N-33561 (NAS 7-100)

MEMORANDUM
RM-3279-JPL
OCTOBER 1962

THE STUDY OF PLANETARY ATMOSPHERES BY STELLAR OCCULTATION

H. L. Weisberg

PREPARED FOR:
JET PROPULSION LABORATORY
California Institute of Technology

The RAND Corporation
SANTA MONICA • CALIFORNIA

N-33561 (NAS 7-100)

MEMORANDUM
RM-3279-JPL
OCTOBER 1962

THE STUDY OF PLANETARY ATMOSPHERES BY STELLAR OCCULTATION

H. L. Weisberg

This research is supported and monitored by the Jet Propulsion Laboratory, California Institute of Technology under Contract No. N-33561 (NAS 7-100) for the National Aeronautics and Space Administration. This report does not necessarily represent the views of the Jet Propulsion Laboratory or the National Aeronautics and Space Administration.

The RAND Corporation
1700 MAIN ST. • SANTA MONICA • CALIFORNIA

PREFACE

This Memorandum, sponsored by the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. N-33561 (NAS 7-100), is one result of a series of investigations of experimental techniques for studying planetary atmospheres. It consists of a review of progress to date in the somewhat obscure technique of stellar occultation, along with proposals for future applications to space probes and conventional astronomy.

ABSTRACT

The vertical structure of a planetary atmosphere can be studied by observing the intensity of light from a star as it passes behind the atmosphere at the limb of the planet. The theory for interpreting such observations is reviewed. With balloon astronomy, this technique may be very useful. A possible study of the Martian atmosphere by observing eclipses of Phobos is also discussed. Finally, use of this technique with a flyby space probe, where it would complement other measurements, is proposed and analyzed.

CONTENTS

PREFACE iii

ABSTRACT v

LIST OF SYMBOLS ix

Section

I. INTRODUCTION 1

II. HISTORY 6
- A. Occultations 6
- B. Eclipses 7

III. THEORY OF OCCULTATION BY A PLANETARY ATMOSPHERE 9
- A. Model Adopted 9
  - 1. Method of Computation 9
  - 2. Approximations 9
- B. Effects of Refraction 11
  - 1. Variation of Refractivity with Height 11
  - 2. Path of the Ray 13
  - 3. Differential Refraction 17
  - 4. Light Curve When Scattering and Absorption are Negligible 17
  - 5. The "Critical Level" 21
- C. Effects of Absorption and Scattering 22
  - 1. Reduced Path Length 22
  - 2. Rayleigh Scattering 23
  - 3. Molecular Absorption 24
  - 4. Absorption by Haze, Clouds and Dust 24

IV. APPLICATION TO BALLOON ASTRONOMY 25

V. OBSERVATIONS OF ECLIPSES OF PHOBOS 26
- A. Instrumental Problems 28
- B. Limb Darkening and Finite Size of Sun 30
- C. Model Calculation 31

VI. OCCULTATION PHOTOMETRY FROM A FLY-BY SPACECRAFT OR ORBITER 35
- A. Utility of Data Obtained 35
- B. Suitable Experiments 36
  - 1. Using the Sun 36
  - 2. Using a Star 36

BIBLIOGRAPHY 41

LIST OF SYMBOLS

B	= Coefficient determining solar limb darkening
ds	= Element of area in unrefracted ray
ds'	= Element of area in ray after differential refraction
D	= Distance from observer to atmosphere
$f(D,x)$	= Intensity observed from a point source
$F(D,x)$	= Intensity observed from an extended source
g	= Acceleration due to gravity
h	= Scale height, $\frac{RT}{\bar{m}g}$
I_1	= Intensity of received light
I_o	= Intensity of incident light
J_1	= Bessel function of first order
k_R	= Coefficient of Rayleigh scattering
ℓ	= Distance from undeviated ray to surface of planet
$\ell(y)$	= A factor determined by shape and limb darkening of Sun
$L(\gamma)$	= Limb darkening of Sun
$\bar{m}$	= Mean molecular weight
n	= Index of refraction
p	= Pressure
R	= Gas constant
T	= Temperature
X	= Reduced length of path
z	= Altitude above surface
β	= Half angle subtended by Sun seen from satellite
γ	= Angle between line of sight and normal to surface of Sun

ϵ = Refractivity

θ = Angular coordinate of point along ray

ξ = Angle between radius vector and tangent to ray

ρ = Density

τ = Optical thickness of path

ω = Deviation of ray

Γ = Logarithmic temperature gradient, $\frac{1}{T}\frac{dT}{dz}$

<u>Subscript</u>

1 Corresponding to altitude of closest approach of ray

r At reference level in atmosphere

s At unit density

I. INTRODUCTION

There are two optical ways to study the atmosphere of a planet without landing on the planet. One is to look directly at the planetary disk and observe reflected sunlight or self emission. The second is to observe light from a distant star or the Sun which has passed at grazing incidence through the atmosphere. The latter method is the subject of this report. This method is distinguished by its ability to give information directly about the vertical structure of the atmosphere, its pressure, its temperature, and perhaps its cloud or dust content.

For observations from Earth, the method can be applied to an occultation of a star by a planet (Fig. 1) and also to the eclipse of a planetary satellite by its primary (Fig. 2). Both kinds of observations have been made to some extent in the past (Sec. II). The mathematical theory upon which rest the inferences made from such observations is given in Section III.

The possibilities of these earthbound techniques are not yet exhausted. Because of atmospheric seeing limitations, only occultations of extremely bright stars have heretofore been observable photometrically. Such occultations seldom occur. However, the introduction of balloon-borne telescopes (Sec. IV) operating above most of the Earth's atmosphere will make frequent and accurate occultation measurements possible.

For a planet having a suitable satellite, the eclipse technique enables an observer on Earth to probe more deeply in the planetary

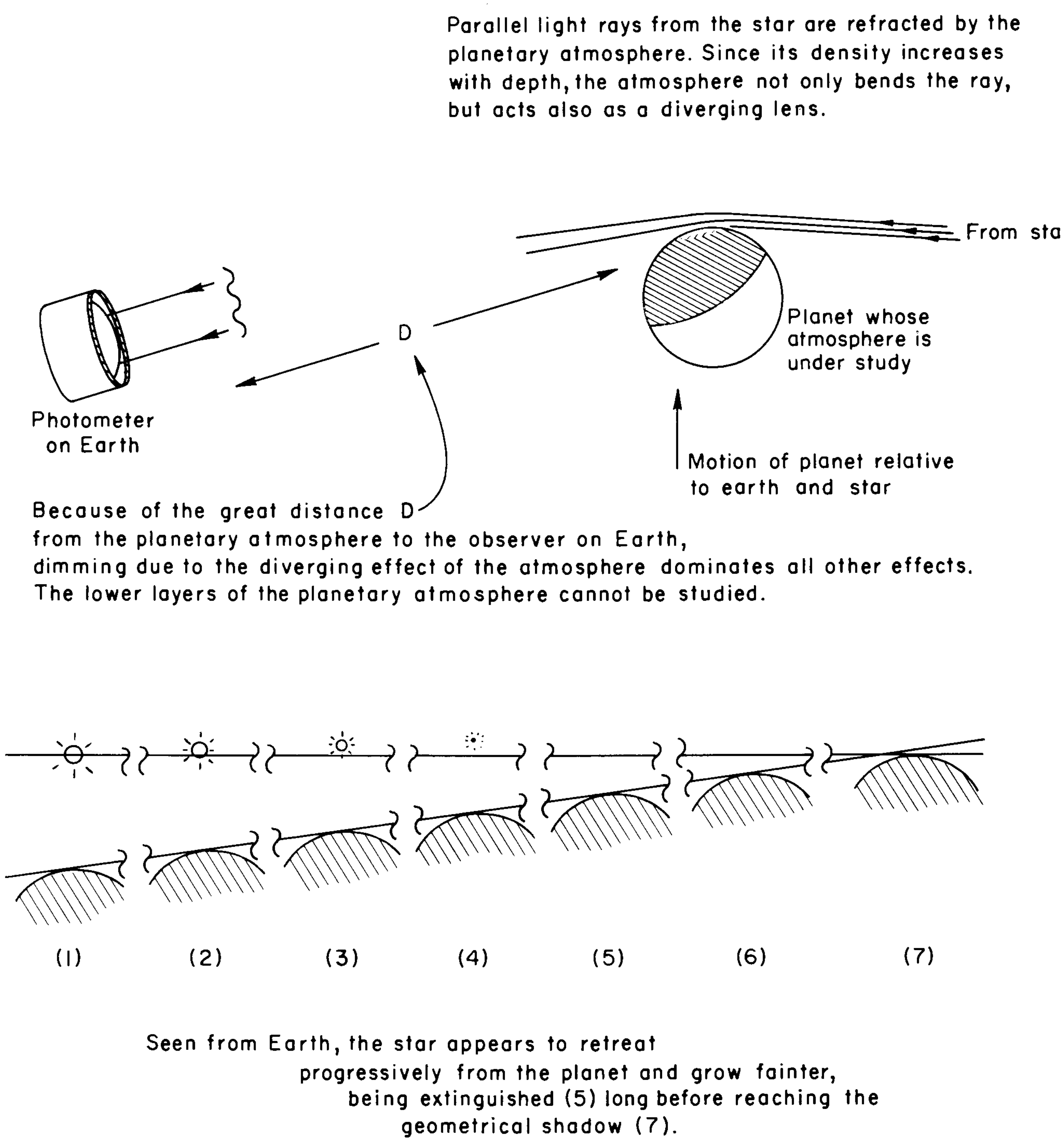
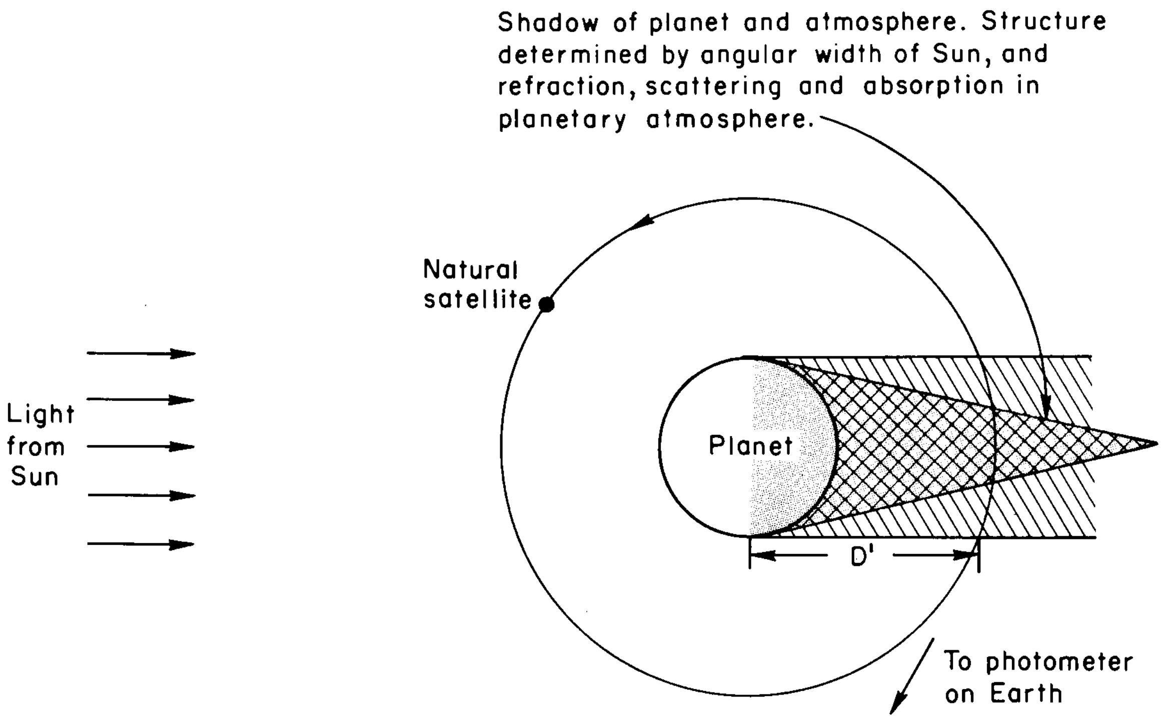


Fig. 1—Conditions for studying a planetary atmosphere by astronomical occultation



Distance D' plays same role in determining importance of refractive effects as does Earth — planet distance D in astronomical occultation. Since D' << D, refractive divergence effects are small and low levels in atmosphere can be studied.

Path of satellite as seen from Earth when configuration of Sun, planet, and Earth are suitable.

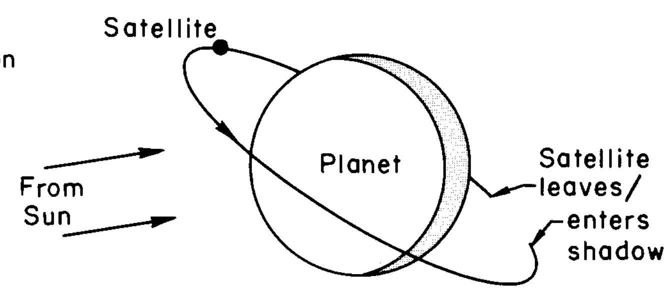


Fig.2 — Conditions for studying a planetary atmosphere by observation of eclipses of a planetary satellite

atmosphere than does the occultation technique. By using astronomical methods now being developed, it may be possible to observe eclipses of Phobos photometrically. An analysis of this instrumentally difficult, but possibly useful, experiment is given in Section V.

With the advent of the instrument-bearing spacecraft, the limitations of earthbound observations can be overleapt by observing stellar occultations from the spacecraft (Fig. 3). The theory of Section III can also be applied directly to such observations. As is discussed in Section VI, stellar occultations will be frequent when the spacecraft approaches the planet, and at the same time, deep levels in the planetary atomsphere can be probed. The instrumental and data-transmission requirements seem reasonable, and the data obtained will complement those from other apparatus aboard the spacecraft.

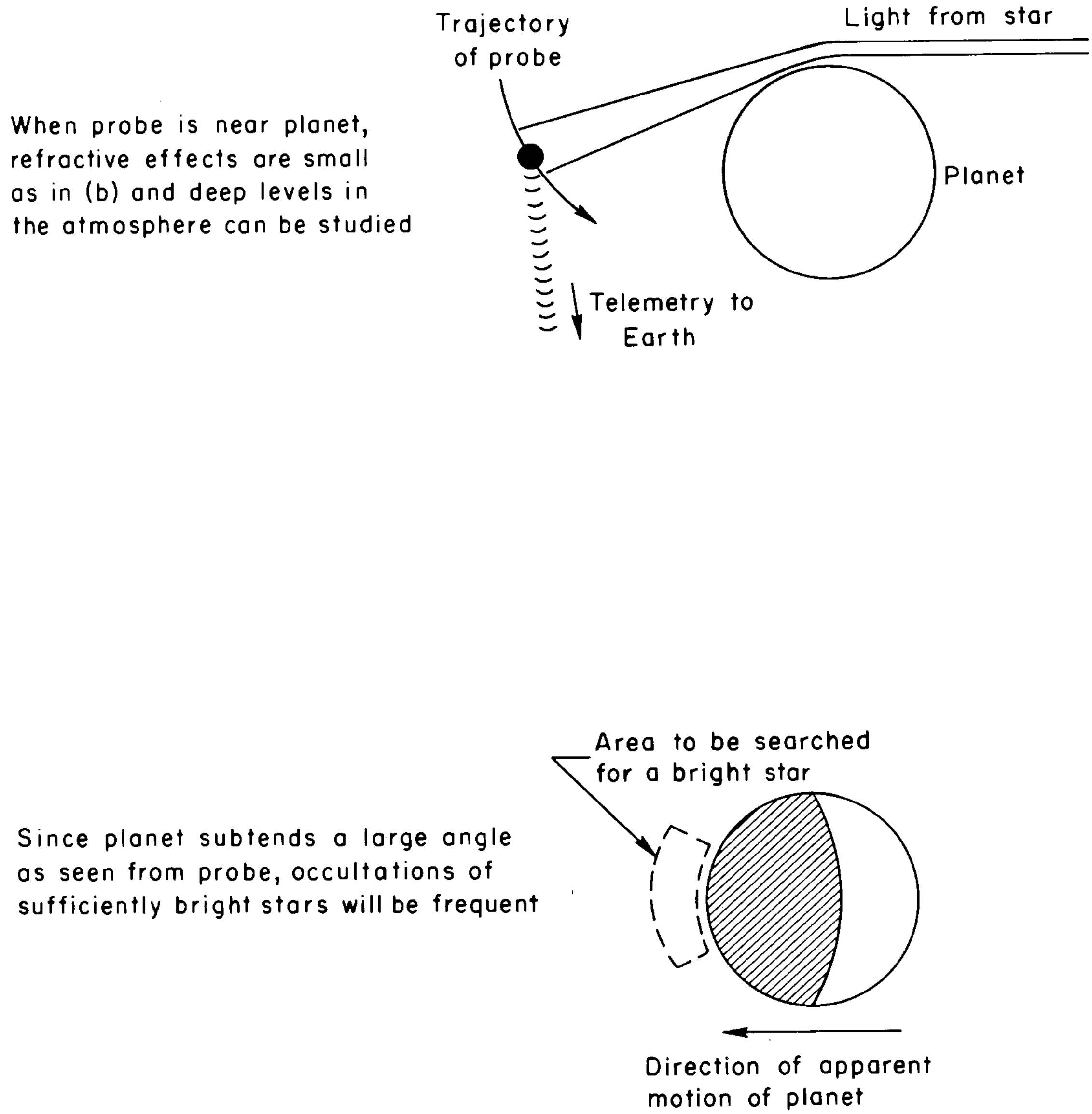


Fig.3 — Occultation observed from a fly-by space probe or planetary orbiter

II. HISTORY

A. OCCULTATIONS

The theory describing the occultation of a star by a planet with an atmosphere was first given by Pannekoek (1903). He showed that, in an occultation observed from Earth, extinction due to differential refraction dominates all other effects and becomes effective while the stellar light is still passing through the rarified upper atmosphere of the planet. From photometry of the star as it is being occulted, one can determine the pressure scale height $h = RT/\bar{m}g$ of the planet's atmosphere. If the main constituents of the atmosphere are known, it is also possible to determine the pressure at the occulting layer. This theory was given again in somewhat elaborated form by Fabry (1929), whose article was intended as an "advertisement to observers."

Application of the theory had to wait for the combination of modern photometric techniques and the planetary occultation of a sufficiently bright star. The occultation of Sigma Arietis by Jupiter was observed photometrically by Baum and Code (1953) at Mt. Wilson when Jupiter was 7.9 photovisual magnitudes brighter than Sigma Arietis. Because of seeing limitations, it was impossible to use a focal-plane diaphragm smaller in diameter than 8 seconds, so 1 or 2 per cent of Jupiter's light was admitted to their detector. They used a spectroscope to observe at the wavelength of the Ca II line. Light of this wavelength is strongly absorbed in the Solar (and hence Jovian) spectrum but not in the spectrum of Sigma Arietis; a three magnitude advantage was thus obtained. The analysis of Baum and Code showed the

scale height in the upper atmosphere of Jupiter to be between 6.3 and 12.5 km. For values of T in the range 70 to 100°K, the measured scale height corresponds to molecular weights in the range 1.8 to 5.1, from which they inferred that the prevailing upper atmospheric constituents of Jupiter are hydrogen and helium. The same occultation was observed visually by Pettit and Richardson (1953), who found it possible to follow the star as it penetrated much further into the Jovian atmosphere. However no quantitative results were obtained.

Observations of an occultation of the first-magnitude star Regulus by Venus are reported by Menzel and de Vaucouleurs (1960), who did not use the spectroscopic technique. Regulus is occulted by Venus four times in the period 600 AD — 2600 AD (Meeus, 1960). Menzel and de Vaucouleurs were able to measure the scale height and also to detect a variation of the scale height with altitude. They obtained, at a height of 60 ± 8 km above the visible surface, a value of 6.8 ± 0.2 km for the scale height, 0.010 ± 0.002 km^{-1} for its logarithmic derivative with respect to altitude and, for an assumed atmospheric composition and temperature, 2.6 ± 0.13 dynes cm^{-2} for the pressure.

Other occultations have occasionally been observed visually, but no numerical results have been obtained (de Vaucouleurs, private communication).

B. ECLIPSES

The theory describing the optical effects observed during the eclipse of a satellite by a planet having an atmosphere was given by Link (1934). It is necessary to include effects of scattering and

absorption in the planetary atmosphere. It is also necessary to take into account the effect of the finite size of the Sun (and its limb darkening) and, in some cases, the finite size of the satellite.

Link's analysis of photographic observations by Eropkin of eclipses of the bright satellites of Jupiter led him to the conclusion that there is an "absorbing layer" at an extreme height above the surface of Jupiter. However, the measurements need to be repeated with more modern techniques.

A book has been written discussing eclipses and occultations (Mikhailov, 1954).

The atmosphere of Earth also may be studied by observations of eclipses of the Moon. An analysis by Link (1932) of a number of observations suggests the presence of an absorbing layer of optical thickness of the order of 0.05 at an altitude of 20 km or more; the question of its reality remains open (van de Hulst, 1956). The data need to be re-analyzed using modern information about the Earth's atmosphere.

Recently, an attempt has been made to study the ozone structure of the Earth's atmosphere by observing eclipses of the artificial satellite Echo I. A preliminary report is given by Venkateswaran, Moore, and Krueger (1961).

III. THEORY OF OCCULTATION BY A PLANETARY ATMOSPHERE

A. MODEL ADOPTED

1. Method of Computation

Given the configuration of source, planet and observer at each time (see Fig. 4), and an assumed set of parameters for the structure of the planetary atmosphere, the light curve (relative intensity versus time at a given wavelength) can be computed as follows. First, considering that the atmosphere acts only through its refractivity, find the path through the atmosphere of the ray from source to observer (Secs. III.B.1 and III.B.2) and compute the extinction due to differential refraction (III.B.3). Next, compute the reduced path length, or total amount of atmospheric gas traversed (III.C.1). Multiply it by the sum of coefficients of absorption due to Rayleigh scattering (III.C.2), molecular absorption (III.C.3) and absorption by clouds, dust and haze (III.C.4) to get the optical thickness τ. Multiply the intensity reduction due to differential refraction by the factor $e^{-\tau}$ to get the over-all extinction.

2. Approximations

It is important to keep the approximations made in mind since, in some cases, one might have to include an effect neglected here.

Some geometrical approximations made are:

(1) the source is sufficiently distant that its light may be considered to be a plane wave,

(2) the planet is a sphere of radius r,

(3) the distance x (Fig. 4) is much smaller than D,

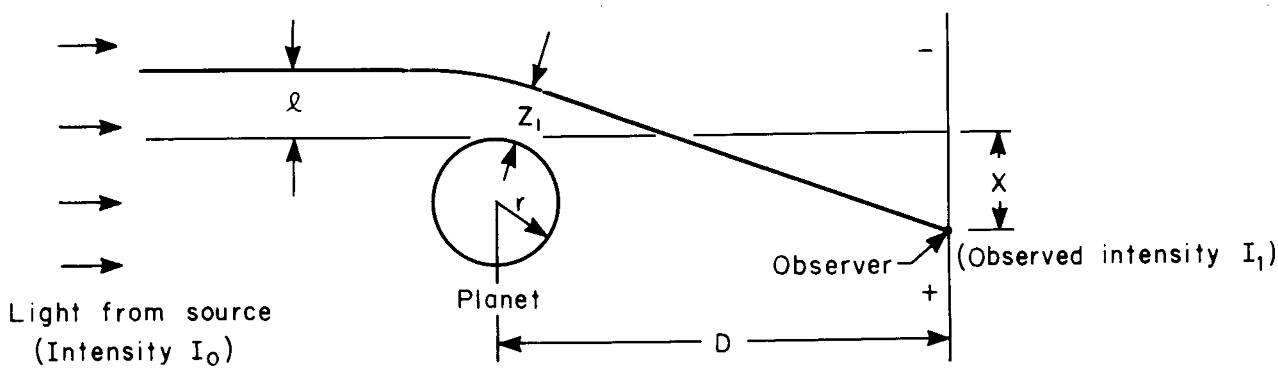


Fig. 4—Configuration of source, planet, and observer

(4) the radius of the planet is much greater than any characteristic height in the atmosphere.

Some approximations about the atmosphere are:

(5) the atmosphere is a perfect gas and hydrostatic equilibrium holds,

(6) the chemical composition of the atmosphere is uniform (but c.f. Sec. III.B.5),

(7) the refractivity of the atmosphere at any point is small compared with 1,

(8) the change in temperature is small over an altitude difference of one scale height.

Finally, an important assumption is made about the extinction process:

(9) multiple scattering is negligible.

Assumption (9) implies that only so-called "direct absorption" need be taken into account. In fact, if the photometer is sensitive to light from a solid angle Ω, then the maximum intensity of scattered light from the star that can be received at any time during the occultation is of the order of $\Omega/4\pi$ times the intensity before occultation, the exact intensity depending on details of the scattering process in the planetary atmosphere. Since $\Omega/4\pi \ll 1$, assumption (9) is justified.

B. EFFECTS OF REFRACTION

1. Variation of Refractivity With Height

The optical properties of the atmosphere are determined by its refractivity ϵ, related to the index of refraction n by $n = 1 + \epsilon$. The refractivity is directly proportional to the atmospheric density ρ,

so if ε_s is the refractivity when the density is one unit, the refractivity at any altitude z is

$$\varepsilon(z) = \varepsilon_s \rho(z) = \varepsilon_s \frac{\bar{m}p(z)}{RT(z)} \qquad (1)$$

Since the pressure p is related to the temperature T by the equation of hydrostatic equilibrium (see below), the variation of refractivity with height is in effect determined by the variation of temperature with height.

Because we assume that the variation of temperature with an altitude change of one scale height is small, an important simplification becomes permissible. The effects of refraction can be computed on the assumption that the temperature varies linearly with altitude; we must use for the temperature gradient its value in the actual atmosphere evaluated at the altitude z_1 of closest approach of the ray, and for the pressure and temperature at z_1 their corresponding values in the actual atmosphere. Although in the actual atmosphere the higher derivatives of T with respect to altitude may not vanish, their contribution to the refractive effects will be negligible.

Thus we consider an atmosphere with temperature

$$T(z) = T_1\left\{1 + \Gamma(z-z_1)\right\} \qquad (2)$$

where Γ is a constant.

Since Γ $(z-z_1)$ is small compared to unity over the region of interest, the equation of hydrostatic equilibrium,

$$\frac{dp}{dz} = - g \frac{\bar{m}p}{RT} \qquad (3)$$

becomes

$$\frac{dp}{dz} = - g \frac{\bar{m}p}{RT_1} [1 - \Gamma (z-z_1)]$$

which can be integrated to give

$$p(z) = P_1 \exp \{- \frac{\bar{m}g}{RT_1} [(z-z_1) - \frac{\Gamma}{2} (z-z_1)^2]\} \tag{4}$$

Using this result and Eqs. (1) and (2), we have

$$\varepsilon(z) = \varepsilon_1 [1 - \Gamma (z-z_1)] \exp \{- \frac{1}{h_1} [(z-z_1) - \frac{\Gamma}{2} (z-z_1)^2]\} \tag{5}$$

For most computations to first order in Γh, the quadratic term in the exponential can be neglected.

2. Path of the Ray

The path of a ray of light passing by the planet can be found from Fermat's principle, $\delta \int n ds = 0$. The path is illustrated in Fig. 5. At the point A, the ray has reached its lowest altitude, z_1; by symmetry its direction is then perpendicular to the radius vector ($\xi = 90^\circ$). Expressing the coordinates of the general point B along the path in polar coordinates $r+z$, θ, and applying the Euler--Lagrange equation with $\frac{\partial n}{\partial \theta} = 0$ because of the assumed symmetry of the atmosphere,

$$\frac{[n(z)](r+z)^2 \frac{d\theta}{dz}}{\left[1 + (r+z)^2 (\frac{d\theta}{dz})^2\right]^{1/2}} = \text{const} \equiv \lambda \tag{6}$$

[redacted]

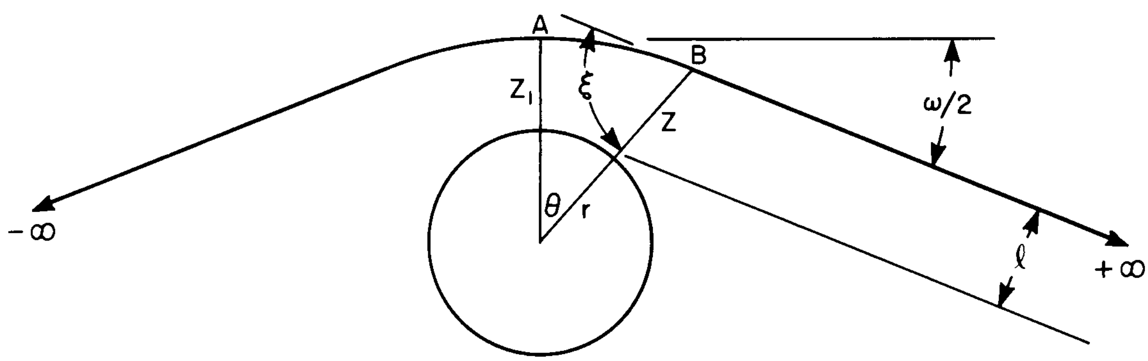


Fig. 5— Path of ray due to refraction in the atmosphere

It is geometrically evident that

$$\sin \xi = \frac{(r+z)\ d\theta}{\left[dz^2 + (r+z)^2\ d\theta^2\right]^{1/2}} = \frac{(r+z)\ \frac{d\theta}{dz}}{\left[1 + (r+z)^2 (\frac{d\theta}{dz})^2\right]^{1/2}} \qquad (7)$$

Putting (7) into (6) and evaluating at $\xi = 90^\circ$ and at $\xi = 0^\circ$ gives

$$\lambda = [n(z)](r+z)\sin \xi = (1 + \epsilon_1)(r+z_1) = r + \ell \qquad (8)$$

so that

$$\boxed{\ell = z_1 + \epsilon_1\ (r+z_1)} \qquad (9)$$

Except for rays penetrating deep into the atmosphere, ℓ is approximately equal to z_1.

Solving Eqs. (6) and (8) for $\frac{d\theta}{dz}$ and integrating,

$$\theta(z) = \int_{z_1}^{z} \frac{(1+\epsilon_1)(r+z_1)\ dz}{(r+z)\left[(1+\epsilon)^2(r+z)^2 - (1+\epsilon_1)^2(r+z_1)^2\right]^{1/2}} \qquad (10)$$

Since

$$\frac{\omega}{2} + \frac{\pi}{2} = \theta(\infty)$$

and

$$\frac{\pi}{2} \equiv \int_{z_1}^{\infty} \frac{(r+z_1)\ dz}{(r+z)\left[(r+z)^2 - (r+z_1)^2\right]^{1/2}}$$

it follows that

$$\omega(z_1) = 2\int_{z_1}^{\infty} \left(\frac{r+z_1}{r+z}\right) \left\{ \frac{\left(\dfrac{(2rz - 2rz_1 + z^2 - z_1^2)^{1/2}}{\left[\dfrac{(1+\epsilon)^2}{(1+\epsilon_1)^2}\ (r+z)^2 - (r+z_1)^2\right]^{1/2}} \right) - 1}{(2rz - 2rz_1 + z^2 - z_1^2)^{1/2}} \right\} dz \qquad (11)$$

With the approximations that $\epsilon_1 \ll 1$ and $|z-z_1| \ll r$ in the region where the integrand is large, the above equation becomes

$$\omega(z_1) = \sqrt{\frac{2}{r}} \int_{z_1}^{\infty} \frac{1}{(z-z_1)^{1/2}} \left\{ \left(1 + \frac{\epsilon-\epsilon_1}{z-z_1} r\right)^{-1/2} - 1 \right\} dz \qquad (12)$$

With the further approximation that $\left|\frac{\epsilon-\epsilon_1}{z-z_1}\right| r \ll 1$,

$$\omega(z_1) = \sqrt{\frac{r}{2}} \int_{z_1}^{\infty} \frac{\epsilon_1-\epsilon}{(z-z_1)^{3/2}} \left[1 + \frac{3}{4}\frac{\epsilon_1-\epsilon}{z-z_1} r + \ldots\right] dz \qquad (13)$$

We can now use Eq. (5) from the previous section giving the variation of refractivity with scale height to get

$$\omega(z_1) = \epsilon_1 \sqrt{\frac{r}{2}} \int_{z_1}^{\infty} \left\{ \frac{1 - \exp\left\{-\frac{z-z_1}{h_1}\right\}}{(z-z_1)^{3/2}} + \frac{\Gamma \exp\left\{-\frac{z-z_1}{h_1}\right\}}{(z-z_1)^{1/2}} + \frac{3}{4}\epsilon_1 r \frac{\left(1 - \exp\left\{-\frac{z-z_1}{h_1}\right\}\right)^2}{(z-z_1)^{5/2}} \right\} dz$$

$$\boxed{\omega(z_1) = \epsilon_1 \left(\frac{2\pi r}{h_1}\right)^{1/2} \left[1 + \frac{\Gamma h_1}{2} + (\sqrt{2}-1)\frac{\epsilon_1 r}{h_1}\right]; \quad \epsilon_1 = \frac{\epsilon_s p_1}{h_1 g}} \qquad (14)$$

(The quadratic term in the exponential from Eq. (5) has been neglected, since it does not contribute to the result, to first order in Γ.) The term including Γh_1 is the first-order correction for the variation with height of the temperature. The term involving $\frac{\epsilon_1 r}{h_1}$ is an additional small correction.

3. Differential Refraction

The ratio of incident to received intensity is equal to the ratio of ds' to ds (Fig. 6), or

$$\frac{I_o}{I_1} = \frac{ds'}{ds} = \frac{ds + Dd\omega}{ds} = \frac{ds + D\left|\frac{d\omega(z_1)}{dz}\right| ds}{ds} = 1 + D\left|\frac{d\omega(z_1)}{dz}\right| \qquad (15)$$

Combining Eqs. (5) and (14) from the previous sections gives

$$\omega(z) = \omega_1[1-\frac{3}{2}\Gamma\ (z-z_1)]\ \exp\{-\frac{1}{h_1}[(z-z_1) - \frac{\Gamma_1}{2}(z-z_1)^2]\} \qquad (16)$$

so that

$$\boxed{\frac{I_o}{I_1} = 1 + \frac{\omega_1 D}{h_1}(1 + \frac{3}{2}\Gamma\ h_1)} \qquad (17)$$

4. Light Curve When Scattering and Absorption are Negligible

When the occultation of a star by a planet is observed from Earth, only the effects of refraction need be considered to compute the light curve. In this case it is possible to give an analytic expression for the light curve, to a good approximation. The expression given in this section applies both when the atmosphere is isothermal, and in the more general case when the temperature varies linearly with altitude.

During the occultation, we can approximate that D remains constant, while x increases linearly with time (see Fig. 4). Also (Sec. III.B.2) ℓ is approximately equal to z_1. Thus

$$z_1 - \omega_1 D = -\dot{x}t + C \qquad (18)$$

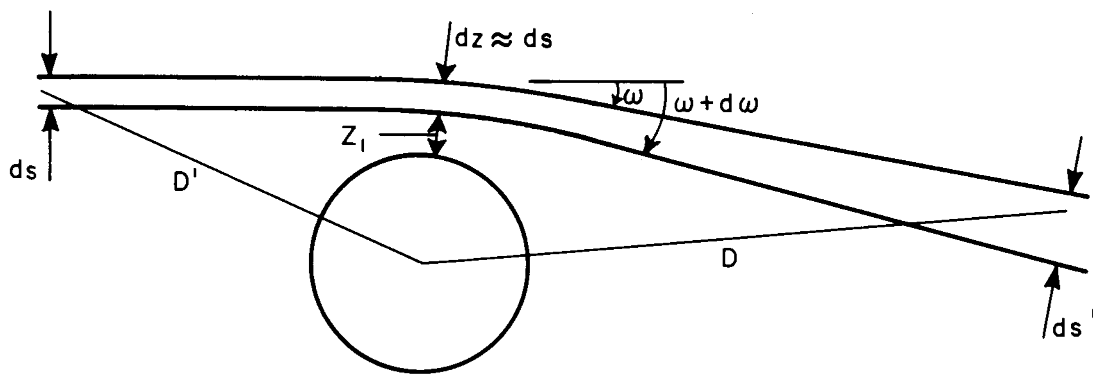


Fig. 6— The effect of differential refraction on the received intensity

In this calculation, the quadratic term in the exponent in (16) again will be neglected. This term only starts to contribute when the occultation has progressed to the point where the received intensity is several orders of magnitude weaker than the initial intensity. In any case, at that point, the analysis may be invalid, since turbulence may have begun to influence the refraction (Baum and Code, 1953). Thus (16) is rewritten as

$$\omega_1 = \omega_r \exp \left\{ - \frac{z_1 - z_r}{h_r} \right\} [1 - \frac{3}{2} \Gamma (z_1 - z_r)] \tag{19}$$

where the subscript r denotes an arbitrary reference level in the atmosphere. Since there is a linear relationship between h_1 and h_r, (17) becomes

$$\frac{I_o}{I_1} = 1 + \frac{\omega_1 D}{h_r} [1 + \frac{3}{2} \Gamma h_r - \Gamma(z_1 - z_r)] \tag{20}$$

Choose such a reference level z_r that $t_r = 0$ and $I_o/I_1 = 2$. In the above three equations, the variables ω and z can be eliminated, to give the relationship between I_o/I_1 and t, as follows.

Evaluating (20) and (18) respectively at z_r gives

$$\omega_r = \frac{h_r}{D} (1 - \frac{3}{2} \Gamma h_r) \tag{21}$$

and

$$C = z_r - h_r(1 - \frac{3}{2} \Gamma h_r) \tag{22}$$

Equation (18) now becomes

$$z_1-z_r - h_r \exp\left\{-\frac{(z_1-z_r)}{h_r}\right\} [1 - \tfrac{3}{2}\Gamma(z_1-z_r) - \tfrac{3}{2}\Gamma h_r] \tag{23}$$

$$= - \dot{x}t - h_r\ (1 - \tfrac{3}{2}\Gamma h_r)$$

Equation (20) becomes

$$\frac{I_o}{I_1} - 1 = \exp\left\{-\frac{(z_1-z_r)}{h_r}\right\} [1 - \tfrac{5}{2}\Gamma\ (z_1-z_r)] \tag{24}$$

from which

$$\ln\left(\frac{I_o}{I_1} - 1\right) = -\frac{(z_1-z_r)}{h_r} - \tfrac{5}{2}\Gamma\ (z_1-z_r) \tag{25}$$

Solving this equation for z_1-z_r/h_r

$$-\frac{(z_1-z_r)}{h_r} = (1 - \tfrac{5}{2}\Gamma h_r)\ \ln\left(\frac{I_o}{I_1} - 1\right) \tag{26}$$

With this result, (23) becomes*

---

*It should be remembered that variation with altitude of mean molecular weight, due to diffusion and photo-dissociation, may be as important as variation of temperature in the upper atmosphere. In that case, not only is it necessary to replace Γ by

$$\Gamma' = \frac{1}{T}\frac{dT}{dz} - \frac{1}{\bar{m}}\frac{d\bar{m}}{dz}$$

for a linear variation with altitude of $\bar{m}$, but also it is necessary to take into account the change with altitude of the refractivity constant ϵ_s (Sec. III.B.1), due to the changing molecular composition. Unfortunately, this has the effect of introducing another parameter,

$$\frac{d\bar{m}}{dz},$$

into the theoretical light curve.

$$\boxed{(1 - \Gamma h_r)\ \ln\left(\frac{I_o}{I_1} - 1\right) + \left(\frac{I_o}{I_1} - 1\right) - \Gamma h_r \left(\frac{I_o}{I_1} - 1\right) \ln\left(\frac{I_o}{I_1} - 1\right) = (1 + \tfrac{3}{2}\Gamma h_r)\ \frac{\dot{x}t}{h_r} + 1} \tag{27}$$

defining, for a given h_r and Γ, a light curve. If the atmosphere is isothermal, (27) reduces to

$$\ln\left(\frac{I_o}{I_1} - 1\right) + \left(\frac{I_o}{I_1} - 1\right) = \frac{\dot{x}t}{h_r} + 1 \tag{28}$$

Choosing the values of h_r and Γ that give the best fit of (27) to the observed light curve, (21) can now be used to find ω_r; if values are assumed for $\bar{m}$ and $\bar{\epsilon}_s$, T_r and p_r can also be determined, from (14). Finally, if the diameter of the planet is known accurately enough, an extrapolation from p_r can yield a value for the pressure at the surface.

5. The "Critical Level"

It has been assumed thus far that $\frac{\epsilon r}{h} \ll 1$; physically this means that the curvature of the rays in the atmosphere is small compared with the curvature of the surface of the planet. However, if the pressure at some altitude is sufficiently great, this condition will not hold. Near the level where $\frac{\epsilon r}{h} = 1$, the two curvatures will be equal; a ray starting in the right direction has a near-circular trajectory, concentric with the planet. The effect has been discussed by Link (1934).

It is impossible, using the occultation technique, to study the atmosphere below this critical level.

C. EFFECTS OF ABSORPTION AND SCATTERING

1. Reduced Path Length

In order to compute the extinction due to absorption and scattering, it is necessary to find the reduced path length, given by

$$X_1 = \frac{p_1}{T_1} \int_{-\infty}^{\infty} \frac{\epsilon}{\epsilon_1}\, ds \equiv 2\,\frac{p_1}{T_1} \int_{z_1}^{\infty} \frac{\epsilon}{\epsilon_1} \left[1 + (r+z)^2 \left(\frac{d\theta}{dz}\right)^2\right]^{1/2} dz \qquad (29)$$

where the integral is to be taken along the path of the ray. If p is measured in atmospheres, T in units of 273°K and ds in centimeters, X will be in units of "cm-NTP." Using Eqs. (6) and (8) for $\frac{d\theta}{dz}$, the above equation becomes

$$X_1 = \frac{2p_1}{T_1} \int_{z_1}^{\infty} \frac{\epsilon}{\epsilon_1} \left[1 - \frac{(1+\epsilon_1)^2\,(r+z_1)^2}{(1+\epsilon)^2\,(r+z)^2}\right]^{-1/2} dz \qquad (30)$$

With the approximation that $\epsilon_1 \ll 1$

$$X_1 = \frac{2p_1}{T_1} \int_{z_1}^{\infty} \frac{\epsilon}{\epsilon_1} \left[1 - (1 + 2\epsilon_1 - 2\epsilon)\,\frac{(r+z_1)^2}{(r+z)^2}\right]^{-1/2} dz \qquad (31)$$

With the further approximation that $|z-z_1| \ll R$ over the region where the integrand is large,

$$X_1 = \frac{2p_1}{T_1} \left(\frac{r}{2}\right)^{1/2} \int_{z_1}^{\infty} \frac{\epsilon}{\epsilon_1} \left[z-z_1 - r\,(\epsilon_1-\epsilon)\right]^{-1/2} dz \qquad (32)$$

Introducing the expression in (5) for ϵ (with the quadratic term in the exponential neglected), and assuming $\frac{\epsilon_1 r}{h_1} \ll 1$,

$$X_1 = \frac{p_1}{T_1} (2r)^{1/2} \left(1 + \frac{\varepsilon_1 r}{2h_1}\right) \int_{z_1}^{\infty} \frac{\exp\left\{-\frac{z-z_1}{h}\right\}}{(z-z_1)^{1/2}} \left[1 - \Gamma (z-z_1)\right] dz$$

$$\boxed{X_1 = \frac{p_1}{T_1} (2\pi r h_1)^{1/2} \left(1 + \frac{\varepsilon_1 r}{2h_1} - \frac{\Gamma h_1}{2}\right)} \qquad (33)$$

The term $\frac{\varepsilon_1 r}{2h_1}$ is the first-order correction for the curvature of the path, and the term $\frac{\Gamma h_1}{2}$ is the first-order correction for the variation of temperature with height.

2. Rayleigh Scattering

Since multiple scattering has been neglected (approximation 9), each kind of scattering and absorption contributing to the intensity reduction of the ray can be treated separately.

The optical thickness due to Rayleigh scattering is determined by the reduced pathlength (33) and by the coefficient of Rayleigh scattering at standard temperature and pressure. The latter is given by

$$K_R = \frac{32\pi^3}{3n_o\lambda^4} \sum_i \nu_i (n_i - 1)^2 f_i \qquad (34)$$

(van de Hulst, 1956). Here n_o is Loschmidt's number, 2.70×10^{19} cm^{-3}; λ is the wavelength, and v_i, n_i, and f_i are the fraction by volume, index of refraction at standard temperature and pressure, and coefficient of depolarization respectively, for the i^{th} gas in the atmosphere.

3. Molecular Absorption

If the intensity of light in an eclipse of a satellite, or in an occultation of a star observed from a fly-by or orbiting probe, were observed at wavelengths absorbed by an atmosphere's components, the variation with altitude of those components could be determined. Already the vertical distribution of ozone in the earth's atmosphere is being studied by observing eclipses of an artificial satellite illuminated by the Sun (Venkateswaran, Moore, and Krueger, 1961).

However, if such detailed knowledge of chemical composition of the atmosphere is not required, observations can be made at wavelengths in the visible region, where molecular absorption is expected to be negligible compared to Rayleigh scattering.

4. Absorption by Haze, Clouds and Dust

No detailed discussion of possible kinds of absorption by haze, clouds and dust will be given, although it is clear that such effects are important. For example, observations of "dust storms" on Mars indicate that the absorption for a grazing ray may be great indeed, while the clouds that are thought to exist on Venus should also have great effects. Once light curves have been obtained, one could compare them with curves computed for pure molecular atmospheres and account for the departure observed by means of absorbing layers in the upper atmosphere, cloud banks, wind-blown surface dust, or some other appropriate mechanism. Quantitative information about the optical properties of these features could then be obtained.

IV. APPLICATION TO BALLOON ASTRONOMY

As has been suggested by de Vaucouleurs (1960), it will be possible to observe occultations with reasonable regularity once small telescopes with suitable guidance can be carried aloft by balloons. Since a telescope thus situated is not limited by atmospheric seeing, a focal plane diaphragm much smaller than is now possible with even the best telescopes could be used. For example, for a planet with brightness of the first magnitude and an apparent radius of 5 sec and a telescope of 20-in. diameter permitting a diaphragm of 0.2-sec radius, it will be possible to observe photometrically the occultation of any star brighter than approximately eighth magnitude.

Occultation measurements could thus be repeated with greater accuracy for Venus and Jupiter, and also extended to other planets. In combination with determinations of atmospheric compositions and of precise visual diameter yielded by other balloon-telescope observations, the occultation measurements could be used not only to determine the scale height at some given altitude in the upper atmosphere of the planet, but also to determine the absolute pressure at that altitude (see Sec. III.B.4).

Finally, if a sufficiently bright star is occulted, it would be possible to follow it even when its intensity has been greatly reduced, the intensity decreasing eventually by a factor of *e* (approximately one magnitude) for each additional scale height penetrated. Thus somewhat lower regions of the atmosphere could be probed.

V. OBSERVATIONS OF ECLIPSES OF PHOBOS

Observations of the eclipses of natural satellites provide a way to obtain from Earth occultation data like that obtainable by an observer near the planet. The finite size of the Sun, however, and in some cases of the satellite, smears the light curve in time and greatly reduces the sensitivity of the method. Also, unfortunately, it is difficult to make rapid photometric observations of a faint satellite located very close to a bright planet.

The most desirable satellites to observe are those of Mars, because they are very close to their primary. Although observations of eclipses of satellites of other planets, particularly Jupiter, may give useful information and may also be much more feasible observationally, they will not be considered further here.

The appropriate time for the observation of eclipses of a satellite is during the period near quadrature, when the angle Earth-planet-Sun has its largest value. The orbital planes of Phobos and Deimos around Mars are tilted with respect to the plane of the orbit of Mars around the Sun, so the satellites will have eclipses only for certain heliocentric longitudes of Mars. However Phobos is so near to Mars that eclipses are frequent. Deimos is further from Mars; its eclipses are both less frequent and, due to the enhanced smearing effect of the finite size of the sun, less interesting. An approximate ephemeris is given graphically in Fig. 7 for the quadrature of 1962, with the data needed to determine the feasibility of making the observations.

Date (1962)	Magnitude			Angular separation, "D"
	Mars	Phobos	Deimos	
Sept 3	1.2	14.6	16.1	3.4
Sept 23	1.1	14.5	16.0	4.0
Oct 13	1.0	14.4	15.9	4.6
Nov 2	0.7	14.1	15.6	5.2
Nov 22	0.4	13.8	15.3	5.9
Dec 12	0.1	13.5	15.0	6.3
Dec 32	-0.4	13.0	14.5	5.6

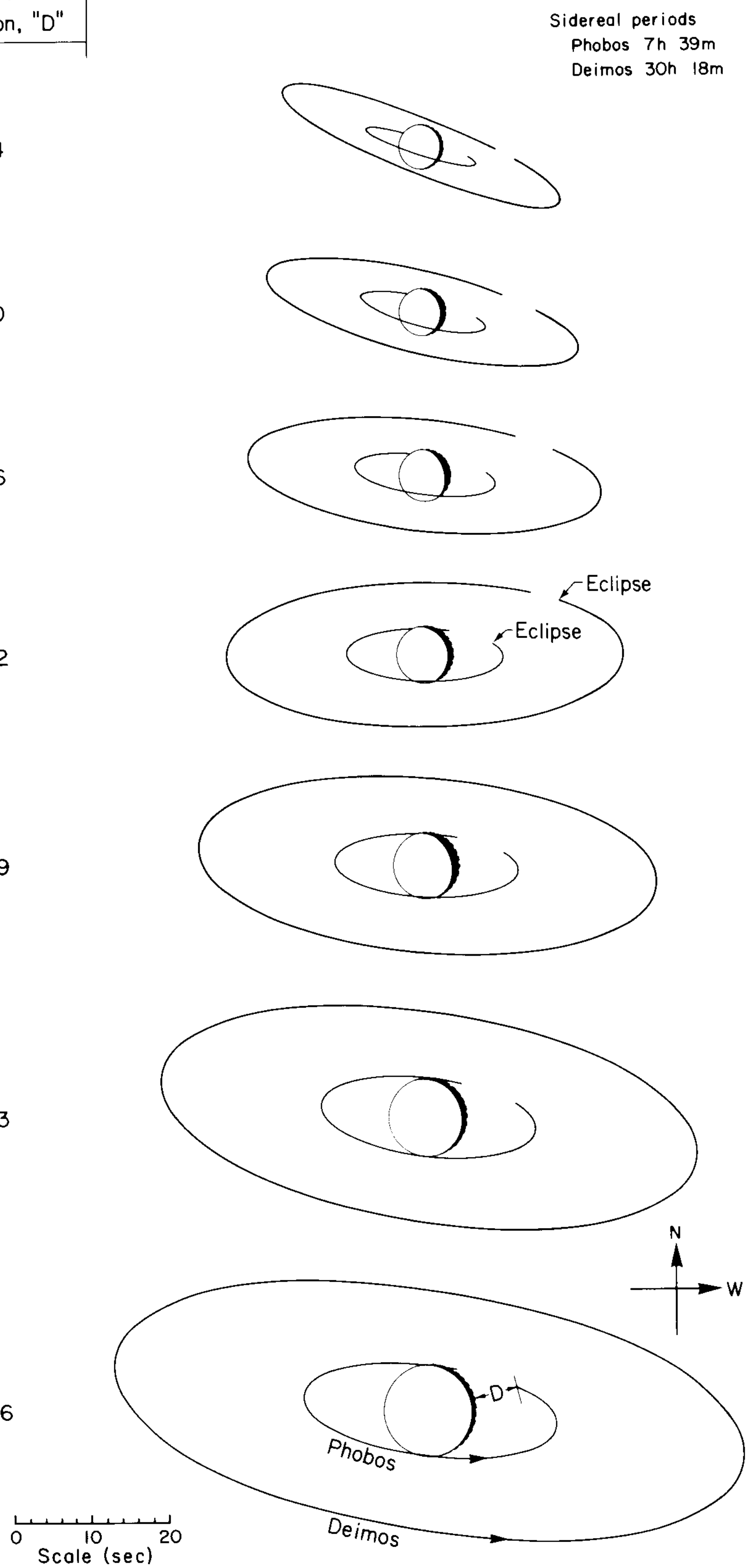


Fig. 7—Approximate ephemeris for quadrature of 1962, Mars, Phobos, and Deimos

A. INSTRUMENTAL PROBLEMS

Phobos is extremely faint. However, in observing its eclipses, the major problem is not sensitivity alone but rather that of discriminating against unwanted light from Mars. The brightness difference between Phobos and Mars is 13.6 magnitudes, or 3.6×10^{-6}, while at the time of eclipse, Phobos is at most 6.3 seconds of arc from the edge of the disk of Mars.

Unwanted light from Mars is received because of diffraction by the telescope. Scattered light is negligible provided the telescope is designed to minimize scattering and is operating under optimum conditions (clear sky and telescope optics free from dust and scratches). The diffraction can be estimated from the normalized expression for Fraunhofer diffraction from a circular aperture,

$$I = \frac{1}{\pi\varphi^2} [J_1(\frac{\pi d\varphi}{\lambda})]^2 \tag{35}$$

where d is the diameter of the telescope, φ the angle from the maximum of the diffraction pattern, and λ the wavelength. For large values of φ the average of this function approaches

$$\frac{\lambda}{\pi^3 d\varphi^3} = \frac{.13}{d(\text{inches})[\varphi(\text{seconds})]^3} \ (\text{sec. of arc})^{-2} \tag{36}$$

for a wavelength of 0.5μ. This expression should be integrated over the disk of Mars; however, if the satellite is sufficiently far from the edge, Mars can be treated as a point source, giving, for $\varphi = 11$ seconds and $d = 40$ inches, $I = 2.4 \times 10^{-6}$ per square second. Thus, the intensity

received from Phobos before occultation is equal to that received from approximately 1.5 sq. sec. of background.

Visual observations could give some useful information. A modification of conventional photometric techniques could be used in which a comparison star of adjustable brightness is projected on the visual field near the satellite being eclipsed. In this way, compensation is made for effects of seeing and background brightness. Unfortunately, Phobos is so faint that, even with a large telescope, the ability of the eye to perceive fractional changes in its intensity is poor. A further drawback is that observations will be limited to the narrow wavelength range of maximum visual sensitivity; as will be seen in Sec. V. C, one actually wants light curves from as many spectral regions as possible.

The observations could be made by means of cinematography, taking perhaps one frame each 5 seconds. An emulsion far faster than that conventionally used for photometry is required, and since the image of Phobos would be formed of so few blackened grains, the photometric accuracy would be poor.

The most promising technique to use is probably photoelectric photometry. Servo-controlled fast guiding should be used, tracking either Mars or the satellite itself. In this way, a small focal plane diaphragm could be used to reduce the "noise" due to diffracted light from Mars; at the same time, the fast guiding would reduce the fluctuations in this noise due to seeing fluctuations.

B. LIMB DARKENING AND FINITE SIZE OF SUN

The half angle β subtended by the Sun when seen from Phobos is approximately 0.003 radians. On the other hand, at the time of eclipse the distance D from Phobos to the atmosphere of Mars (Fig. 4) is 8700 kilometers, so a characteristic distance of 18 km, or approximately one scale height, in the atmosphere of Mars subtends an angle of 0.002. Therefore, the light which reaches Phobos is a sum of contributions from various parts of the disk of the Sun, each passing through a different layer of the Martian atmosphere. The effect is a spreading of the light curve and a smoothing of the discontinuities due to fine structure in the atmosphere.

To take this effect into account, we may consider the distance D as fixed for all light received from the Sun, but the distance x as variable, depending on the part of the Sun from which the light under consideration issues. Thus, if $f(D,x')$ is the intensity for a point source at the center of the sun, the integrated intensity taking into account the finite size of the Sun is of the form

$$F(D,x') = \int_{-1}^{1} f(D,\ x' + \beta\, Dy)\ \ell(y)\ dy \qquad (37)$$

where $\ell(y)$ takes into account both the circular shape of the disk and its limb darkening. (Neglecting limb darkening, $\ell(y)$ is simply $\sqrt{1 - y^2}$ times a normalizing constant.)

Let $L(\cos\gamma)$ be the factor giving the solar limb darkening, where γ is the angle between the line of sight and the normal to the surface of the Sun. Then the normalized value of $\ell(y)$ is

$$\ell(y) = \frac{\int_{-\sqrt{1 - y^2}}^{\sqrt{1 - y^2}} L(\sqrt{1 - y^2 - x^2})\, dx}{\int_{-1}^{1} \int_{-\sqrt{1 - y^2}}^{\sqrt{1 - y^2}} L(\sqrt{1 - y^2 - x^2})\, dx\, dy} \qquad (38)$$

A simple approximation to the solar limb darkening is

$$L(\cos \gamma) = B + (1 - B) \cos \gamma \qquad (39)$$

Then $\ell(y)$ becomes

$$\ell(y) = \frac{3}{\pi(B+2)} \left\{ 2B \sqrt{1 - y^2} + (1-B) \frac{\pi}{2} (1 - y^2) \right\} \qquad (40)$$

If the satellite is large, a similar integration should be done over its disk; its law of limb darkening would have to be known, or guessed. However the satellites of Mars are sufficiently small (~ 16 km in diameter for Phobos) that this need not be done. Also the eclipse is sufficiently brief that the tumbling of the satellite on its axis, and resulting variation in its reflectivity, can be neglected.

C. MODEL CALCULATION

To get some idea of the expected effect of the Martian atmosphere, consider a dust-free Martian atmosphere with the following parameters, corresponding roughly to reasonable guesses based on present data:

$T = 230^{\circ}K$ = constant

$P_{surface} = 94$ mb

composition 96 per cent N_2, 2 per cent A, 2 per cent CO_2 by volume.

With this model, the only effects that need be considered are differential refraction, direct Rayleigh scattering, and finite size and limb darkening of the Sun. Clearly the neglect of dust and haze makes the model unrealistic; however, there are not at present data enough to make possible a more realistic model. With the above model, the light curve is determined by

$$F(x') = \int_{-1}^{1} \left\{ \begin{array}{ll} \dfrac{\exp[-K_r X_1 \exp(-z/h)]}{1 + (\omega_1 D/h)\exp(-z/h)} \quad \ell(y) \ , & z > 0 \\ \\ 0, & z \leq 0 \end{array} \right\} dy \qquad (41)$$

and

$$x' + \beta D y = - z + \omega_1 D e^{-z/h}$$

with ω_1 given by Eq. (14), k_r by Eq. (34), X_1 by Eq. (33), and $\ell(y)$ by Eq. (40).

With the additional constants D = 8700 km, β = 0.00305, B = 0.4, g = 3.7 m $\sec^{-2}$ and r = 3400 km, we have

$$F(x') = \int_{-1}^{1} \left\{ \begin{array}{ll} \dfrac{\exp[(-0.074/\lambda^4)\exp(-z/18)]}{1 + 0.505 \exp(-z/18)} \left[\frac{1}{\pi} \sqrt{1-y^2} + \frac{3}{8}(1-y^2)\right], & z > 0 \\ \\ 0, & z \leq 0 \end{array} \right\} dy$$

$$(42)$$

and $x' + 26.5\, y = - z + 9.6e^{-z/18}$, with λ in microns and x' in kilometers. Light curves obtained from these equations for several wavelengths are given in Fig. 8. For comparison, a curve is given for the case of no atmosphere, where the only effect is the finite size and limb darkening of the Sun.

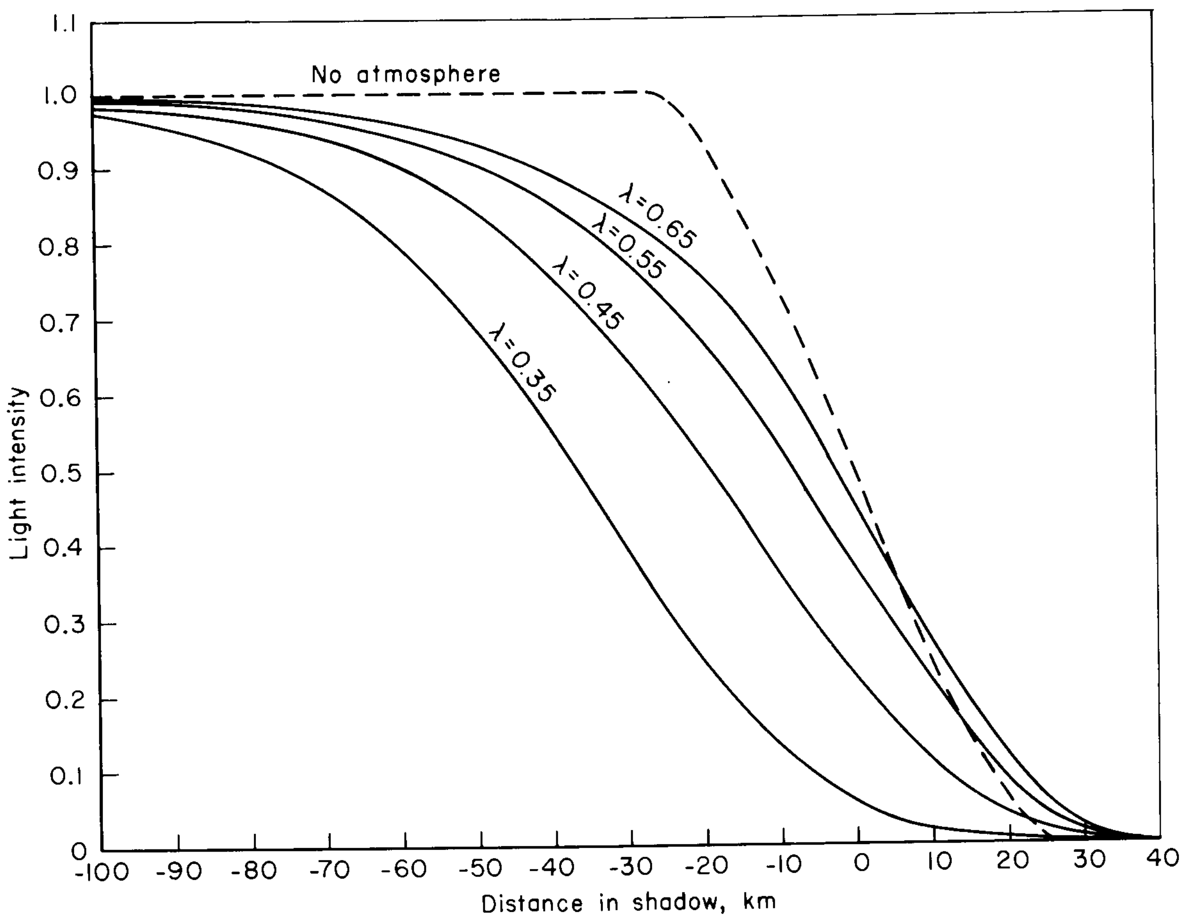


Fig. 8—Computed curves of the light from Phobos entering an eclipse

The projected velocity of Phobos is 2.0 km/sec, so each 10 km division corresponds to a time of 5 seconds.

VI. OCCULTATION PHOTOMETRY FROM A FLY-BY SPACECRAFT OR ORBITER

A. UTILITY OF DATA OBTAINED

Balloon-mounted and Earth-satellite—mounted telescopes, and optical instruments aboard fly-by and orbiting space probes will permit us to answer many questions about planet surfaces and atmospheres. Many of the data will be obtained by looking directly at a part of the planet's disk; the light seen will be a composite of light reflected from the surface and from atmospheric gases, clouds, and dust. For most planets, the effect of multiple scattering will be important.

The intensity and polarization of light reflected by a planet with a pure Rayleigh atmosphere and a diffusely reflecting surface can be determined numerically by means of a large computer. The same characteristics for an atmospheric model with dust or haze undoubtedly also can be determined, although the problem is much more complicated. However, it is not clear that the inverse problem, that of inferring the structure of the atmosphere from measurements of the reflected light, can be solved uniquely without other independent information. For example, it would be desirable to know the reflective properties of the surface taken alone or the distribution with altitude of the clouds and haze.

However, an occultation observation gives data that (1) refer to various specific altitudes in the atmosphere rather than to the whole atmosphere and surface combined, and (2) can be interpreted without knowledge of the surface properties of the planet.

Thus a program of planetary photometry should include, if possible, some occultation measurements observed from a point near enough to the planet that low levels in the atmosphere are probed. With the partial exception in a few special cases of observations of the eclipses of a planetary satellite, as discussed in Section V, such an observation is possible only by means of a space probe.

B. SUITABLE EXPERIMENTS

1. Using the Sun

If the trajectory of the space probe can be so controlled that the Sun is occulted by the planet, then a very powerful way to study the atmosphere of the planet becomes available. A simple radial scan of intensity at various wavelengths can be analyzed with a theory similar to that discussed in Section III. If a TV pickup with various color filters were available, the structure of the atmosphere, including clouds, could be studied in great detail indeed. Such an experiment might also be a good one for a survey of the atmosphere from a planetary orbiter. However, a complete picture of detailed atmospheric structure would not be obtained, since all data would refer only to atmospheric conditions at local sunrise and sunset.

2. Using a Star

From a space probe near a planet, occultations of stars bright enough to yield useful information will be frequent. It is not necessary to wait for a prearranged occultation. Instead, a given area of sky off the limb of the planet, such as the dotted area in Fig. 3, can be searched until a sufficiently bright star enters it, and that

star can then be tracked as it is occulted. The frequency with which suitable stars occur in this area is determined by its angular extent, the distribution of stars of various brightnesses in the sky, and the minimum stellar brightness which will be acceptable for the measurement. The latter, in turn, is determined by the precision to which measurements are desired, the orbital parameters of the probe, and the sensitivity of the photometer.

The area of sky to be searched might be of the order of 100 square degrees. For example, this would ensure that, for a low altitude orbiter designed for synoptic measurements, a measurement could be made every few degrees along the orbit; for a "fly-by" probe passing within a radius or so of the planet it would ensure that several useful occultations would take place during the period of close approach.

The average number of stars at 0° and 90° galactic latitude in 100 sq. deg. of sky is given in the table below.

Table [redacted]

[redacted]	Magnitude			
Latitude	4	6	8	10
0°	1.5	13	100	770
90°	.5	3.7	28	180

Thus the experiment must be designed for stars as faint as approximately fifth magnitude.

The photometer sensitivity needed is determined ultimately [redacted] by the requirement that a statistically significant number of photons be received during a time short enough that the change in received intensity

is negligible. The equivalant number of 0.5μ photons received from a star of magnitude m by a photometer in space of diameter d in. is

$$\frac{dN}{dt} = 1.4 \times 10^{8} \times 10^{-0.4m} d^{2} \tag{43}$$

Only a fraction of these are useful for a given measurement since it is desirable to observe in a narrow spectral range. Still, it is clear that, with a photometer as small as 1 in. in diameter, useful information can be obtained.

Because of scattered sunlight, the occultation technique is severely limited in its ability to assist in the study of the atmosphere on the day side of the planet. In fact, the brightness of the lower part of the sunlit atmosphere, seen edge on, will be comparable to that of the planetary disc. Thus the occultation technique will, in most experiments, be useful only for study of the nighttime atmosphere.

A suitable experiment would have to be performed in the following sequence:

(1) Locate the region to be searched (Fig. 3);

(2) Find, in the region, a star of a given magnitude;

(3) Lock on the star, stop down the solid angle; and track the star as it is occulted, telemetering back to Earth the intensity of light versus time at various wavelengths.

With proper design the instrument for this experiment could be small and reliable; much of it could be utilized in other photometric experiments.

For a single occultation, suitable data would be intensities at perhaps four wavelengths and ten times, starting when the star first

begins to fade. If each point has an accuracy of one part in 128, then a transmission capacity of 320 bits per occultation of information is required.

The occultation technique is particularly suited for the following experiments from space vehicles:

(1) synoptic studies from an earth satellite of

(a) nocturnal ozone distribution in the Earth's atmosphere

(b) height of cloud tops

(2) preliminary study of gross properties of a relatively unknown planetary atmosphere from an advanced (by 1962 standards) fly-by probe

(3) synoptic studies from an orbiter of a remote planetary atmosphere whose gross properties are already known, possibly including pressure and temperature variations.

BIBLIOGRAPHY

Baum, W. A., and A. D. Code, "A Photometric Observation of the Occultation of Sigma Arietis by Jupiter," Astron. J., Vol. 58, No. 1208, May, 1953, pp. 108-112.

Fabry, C., "Le Rôle des Atmosphères dans les Occultations par les Planètes," J. Observateurs, Vol. 12, No. 1, January 15, 1929, pp. 1-10.

Link, M. F., "Théorie Photométrique des Éclipses de Lune," Bull. Astron., Vol. 8, 1932, pp. 77-108.

----, "Occultations et Éclipses par les Planètes," Bull. Astron., Mémoires et Variétés, 1934, pp. 227-249.

Meeus, J., "Occultations of Bright Stars by Planets," J. Brit. Astron. Assoc., Vol. 70, No. 4, 1960, pp. 174-183.

Menzel, D. H., and G. de Vaucouleurs, "Results from the Occultation of Regulus by Venus, July 7, 1959 (abs.)," Astron. J., Vol. 65, No. 6, August, 1960, p. 351.

Mikhailov, A. A., Teoria Zatmenii (Theory of Eclipses), Moscow, 1954.

Pannekoek, A., "Über die Erscheinungen, welche bei einer Sternbedeckung durch einen Planeten auftreten," Astron. Nachrichten, Vol. 164, No. 3913, 1903, pp. 5-10.

Pettit, E., and R. S. Richardson, "Motion Pictures of the Occultation of Sigma Arietis by Jupiter on November 20, 1952," Publ. Astron. Soc. Pacific, Vol. 65, April, 1953, pp. 91-92.

van de Hulst, H. C., "Scattering in the Atmospheres of the Earth and Planets," in Kuiper, G. P. (ed.), The Atmospheres of the Earth and Planets (rev. ed.), University of Chicago Press, Chicago, 1956.

de Vaucouleurs, G., "A Survey of Physical Problems of the Nearer Planets and a Review of Observational Techniques Applicable to Balloon-Borne Telescope Systems," Chap. 2 in Strong, J. D., G. de Vaucouleurs, and P. Zwicky, Planetary Astronomy from Satellite-Substitute Vehicles, Missile Development Center, Holloman Air Force Base, New Mexico, Report TR-60-6, March, 1960.

Venkateswaran, S. V., J. G. Moore, and A. J. Krueger, "Determination of the Vertical Distribution of Ozone by Satellite Photometry," J. Geophys. Res., Vol. 66, No. 6, June, 1961, pp. 1751-1771.