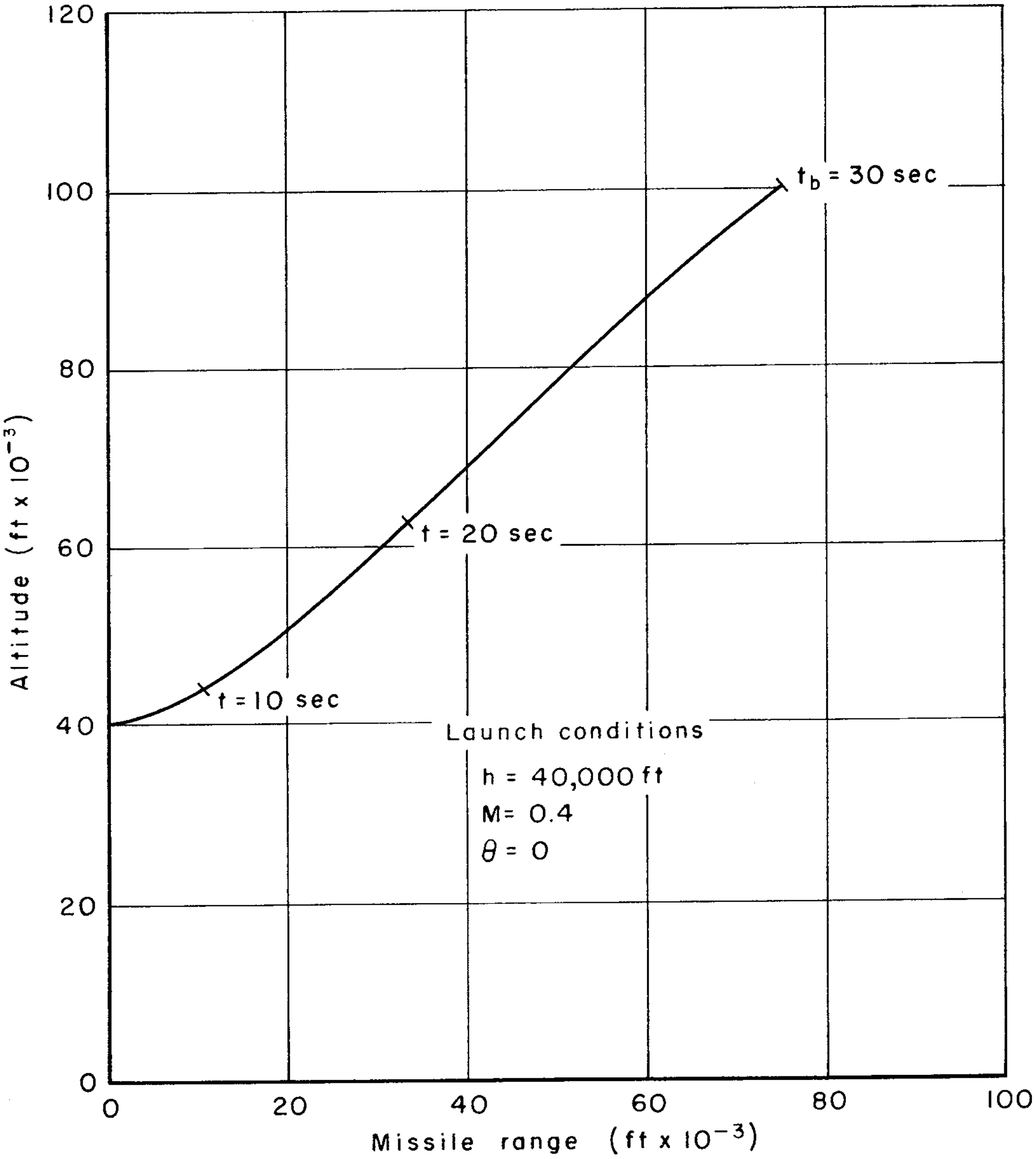

Fig. 1 — Typical boost trajectory

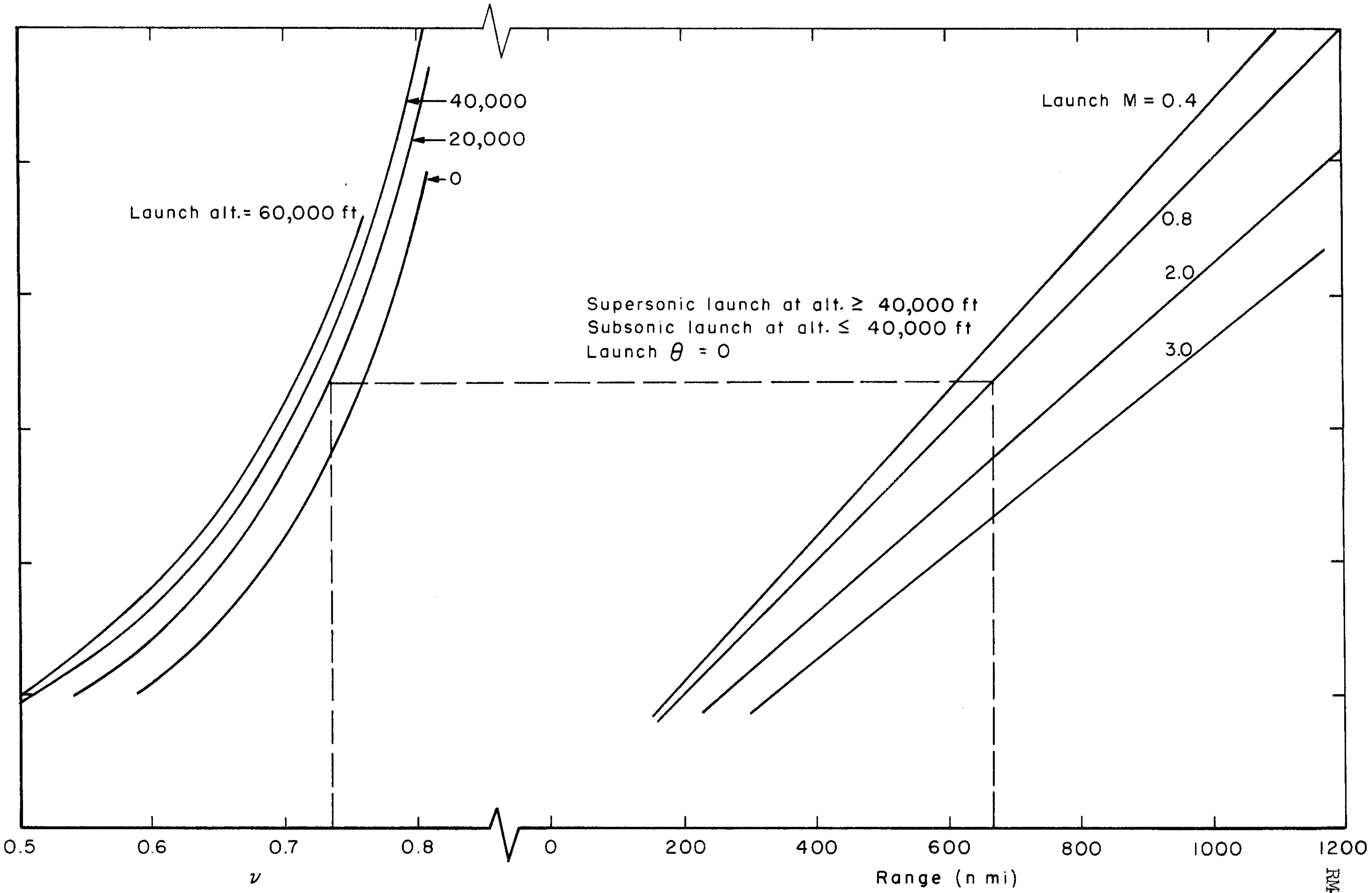

Fig. 2 — Ballistic ASM performance (single stage)

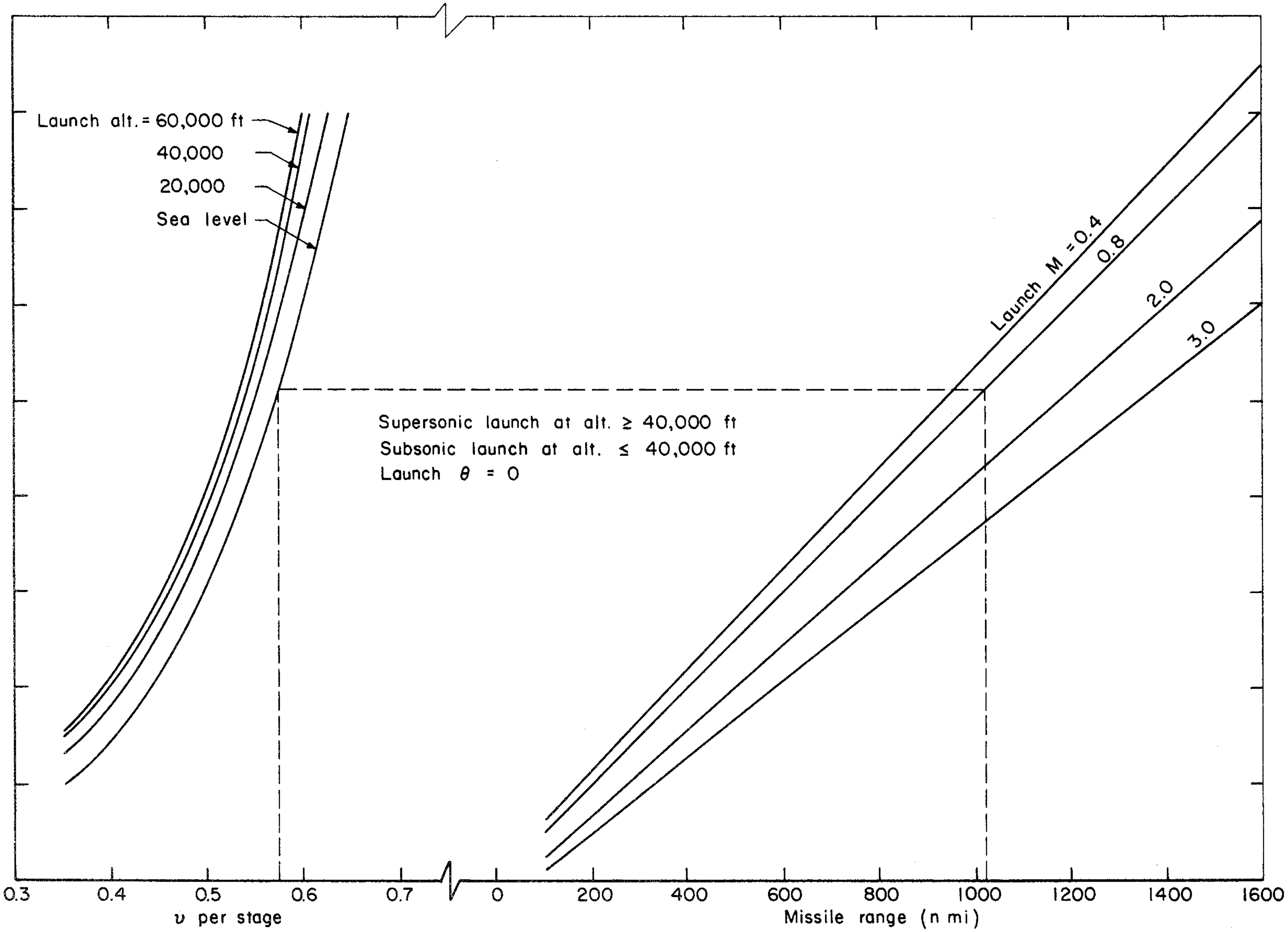

Fig. 3 — Ballistic ASM performance (two stages)

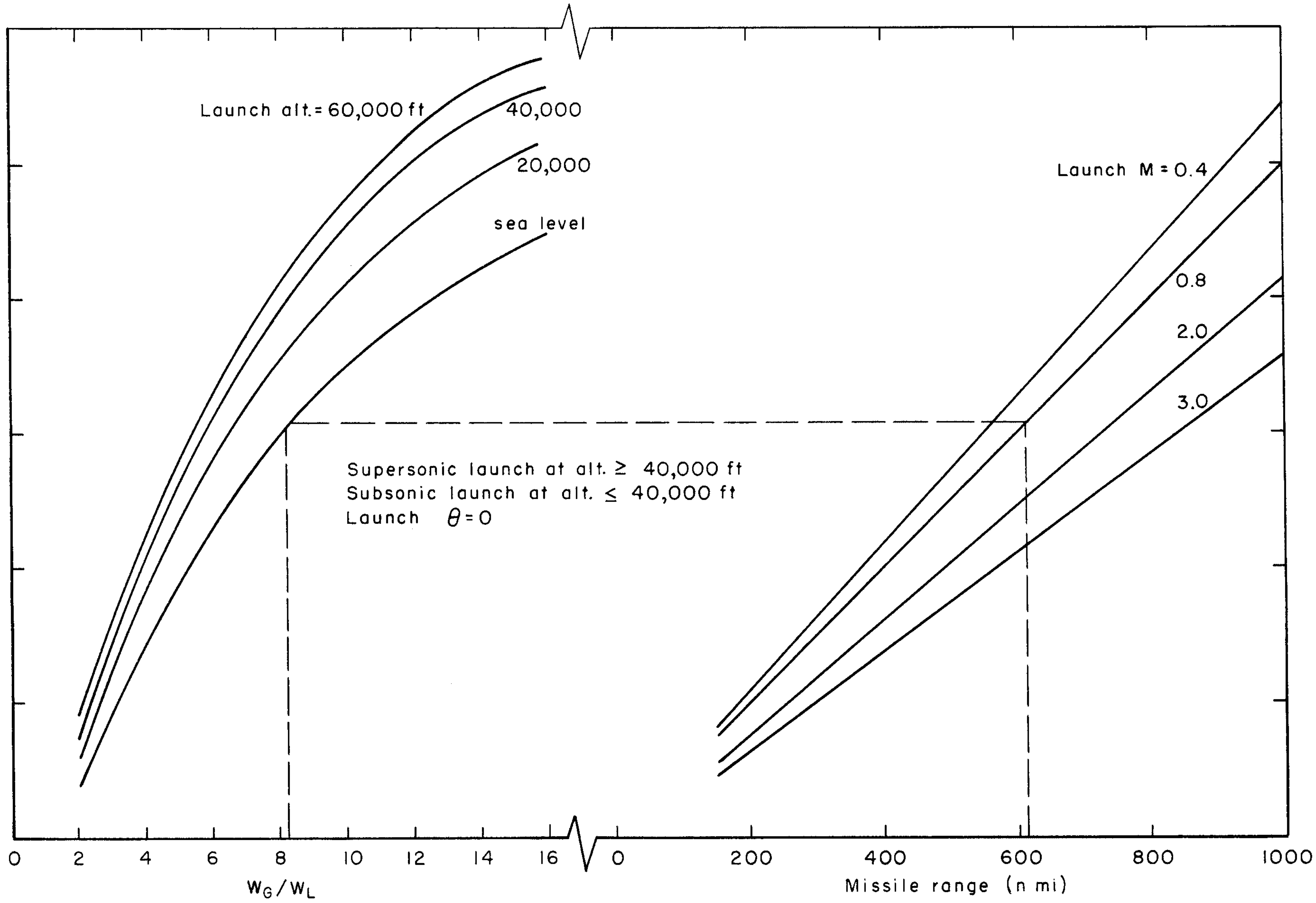

Fig. 4 — Ballistic ASM performance (single stage)

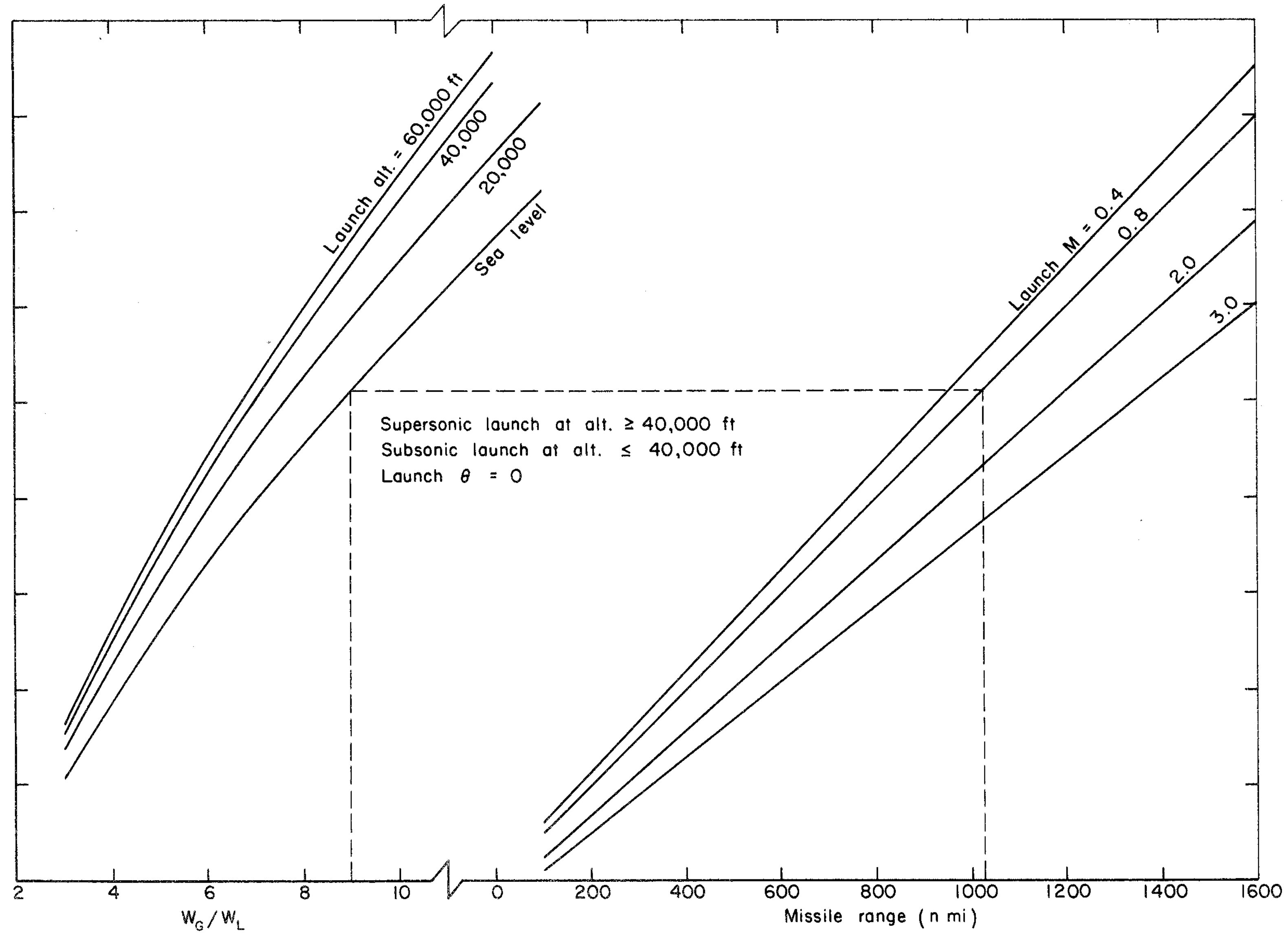

Fig. 5 — Ballistic ASM performance (two stages)

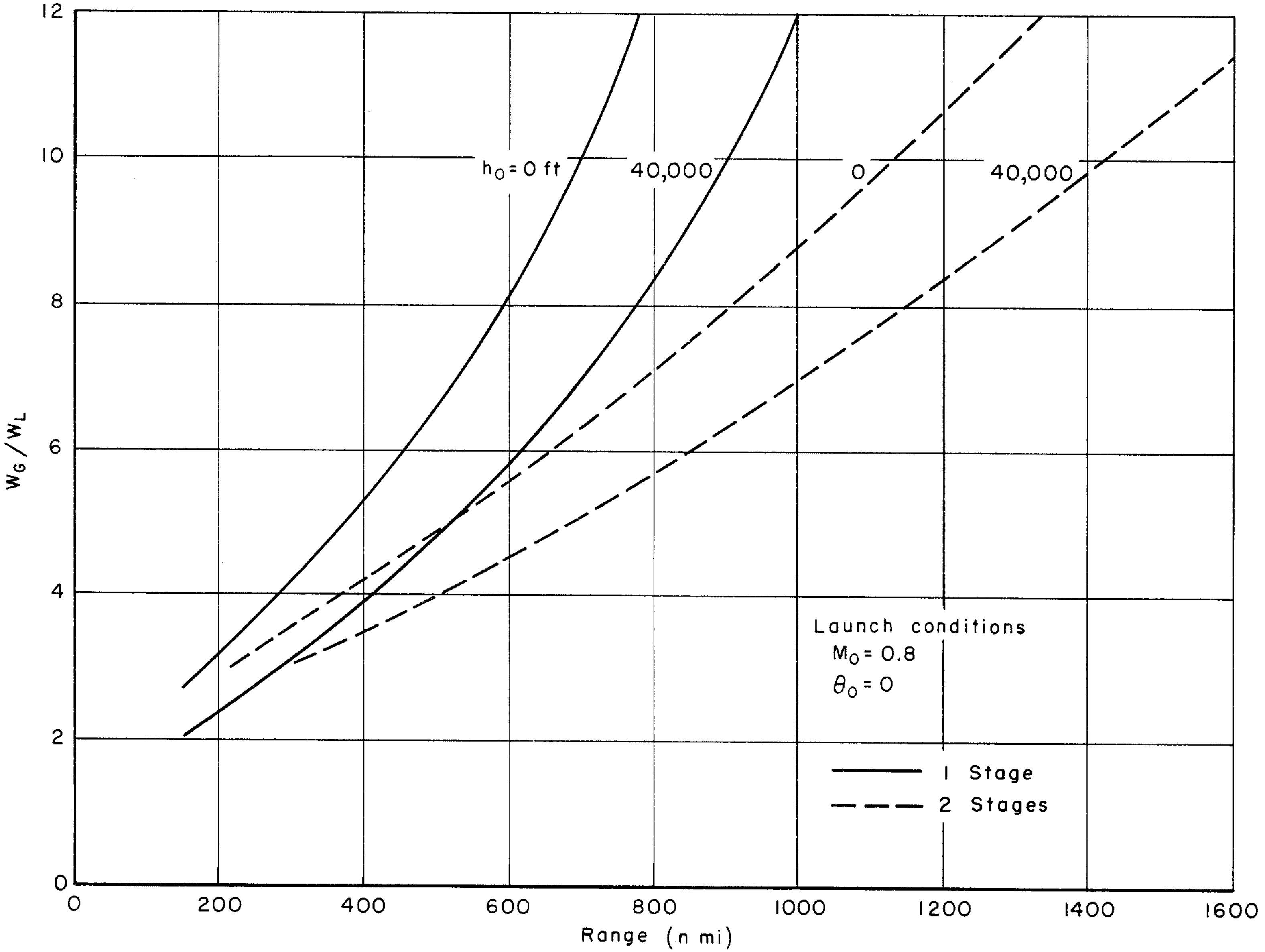

Fig. 6 — Missile payload — ratio variation

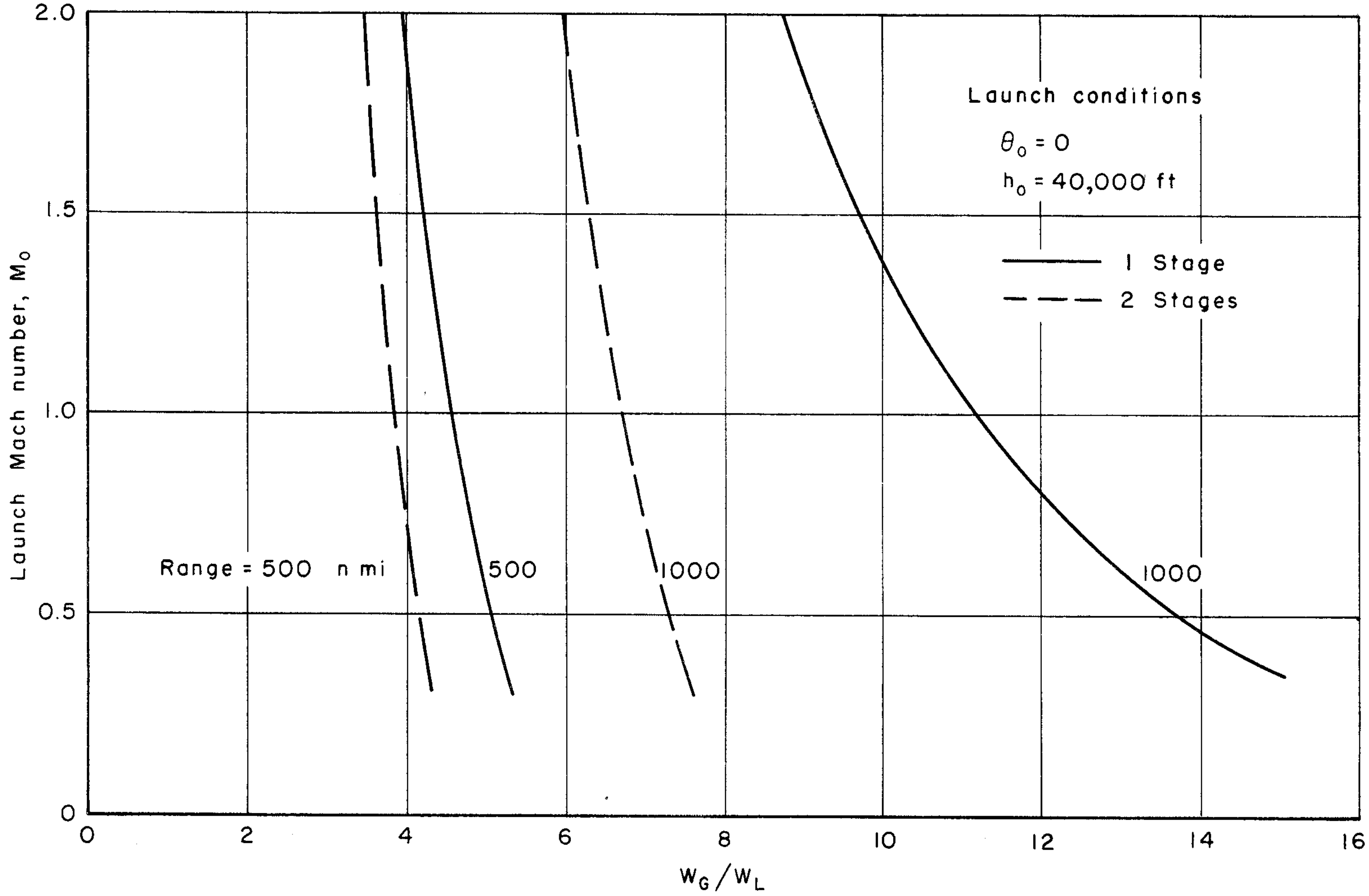

Fig. 7 — Effect of launch Mach number on missile performance

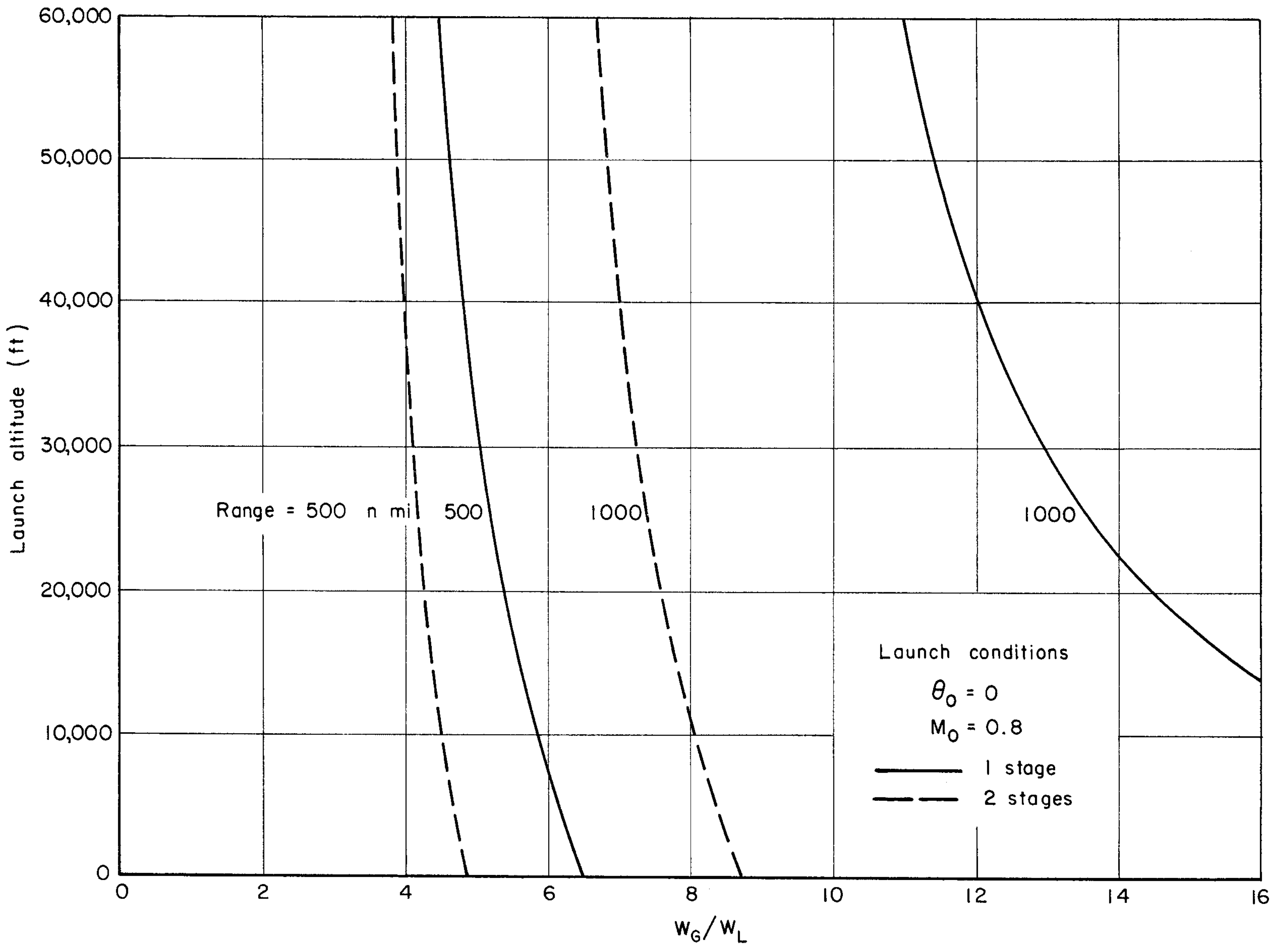

Fig. 8 — Effect of launch altitude on missile performance

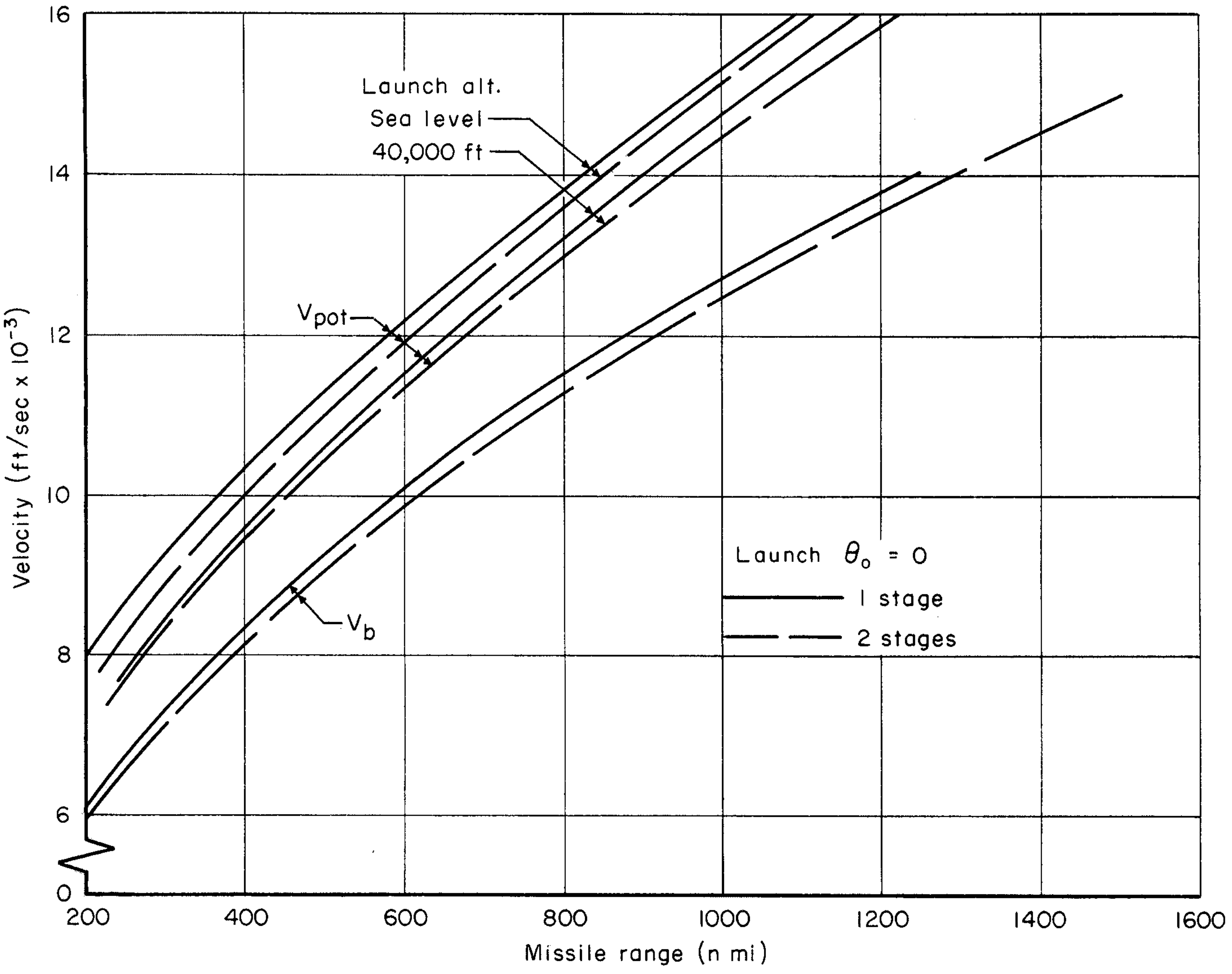

Fig. 9—Required theoretical and burnout velocity

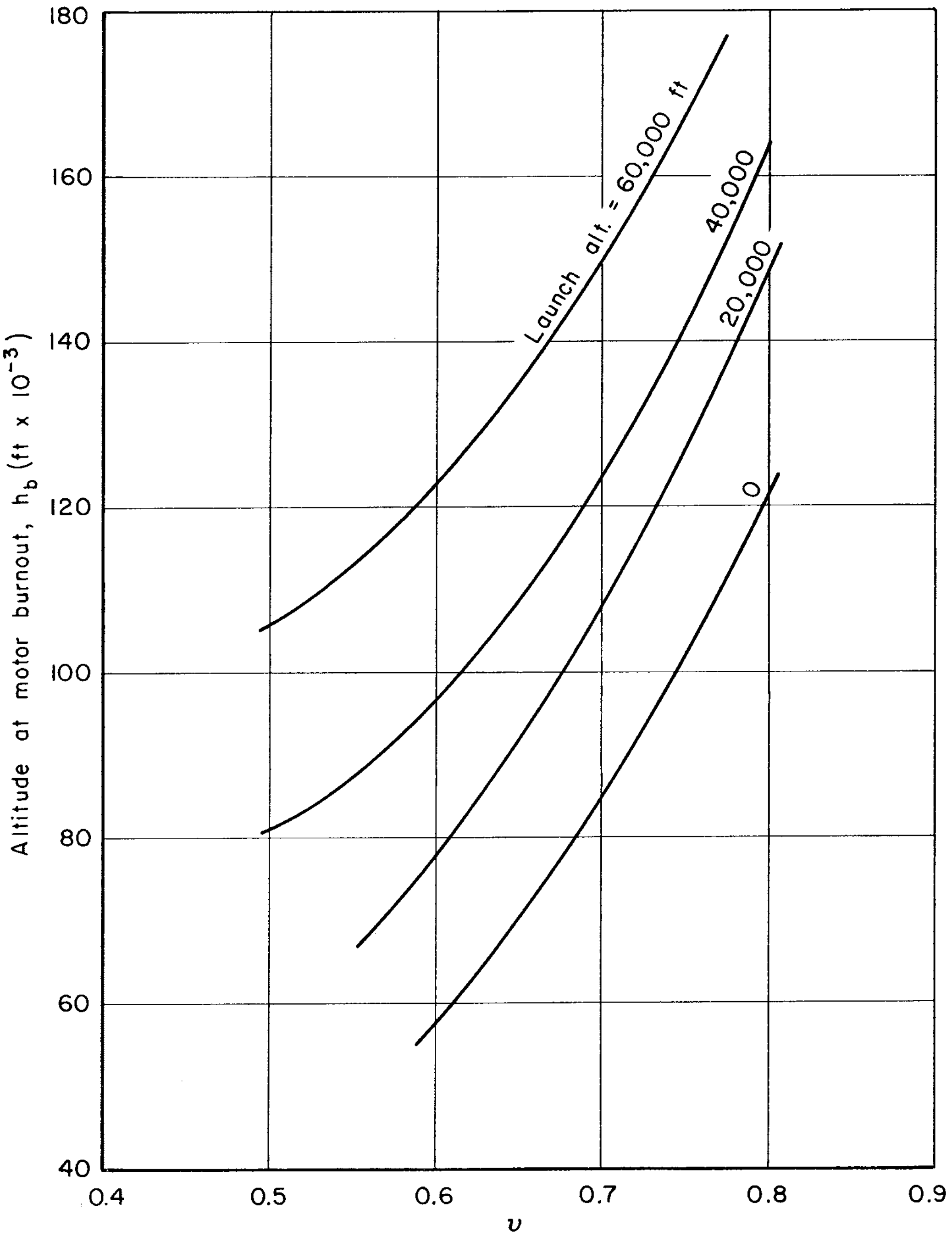

Fig. 10 — Altitude at motor burnout (one-stage missile)

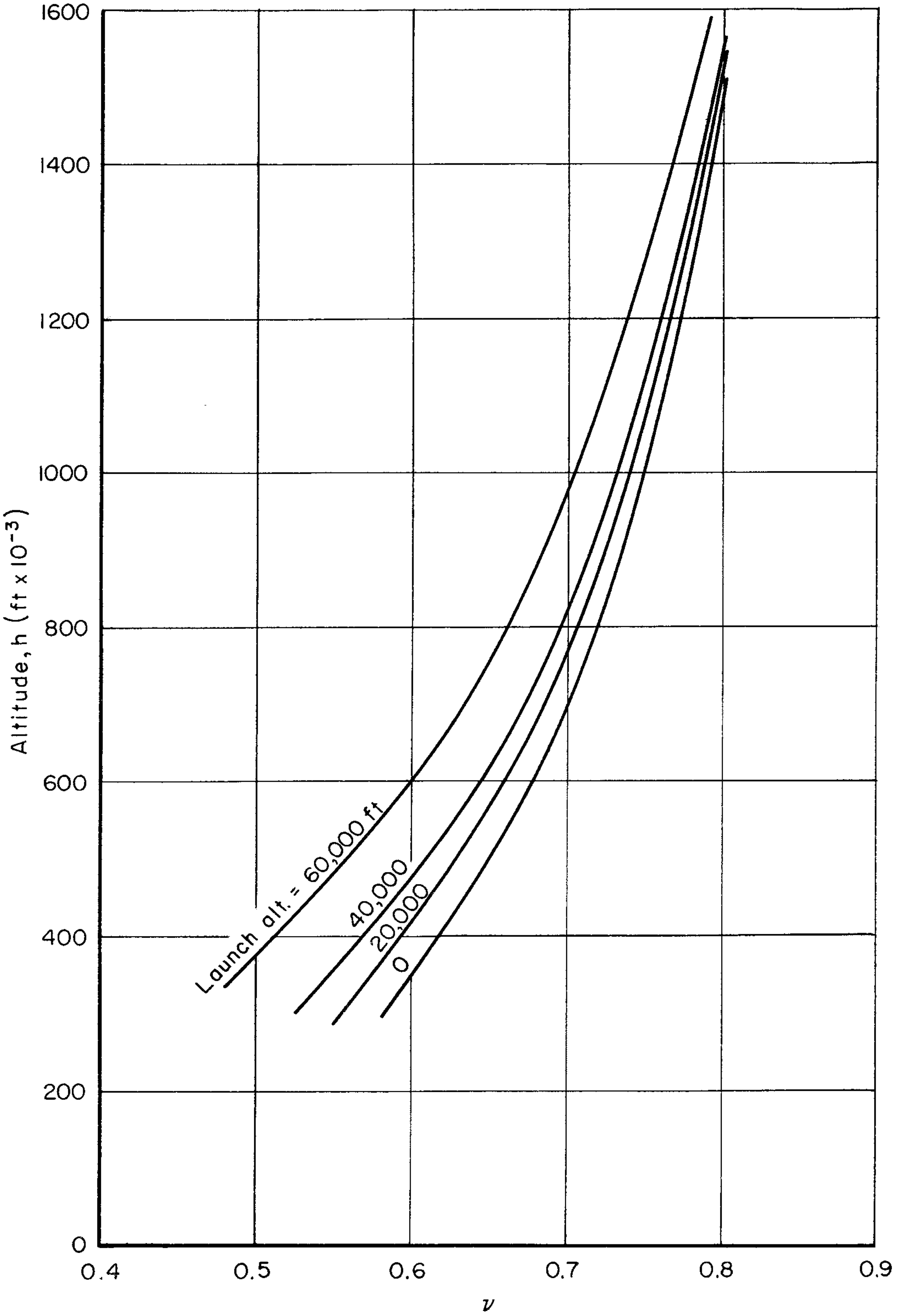

Fig. 11—Altitude at apogee
(One-stage missile)

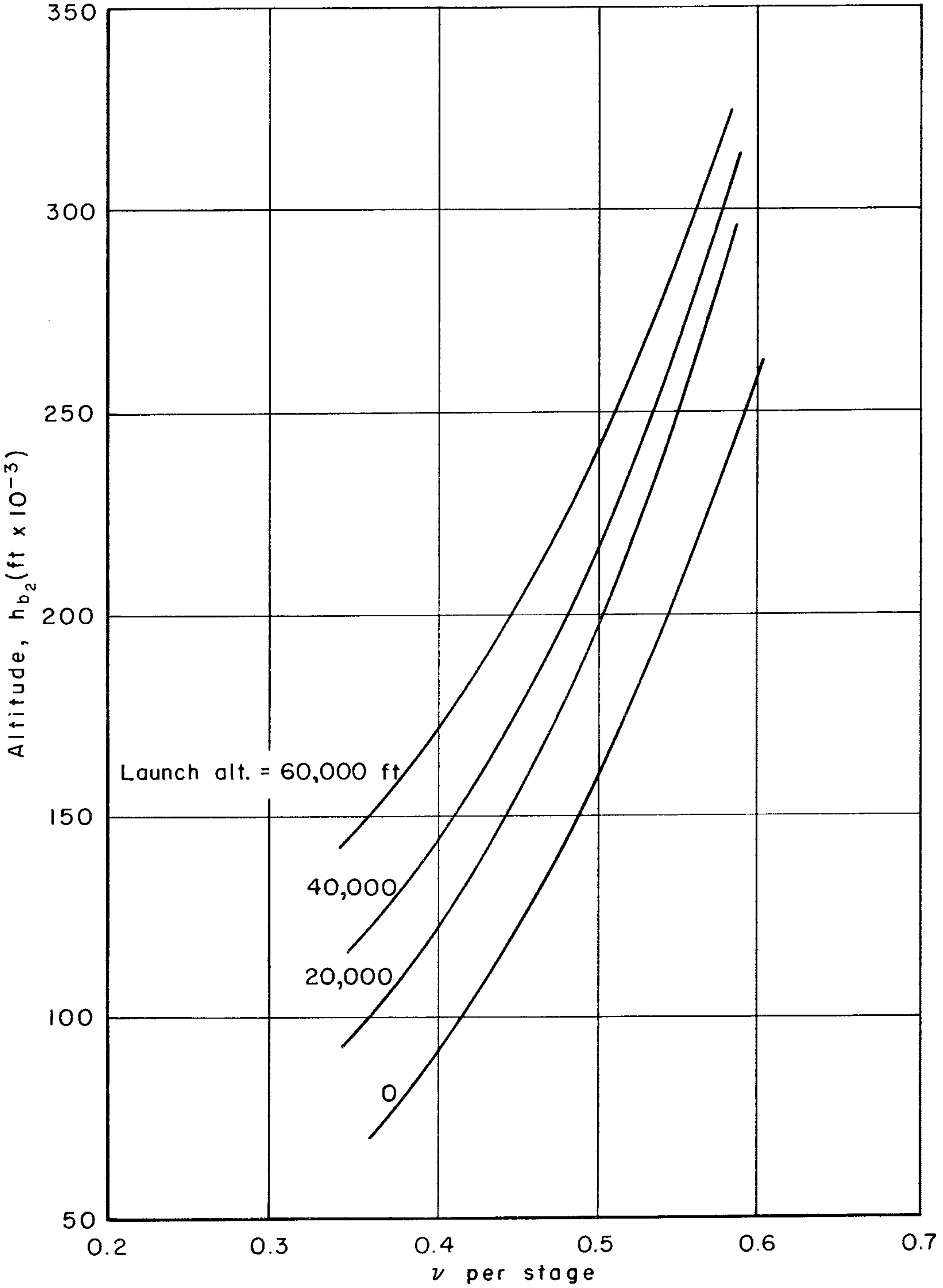

Fig. 12 — Altitude at second-stage burnout

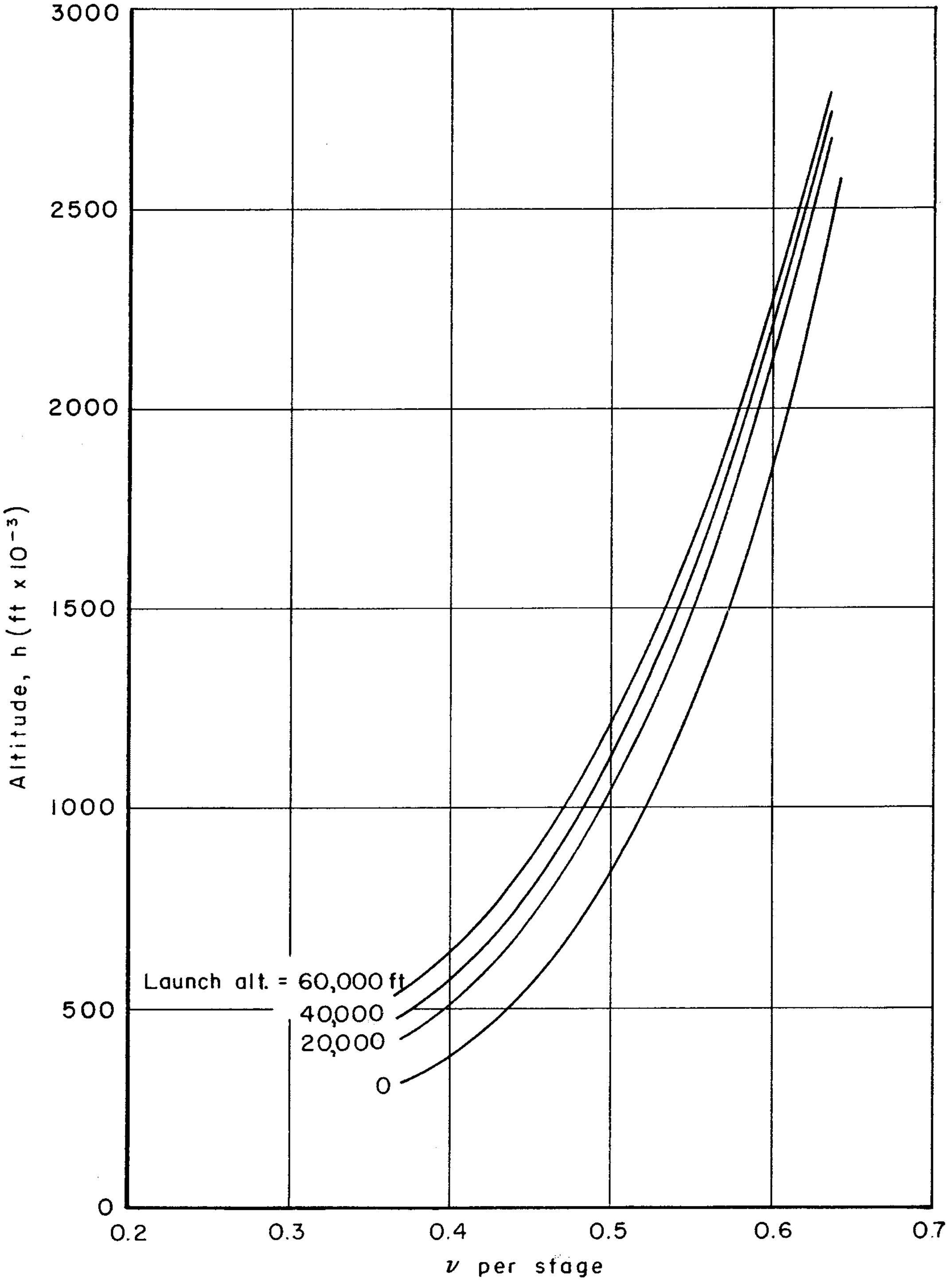

Fig. 13—Altitude at apogee (two-stage missile)

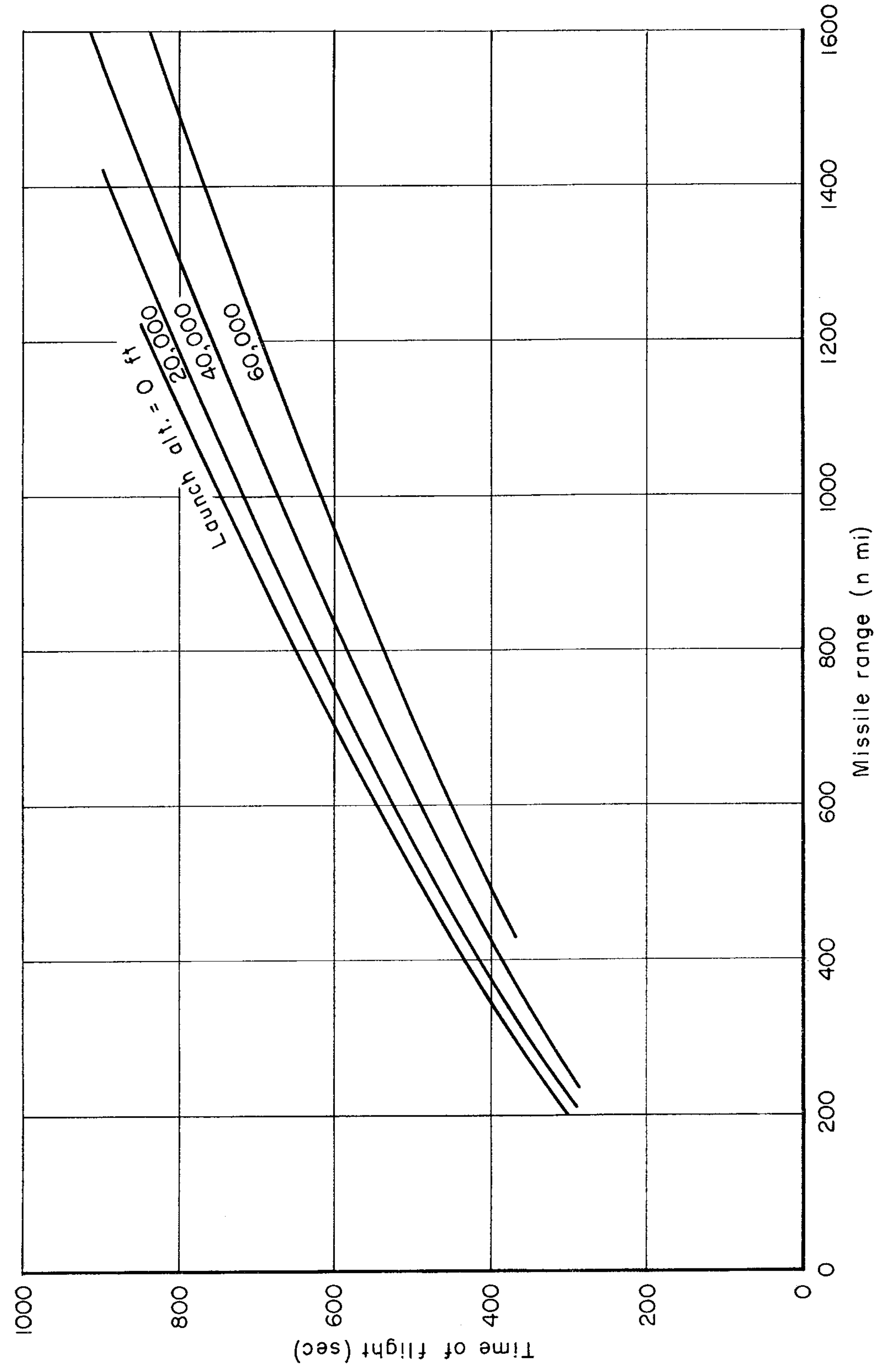

Fig. 14—Approximate flight time

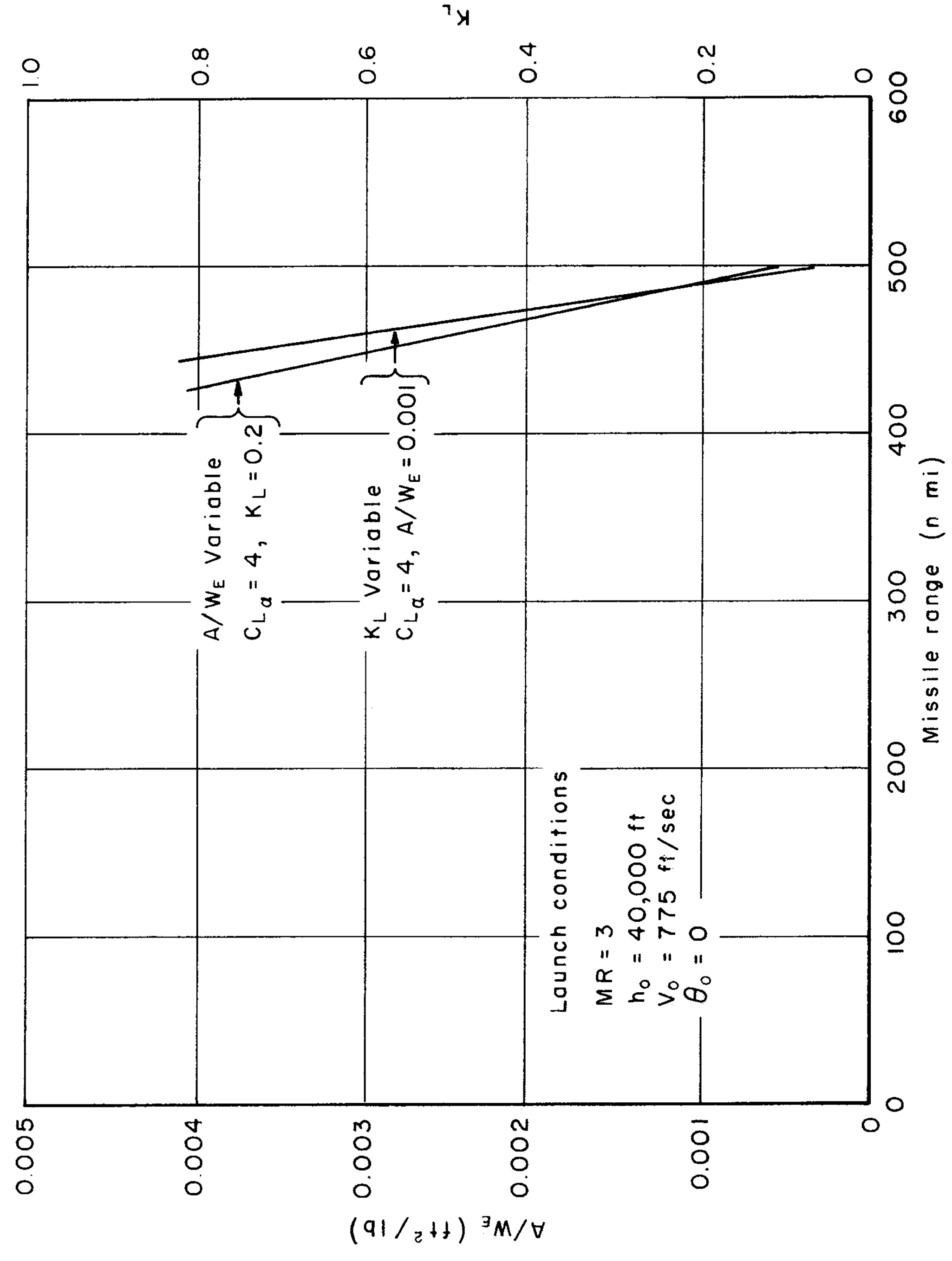

Fig. 15 — Variation of missile range with configuration (single stage)

Appendix B

TRAJECTORY CALCULATION PROCEDURE

The equations of motion were written in the usual form, assuming a spherical, nonrotating earth.

$$V' = \frac{T - D}{m} - g \sin \theta \tag{1}$$

$$V \theta' = \frac{L + T \sin \alpha}{m} + \frac{V^2 \cos \theta}{R + h} - g \cos \theta \tag{2}$$

$$h' = V \sin \theta \tag{3}$$

$$\phi' = \frac{V \cos \theta}{R + h} \tag{4}$$

where

$$g = g_s \left(\frac{R}{R + h}\right)^2$$

$$()' = \frac{d()}{dt}$$

and m is the instantaneous mass.

The missile is launched at a given altitude, velocity, path angle, and angle of attack (h_o, V_o, θ_o, α_o). During first-stage burning, the velocity vector is rotated upward by means of aerodynamic lift to a specified burnout value (θ_b). This rotation is forced by the function

$$\theta = (\theta_b - \theta_j' t_b - \theta_o)\left(\frac{t}{t_b}\right)^2 + \theta_j' t + \theta_o \tag{5}$$

where θ_j' is determined by solving Eq. (2) for the initial values of h, V, θ, and α. The shape of the boost trajectory can be controlled by changing the value of α_o.

In the calculation procedure, Eq. (5) is used to determine θ and θ' and then Eq. (2) is used to determine α. To keep the missile flight path within reasonable bounds a maximum value (n) is specified for the term L/W. If during the calculation procedure this maximum value of n is exceeded, the procedure is changed so that α is determined from n and the other parameter values existing at that moment. This new value of α is then used in Eq. (2) to determine θ' and θ. The actual value of α never exceeded 25 deg in the maximum-range trajectories.

After first-stage burnout all lift terms are dropped and the missile follows a ballistic path until $h = 0$. In the case of a two-stage missile, the second-stage burning is initiated at a specified time t_2 and thrust terms are added to the basic ballistic terms in the equations.

Missile drag is calculated during both ascent and descent by the following equation

$$\frac{D}{W_E} = 0.7 C_D P M^2 \frac{A}{W_E}$$

For altitudes between 0 and 36,000 ft

$$p = 2116 \left[\frac{518.7 - 0.003566h}{518.7}\right]^{5.256}$$

and for altitudes between 36,000 and 250,000 ft

$$\log_{10}\left(\frac{p}{2116}\right) = -0.65123 - 0.20874 \cdot 10^{-4}\,(h - 36{,}000)$$

Above 250,000 ft altitude, $p = 0$.

A parabolic lift-drag relationship is assumed, so

$$C_D = C_{D_o} + K_L\, C_L^{\,2}$$

The variation of C_{D_O} with Mach number is shown in Fig. 16.

The variation of motor specific impulse (I) with altitude is shown in Fig. 17. This motor performance curve was chosen arbitrarily from the wide range of possible design compromises and does not represent the performance of any particular motor. In choosing this particular motor design, it was assumed that the missile might be launched at any altitude between sea level and 60,000 ft. If, for example, it was believed that the missile would not be launched at altitudes below 40,000 ft, a different combination of chamber pressure and nozzle-expansion ratio would be used.

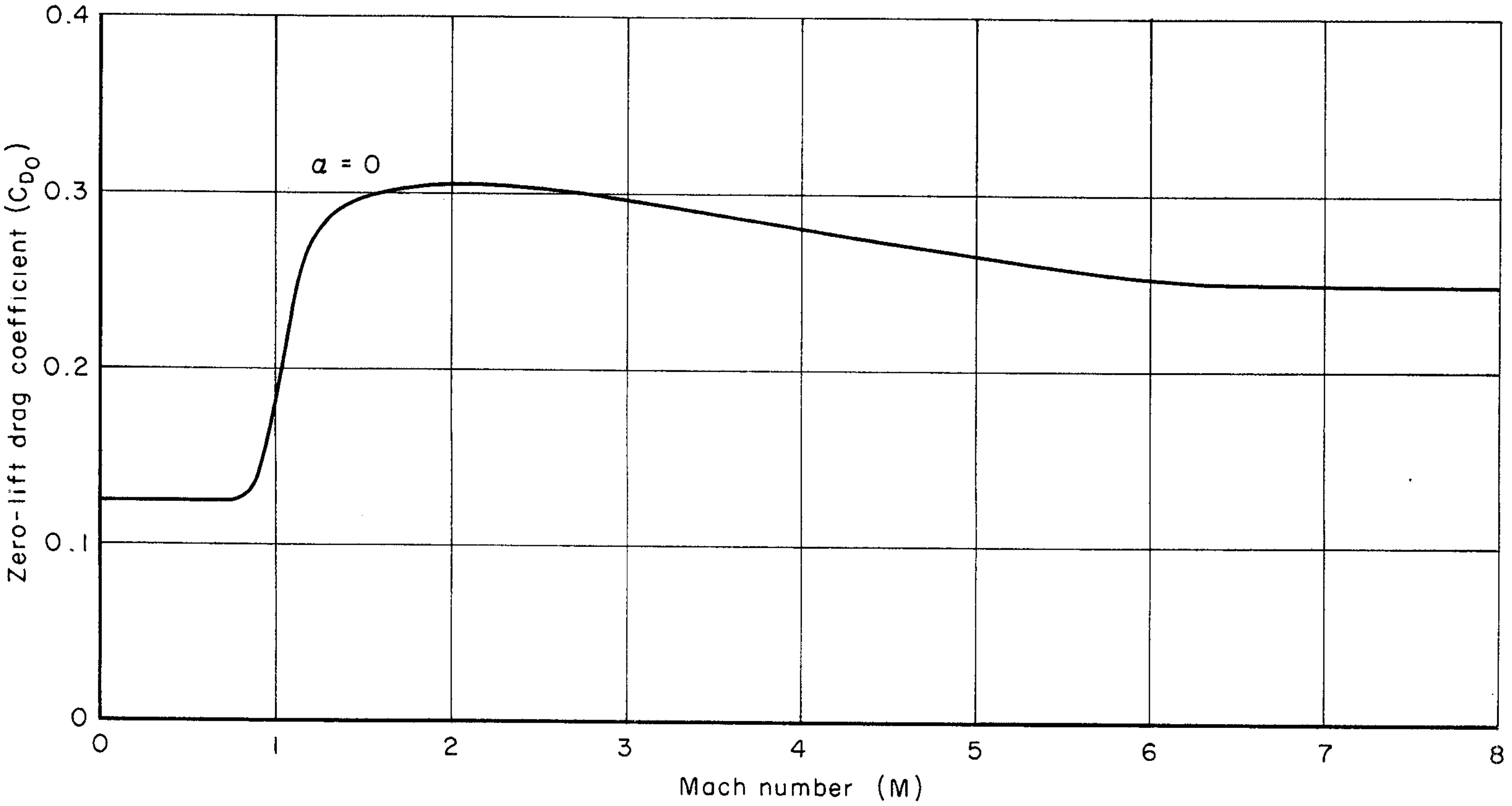

Fig. 16—Missile drag coefficient

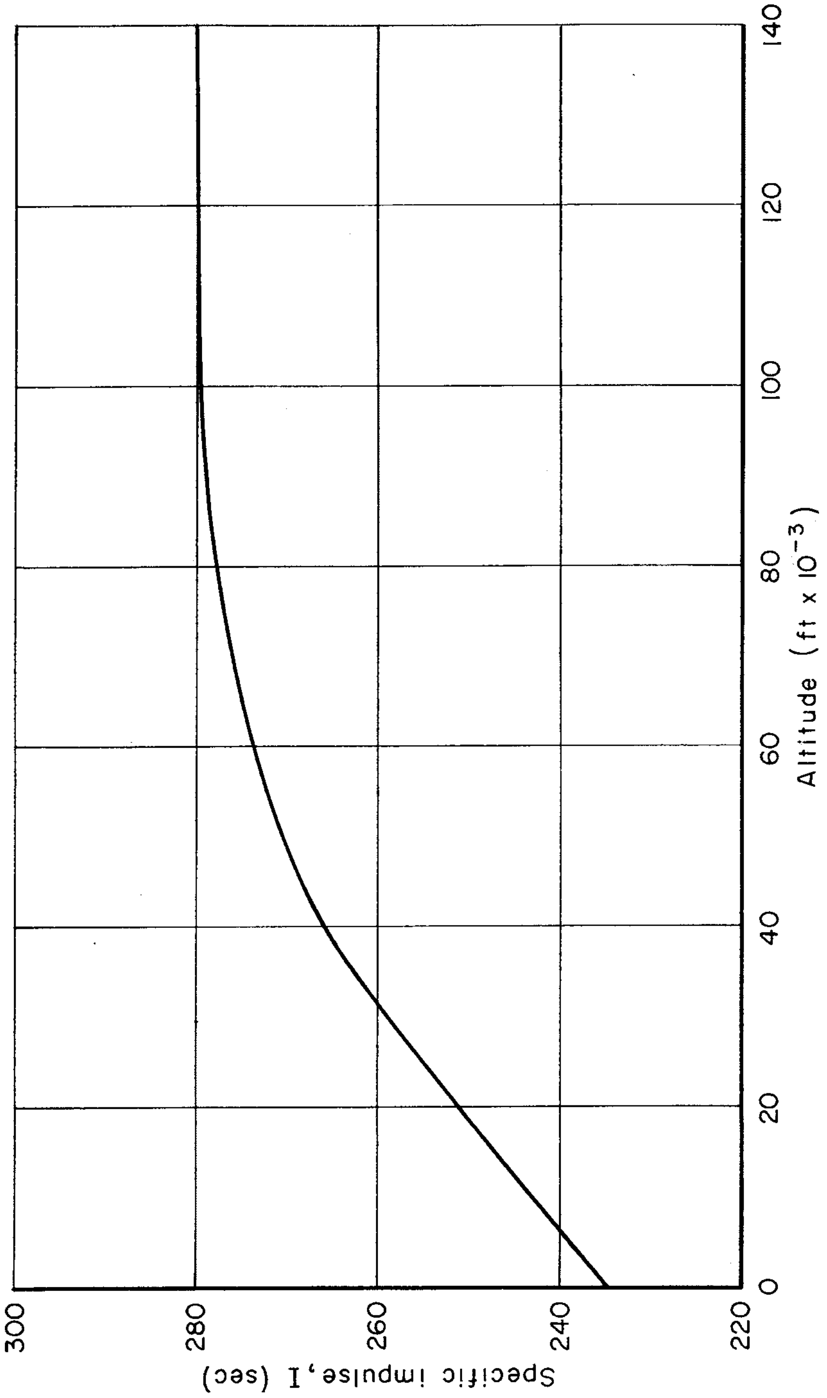

Fig. 17 — Motor performance