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AN EXPERIMENTAL INVESTIGATION OF PRIORITY ASSIGNMENT IN A JOB SHOP

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PREFACE

Priorities and priority systems are a basic part of the Air Force environment. At base level, the maintenance organization often must decide which aircraft to fix next; at depot level, the repair activity must decide which parts to repair next. Whenever resources are inadequate to meet all demands immediately, a priority system, either implicit or explicit, is required. Priority rules are, in many situations, an important aspect of efficient management.

This Memorandum appraises a large number of priority rules in the context of a queueing network. Priority rules seldom receive much analysis, yet priority can make a substantial difference in performance. Switching from the worst to the best priority rule can reduce shop processing time (total repair cycle time) by a factor of two or more under some conditions, which implies that the payoff from devoting more attention to priority rules is large.

This Memorandum will be of interest to those individuals in the Air Force responsible for the management or analysis of maintenance activities. It should have relevance to base-level problems of operating commands and to the depot activities of AFLC.

SUMMARY

This Memorandum covers a study of priority disciplines in a queueing network. Such a network is a model for a simplified job-shop process, which, in turn, is the basic model for a large proportion of multi-product manufacturing activity or multi-skill maintenance activity. In fact, a queueing network model is relevant for any situation having a set of different types of facilities, and a sequence of tasks that consecutively require the services of one or more of these facilities.

The investigation employs Monte Carlo simulation on a digital computer to compare and evaluate a large number of priority disciplines. It represents a consolidation and continuation of a number of previous similar studies. It considers a far greater number and variety of disciplines and much longer and more carefully controlled tests than its predecessors. The measures of performance considered include both measures of inventory -- number of jobs in the system and work-content of these jobs -- and measures of individual job progress -- time in the shop, and lateness against an assigned due-date.

The study principally concerns a discipline in which, at each point of decision, the job with the shortest processing time from among those available for assignment is chosen. Such a procedure is known to have optimal properties for certain simple, small-dimensional systems. It is shown in this study to be of dominating importance in networks as well. Regardless of the measure of performance, the simple version of this discipline is among the best procedures of the many tested. Moreover many variations of the basic discipline, and

combinations with other disciplines are exhibited, demonstrating the possibility of designing an appropriate discipline for any stated measure of performance. The magnitude of the difference in performance between this "shortest-operation" discipline and the nominal standards of comparison -- such as "first-come-first-served" or "random" selection -- depends upon the particular measure of performance and such conditions as the level of utilization in the system, but factors of two or more are frequently displayed. The performance improvements that can be obtained with shortest-operation and related disciplines appear to be of important practical value, and there would not appear to be prohibitive implementation problems.

While it is unlikely that the shortest-operation discipline is an optimum procedure for any reasonable measure of performance, the rules which appear to be marginally better in this test are tremendously more complicated and would require rather elaborate communication and computing facilities for implementation. The shortest-operation selection procedure does, of course, require some a priori information about processing times, but it does not appear to be particularly sensitive to errors in estimating these times, so that this requirement is not likely to be restrictive.

On one hand, the investigation could be considered theoretical, exploring for a network configuration of queues the validity of certain results that have been analytically established for a single queue model. At the same time it could be considered a practical comparative study of many of the procedures actually employed in manufacturing and elsewhere to sequence tasks over a number of separate facilities. Some of the results appear capable of immediate

application. Perhaps more importantly, the work should suggest a number of modifications and combinations of procedures appropriate for particular circumstances. It is the writer's intent to report the work in sufficient detail to provide a point-of-departure for further work, in order to spare future investigators the necessity of retracing some of these obvious steps.

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I. INTRODUCTION

The "job shop scheduling problem" has received considerable attention in recent years. The name generally invokes an image of a factory composed of general-purpose machine tools, doing diverse processing operations on lots or orders of metal parts which are moved in "tote boxes" from one machine to another. In the purest form of job shop each job is unique -- a specified quantity made to special customer order which will never be repeated. There is perhaps also a feeling that this type of manufacturing is an anachronism -- the obsolescent remnant of another generation's technology. Certainly from a practical point of view, a major effort toward obtaining a solution would not be warranted if the problem itself were to disappear in the trend toward line-organized production and automated processing equipment. But this is hardly the case. Although the trend certainly exists and may accelerate, the classical job shop, with its inherent flexibility, will remain a significant factor in the manufacturing industries. Moreover, the new technology still represents a form of job shop, on a larger scale and with slightly different parameters, for which a satisfactory scheduling procedure is all the more important.

Actually the job-shop process is a phenomenon which is more general, more common and more important than the general purpose machine shop. The term "job shop" has become a convenient identifier of a general class of problems, rather than an exhaustive description of the area of application. The process involves a set of distinct, diverse processing facilities and a set of tasks, each requiring work

to be done in sequence, at one or more of the processing facilities. In general, the order in which a task passes from one facility to another is not the same for all tasks in the system. The objective of scheduling is to array the tasks over the facilities in time, subject to occupancy and precedence constraints, so as to optimize some temporal characteristic of the tasks and/or the facilities. Clearly, the tasks may be job-lot quantities of piece-parts, with sequences of machining operations to be performed. They could, however, just as well represent messages to be transmitted by a communications network, vehicles to be passed by a highway system, research and development projects under contract to a functionally-organized engineering firm, or post-flight maintenance work at an Air Force base.

The underlying analytical model for the job-shop process is a queueing system having two or more non-identical servers, with each customer required to visit a specified subset of the servers, in specified sequence, before being discharged from the system. The usual queueing measures are of interest -- distribution of state probabilities, distribution of waiting times, and server utilization. In a practical situation one is concerned with optimizing or at least improving the process, rather than just describing its performance. In a systems design problem one is concerned with specifying the best type, number and configuration of facilities to service an imposed load. In an operating or scheduling problem one is given a system configuration and is concerned with specifying disciplines or operating procedures that will optimize its performance.

The following terminology describes the job-shop process. The shop consists of a set of machines, divided into mutually exclusive, exhaustive subsets called machine groups. The several members of a particular machine group are considered indistinguishable with regard to their ability to perform the work expected of the shop. The work the shop is to perform consists of a sequence of jobs. Associated with each job is a routing which is an ordered list of operations. Each operation is a simple task to be performed by a single machine. The operation is described by the specification of the machine group from which the machine to do the work must be drawn, and by the length of time required to do the work. This latter may, of course, be a random variable whose actual value is not known in advance of execution. The time to do the work consists of a set-up time -- time required to prepare the machine to do the work -- and processing time -- time to do the actual work. The number of identical pieces constituting the job is known as the lot-size; the processing time for each operation of a job can be considered the total time to perform the work task upon each piece of the job.

A job may have other attributes of interest. Two important ones are a job's index of value or relative importance, and a due-date that indicates when job completion (by completion of processing of the last operation) is desired. If a job has an attribute of due-date, then it also acquires an attribute called lateness during its journey through the shop, depending upon the comparison of the actual and desired completion times.

A simple job shop might be considered to have the following properties:

A. Single Limiting Resource -- there is a single limiting resource called "machine," for which the jobs compete and wait. There is no circumstance (such as the lack of an operator) that will prevent work being done when a suitable machine becomes available. (Note that it is the single resource, rather than the name that is important. In many applications, manpower rather than equipment is limiting, but this is still a single resource limitation.)

B. Independence of Machines -- the "cooperation," or synchronization of two or more machines is never required to perform an operation.

C. Simple Source of Jobs -- all jobs arriving at the shop are "similar" at least in the sense of having come from a common generating mechanism.

D. Strictness of Routing -- operations on each job must be performed in the exact order indicated and by a machine of the type specified.

E. Occupancy Constraint -- each machine can work on only one operation of one job at any particular point in time. A job may be in process on only one operation, on one machine, at any particular point in time.

F. Sequence Independence of Set-up -- the time to prepare a machine to perform a particular operation depends only upon that operation and not upon the nature of the operation that preceded upon this machine.

G. Reliability of Equipment -- the machines of the shop are not subject to random failures that make them unavailable for work. Neither do they perform improper work that causes pieces to be discarded, and hence lot-size of a job to decrease from one operation to the next.

H. Constant Lot Size over all Operations of a Job -- lots cannot be split or combined for any purpose. The jobs are distinct and indivisible. This assumption, along with F and G, allows one to ignore the number of pieces per lot and the distinction between set-up and processing times, and consider the total time for the operation as a single indivisible quantity.

I. Continuous Stationary Operation in Time -- the system operates continuously and time is not divided into shifts, days or weeks. The same array of machines is continuously available. External circumstances (including job arrival pattern) are stationary.

It is highly doubtful that there are any interesting real systems which are simple job shops in this strict sense. Real "shops" are much more complex phenomena, and lack one or more of these assumed properties. For example, real shops have machines that sometimes break down and sometimes spoil work. Also, there is usually some latitude in the order in which operations must be performed. Nevertheless, it would seem that this simplified model is the common core of the model for a very large number of more complex systems. This generality and relative simplicity suggest that this is a subject for basic investigation. If the properties of such a simplified shop were well understood, and good design criteria and good operating procedures known, this would be a tremendously useful starting point for

investigators who have to concern themselves with a more complex model, reflecting the idiosyncracies of a particular application.

Unfortunately, even this basic core problem has not yielded to analytical investigation, and there is no indication that significant progress is imminent. Compared to the systems that have been successfully studied and for which optimal procedures are known, even the simple job shop is an extremely complex process. As described here, it is clearly a queueing process involving a network configuration of servers. What is described in the literature as the "m by j job shop problem" is a finite and deterministic approximation to this process. This further abstraction would be instructive, although not particularly realistic, if the stronger assumptions would allow more powerful results to be obtained. This has not been the case, and the only positive results have been obtained for "shops" of one, two and three machines.[1-4] These results are not particularly instructive for larger shops, as they have been obtained precisely because the one- and two-machine system does not possess the complexity and scheduling opportunities of larger systems.

The scheduling of the customers of a single-channel queue is reasonably well understood.[5-13] Comparably strong results for a network configuration of queues would be a major step toward a solution to the job-shop scheduling problem, although priority classification systems would not represent as complete an answer to the network problem as they do to the single-channel queue, since a much richer set of procedures are available in the network. Jackson has given sufficient conditions for the several queues of a network to be

treated independently [14] and has also given expressions for the state probability distribution of certain network systems.[15] Unfortunately, Jackson's results do not hold for the type of priority classifications known to be interesting in the single-channel case.

Faced with formidable obstacles in both the deterministic and queueing approaches to the problem, a number of investigators have resorted to experimental investigation by means of computer simulation. Hopefully this could provide good, although not optimal, procedures with immediate utility, and potentially could provide insight which could guide future analytical investigation. Quite a number of such experimental investigations have been performed and a few have been reported in the literature.[16-20] Although the individual sponsoring organizations may have received significant benefit from the results of these studies, the general knowledge obtained has been less impressive than one might have expected.

Part of this lack of accomplishment has certainly been due to the considerable difficulty of producing a computer program to represent a reasonably complex job shop, and to the equal difficulty of modifying this program to explore alternative structures and disciplines. These studies have tested different types of procedures under different sets of conditions so that the results are hardly comparable. Moreover, the sample sizes or length of runs in most cases were so small, considering the variability of the process, that the work can only be considered exploratory.

This Memorandum describes a continuation and expansion of this line of investigation. It can be viewed in either of two lights:

1) An attempt to compare, under common conditions, most of the procedures tested by other experimental investigators, as well as others suggested by theoretical considerations and examination of early results of the study; this to be done with sample sizes large enough and under sufficient control that formal statistical statements could be made.

2) An attempt to establish or disprove experimentally, for the network case, results comparable to those obtained analytically for priority classifications in single-channel cases.

The writer had the incomparable advantage of being able to use the SIMSCRIPT language [21] for the required programming. Although this was the first actual use of SIMSCRIPT, which was still in development at the time and presented some difficulties, the work could not have been done without it. Generous time on the RAND IBM 7090 Computer was also made available, so that the writer's ingenuity was the limiting factor in this study.

II. DESCRIPTION OF THE EXPERIMENT

DESCRIPTION OF THE SYSTEM STUDIED

The system was simply a network of queues -- a set of identical, single-server queues ($M/M/1/\infty$) with exponential arrival times and exponential service times and with customers randomly routed from one server to another. This could be considered a completely symmetrical and completely balanced simple job shop (as defined earlier), with one machine in each group. The processing time distribution for each machine was identically the same: an exponential distribution with mean of one. Each processing time was expressed as a three-digit number of the form (X.XX). Each of the exponential random variables was rounded to the closest number of this form, except that variables that rounded to 0.00 were omitted and variables greater than 9.99 were taken equal to 9.99. The routing of the operations was strictly random. A job was equally likely to find its first operation on any of the m machines in the shop. The transition matrix assigned a probability of $1/m$ to movement to each of the other $m-1$ machines in the shop and to the possibility of leaving the shop. (A job was allowed to return to a machine but not to have two consecutive operations on the same machine.) It is easily shown that with this generation process, the distribution of the number of operations per job is given by:

$$\text{Prob}(n) = \frac{1}{m}\left(\frac{m-1}{m}\right)^{n-1} \qquad \text{for } n = 1,2,3,\ldots,$$

and the mean number of operations is m. (In actual use, the number of operations was truncated at 39 for convenience of tape format, which reduced the actual mean number of operations very slightly.)

The jobs were released to the shop one at a time, in the order generated. The interarrival times were obtained from an exponential distribution whose mean was set to yield a nominal utilization of 90 per cent in the shop (and therefore 90 per cent on each machine). To accelerate the initial loading of the shop, the first 49 interarrival intervals were set equal to zero so that the first 50 jobs arrived simultaneously at the beginning of the run. A set of jobs was generated by a separate (FORTRAN) program in advance of the experimentation and preserved on magnetic tape. This means that identically the same set of jobs -- arrival times, routings and processing times -- was presented to the shop to compare different priority classification systems.[22] Four sets of 10,000 jobs each were used during the course of the experiment. These sets are described and named in Table 5.* Each of the job sets was obtained from exactly the same distributions and transition matrix. Different jobs were obtained by providing a different initial value (seed) to the pseudo-random number generation routine in each case.

The random routing was used throughout the investigation. Although there had been conjecture that the general flow pattern would have significant bearing on the performance of different scheduling procedures, a previous study [18] found no significant difference, and this factor was held constant throughout the present study.

The number of machines was set at nine through most of this investigation. One previous study had indicated that this would be a sufficient number to exhibit full shop complexity [18] and another

*Tables are in the Appendix.

study had indicated no significant change in increasing the number of machines to twenty-seven.(16) Since execution time of the experiments would vary approximately as the square of the number of machines, it was worthwhile using as small a system as possible. A few runs were made with a shop size of three, five and seven machines. Actually no change was made in the shop -- the nine machines were still available, and the same job tape was used. A routine that would delete all the operations on certain machines was simply added to the job arrival section. For example, for a seven-machine shop the entry routine simply deleted all operations marked for machines 8 and 9. This was not only the simplest way to change shop size, it also preserved a maximum comparability between runs.

CONDITIONS FOR THE EXPERIMENTAL RUNS

Every run consisted of the release of 10,000 jobs, with inter-arrival intervals obtained (in advance in job generation) from an exponential distribution. Remember that in each case the shop was initially empty and was artificially pre-loaded by having the first 50 jobs arrive simultaneously at the beginning of the run. Each run terminated at the time that would have represented the arrival time of the $10{,}001^{st}$ job. In every run the objective was to estimate equilibrium performance. The conditions were completely stationary -- the same conditions and procedures were used throughout a run.

The following indicates the factors varied in the experiments, and the number of "levels" of each employed:

1. 92 "different" priority assignment procedures;
2. 4 methods of assigning due-dates to the jobs (due-date assigned on arrival to shop);

3. 4 shop sizes;

4. 3 types of processing time distribution (time was exponential on each tape; could be transformed on arrival to shop);

5. 2 levels of shop utilization (processing times of each job could be multiplied by a constant scale factor on arrival to shop);

6. 4 sets of 10,000 jobs.

Each experimental run represented a particular choice from each one of these six categories. It is important to realize that "replication" has a somewhat unusual meaning in experimentation conducted on a computer. If one were to replicate a run, using the same specification from each of the six categories, he would obtain identically the same results. There is no residual or random variation in that sense. Replication is obtained within each run -- by examining subsets of the performance statistics, and by testing a given specification from categories 1 to 5 under different choices of job set (category 6).

A total of 130 "production" runs was made. A run required anywhere from 15 to 35 minutes of computer (7090) time, depending primarily on the selection of priority assignment procedure. Including time for testing and preliminary investigation to determine appropriate measurement procedures, approximately 60 hours of computer time were expended in the investigation reported here.

Thirteen different combinations of choice from categories 2 to 5 were used. Each of these combinations is described, and assigned an identifying name, in Table 6. Greater detail of the choices in category 2 (the assignment of due-dates) and category 6 (the choice of job tape) is given in Tables 4 and 5 respectively.

PRIORITY ASSIGNMENT PROCEDURES

The priority assignment procedure was a subroutine, readily changed from one run to the next, that assigned numeric value to an attribute named priority, or priority index, for each operation of each job. Actually the routine provided only the high-order part of the computer word that represented the priority, the low-order part being formed by the time of arrival of the job at the current queue. Since in every case the simulator selected for assignment the operation with the smallest value of priority, ties in the assigned priority were resolved in order of the job's arrival at the current queue.

Using notation defined in Table 1, Table 2 defines the assignment rules used in the investigation. There are 17 basic rules (Table 2a), and 23 rules that are variations and combinations of these basic rules (Tables 2b-d). A parameter (in some cases, two parameters) controls the operation of many of these rules. With different values of the control parameters, one has, in effect, different assignment rules. Including this type of variation, a total of 92 different rules were tested.

It is impossible to cite the origin of most of these rules. They include some obvious standards of comparison (RANDOM, FCFS); rules suggested by previous analytical work (FOPNR, SHOPN, WINQ); antithesis rules which, although of no interest themselves, serve to exaggerate observed effects (MOPNR, LOPN); and rules thought to be more or less representative of procedures actually in frequent use (DDATE, SLACK). The set includes most of the rules that have been considered in previous experimental work. The writer's associates at Cornell, General Electric, and RAND suggested many of the most interesting and important rules.

MEASURES OF PERFORMANCE

Basically, there are four measures of performance of interest in a job shop. Two of these -- the utilization of facilities, and the amount of work-in-process inventory -- are essentially attributes of the shop. The other two -- the total time in the shop, and the lateness with respect to an assigned due-date -- are attributes of jobs passing through the shop. It should be intuitively obvious, and it can be rigorously shown[23] that, at least with respect to their mean values, these measures are closely interrelated. As long as a shop is unsaturated (utilization is <1) the following relationship holds:

$$\bar{L} + \bar{A} = \bar{S} = \frac{\bar{W}\,\bar{N}}{m\,\bar{U}} = \frac{\bar{W}}{m\,\bar{U}}\,(\bar{N}_q + m\bar{U}),$$

where:

$\bar{L}$ is the mean lateness,

$\bar{A}$ is the mean allowable time in the shop,

$\bar{S}$ is the mean total time in the shop,

$\bar{N}$ is the mean number of jobs in the shop,

$\bar{N}_q$ is the mean number of jobs in queue (total of all queues),

$\bar{U}$ is the mean shop utilization,

$\bar{W}$ is the mean work content (sum of the processing times) of the jobs,

m is the number of machines in the shop.

As long as the shop is asymptotically unsaturated, this relationship holds regardless of the scheduling procedure employed, the shop configuration, or work load imposed.

Most of the runs that comprise this investigation were conducted in such a manner that $\overline{U}$, $\overline{W}$, $\overline{A}$ and m remained constant. This means that $\overline{S}$ is always directly proportional to $\overline{N}$, that $\overline{N}_q$ differs from $\overline{N}$ by a constant, and that $\overline{L}$ differs from $\overline{S}$ by a constant. The relationship constrains only the means of the distributions of these measures; it does not say anything about the variance or shape of the distribution, or the values pertaining to individual jobs.

In a single-machine shop (single-channel queue) there are two obvious measures of work-in-process inventory:

1. the number of jobs in queue, and
2. the work-content, or sum of the processing times of the jobs in queue.

Either of these measures may, of course, be weighted by some index of "value" of the individual jobs. The generalization to a larger shop is obvious for the first measure, and this -- the total number of jobs in all queues in the shop -- is used as the principal measure of performance throughout the investigation. It is apparent from the relationship given above that a scheduling procedure that minimized the mean number of jobs in queue for a particular shop and load would also minimize the mean number of jobs in the shop (including those on machines), the mean time spent in the shop by the jobs, and the mean lateness.

The generalization of the work-content measure of the single-machine shop to a larger shop is less obvious. In the more general case, each job in queue not only has the work-content of the particular operation for which it is waiting, it also has the processing times of the preceding operations (which have already been completed) and of

the succeeding operations (which have yet to be done). Hence four different variations of work-content, work-in-process inventory are recorded (or computed):

1. total processing time on all operations of jobs in the shop;
2. processing time on operations completed on jobs in the shop;
3. processing time on operations remaining to be done on jobs in the shop; and
4. processing time of the specific operation for which the job is waiting in queue, called "imminent-operation work-in-queue" in the tables (I-O WinQ).

The interest and implication of these different measures of inventory will be discussed later. It might, however, be noted, that the general strategy of the investigation was to seek priority assignment procedures that would minimize the number of jobs in queue and simply to observe the resulting values of the work-content measures.

Each run of the experiment was under stationary conditions, and for every measure of performance, the objective was to estimate the equilibrium or steady-state value. To accelerate the attainment of this condition and minimize the effect of starting the shop in an "unnatural" condition -- idle and empty -- the shop received an initial pre-load of 50 jobs. Preliminary runs with very detailed display of performance were made to investigate the transient conditions induced by the artificiality of the starting conditions. In each case it was obvious that after 100 or 200 jobs had been released to the shop, there remained no visual evidence of the effect of starting. For insurance against the possibility that some more complicated assignment procedure that was not observed in this preliminary test

would take longer to "settle down," the first 400 jobs were considered "run-in". Statistics for these jobs -- total time in the shop, and lateness -- were not included in the data reported; measurement of shop statistics -- number of jobs in queue, and other measures of work-in-process -- did not begin until the run was 400 time-units old, which was roughly concurrent with the arrival of the 400^{th} job.

In order to give some idea of the variability of the shop statistics, the run was subdivided into intervals of 400 time units each, and a mean value for each statistic was obtained for each 400-unit interval. Each run was long enough that measurements for 27 such intervals could be obtained. The first was discarded; the remaining 26 were reported.

Similarly, for purposes of assessing variability, the job statistics were collected for sets of 100 jobs. The sets were defined based on the arrival order of the jobs. When a job was completed and left the shop, its attributes of time-in-shop and lateness were posted to accumulators for its particular 100-job set. The jobs always arrived in the same order, but their order of completion depended on the priority assignment procedure used, so it was never the same as the arrival order and was not the same for any two runs. If the statistics of all completed jobs (except those discarded for run-in) were reported and used to evaluate rules, some rules obtained an unfair advantage by reporting out their "successes" early for inclusion in the sample, and leaving their "failures" uncompleted in the shop at the end of the run so that they would never be counted. This was a most important point of experimental procedure, and in several previous

investigations[17] failure to protect against this "end effect" completely obscured any real performance difference between the rules tested. The procedure employed here was to consider only sets of 100 jobs that were entirely completed in every run of the investigation. The 91st set of 100 jobs was the last one to be entirely completed in every run. Under some of the priority assignment procedures (first-come, first-served for example) five or six more sets of jobs were completed, but these data were discarded, nonetheless, to ensure comparability between runs. Thus, job statistics for each run were based on 87 sets of 100 jobs each -- jobs numbered 401 to 9100 out of the full set of 10,000.

The measures of performance and the methods of measurement are precisely defined in Table 3. For shop statistics the sample variance of the 26-interval means is reported. For job statistics the variance of the sample distribution of 8700 job values is reported (rather than the variance of the means of 100-job sets).

III. RESULTS OF THE EXPERIMENT

The results of the experiment are presented in Tables 7, 8, 9, and 11 through 25. The summary statistics for each of the 130 runs are presented in Table 7 --the Shop Statistics and Job Statistics are presented in Tables 7a and 7b, respectively. More detailed results with regard to lateness for certain of the runs are given in Table 25. Tables 12 and 13 provide a detailed comparison of the number of jobs in queue for runs using the different job tapes. None of the other tables present additional information; they simply represent various ways of extracting, summarizing and comparing the information contained in Table 7.

IV. ANALYSIS AND DISCUSSION OF RESULTS

There were two principal areas of investigation. The first was the minimization of work-in-process inventory. Of the several different ways of measuring this inventory, attention was concentrated on the number of jobs in queue, although the other measures were recorded in each case and some effort was expended toward devising rules that would reduce the other measures. The second major area was the study of job lateness, including the interaction between the methods of assigning due-dates and the methods of assigning job priority.

As indicated in the previous discussion, there is an important relationship between these measures of performance, and a priority assignment procedure that minimized the mean number of jobs in queue and would also minimize the mean value of job lateness (using a particular definition of "lateness"). But there is much more to the question of good lateness performance than this -- the minimization of this mean lateness is desirable, but not sufficient.

This discussion will consider first the question of minimization of inventory and the concomitant minimization of mean lateness, and then the further study of the lateness question.

THE MINIMIZATION OF WORK-IN-PROCESS INVENTORY

The Number of Jobs in Queue

Of the many ways of measuring work-in-process inventory, attention was focused on the number of jobs, rather than some measure of work content. This was done for a theoretical, rather than a practical

reason. In a manufacturing shop the reduction of the number of jobs in process is probably desirable, but reduction of the dollar value of inventory is certainly more important, and in most cases this is probably more closely correlated with work-content, than simple job count. In certain special cases, however, whose jobs are very similar; the number of jobs in the shop is the direct and important measure. In an Air Force base maintenance situation, for example, the appropriate objective is the minimization of the number of aircraft awaiting maintenance (equivalent to maximizing the number available for service).

The effect of different means of assigning priorities is reasonably well understood for the special case of a single-channel queue. The following is a brief summary of these results. In these statements a "priority class" is considered to be a set of jobs that have identical values of the priority index. The implied selection mechanism is that assignment will be made from the lowest-numbered priority class represented in the queue. The machine will never be held idle when one or more jobs are in queue.

1) If jobs are assigned to priority classes in such a manner that the mean processing time is the same for each class, then the numbering of the priority classes has no effect on the mean number of jobs in the queue. The mean number will be the same as that given for a first-come-first-served discipline.(6)

2) If the mean processing time for two priority classes is different, the mean number of jobs in queue will be reduced by assigning a lower priority index to the class with the smaller mean processing time. It will be increased by assigning a lower index to the class with the greater mean processing time. (6) (11)

3) The mean number of jobs in queue will be minimized by assigning each job to a priority class whose index is equal to the

processing time of the job. The mean number of jobs in queue will be maximized by assigning each job to a priority class whose index is equal to the reciprocal of the processing time of the job. This says that the optimal procedure in a single-channel queue (with respect to minimization of mean number of jobs in queue) is to choose the job with the shortest processing time each time that an assignment is to be made. (11) The amount of reduction that can be achieved is very significant. Comparative values for exponential distributions and different levels of utilization are given in Table 26; these values are asymptotic and theoretical, not experimental.

4) The mean work-content of a single-channel queue (work-content being the sum of the processing times of the jobs in queue) is independent of priority assignment -- it depends only on the characteristics of the server and of the jobs that must be processed, and cannot be changed by altering the order in which the jobs are processed. This says that the product of the mean number of jobs in queue and the mean work-content per job in queue is constant. A priority assignment rule that reduces the mean number of jobs causes an offsetting increase in the mean work-content -- the queue will contain fewer, but longer jobs. (5)

These are very powerful results and one can fairly say that the problem of scheduling a single-channel queue has been "solved." The extension of these results to a more general network configuration would not constitute a solution in the same strong sense since there are presumably better ways than priority assignment for scheduling networks, but it would nonetheless represent tremendous progress. Unfortunately this is an extremely difficult analytical task and such extension does not appear imminent. The most important result obtained is the demonstration that under certain circumstances the individual queues of a network are, in effect, independent, and that their performance can be predicted using the well-developed, single-channel results.(14), (15) Unfortunately, the conditions that allow this decomposition do not include the interesting priority assignment procedures -- they are limited to rules for which the mean processing time

of each priority class is the same, so that the mean number of jobs in each individual queue and hence in the entire network is unaffected by the numbering of the classes. There are essentially no analytical results that apply to the problem of sequencing the work of a network of two or more queues.

From a theoretical point of view this investigation could be considered to be an attempt to extend experimentally certain of these single-channel results to the general network case. In this light, the principal objective of the investigation is the consideration of the following theorem:

> Conjecture: The optimal (with respect to the minimization of the number of jobs in queue) method of assigning jobs to priority classes in a general job shop is such that in each queue the priority index of the job is equal to the processing time of the operation for which it is waiting.

One can give intuitive or heuristic arguments both for and against this proposition. If one focuses attention on any individual queue in the shop, this procedure is known to minimize the mean number of jobs in that queue. Since the measure for the shop is simply the sum of the measures for the individual queues that comprise the shop, one could expect that this would be minimized by minimizing the individual components. This would certainly be true if the shop were a "loose confederation" of queues -- if jobs needed only a single operation, so that they entered the shop at a particular queue and left the shop after each operation was performed. It is somewhat less clear what will result when the jobs move from one queue to another in the shop. In the random-routed case, only one operation completion out of m (where m is the number of machines in the shop) results in a job actually leaving

the shop. The rest of the time the job simply moves to another queue and the job count for the whole shop does not change. In minimizing the number of jobs in a particular queue one is, in large measure, shifting jobs to another queue (by advancing them in their routing), and, if this is done simultaneously at each queue, it is not at all clear what the aggregate effect will be. There is at least reasonable doubt that this local suboptimization results in optimal performance for the system as a whole. It would seem likely that there exists some form of coordinated scheduling procedure for the entire network that would be able to surpass the performance of this simple and local priority assignment procedure. However, even if this is the case, one could still salvage an interesting and useful theorem by establishing the optimality of the shortest-operation-selection procedure with respect to a more restricted class of procedures.

There is, of course, a procedural difficulty in that one can never prove a positive assertion experimentally -- one can only fail to disprove an assertion. In this case one could disprove optimality simply by exhibiting a counterexample and offering statistical evidence of the high likelihood of its superiority. Failure to exhibit a counterexample is, at best, inconclusive. One can actually only assert superiority for the candidate rule over the alternatives explicitly tested, and the result is interesting only if the set of alternatives is in some sense rich and exhaustive.

The rather substantial advantage of the shortest-operation discipline in a job shop was demonstrated in previous work,[16,17,18] and this is substantiated here. Under STD (standard) conditions, the mean number

of jobs in queue is 59 for first-come-first-served (FCFS) and 23 for shortest-operation (SHOPN) (Table 7a). Presumably SHOPN would be superior for at least any symmetrically balanced shop, but the magnitude of the advantage would depend upon the level of utilization. This ratio (SHOPN/FCFS) of almost 0.4 is slightly greater than the ratio for a single-machine shop at a comparable level of utilization (0.89, Table 26). More will be said about the effect of shop size, and other determinants of relative performance later. It is worth noting at this point, however, that the differences between procedures is very appreciable -- a factor of 2.5 between SHOPN and what might be considered nominal or standard rules (FCFS, RANDOM), and a factor of 7.5 between SHOPN and the antithetical longest-operation rule (Table 7a).

To the best of the author's knowledge, no procedure superior to SHOPN has been previously exhibited, although the attempt has been made.(18) Hence, in a real sense the 23.25 performance of SHOPN became the target value for the investigation. The first class of challengers were rules based on the concept that it did little good to give preference to a job that would then move on into a congested queue; the priority-assigning mechanism should look ahead to where the jobs will go on leaving this machine and give preference to those that will move to relatively empty queues. It is important to realize this is not done in the interest of reducing idle-time. Idle-time is just the complement of utilization, an element completely determined by the specification of the shop and the jobs, and one which cannot be affected by priority assignment. It is possible, however, to determine when the idle-time occurs and, by doing so, to reduce the queue lengths. Rules

15 NINQ, 16 WINQ and 17 XWINQ represent three different ways of implementing this concept. WINQ looks at the sum of the processing times of the operations already waiting in the next queue to which each of the current candidate jobs will move. XWINQ is similar, but adds to the work-content of the jobs already waiting in each queue the work-content of jobs whose arrival in that queue is imminent, because they are currently being processed on their preceding operation. NINQ is analogous to WINQ, but considers only the job count and not the work content.

The comparative results are (from Table 7a):

Rule	Mean Number of Jobs in Queue
15 NINQ	31.20
16 WINQ	40.43
17 XWINQ	34.03
9 SHOPN	23.25
2 FCFS	58.87

The performance of these "subsequent operation" rules is good, relative to FCFS, but none of them seriously challenges SHOPN. The advantage of XWINQ over WINQ evidences the value of the additional information that is provided to the priority mechanism under XWINQ. The fact that NINQ has better performance than either XWINQ or WINQ is surprising considering that the number of jobs is a much cruder measure of queue congestion than work-content. Actually, NINQ takes unfair advantage of a peculiarity in the definition of these rules. In each of these three rules SHOPN resolves ties. Work-content is a sum of three digit numbers, and ties must be extremely rare. Number in queue is always a small integer and ties would be relatively common, so that frequently NINQ is actually using SHOPN operation.

Although these procedures did not themselves surpass SHOPN performance, the fact that they achieved appreciably better performance than FCFS, and did so without using information on the processing time of the imminent operation, was tremendously informative. Consider that in the single-machine case, the processing time is completely sufficient information and SHOPN makes full use of this information. If SHOPN was to be surpassed in a network it would have to be by a procedure that found some additional information. WINQ and XWINQ showed the existence and nature of such information.

None of the other Basic Rules (Rules 1-17, as defined in Table 2a) was a serious challenger, but several were interesting, informative and worth comment. Rule 5 (FOPNR) gave preference to the job with the fewest operations remaining. This achieved a modest advantage over FCFS (52 versus 59), again using information other than imminent operation processing time. In particular, the idea of giving preference to jobs waiting for their last operation -- to get them out of the shop and out of the measure of performance -- had intuitive appeal. Rule 12 (LWKR) has a similar intent -- preference is given to jobs with the least work remaining, work being measured by the sum of processing times rather than the number of operations. This rule uses more information than FOPNR, and at least on the last operation of a job is equivalent to SHOPN, and its performance is somewhat better (47.5). Rule 4, which gave preference in order of due-date, gave performance similar to LWKR simply because in this case the due-dates were assigned to give allowances directly proportional to the work-content of the jobs. Rule 11 (LSOPN) gave preference to the job for which the subsequent operation was the longest, and was designed

to "move work downstream" rapidly. This rule showed slightly better performance than FCFS (54 versus 59), most likely due to the fact that a job waiting for its last operation received preference under this rule.

In an attempt to surpass SHOPN performance, the investigation then turned to "combination rules." Since none of the basically different types of rules had come very close to SHOPN performance, it seemed apparent that a good rule should be similar to SHOPN. The strategy employed in the design of rules was to modify SHOPN to use various forms of additional information. The selection between jobs would be based upon SHOPN if there was a large difference between the processing times, and would be influenced by other considerations if the processing times were nearly equal. While this led to the construction of rules whose performance was superior to that of SHOPN, it probably also precluded finding any rule which had a great margin of superiority. Each combination rule was, in fact, a family of rules, parameterized by a weighting coefficient, or exponent. In each case, a series of runs was made to see how the performance of the rule changed with different values of the control parameter. None of these sequential investigations went far enough to determine the exact value of the parameter that yielded the best performance, but hopefully in each case went sufficiently far enough to indicate the potential performance of the combination. The following combinations were considered (Table 2b):

906 P/OPNR(a) Ratio of processing time to number of operations remaining; parameter is exponent of denominator.

913 P/WKR(a) Ratio of processing time to total work remaining; parameter is exponent of denominator.

911 P-SP(a) Linear combination of imminent-operation processing time and negative of subsequent-operation processing time; parameter is coefficient of P, the imminent-operation processing time. a=1 gives SHOPN.

912 P+WKR(a) Linear combination of P and total work remaining; parameter is coefficient of P.

915 P+NINQ(a) Linear combination of P and number of jobs in subsequent-operation queue; parameter is coefficient of P.

916 P+WINQ(a) Linear combination of P and work-content in subsequent-operation queue; parameter is coefficient of P (note actual rule had additional term and parameter, but these were inoperative in runs considered here).

917 P+XWQ(a) Linear combination of P and XWINQ; parameter is coefficient of P.

The results for the best runs of each of these rules (under STD conditions) are the following (from Table 7a):

Rule	Mean No. of Jobs in Queue
906 P/OPNR(.5)	29.17
913 P/WKR(.1)	23.37
911 P-SP(.95)	23.15
912 P+WKR(.9852)	22.98
915 P+NINQ(.9)	22.77
916 P+WINQ(.97)	22.83
917 P+XWQ(.96)	22.67
9 SHOPN	23.25

It is apparent from the values of the parameters that all of these rules are very close to SHOPN in operation; they simply find different ways of resolving near-ties in processing time. There

appears to be some slight virtue in favoring jobs nearing completion (P+WKR, P-SP), and some slight further benefit in favoring jobs destined for uncongested queues. It would not appear to make a great deal of difference how that congestion is defined and measured. The best performance (by a narrow margin) was exhibited by the rule that required the most information (P+XWQ); it represents a 2.5 per cent improvement over SHOPN.

Statistical Significance of Results

In the finite and deterministic problem where n jobs are to be sequenced on one machine, the mean number of jobs in queue is minimized by using a SHOPN sequence.(3) If this were not true, a sufficient disproof would be the exhibition of a single superior sequence for one particular set of jobs. In this finite and deterministic sense, then, several counterexamples have been offered. The conjecture has been shown to be false that "in a 9 x 10,000 job-shop problem, the number of jobs in queue is minimized by SHOPN sequencing." Although SHOPN sequencing may be "good" for all sets of jobs, and may be "best" for "most" sets of jobs, the conjecture is not generally true, since a superior sequence (in fact, several superior sequences) has been exhibited for one set of jobs.

However, the intent has been to view the job-shop process as continuous and stochastic. Each priority assignment procedure has an asymptotic equilibrium value for the expected number of jobs in queue; and the experimental runs were conducted to enable us to estimate the asymptotic value of performance. The experimental run is a sample drawn from an infinite population, and the measurements obtained are

considered subject to sampling error. Under this interpretation, to establish the counterexample required to disprove the conjecture, it is necessary to establish the statistical significance of the difference between the procedures. One must show that the claimed superior performance is most unlikely to be a chance phenomenon.

Since the attribute of interest is a function of time, say $N(t)$, and is measured continuously during the interval from t_1 to t_2, the statistic obtained from a run is:

$$\frac{1}{(t_2-t_1)} \cdot \int_{t_1}^{t_2} N(t)dt.$$

If $N_a(t)$ is the value of the attribute when rule a is being used, and $N_b(t)$ when rule b is being used, then one needs to show that there is a significant difference between:

$$\frac{1}{(t_2-t_1)} \cdot \int_{t_1}^{t_2} N_a(t)dt, \quad \text{and} \quad \frac{1}{(t_2-t_1)} \cdot \int_{t_1}^{t_2} N_b(t)dt;$$

or equivalently, that:

$$\frac{1}{(t_2-t_1)} \cdot \int_{t_1}^{t_2} [N_a(t) - N_b(t)]\,dt$$

is significantly different from zero. If there is a statistical procedure for doing this directly, the author is unaware of it. What was done amounts to a discrete approximation to such a test. As detailed in Table 3, each run was subdivided into 26 intervals, and a separate measurement obtained for each interval. For each run this gave a sequence of 26 random variables, presumably identically distributed, but not mutually independent. For comparative tests, a sequence of 26 differences was obtained from the corresponding pairs of observations in the sequences for the two procedures being compared.

One would expect much smaller amounts of autocorrelation in such a sequence of differences, and this was indeed observed to be the case.

Considering the comparison of performance of SHOPN and P+WINQ (0.97), the difference between 23.25 and 22.83 of 0.42, or 1.8 per cent, does not represent overwhelming evidence that SHOPN has been surpassed. The 26 differences range from 2.24 in favor of SHOPN to 2.75 in favor of P+WINQ (data given in Tables 12 and 13). In 8 of the 26 intervals, a smaller mean was reported for SHOPN. The sample standard deviation was 0.186. The coefficient of autocorrelation for successive differences was -0.26 which, for 25 degrees of freedom,* is not significant at the 5-per-cent level. One can hopefully assume independence and calculate the sample standard deviation to be 0.186. The mean difference of 0.42 is 2.24 standard deviations from zero. Appropriate values of the "t" statistic for 25 degrees of freedom are:

Significance level (%)	"t"
10	1.32
5	1.71
2.5	2.06
1	2.49
0.5	2.79

Formally, one can test the hypothesis that the difference in mean number of jobs in queue (subtracting P+WINQ from SHOPN) is greater than zero against the alternative that the difference is less than or equal to zero. If the assumptions of the "t test" were strictly

*The autocorrelation analysis performed here is described on pages 337 and 136 of Ref. 24.

satisfied, this would be strong evidence to reject the alternative hypothesis, since it is rather unlikely (approximately one chance in fifty) that these results would be observed if the true difference were zero or less. One could thus conclude that the true performance of P+WINQ gives a lower value of mean jobs in queue than SHOPN; that although the absolute numeric difference is small, the runs were sufficiently long to establish its significance (from a statistical, if not practical, point-of-view).

A comparison of P+XWQ(0.96) and SHOPN gives even stronger evidence of the existence of the required counterexample. The coefficient of autocorrelation is 0.06, which is not significant. Assuming independence for the calculation of the sample standard deviation, the mean of the 26 differences of 0.586 is 3.37 standard deviations from zero.

There is, however, a subtle but important flaw in this argument. If the objective of the investigation had been, from the outset, to compare the performance of P+WINQ(0.97) and SHOPN, or P+XWQ(0.96) and SHOPN, these results would constitute very strong evidence. This of course was not the case -- the investigation undertook, through repeated runs on the same set of jobs, to find or develop a procedure that would surpass SHOPN. If one were to test a large number of procedures which were, in fact, exactly equal to SHOPN in performance, one would expect that sooner or later, entirely by chance, a "significant" result would be indicated. The test of hypothesis described above was an ex post test, clearly suggested by an exmination of the data. Moreover the sequential nature of the investigation, in which the results of each run were used to contrive new candidate rules,

also contributes suspicion that the rules that evolved to eventually surpass SHOPN performance represent an adaptation to the peculiarities of this particular set of jobs and are not indicative of general performance.

Clearly replication, in which the rules developed on the original job set were compared on an independent set of jobs, was required. Comparative data for shop statistics from the replication runs are summarized in Table 11. The most important of these data are those for mean number of jobs in queue:

Job Set	SHOPN	P+WINQ(0.97)
Original (STD)	23.25	22.83
Rep 2	28.46	27.91
Rep 3	33.08	33.92
Rep 4	26.50	26.54

The comparison of SHOPN and P+WINQ(0.97) on the second set of jobs seemed fairly conclusive. The coefficient of autocorrelation was 0.11 which is not significant. Assuming independence for the calculation of the sample standard deviation, the mean difference of 0.550 represented 3.40 standard deviations -- a highly significant result. This seemed a sufficient counterexample, even though it was likely that the control parameter of 0.97 was not precisely the optimum value, and that the P+XWQ rule would do even better.

What was surprising about the Rep 2 runs was the absolute value of the results -- SHOPN performance of 28 as compared to 23 in the first run. This is entirely explained by the different shop utilizations obtained with the different job sets -- 0.884 with the first set

and 0.904 with Rep 2; but this is a surprisingly large difference in utilization for a large set of jobs obtained from sources that were identical except for different seeds in a pseudo-random number generator. One should note that although the variability with respect to job sets in this case is an order of magnitude larger than the difference to be measured, appropriate experimental procedures permit comparisons to be made.

The third and fourth job sets were produced to see what further variation in utilization would result, and not because of any doubt with regard to the comparison of SHOPN and P+WINQ. The results were most revealing. The third job set provided even higher utilization (0.919), correspondingly higher values for number of jobs in queue, and a reversal of the comparative performance of SHOPN and P+WINQ(0.97). Moreover, the coefficient of autocorrelation in this case is 0.42, which is highly significant, and one must reject the hypothesis of independence. The fourth job set provided utilization of 0.893 and exhibited a virtual tie between SHOPN and P+WINQ (0.97). The mean number of jobs in queue for each 400 time-unit interval for each job set is given for SHOPN in Table 12 and for P+WINQ in Table 13. The queue with the greatest mean number of jobs is also identified for each interval in these tables, and the overall mean value for each of the nine queues is given. These data provide at least partial explanation for the reversal of comparative performance. The generation program from which all of the job sets were obtained was constructed to provide a work load that was asymptotically symmetric and balanced over the nine machines of the shop. There would, of course, be short-run chance variations in balance. The third job set was notable in the

nature and extent of the imbalance that developed. Consider the following comparison, drawn from the summary data of Table 12.

	STD	Rep 2	Rep 3	Rep 4
Ratio of max q to min q (total run means)	1.25	1.60	2.78	1.54
Ratio of max q to all q's (total run means)	.12	.15	.23	.14
Max single interval single q mean	8.70	14.40	20.69	16.21
Greatest run; no. consecutive intervals one q is max	3	5	8	3

In the Rep 3 run it is apparent that SHOPN exhibits a marked advantage over P+WINQ during the period of high workload and serious imbalance -- the nine intervals beginning at time 2000. P+WINQ is generally superior through the rest of the run. One can readily theorize that during periods of imbalance one or several queues tend to dominate the shop -- and SHOPN is known to be optimum with respect to a single queue. This would suggest that a rule such as P+WINQ would be superior during periods of relative balance; while SHOPN would be superior during periods of imbalance. One need not, of course, ponder which condition is most likely to occur in order to choose between these alternatives. P+WINQ is already heavily based on SHOPN -- it is a simple extension to allow the weighting coefficient to vary dynamically according to some function of current shop balance. This hypothesis is not entirely convincing, for while it provides a convenient ex post rationalization of the results for the

STD and Rep 3 runs, it does not help to explain the difference between the results of the Rep 2 and Rep 4 runs for which the balance conditions were very similar. It does provide an interesting point of departure for further work, however.

If the Rep 3 and Rep 4 runs had never been executed, the author would have had no hesitation in making a strong assertion that one, in fact several, counterexamples to SHOPN had been exhibited and statistically proven. Once run, the evidence of Rep 3 and Rep 4 comparisons cannot be ignored, and the Scotch verdict of "not proven" must be returned. Moreover, this situation casts some doubt on the validity of the statistical procedures that would have led to an unwarranted strong conclusion at the end of the Rep 2 runs. Apparently, either the independence, and/or the normality required for the "t test" was lacking in this case.

The author can recall an earlier failure to disprove the optimality of SHOPN and can appropriately reiterate his conclusion on that occasion:

> "This result greatly increased our respect for the ability of the shortest-operation discipline. While we still believe that a superior rule can be devised, it is not going to be an easy task and the rule may well have to be a very complex one."*

A Procedure That Does Not Always Make Available Assignments

Procedure 99 SHOPNH provided the only exception to the rule that the best job would be selected from queue whenever the machine became available -- the machine was never held idle when there was work waiting in queue. Under SHOPNH, whenever an assignment was to be made,

*Page 72 of ref. 18.

the procedure examined the jobs that were currently being processed on other machines in the shop to see if any of them would have their next operation on the machine open for assignment. If so, and if the completion was "imminent" and the subsequent operation was sufficiently shorter than the shortest operation waiting in queue, then no assignment would be made. The machine would be held idle to await the arrival of the shorter operation. The propensity to hold off assignment can be varied by setting two control parameters in the procedure.

Only three tests were made, but the results were not particularly encouraging. When an assignment was not made whenever the subsequent arrival would complete its operation on this machine sooner than the shortest operation currently in queue (SHOPNH(100,0)), the mean number of jobs in queue was 50.7 as compared to 23.3 for straight SHOPN. (One interesting result of this run was the large value of imminent-operation work-in-process -- 206, while 74 was the greatest value produced by any other rule.) In a run in which the "hold" was allowed only if there was a single job in queue (SHOPNH(1,0)), the mean value was 24.0. If the "hold" took place only when there was a single job in queue, and the anticipated arrival would be completed, "considerably" earlier (by 1 time unit, equal to the mean processing time), then the mean value of jobs in queue was reduced to 23.5 (SHOPNH(1,1)). It would appear that this procedure can approach the performance of SHOPN as the controls are set, so that the "hold" would rarely be employed. It seems unlikely that it would ever attain an interesting superiority over SHOPN.

Classification of Priority Assignment Procedures According to Information Requirements

There is a great disparity in the amount and type of information required to implement the different types of priority assignment procedures listed in Table 2, and it is instructive to classify the procedures according to their requirements. Four different (information) types are defined:

I. A LOCAL, OPERATION rule bases priority, and hence selection, only on information available in the queue from which selection is to be made. It considers only jobs that are waiting in queue at the moment selection is required, and from these jobs only attributes of the particular operation for which the job is waiting.

II. A LOCAL, JOB rule can use any information available to a type I rule, plus any other attributes of the jobs in the queue from which selection is to be made.

III. A GLOBAL, CURRENT STATUS rule can also use information regarding attributes of any job in the shop, and the information relevant to the current status of any machine, queue or job in the shop.

IV. A GLOBAL, PREDICTED STATUS rule can attempt to use the information on current status to lay out a "program" some distance in the future predicting job or shop status.

In Table 9, each of the priority assignment procedures defined in Table 2 is assigned to one of these Information Classifications. In Table 10, the best performance within each Information Classification is summarized for each of the various measures of performance.

For the mean number of jobs in queue, the best local-operation rule is, of course, SHOPN. P+WKR, a local job rule, makes a slight improvement by considering the amount of work remaining as well as the current processing time. P+NINQ makes a further improvement by considering the state of the next queue downstream for each candidate job; and P+XWQ represents still a further improvement by better predicting the state of the next queue downstream.

If the information available to a local operation rule is defined so strictly that an operation is not allowed to "know" that it is last on a job's routine, there seems little question that SHOPN is the optimal procedure with respect to all such rules. If one is able to give preference to jobs on their last operation (as is done in P+WKR), a marginal improvement can perhaps be obtained. What is most impressive about these data is the very slight improvement indicated for very substantial increases in information required. SHOPN has very modest information requirements -- it does not require any computing, or even recording of information, at the decision site. (Moreover, it is demonstrated below that it is not even particularly sensitive to errors of estimate in the information that it does require.) The global rules would certainly require rather elaborate communication and storage equipment for implementation, and it is rather doubtful that their 2.5-per-cent improvement in performance will justify this cost. There is currently a great deal of interest in sophisticated computing-communication systems for the "real-time control of manufacturing operations." While these data certainly do not disprove the utility and economy of such systems, they at least place upon their advocates the responsibility to show that they possess a sufficient performance advantage over a well-conceived local system.

The Effect of Level of Utilization and Shop Size upon SHOPN Performance

The data provide an opportunity to compare SHOPN and FCFS performance at four different levels of utilization, as shown in Table 14. Although the expected utilization of each job tape was 90 per cent, the variation in observed value has been previously noted. In

addition, a set of runs was made using the first job set (STD), in which all of the processing times were multiplied by 0.877 on arrival at the shop. These runs yielded the data described as "Scale 0.877" in the tables. The experimental data for a nine-machine shop can be compared to the theoretical results for a one-machine shop (from Table 26):

Ratio of mean number of jobs in queue: $\frac{\text{SHOPN}}{\text{FCFS}}$

Util.	Theor. 1 Mach	Exper. 9 Mach
78%	0.49	0.51
88%	0.38	0.39

The results of the runs in which different numbers of machines were deleted from the full nine-machine shop (all at 88-per-cent utilization) are summarized in Table 15. The key points are the following:

Number of Machines Operating	Mean No. of Jobs per Queue	Ratio of Mean Jobs-SHOPN (experimental) to Mean Jobs-FCFS (theoretical)
3	2.801	0.417
5	2.626	0.391
7	2.593	0.386
9	2.583	0.384

(Recall that all rules that are completely independent of processing time have the same expected mean number of jobs as FCFS.) These results provide no reason to question that the relative advantage of SHOPN over the FCFS-class rules in a job shop is of comparable magnitude to the theoretical results available for the single-machine case, and that this advantage increases markedly with utilization as in the single-machine case.

The experimental study of the effect of shop size gives some indication that the advantage of SHOPN slowly increases with the number of machines. The differences are not great, but the manner of conducting these runs -- using identically the same set of jobs and deleting operations according to the specified number of machines -- yields data that are highly correlated between runs, but which should provide very precise comparison. The theoretical results for the comparison in a single-machine case are strangely out of line with these experimental data. This may simply be the result of experimental error, or it could be that this ratio jumps as a second machine is added to the simple single-machine shop and then decreases monotonically as machines are added. In any event, the differences are not large and there would seem little reason to pursue the matter further.

However, the mean number of jobs per queue is of considerable practical interest. Since Jackson has shown that the queues can be considered independent,[14] it is known under any FCFS-class rule (with some further restrictions on distributions and routing) that the number of jobs per queue are independent of the number of queues in the system. It was known that the queues are not strictly independent for rules that consider processing time (or any measure correlated with processing time), but it was not known how serious an error was being made when independence was assumed in these cases. These data suggest that the assumption of independence is a reasonably good approximation, and that it is conservative -- a shop will actually perform somewhat better than an equivalent number of independent machines. This means that the work of such investigators as

Reinitz and Holt,[25] which depends crucially upon the assumed independence of queues, is well justified and can be of great practical value.

The Effect of Processing Time Distribution and Errors of Estimate

Runs were made to compare SHOPN and FCFS under three different processing time distributions -- exponential, rectangular, and constant. In each case the STD job tape was used and the processing times, which were actually exponential on the tape, were transformed on arrival at the shop. Thus the job routings and arrival times were identically the same between runs, and the processing times were correlated. The mean of the distribution was the same in each case so that the shop utilization was also held constant. The results are summarized in Table 16. It is known that under FCFS the number of jobs in queue is directly proportional to the variance of the processing time distribution (assuming exponentially distributed interarrival intervals, which was the case here). SHOPN performance of course coincides with FCFS for constant processing time, since all jobs are tied with respect to processing time and ties are resolved in the order of arrival. It is interesting, although not particularly surprising, to note that the absolute performance of SHOPN (not just relative to FCFS) improves with increasing variance of the processing time distribution. As the variance increases, there are more frequent occasions when alternative candidates have significantly different processing times and the choice becomes important.

If the performance of SHOPN were highly sensitive to errors in estimating the processing time of the individual operations, this would be a crucial limitation and deterrent to the use of the above

procedure, since there are probably few situations that have perfect information available in advance of execution, and many situations that have only very crude estimates available beforehand. Clearly, a bias in the estimating procedure cannot adversely affect SHOPN since only comparative information is used and the only issue is the effect of the precision or the procedure. Runs indicating the sensitivity, or lack of sensitivity to the precision of the estimating procedure are summarized in Table 17. In each of these runs, priority was based on an estimate of the true processing time. The percentage error of these estimates was uniformly distributed between the limits indicated in the table. There was no significant degradation of performance if the errors of estimate were less than $\pm$ 10 per cent. In an industrial environment this is a significant error level since experienced practitioners of the art of work measurement can allegedly attain this precision. Whether they can or not is almost beside the point, since such precision is hardly required -- at least not from a sequencing point of view. Table 17 shows that errors uniformly distributed between $\pm$ 100 per cent increase the mean number of jobs in queue by only 17 per cent to a level still substantially better than any of the other basic rules. This 17-per-cent figure is approached only by combination rules which also depend on processing time and presumably would be affected by the same errors of estimate.

As an indication of the tremendous value of even crude information on processing time, a run was made in which operations were considered "short" or "long" depending upon whether their time fell below or above the mean of the processing time distribution. Giving

preference to the "short" operations in an otherwise FCFS operation gave a mean number of jobs in queue of 35 -- still a very respectable level of performance. When the probability of a correct classification in this scheme was set to 0.75 instead of 1.00, appreciable degradation of performance did occur, but it should be noted that each operation had an equal probability of being misclassified. In practice, misclassification would be most likely for jobs with time near the mean, for which the consequences of misclassification are not serious.

Modifications of the Shortest-Operation Procedure

Several further modifications of SHOPN were tested, not in the interest of surpassing SHOPN performance, but rather to see how much of its virtue would be retained by variations intended to overcome its basic deficiency. The characteristic that seems to arouse suspicion about the value of strict SHOPN operation is the rule's complete indifference to the plight of individual jobs, which, through a combination of bad fortune and several long operations, may wait intolerably long in queue.

An obvious compromise is to use SHOPN for selection until any job has been "too long" passed over in favor of shorter operations; then to give that job precedence regardless of its processing time. This rule (TSHOPN(a); 902) can be parameterized by the amount of time that a job is allowed to wait in a queue before being forced out. The performance of this rule for different severities of "truncation" is given in Table 18. In interpreting the severity of truncation one should know that the mean amount of waiting time per operation under

FCFS is 7.27 time units (under SHOPN, 2.78 time units), and that the mean processing time is 1.00 time units. Against these values, truncation at 32 time units would seem like a rather mild modification that would be infrequently operative, but it results in a 41-per-cent degradation of aggregate performance. This suggests that SHOPN is very sensitive to this form of truncation.

A second method of attempting to obtain the power of SHOPN without its problems is to use SHOPN as an "overload" or "emergency" procedure in a system that "normally" operates under FCFS (FCFS*S(a); 2-9). This rule makes selections in arrival order until a queue reaches a certain critical size, then makes selection by SHOPN until the queue is brought back under control. Under this rule the selection process for a particular queue will alternate between FCFS and SHOPN, depending upon the state of the queue; at any given instant some queues in the system may be operating under FCFS while others are under SHOPN. Performance of this alternating rule for transition points of 5 and 9 jobs is given in Table 18. The mean number of jobs per queue under SHOPN is 2.58 and under FCFS is 6.54. Such results are quite encouraging. This may well be a very practical way of obtaining some of the benefit of SHOPN sequencing in situations where the strict rule is unacceptable.

The performance of a third method of combining FCFS and SHOPN has been reported in a previous study.* The entire shop shifted back and forth between the two rules, changing at regular periodic intervals. It was discovered that the degradation of SHOPN performance was directly proportional to the fraction of time that the shop spent under FCFS operation.

*Page 69 of ref. 18.

In some situations there would be a hierarchy of job value or importance that could not be ignored in making priority assignments. This effect was explored in Rule 929 2CSHOPN(a) that assumes the jobs are of two different types, one of which always takes preference over the other in selection. Each type has the same processing time distribution (exponential), and the basis of selection among jobs of the same type is SHOPN. The performance of this rule with different proportions of jobs of the preferred type is given in Table 19. The interesting and surprising aspect of this rule is the substantial asymmetry -- performance with 10 per cent of the jobs considered preferred is little different from strict SHOPN (24.26 compared to 23.25), while performance with only 10 per cent of the jobs in the less-preferred category causes a substantial degradation (36.11).

Subsequent to these observations, Professor W. L. Maxwell of Cornell and the author have conducted a theoretical investigation (as yet unpublished) of this type of procedure in a single-channel queueing system. We found the same asymmetry to exist. The situation becomes symmetrical as utilization goes to zero; the asymmetry increases with increasing utilization. However, analysis of 2-class SHOPN sequencing in a deterministic n-job, 1-machine model (as considered by Smith[4] and McNaughton[3]) indicates complete symmetry with respect to the proportion of jobs in the two classes.

Other Measures of Work-in-Process Inventory

As described earlier, there are several different ways of measuring the work-content of the jobs in the shop. One can consider:

1. the total of all operations on all jobs in the shop,

2. the total work that has been completed,

3. the total work remaining to be done, or

4. the processing time of the operations for which the jobs are in queue.

The fourth measure, called Imminent-Operation Work-in-Process, is interesting only in that the comparable measure in a single-channel queue is completely unaffected by discipline -- all disciplines are equivalent with respect to this measure. There is just no question that this property does not hold for a queueing network -- that this performance measure, like every other, can be significantly affected by the choice of priority assignment procedure. This measure is unusual in that the performance of SHOPN and LOPN is very similar and reduction is achieved by the "subsequent-operation rules" (WINQ 16 and XWINQ 17). While these differences are undoubtedly statistically significant, the author cannot think of any practical interpretation or value for this measure.

The other measures all have physical and economic significance; there are situations in which each of them would be the dominant criterion. These measures are compared for a number of the more interesting rules in Table 20. (The mean number of jobs in the shop given in this table differs from the statistics previously cited -- mean number of jobs in queue -- by including the jobs which are being processed on machines. The mean number of jobs on machines is, of course, the same for each procedure, so that the mean number in the shop and mean number in queue differ by a constant.) The SHOPN procedure and its derivatives minimize the number of jobs in the shop, but do not minimize the various measures involving work-content, although their

performance is still relatively good. The extreme values for each of these measures is cited in Table 8. The mean total work and mean work completed are minimized (at least with respect to the set of rules tested) by a procedure whose priority is equal to the processing time for the imminent operation divided by the sum of the processing times for all of the operations on the job. The mean work remaining is minimized by setting the priority equal to the processing time for the imminent operation divided by the sum of the processing times for all remaining operations.

These combination rules combine processing time with other information by means of a ratio, and are particularly interesting because the combinations are markedly better than the extremes that use the information in the numerator or denominator alone. For example, basing priority on processing time alone (SHOPN) yields a mean work remaining of 297; work remaining alone (MWKR) yields a mean of 276; the ratio combination (913 P/WKR(1.0)) produces a mean of 220. This is true for all of the work-content measures of performance but not for the number of jobs. It is also not observed for combination rules in which the same information is combined by weighted summation rather than ratio. With rules that are a linear combination of basic rules, one tends to obtain a weighted combination of the performance of the component basic rules. With other methods of combination it is apparently possible to contrive rules that perform substantially better than the component basic ones, at least with respect to certain measures of performance. Table 21 summarizes the results for the different "processing time ratio" rules.

THE INVESTIGATION OF JOB LATENESS

The job attributes of principal interest were the total amount of time the job spent in the shop and the lateness -- defined as the difference between the actual and the allowable shop time. (Lateness can take on negative values, indicating that the job was completed in less than the allowed time.) That is, the shop time and the lateness of a particular job differ by a value called "allowable shop time" assigned to the job as it arrives at the shop. The mean shop time differs from the mean lateness by a constant -- which is the mean of these allowances. Note in particular that this constant difference is in no way affected by the priority procedure employed; a procedure that minimizes the mean shop time will simultaneously minimize the mean lateness. (This is not true for various alternative definitions of lateness.) Moreover, as described earlier, both the mean shop time and the mean lateness are directly related to the mean number of jobs in the shop so that a procedure that minimizes the mean number of jobs in the shop also minimizes mean shop time and mean lateness. This indicates that most of the previous discussion of the virtues of the SHOPN applies identically to the means of these job attributes. These means present essentially no new information and are reported simply for convenience of reference when considering the other measures of the job attributes.

The Variance of the Distribution of Shop Time Values

The variance of the distribution of shop times is not directly related to the mean (and to the mean number of jobs in shop); it represents new information that has both theoretical and practical interest. These results are given in Table 7b.

Of all the procedures tested, the smallest variance was provided by a variation of the "first-come-first-served" discipline which gave precedence to jobs in their order of arrival at the shop, rather than by order of arrival at the particular queue as was done in the normal FCFS. The two alternative definitions of "first-come-first-served" are equivalent with respect to the mean time in shop (and mean number of jobs in shop), but differ by a factor of more than three in the variance of shop time.

The surprising and important result, however, is the very excellent performance of the SHOPN procedure in reducing this variance. It is generally held that while SHOPN may reduce the mean shop time (as compared to, say, FCFS or RANDOM), its price for doing so is a higher variance. In fact, it appears to have a substantially smaller variance than either FCFS or RANDOM and is surpassed only by FASFS (first-arrival-at-shop).

This is not in real contradiction to an earlier study[18] in which the author and W. L. Maxwell found SHOPN to have a greater variance than FCFS, but under a completely different set of experimental conditions. In that test, the release rate was controlled so that each procedure experienced the same mean number of jobs in the shop (and mean shop time) and the utilization was allowed to vary. Running at different utilizations, so that the mean shop times were equal, SHOPN exhibited a variance approximately six times as great as FCFS. In the present investigation, running with equal utilization and different mean shop times, SHOPN exhibits a variance of less than half that of FCFS. This latter seems like a much "fairer" basis of comparison.

This variance is important from a practical point of view in that it is related to the predictability of individual job completion times. There are probably many situations in which the length of time that a job spends in the shop is actually less important than the ability to predict when that time will end. The smaller variance indicates that SHOPN is not inherently less predictable than FCFS. In fact, it is probably much more predictable. Jobs spend more time in the shop waiting than being processed; under FCFS one has only the number of operations on the job as a basis for estimating the amount of time that will be spent waiting in queue. Under SHOPN, the number of operations and their processing times should have high predictive value for estimating queueing time. Further investigation of the prediction of individual job shop times would be most instructive. The variance of the distribution of shop time per operation under SHOPN and FCFS would be interesting, as would separate distributions of shop time for jobs with the same number of operations. Conditional distributions of shop time depending upon some measure of the length of the operations would also be informative. It seems clear that with such additional work, prediction would be much superior under SHOPN than FCFS.

Due-Dates Related to Total Processing Time

The due-dates were assigned to the jobs at the time of their arrival to the shop -- the job tapes themselves contained no due-dates. For most of the investigation, the due-dates were assigned by a routine that made the allowable time in the shop proportional to the sum of the processing times of the operations on the job. This was thought to be one reasonable characterization of due-dates as might be encountered

in actual operations. The constant of proportionality was selected to give a nominal mean lateness of zero, with approximately half of the jobs early and half late under the FCFS procedure. Three statistics were reported to give information about the shape of distribution of latenesses: the mean, the variance (or the standard deviation), and the number of jobs with positive lateness. The investigation centered around the attempt to reduce the proportion of jobs with positive lateness.

There are a number of obvious and well-known "due-date" rules which many firms claim to employ. These are:

4 DDATE; jobs given preference in order of their ultimate due-date.

405 OPNDD; allowable shop time divided equally among the operations to obtain a "due-date" for each operation. Precedence in a particular queue given in order of the "operation due-dates"for the operations in that queue.

413 SLACK; preference given to the job with the least time remaining until due-date after deducting the remaining processing time.

945 S/OPN; "slack" (as computed for 413) is divided by the number of remaining operations and preference given to the job with the smallest ratio.

Using the standard job tape, the results with these procedures are the following (taken from Table 7b):

Procedures	Mean Lateness	Variance	% Jobs Late
4 DDATE	-15.5	432	17.8
405 OPNDD	-9.9	14560	10.4
413 SLACK	-13.1	433	22.0
945 S/OPN	-12.8	226	3.7
2FCFS	-4.5	1686	44.8
9 SHOPN	-44.9	2878	5.0

It is apparent that these rules using due-date information provide a substantial improvement over FCFS in the reduction of the proportion of jobs that are late. This is accomplished in part by a decrease in the mean, but largely by a reduction in the variance of the distribution. (The writer has no explanation or rationalization for the high variance of the OPNDD, but it does not appear to be a computational error. Note that rule 5 FOPNR has very similar performance.) The decrease in the mean is at first surprising, since rules that do not consider processing time should have the same mean as FCFS; but recall that these rules consider due-date, which, in turn, depends upon processing time. The strength of the dependence is similar to that of rules 5 FOPNR and 12 LWKR, and the mean is reduced by a comparable amount.

The "slack per operation" rule (S/OPN) is considerably better than rules that depend upon the due-date, as far as lateness is concerned. S/OPN allows less than 4 per cent of the jobs to be late, by greatly reducing the variance of the lateness distribution. It is interesting to note that SHOPN does almost as well (5 per cent of the jobs late) by employing an entirely different strategy. It depends entirely on a reduction of the mean lateness to offset an increase in variance and still reduce the area of the positive tail of the distribution. This suggested that some combination rule that could reduce both the mean and the variance might surpass the performance of both S/OPN and SHOPN.

One candidate rule -- use shortest operation discipline unless the job is behind operation due-date on entering queue (4059 OPNDDP) -- proved to be a rather complete failure, giving almost as many jobs late as FCFS. A combination of DDATE and NINQ (415 DDNINQ) provided

an improvement over simple DDATE, but nothing comparable to S/OPN, and was dropped. Another possibility was to change the relative importance of the number of operations remaining by raising to a power the denominator of the S/OPN fraction. Powers from 0.5 to 1.2 were tested without reducing the proportion of jobs late below that obtained by the simple ratio (Table 22).

A successful rule was the simple linear combination of SHOPN and S/OPN. The best weighting obtained left only 132 out of the 8700 jobs (1.5 per cent) with positive lateness (Table 22).

Another refinement that improved the performance of SHOPN with regard to the proportion of jobs late (without serious adverse effect on the mean lateness and mean number of jobs in shop) was to take exception from shortest-operation when a job was waiting for its last operation; lateness could be avoided by immediate processing (9-413 P*SLACK). This resulted in 364 jobs late compared to 437 for SHOPN.

One might also note that the variations that do marginally better than SHOPN in the reduction of the mean lateness (917 P+XWQ) also do slightly better with respect to the proportion of jobs late. The best performance of this type was 374 out of the 8700 jobs (4.3 per cent) with positive lateness.

Replication of the Comparison Between SHOPN and S/OPN

While SHOPN and S/OPN exhibit comparable performance for the standard set of jobs, they achieve this performance by two different strategies -- reduction of mean lateness for SHOPN and reduction of variance for S/OPN. It is apparent from the tests using the second and third sets of jobs that these two strategies do not always yield

comparable performance (Table 23). As noted in the earlier discussion, although the same pseudo-random procedure generated all job sets, the second and third sets happen to provide a greater utilization in the shop and a load that is less well-balanced between the different machines. The result is that the shop is more highly congested and the allowable shop times, assigned in the same way (proportional to total work) and with the same constant as for the standard set of jobs, are less adequate. The proportion of jobs late for different rules and different sets of jobs is as follows:

	2 FCFS	9 SHOPN	945 P+S/OPN(0,1)
Standard	44.8%	5.0%	3.7%
Rep 2	57.7	6.1	30.9
Rep 3	67.3	7.3	54.8

The compact (low variance) distribution under S/OPN is extremely sensitive to the adequacy of the mean allowable shop time; a relatively small displacement of the distribution puts a larger portion of its area in the positive region. Although the distribution under SHOPN is similarly displaced, this only gives positive value to slightly more of its attenuated tail, and the overall effect is not great.

There seems no question that SHOPN is less sensitive than S/OPN to variations in load and in determination of allowances. In general, this would probably provide an advantage to SHOPN, but it would also mean that under a slight reduction in load (or an increase in mean allowance) lateness could be virtually eliminated under S/OPN -- under SHOPN it would not be much less than 5 per cent.

Alternative Methods of Assigning Due-Dates

Several alternative methods of assigning due-date were employed (using the standard set of jobs) in an attempt to characterize what might be the most common types of due-dates encountered in actual operations. In method II, the allowance was made proportional to the number of operations on the job rather than the sum of the processing times. In method III, each job was given identically the same allowance, while in method IV the allowance was chosen at random, independent of any characteristic of the job (Table 4). In each case, the controlling parameters were set so that the mean lateness under FCFS would be the same. The proportion of jobs late for the different types of due-dates and different priority procedures is summarized below (and in Table 24):

Rule	I (work)	II (no.opns)	III (constant)	IV (random)
2 FCFS	44.8%	39.9%	33.8%	41.2%
4 DDATE	17.8	26.7	43.9	48.8
9 SHOPN	5.0	6.2	11.0	19.8
945 P+S/OPN(0,1)	3.7	21.6	48.1	52.6

Under DDATE, SHOPN or S/OPN the sequence I, II, III, IV represents progressively poorer ways of assigning due-dates. SHOPN, which does not consider the due-date in its operation, is clearly less sensitive to the method of assigning due-date than the other procedures. The procedures that use due-date information -- DDATE and S/OPN -- are strongest when the due-dates are rational and bear some relationship to the job characteristics. They do not exhibit impressive ability to enforce an arbitrary set of due-dates. When either type III or IV

due-dates were used, FCFS provided better performance with respect to proportion of jobs late than either DDATE or S/OPN. It should be noted, however, that the variance is still substantially smaller with DDATE and S/OPN, regardless of which type of due-date is employed -- this is an important characteristic. An increase in the mean allowance would presumably benefit these rules much more than FCFS or SHOPN, and the proportion of jobs late could be made satisfactorily small.

Although types III and IV due-dates may seem unreasonable and of no theoretical interest, the writer suspects that the procedures actually used in practice can more often be characterized this way than, say, by method I or II. Moreover the combination of these due-dates with priority procedures such as DDATE is not uncommon, so that the revelation of the surprisingly poor performance of this combination is of some practical significance.

In Table 25 the actual number of jobs late from each set of 100 consecutive jobs (consecutive in arrival order) is given for each method of assigning due-date and for four different priority procedures. Some interesting observations can be made from these data. For example, consider the number of sets of 100 jobs that had no jobs late:

Rule	I (work)	II (no.opns)
9 SHOPN	5	2
945 P+S/OPN(0,1)	17	36

Although the overall performance of S/OPN is much poorer under type II due-dates than type I (21.6 against 3.7 per cent late), there is more to it than that. During the periods when the shop load is relatively

light, the S/OPN performance is actually better with type II due-dates, but during periods of congestion the procedure breaks down rather completely. In practice, of course, there is no reason why the methods of assigning due-dates and priority could not be varied from time to time according to conditions in the shop.

Similarly it is apparent from these data that the comparative performance of SHOPN and S/OPN is far from uniform. During periods of relatively light load S/OPN has an advantage, but during periods of congestion SHOPN is much superior. For example, with type II due-dates, S/OPN is superior to SHOPN in 47 sets of 100 jobs, SHOPN is superior in only 36 sets. But in overall performance, S/OPN causes 1878 jobs to be late as opposed to 537 for SHOPN.

The problem of assigning due-dates is closely related to the problem of predicting individual job shop times. If one could predict exactly the shop times that various jobs would experience under a certain priority procedure, one could, of course, assign an allowance equal to the actual time so that the mean lateness and also, the variance would be zero, and no jobs would be late. However, since the shop time depends not only on characteristics of the individual job and the particular priority procedure in use, but also on the nature and status of the other jobs coexistent in the shop during the time that the given job is present, perfect prediction is not attainable.

There is really more to the problem than prediction. Even if one could predict exactly the times when jobs would be completed, it is not always clear that these times would constitute an acceptable set of due-dates. Due-dates often reflect some exogenous relative

urgency associated with the different jobs, and their existence is intended to affect the sequence in which shop work is done. They are, in effect, intended to be independent variables that affect the shop performance, rather than dependent variables. This investigation has suggested that there is some limit in how far priority procedures can go in "enforcing" an arbitrary set of due-dates, but something of this spirit is inherent in the very nature of a due-date. For this reason, there is something basically unreasonable about a procedure, such as SHOPN, that does not consider due-date information even though it exhibits considerable ability to deliver jobs on time.

There is a further difficulty with using SHOPN to reduce the proportion of late jobs. It seems very unlikely that the process by which the due-dates are assigned could remain stationary when the average lateness is continuously a large negative number and the great majority of jobs are completed well in advance of due-date. It seems inevitable that the allowances would eventually be decreased so that they would bear some relationship to the true anticipated shop time, and the proportion of jobs late would then increase accordingly.

The problem is one of finding a combination of due-date assignment procedure and priority sequencing procedure such that:

a. the "natural" shop time of a particular job can be predicted with some precision,

b. a due-date can be set based on both this natural passage time and exogenous relative urgency, and

c. the priority procedure can react to this due-date to accelerate or retard a job with respect to its natural rate of progress.

V. CONCLUSIONS

1. There do not appear to be any obvious and natural measures of performance for a job shop that are invariant to scheduling procedure. The mean number of jobs in the shop, along with the mean shop time and mean lateness, may be altered by the selection of scheduling procedures very much as in a single-channel queueing model. Unlike the single channel, queue measures involving work content are also dependent upon scheduling procedure in the network model.

For every measure of performance considered, the differences between good procedures and nominal standards of comparison -- first-come-first-served, or random selection -- are of practical economic significance.

2. Priority procedures, a subset of all the scheduling procedures that might be employed in a job shop, have interesting and important differences. Probably the priority procedure category does not contain the optimum procedure for any given measure of performance. The best priority procedures are relatively good and, in fact, for non-trivial situations, computationally feasible procedures superior to the best priority procedures have not yet been identified.

Priority procedures are especially important in a practical sense because of the relative ease with which many of them may be implemented.

3. Simulated experimentation, such as employed in the investigation reported here, can be a reasonably effective method of testing, screening, and comparing scheduling procedures. When used in a sequential manner, this technique can be a powerful tool for the design

of a procedure that is good for any particular measure of performance. In some cases, the simulation will simply provide quantitative estimates of differences whose sign was rather intuitively obvious at the outset. In cases where the competitive procedures are more evenly matched, the simulation can still be employed, but careful experimental design and large sample sizes may be required if the difference is to be established with any degree of confidence. There will also be cases where the experimental results will reveal that the investigator's intuition was somewhat inadequate. Examples of the latter in the present study include the surprising asymmetry of the 2 class shortest-operation procedure and the surprisingly poor performance of rule 4059 OPNDDP.

It is clear that a major breakthrough in the simulation technique has been provided by the advent of SIMSCRIPT[21] and other similar languages. These carry simulation some distance from its previous position as a desperate last resort to one of a reasonably practical procedure that can produce results in a finite period of time. This writer would not have had the courage to attempt to code a priority rule like 917 P+XWQ or 99 SHOPNH in anything less than SIMSCRIPT.

4. The independence of the several queues of a network, which Jackson has established to hold for a limited class of priority procedures,[14], [15] is a reasonably good approximation for a much broader class of procedures. This should include virtually any procedure that can be studied in a single-channel queueing model. At least for the shortest-operation discipline, the effect of extrapolating single-channel results to a symmetric network is conservative -- the mean state of an n queue network is slightly less than n times the

mean state of a single-channel queue. Hopefully this is representative of other "non-Jackson" procedures.

5. Selecting operations from queue in order of increasing processing time (SHOPN) is a dominatingly important priority procedure.

a. This is known to minimize the mean number of jobs in queue in a single-channel model.[11]

b. SHOPN is an extremely good procedure in a network of queues. While a counterexample to its optimality, with respect to the mean number of jobs in queue, was not established with complete satisfaction in the present investigation, the writer is convinced that a rule of the form 917 P+XWQ, perhaps with the weighting allowed to vary with time and conditions in the shop, can be shown to be superior to SHOPN by a sufficiently long run. There may, of course, exist procedures less closely related to SHOPN that give a much superior performance so that establishment of a counterexample will be less difficult; but to this writer's knowledge no such procedure that is also computationally feasible has been advanced.

If any consideration at all is given to the difficulty and cost of implementation, however, then SHOPN is certainly the best procedure of those tested in this study, with respect to the minimization of the mean number of jobs in queue. Concomitantly, of sourse, SHOPN minimizes the mean number of jobs in the shop, the mean time in the shop, and the mean lateness.

c. While SHOPN is not the best procedure with respect to any of

the work-content measures, it is quite good in each case; for each of these measures the best procedure of those tested is a combination rule that involves the processing times.

d. A byproduct of the minimization of mean lateness is that the number of jobs with positive lateness is almost as small as for the best of the true due-date rules. For three out of the four methods considered for setting due-dates, SHOPN caused fewer jobs to be late than any other procedure tested.

e. The variance of the distribution of shop times is smaller for SHOPN than for any other basic rule tested, except for a variation of first-come-first-served to which jobs are taken in order of their arrival at the shop. In particular, the variance is smaller than for the normal first-come-first-served (jobs taken in order of their arrival at the particular queue) and the various due-date rules.

f. The virtue of SHOPN does not depend upon the carefully balanced and symmetric conditions employed in these tests. In fact, there is strong evidence to indicate that any lack of symmetry or balance, as would certainly be encountered in practice, would further increase the advantage of SHOPN since one or more queues would tend to dominate the system, and SHOPN is known to be optimum for a single queue.

Also, the worse the situation becomes with respect to the variance of the processing time distribution, the greater the advantage of SHOPN-related rules becomes over those that do not consider processing time.

g. SHOPN is as easily implemented as almost any of the other procedures, and is certainly easier than many of the combination rules. While it obviously requires some a priori knowledge of the processing times, it is not particularly sensitive to errors or imprecision in estimating the processing times, and even crude information provides a substantial advantage in performance.

There certainly must be many situations in which the shortest-operation procedure can be employed directly with very little difficulty in conversion or implementation, and with a tremendous improvement in system performance. There must be an even larger number of situations in which some variation or derivative of this procedure could be employed at very little cost and with at least some beneficial effect on performance. This investigation exhibited many examples of combination procedures involving SHOPN in which some fraction of SHOPN performance was preserved in spite of the dominating influence of other factors. Operating people should consider SHOPN as a standard and desirable procedure, to be used or approached whenever circumstances would allow. Even if it can only be used as a "tie-breaker" -- select in order of processing time, all other things being equal -- it will provide a minor, but free, improvement in performance.

In theoretical investigations SHOPN should be accorded the status of a "standard" of at least the same importance as the traditional first-come-first-served and random methods of selection. It is clear that proposed scheduling procedures should be required to demonstrate their value against the performance level of SHOPN, rather than the easily surpassed performance levels of FCFS or RANDOM selection.

6. In the present investigation, there is no evidence to support the increasingly popular assertion that a major improvement in the

performance of job-shop control procedures can be achieved by the installation of sophisticated (and expensive) data communication, storage and processing systems. The procedures tested that have great information requirements, such as 917 P+XWQ, hardly demonstrate sufficient advantage in performance over SHOPN to justify any serious investment in information processing equipment. The assertion of the value of information processing systems may be true, but procedures have not yet been exhibited that could make proper use of the information such systems would make available. It should not be necessary to point out that the proposed systems should justify their installation against the best that could be done without the elaborate equipment, rather than against something that might be considered "current general practice."

7. A procedure in which priority is assigned according to the amount of "slack time" remaining for each of the operations not yet performed (945 S/OPN) is clearly superior to the other obvious due-date dependent rules, yet even this is somewhat disappointing in its ability to enforce an arbitrary set of due-dates. Combinations of this procedure with SHOPN would seem to have real potential as practical procedures that can simultaneously satisfy the numerous measures of performance important in manufacturing concerns.

Where due-dates can be assigned at the time of job arrival, an allowance proportional to the total processing time appears to be better than the other bases tested. It is conjectured (but was not tested) that a superior method of assignment would be to make the coefficient (by which the total processing time is multiplied to give the allowance) an increasing function of the number of jobs in the

shop at the time of arrival, rather than a constant. This should make additional allowance for the periods of congestion during which the methods tested did not perform well.

A basic limitation of experimental research, shared by the present study, is the difficulty in generalizing results and conclusions beyond the specific circumstances under which the experiments were performed. Strictly speaking, one must limit the conclusions of the present study to perfectly balanced and symmetric, randomly-routed networks of nine queues, in which the interarrival intervals and service times are exponentially distributed with a utilization of approximately 90 per cent. There was some minor exploration of the effect of changing the level of utilization, the number of machines, and the distribution of service times. Previous studies[16], [18] indicated the relative insensitivity of results to the number of machines and the routing pattern.

In fact, the whole point of the study was that the writer believes that results obtained for such a simplified system have much wider applicability. At least the grosser effects demonstrated in this study should indicate the performance attainable for many systems that are dimensionally much larger and logically much more complex. Even if the results and conclusions are not directly applicable, they should provide a useful point of departure for investigations oriented to the idiosyncracies of a real situation.

APPENDIX

Table 1

DEFINITION OF SYMBOLS USED IN PRIORITY RULES

t	time at which a selection for machine assignment is to be made
i	index over the jobs to be processed by the shop
j	index over the sequence of operations on a job
J	specific value of j; the operation for which a job is in queue
M_i	the total number of operations on the i-th job $1 \leq j \leq M_i$
$T_{i,j}$	the time at which the i-th job became ready for its j-th operation (time at which j-1 operation was finished) $T_{i,1}$ is the time at which the job arrived at the shop
D_i	the "due-date" (desired completion time) for the i-th job
$P_{i,j}$	the processing time for the j-th operation of the i-th job (including set-up and tear-down time, if any, assumed to be sequence independent)
$E_{i,j}$	an <u>a priori</u> estimate of $P_{i,j}$
$R_{i,j}$	a random variable, uniformly distributed between 0 and 1, assigned to the j-th operation of the i-th job
$N_{i,j}(t)$	the number of jobs, at time t, in the queue corresponding to the j-th operation of the i-th job
$W_{i,j}(t)$	the total work, at time t, in the queue corresponding to the j-th operation of the i-th job (total work is the sum of the imminent-operation processing times of the $N_{i,j}(t)$ jobs in the queue)
$X_{i,j}(t)$	the total work, at time t, which will "soon" arrive in the queue corresponding to the j-th operation of the i-th job (arrival of these jobs is imminent in the sense that, at time t, their preceding operation is being performed)

Table 1 -- Continued

$Q_k(t)$ the set of jobs in the k-th queue at time t

$V_i(t)$ the priority value of the i-th job at time t

The queueing discipline specifies that when a selection for machine assignment is to be made at time t from the k-th queue, a job I is chosen such that:

$$V_I(t) = \text{minimum}_{i \text{ in } Q_k(t)} (V_i(t))$$

In the event of a tie the job with minimum value which arrived in queue earliest is selected.

Table 2a

BASIC PRIORITY RULES

No.	Abbre-viation	Name	Definition
1	RANDOM	Random	$V_i(t) = R_{i,j}$
2	FCFS	First come, first served	$V_i(t) = T_{i,j}$
3	FASFS	First arrival at shop, first served	$V_i(t) = T_{i,1}$
4	DDATE	Due-Date	$V_i(t) = D_i$
5	FOPNR	Fewest operations remaining	$V_i(t) = M_i - J + 1$
6	MOPNR	Most operations remaining	$V_i(t) = -(M_i - J + 1)$
7	LHALF	Last half preference	$V_i(t) = 0$ if $(M_i - J + 1) \leq M_i/2$ $= 1$ if $(M_i - J + 1) > M_i/2$
8	FHALF	First half preference	$V_i(t) = 1$ if $(M_i - J + 1) \leq M_i/2$ $= 0$ if $(M_i - J + 1) > M_i/2$
9	SHOPN	Shortest operation	$V_i(t) = P_{i,J}$
10	LOPN	Longest operation	$V_i(t) = -P_{i,J}$
11	LSOPN	Longest subsequent operation	$V_i(t) = -P_{i,J+1}$ if $J < M_i$ $= -10.0$ if $J = M_i$
12	LWKR	Least work remaining	$V_i(t) = \sum_{j=J}^{M_i} P_{i,j}$
13	MWKR	Most work remaining	$V_i(t) = -\sum_{j=J}^{M_i} P_{i,j}$
14	TWORK	Greatest total work	$V_i(t) = -\sum_{j=1}^{M_i} P_{i,j}$
15	NINQ	Fewest jobs in next queue (resolve ties with SHOPN)	$V_i(t) = N_{i,J+1}(t)$
16	WINQ	Least work in next queue	$V_i(t) = W_{i,J+1}(t)$
17	XWINQ	Least expected work in next queue	$V_i(t) = W_{i,J+1}(t) + X_{i,J+1}(t)$

Table 2b

COMBINATION PRIORITY RULES

No.	Abbreviation	Description	Definition	Related to Basic Rules	Special Cases
405	OPNDD	Operation due-date rule; equally spaced due-dates are assigned to each operation at time job enters shop	$V_i(t) = T_{i,1} + (D_i - T_{i,1})(J/M_i)$	3,4,5	
4059	OPNDDP	Shortest operation unless job is behind operation due-date on entering queue	$V_i(t) = P_{i,J}$ if $T_{i,J} < (T_{i,1} + (D_i - T_{i,1})(J/M_i))$ $= 0$ otherwise	3,4,5,9	
413	SLACK	Slack Time Rule	$V_i(t) = D_i - t - \sum_{j=J}^{M_i} P_{i,j}$	4,13	
415	DDNINQ(a)	Earliest due-date which will move job out of shop or into a short queue; if none then simple due-date	$V_i(t) = D_i$ if $J = M_i$ $= D_i$ if $N_{i,J+1}(t) < a$ $= D_i + 10^6$ otherwise	4,15	$a \to \infty$, $\to$ DDATE
902	TSHOPN(a)	Truncated Shortest operation	$V_i(t) = P_{i,J}$ if $(t - T_{i,J}) < a$ $= 0$ if $(t - T_{i,J}) \geq a$	2,9	$a = 0$, FCFS $a \to \infty$, $\to$ SHOPN
906	P/OPNR(a)	Ratio: Processing time to number of operations remaining	$V_i(t) = P_{i,J}/(M_i - J + 1)^a$	6,9	$a=0$, SHOPN $a \to \infty$, MOPNR
911	P-SP(a)	Difference: Processing time less subsequent operation processing time	$V_i(t) = a \cdot P_{i,J} - (1-a) \cdot P_{i,J+1}$ if $J < M_i$ $= a \cdot P_{i,J} - (1-a) \cdot 10.0$ if $J = M_i$	9,11	$a=1$, SHOPN $a=0$, LSOPN

Table 2b -- Continued

912	P+WKR(a)	Sum: Processing time and work remaining	$V_i(t) = a \cdot P_{i,J} + (1-a) \sum_{j=J}^{M_i} P_{i,j}$	9,12	a=1, SHOPN a=0, LWKR
913	P/WKR(a)	Ratio: Processing time to work remaining	$V_i(t) = P_{i,J} / \left(\sum_{j=J}^{M_i} P_{i,j} \right)^a$	9,13	a=0, SHOPN a → ∞, → MWKR
914	P/TWK	Ratio: Processing time to total work on job	$V_i(t) = P_{i,J} / \sum_{j=1}^{M_i} P_{i,j}$	9,14	
915	P+NINQ(a)	Sum: Processing time and number of jobs in next queue	$V_i(t) = a \cdot P_{i,J} + (1-a) \cdot N_{i,J+1}(t)$	9,15	a=1, SHOPN a+0, NINQ
916	P+WQ/P(a,b)	Ratio: Sum of processing time and work in next queue to processing time on next operation	$V_i(t) = \dfrac{a \cdot P_{i,J} + (1-a) \cdot W_{i,J+1}(t)}{P_{i,J+1}^b}$ (If $J=M_i$, let $P_{i,J+1}=10.0$)	9,11,16	a=1, b=0, SHOPN a=0, b+0, WINQ
917	P+XWQ(a)	Sum: Processing time and expected work in next queue	$V_i(t) = a \cdot P_{i,J} + (1-a) \cdot (W_{i,J+1}(t) + X_{i,J+1}(t))$	9,17	a=1, SHOPN a=0, XWINQ
945	P+S/OPN(a,b)	Sum: Processing time and slack per operation	$V_i(t) = a \cdot P_{i,J} + (1-a) \cdot \dfrac{D_i - t - \sum_{j=J}^{M_i} P_{i,j}}{(M_i - J+1)^b}$	4,5,9,13	a=1, SHOPN a=0, b=0, SLACK
945f	P+S/OPNf(a,b)	Sum: Processing time and slack per operation; weighting a function of number of operations remaining	$V_i(t) = c \cdot P_{i,J} + (1-c) \cdot \dfrac{D_i - t - \sum_{j=J}^{M_i} P_{i,j}}{(M_i - J+1)^b}$ where: $c = a + \min((1-a), (M_i - J+1)/b)$	4,5,9,13	a=1, SHOPN b → 0, → SHOPN b → ∞, → P+S/OPN(a,1)

Table 2c

ALTERNATING PRIORITY RULES

No.	Abbrev.	Description	Definition	Component Rules	Special Cases
2-9	FCFS*S(a)	First come, first served with shortest operation overload protection	$V_i(t) = T_{i,J}$ if $N_{i,J}(t) < a$ $= P_{i,J}$ otherwise	2,9	a=1, SHOPN a→∞, →FCFS
10-9	LOPN*S(a)	Longest operation with shortest operation overload protection	$V_i(t) = -P_{i,J}$ if $N_{i,J}(t) < a$ $= P_{i,J}$ otherwise	9,10	a=1, SHOPN a→∞, →LOPN
15-9	NINQ*S(a)	Fewest jobs in next queue, if that queue is short enough (resolve ties by shortest operation); otherwise shortest operation	$V_i(t) = N_{i,J+1}(t)$ if $\min_{i \text{ in } Q} (N_{i,J+1}(t)) < a$ $= P_{i,J}$ otherwise	9,15	a=0, SHOPN a→∞, →NINQ
9-413	P*SLACK	Shortest operation, except that if a job is waiting for its last operation and lateness can be avoided by taking it out of order this is done	Let $U_i = D_i - t - P_{i,J}$ Let $P = \min_{i \text{ in } Q} (P_{i,J})$ Let $G = \min_{\text{overall } i \text{ with } J=M_i \text{ and } U_i > 0} (U_i)$, minimum If $G < P$ then $V_i(t) = U_i$, considering only i with $J = M_i$ and $U_i > 0$ If $G \geq P$ (or if G does not exist) $V_i(t) = P_{i,J}$ for all i	9,413	

Table 2d

SPECIAL PRIORITY RULES

No.	Abbrev.	Description	Definition	Related to Basic Rule	Special Cases
90	SHOPNE(±a %)	Selection based on an a priori estimate of processing time	$V_i(t) = E_{i,J}$ $E_{i,J} = P_{i,J} \cdot (1+.002a(R_{i,J}-.5))$	1,9	a=0, SHOPN a→∞, →RANDOM
92	2CLASS(a,p)	2 class, with chance of misclassification; FCFS within class, SHOPN between classes	$Prob(V_i(t) = 0 \mid P_{i,J} < a) = p$ $Prob(V_i(t) = 1 \mid P_{i,J} < a) = 1-p$ $Prob(V_i(t) = 0 \mid P_{i,J} \geq a) = 1-p$ $Prob(V_i(t) = 1 \mid P_{i,J} \geq a) = p$	2,90	a=0, p=0, FCFS a=0, p=1, FCFS a > 9.99, p=0, FCFS a > 9.99, p=1, FCFS
929	2CSHOPN(a)	2 class shortest operation	Let K b low-order digit of i, (the job number) $V_i(t) = P_{i,J}$ if $K < a$ $V_i(t) = 10.0 + P_{i,J}$ if $K \geq a$	9,92	a=0, SHOPN a=10, SHOPN
99	SHOPNH(a,b)	Shortest operation, with a "hold" if there are few jobs in queue and a job with shorter processing time will arrive soon	$V_i(t) = P_{i,J}$ Let $P = \min_{i \text{ in } Q} (P_{i,J})$ Make no assignment (hold machine idle) if $N_{i,J}(t) \leq a$ and if there is a job I now being processed for its J-1 operation that will come to this queue for its J-th operation with: $(T_{I,J} - t + P_{I,J}) < P-b$	9	a=0, SHOPN b > 9.99, SHOPN

Table 3a

MEASURES OF PERFORMANCE: JOB CHARACTERISTICS

Abbrev.	Description	Definition
Lateness	Job lateness	$L_i = C_i - D_i$
Mean	Mean of latenesses of sample jobs	$ML = \sum_{i=401}^{9100} L_i/8700$
Var	Variance of latenesses of sample jobs	$VL = \sum_{i=401}^{9100} L_i^2/8700 - ML^2$
NJL	Number of jobs late	Let $A_i = 1$ if $L_i > 0$ $= 0$ if $L_i \leq 0$ $NJL = \sum_{i=401}^{9100} A_i$
Shop Time	Total time of job in shop	$S_i = C_i - T_{i,1}$
Mean	Mean of shop times of sample jobs	$MS = \sum_{i=401}^{9100} S_i/8700$
Var	Variance of shop times of sample jobs	$VS = \sum_{i=401}^{9100} S_i^2/8700 - MS^2$

DEFINITION OF SYMBOLS:

i	index over the jobs to be processed by the shop.
j	index over the sequence of operations on a job.
J	specific value of j; the operation for which a job is in queue.
M_i	the total number of operations on the i-th job.
$P_{i,j}$	the processing time of the j-th operation of the i-th job.
D_i	The due-date of the i-th job.
C_i	the time at which the i-th job completes operation M_i and leaves the shop.
$T_{i,1}$	The time at which the i-th job arrives at the shop.

Table 3b

MEASURES OF PERFORMANCE: SHOP CHARACTERISTICS[a]

Abbrev.	Description	Definition
Jobs in Q	Total number of jobs in queue	$N(t)$ = total number of jobs in queue at time t.
Total work	Total processing time of all operations on all jobs in shop.	$TW(t) = \sum_{i} \sum_{j=1}^{M_i} P_{i,j}$ summing over all jobs in shop at time t.
Work rem	Total processing time of all operations not yet started on all jobs in shop.	$WR(t) = \sum_{i} \sum_{j=J}^{M_i} P_{i,j}$ summing over all jobs in shop at time t.
I-O WinQ	Sum of the imminent-operation processing times of all jobs in queue.	$WQ(t) = \sum_{i} P_{i,J}$ summing over all jobs in queue at time t.

[a]Each of these measures of performance is a function of time. In each case a "mean" and a "variance" are reported. These are obtained in the following manner:

If Z(t) is the basic measure of performance, a function of time, let

$$IZ(a) = \frac{\int_{400a}^{400(a+1)} Z(t)dt}{400} \quad \text{for } a = 1, 2, \ldots, 26.$$

Then $$\text{MEAN } Z = \sum_{a=1}^{26} IZ(a)/26;$$

$$\text{VAR } Z = \sum_{a=1}^{26} IZ(a)^2/26 - (\text{MEAN } Z)^2.$$

Table 4

METHODS OF ASSIGNING DUE DATES

Designation	Description	Definition
I	Allowance proportional to the total work to be done on the job.	$D_i = T_{i,1} + 9 \sum_{j=1}^{M_i} P_{i,j}$
II	Allowance proportional to the number of operations on the job.	$D_i = T_{i,1} + 8.883 \cdot M_i$
III	Constant allowance, the same for all jobs	$D_i = T_{i,1} + 78.7985$
IV	Random allowance	$D_i = T_{i,1} + 157.597 \cdot U_i$

DEFINITION OF SYMBOLS:

D_i the due-date of the i-th job.

$T_{i,1}$ time at which the i-th job arrived at the shop.

M_i the number of operations on the i-th job.

$P_{i,j}$ the processing time for the j-th operation on the i-th job.

U_i a random variable, uniformly distributed between 0 and 1, assigned to the i-th job.

Table 5

DESCRIPTION OF JOB TAPES

Name	No. of Jobs	Routing	Processing Time Distribution	Interarrival Intervals		Time at End Tape	Seed
				1-49	50-		
R1	10,000	Random	Trun. Exp.[a] (1,1) 0.01-9.99		Exp. (1.0901, 1.0901)	10,955.69	13579
R2	10,000	Random	Trun. Exp.	0	Exponential	10,847.61	31995
R3	10,000	Random	Trun. Exp.	0	Exponential	10,902.93	11111
R4	10,000	Random	Trun. Exp.	0	Exponential	10,860.22	13531

[a]Truncated exponential.

Table 6

DESCRIPTION OF TEST CONDITIONS

Name	Job Tape	Type of Due Date	Processing Time Distribution	Scale Factor	No. of Machines
Std	R1	I	Exponential (1,1)	1.0	9
Rep 2	R2	I	Exponential	1.0	9
Rep 3	R3	I	Exponential	1.0	9
Rep 4	R4	I	Exponential	1.0	9
DD II	R1	II	Exponential	1.0	9
DD III	R1	III	Exponential	1.0	9
DD IV	R1	IV	Exponential	1.0	9
Scale 0.877	R1	I	Exponential	0.877	9
Rect PTM	R1		Rectangular (1,0.33)	1.0	9
Cons PTM	R1	I	Constant (1,0)	1.0	9
3 Mach	R1		Exponential	1.0	3
5 Mach	R1		Exponential	1.0	5
7 Mach	R1		Exponential	1.0	7

Table 7a

EXPERIMENTAL RESULTS -- SHOP STATISTICS

	Rule	Conditions	Jobs in Q		Total Work		Work Rem		1-QWINQ	
			Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
1	RANDOM	Std (Seed=1)	56.86	317.89			537	23548	57.09	335.46
1	RANDOM	Std (Seed=1357)	59.42	372.84	1080	100403	554	24494	58.39	392.22
2	FCFS	Std	58.87	335.87	1078	101639	559	27104	58.29	325.70
3	FASFS	Std	57.51	259.15	777	34327	563	23557	57.44	276.61
4	DDATE	Std	49.45	138.41			842	33027	53.72	180.16
5	FOPNR	Std	52.23	96.19			1019	18493	51.83	84.38
6	MOPNR	Std	72.30	1158.70			252	3916	70.76	1148.95
7	LHALF	Std	58.63	211.94			722	34741	58.53	210.12
8	FHALF	Std	62.49	542.62			391	14374	61.98	556.40
9	SHOPN	Std	23.25	41.45	545	19129	297	6730	62.54	635.52
10	LOPN	Std	167.44	1021.87	2709	232933	1369	72087	59.08	79.86
11	LSOPN	Std	54.30	256.54	1007	92057	527	24367	53.72	275.74
12	LWKR	Std	47.52	85.14			989	48217	51.81	93.53
13	MWKR	Std	109.97	3178.06			276	5595	69.07	951.49
14	TWORK	Std	82.91	1445.38	587	25452	323	8964	65.16	730.29
15	NINQ	Std	31.20	101.77			358	11607	42.20	257.35
16	WINQ	Std	40.43	170.12	801	59873	421	16766	39.16	165.69
17	XWINQ	Std	34.03	105.08	709	41563	379	12788	41.26	244.00
405	OPNDD	Std	55.05	333.19			483	26751	57.62	340.51
4059	OPNDDP	Std	58.78	306.40			557	24809	58.29	296.08
413	SLACK	Std	51.25	172.97			866	3869	55.56	228.44
415	DDNINQ(1)	Std	40.25	113.02			598	25357	42.35	135.21
902	TSHOPN(4)	Std	55.67	408.13			534	25096	57.77	295.28
902	TSHOPN(8)	Std	53.50	309.81			520	24955	60.32	298.42
902	TSHOPN(16)	Std	44.20	333.85	863	98135	452	26550	60.54	386.37
902	TSHOPN(32)	Std	32.85	197.00	664	98229	368	17672	61.36	521.18
906	P/OPNR(0.5)	Std	29.17	97.39			238	3432	69.33	969.30
906	P/OPNR(1.0)	Std	35.42	168.55			227	2967	70.59	992.32
911	P-SP(0.5)	Std	26.11	52.91	593	23787	324	8188	60.32	677.88
911	P-SP(0.9)	Std	23.72	44.31	560	21960	309	7476	63.60	737.77
911	P-SP(0.95)	Std	23.15	41.64	547	20037	302	7258	61.94	652.05
911	P-SP(0.97)	Std	23.25	41.01	545	20451	298	6772	62.37	656.01
912	P+WKR(0.8264)	Std	29.75	46.66	869	38648	604	24617	54.73	287.63
912	P+WKR(0.9091)	Std	25.05	35.61	681	23931	438	12381	57.09	403.73
912	P+WKR(0.9524)	Std	23.59	39.09	603	22424	359	9834	60.39	577.68
912	P+WKR(0.9756)	Std	23.06	37.07	565	19688	321	7766	60.92	570.75
912	P+WKR(0.9852)	Std	22.98	40.12	552	19800	309	7556	61.03	632.61
913	P/WKR(0.05)	Std	23.48	42.34	539	18817	290	6239	63.80	658.94
913	P/WKR(0.1)	Std	23.37	44.48	530	19100	279	5732	62.89	693.77
913	P/WKR(0.25)	Std	24.85	54.98	524	19060	260	4748	65.35	797.14
913	P/WKR(0.5)	Std	29.63	94.14	540	21860	235	3348	70.58	966.83
913	P/WKR(0.75)	Std	37.99	195.45	597	32649	224	2936	73.77	1134.64
913	P/WKR(1.0)	Std	62.94	878.94			220	2765	73.79	1106.22
913	P/WKR(1.2)	Std	83.23	1979.16	941	177093	222	2988	74.21	1147.07
913	P/WKR(2.0)	Std	90.59	2225.59	1010	201555	229	3221	72.95	1108.34
914	P/TWK	Std	42.39	298.72	435	9138	244	3796	68.60	923.16
915	P+NINQ(0.5)	Std	28.28	83.00	622	33303	335	10052	46.07	280.90
915	P+NINQ(0.87)	Std	22.94	37.45	543	18367	298	6614	59.20	529.90
915	P+NINQ(0.90)	Std	22.77	36.62	543	17741	299	6483	60.00	539.69
915	P+NINQ(0.9232)	Std	22.78	39.07	539	18576	295	6506	60.50	581.90
915	P+NINQ(0.97)	Std	23.24	41.54	544	19295	298	6939	62.54	644.67
916	P+WQ/P(0.5,0)	Std	30.14	103.34	649	39059	346	11364	43.35	259.93
916	P+WQ/P(0.667,0)	Std	28.68	96.61	627	36729	336	10690	45.12	277.19
916	P+WQ/P(0.9,0)	Std	23.76	49.09	552	21945	303	7540	55.22	427.44
916	P+WQ/P(0.95,0)	Std	23.00	41.78	545	20334	299	7307	58.49	509.34
916	P+WQ/P(0.96,0)	Std	23.13	43.73	544	20371	300	7231	60.25	597.59
916	P+WQ/P(0.97,0)	Std	22.83	39.59	539	19263	294	6669	59.98	562.86
916	P+WQ/P(0.5,1.0)	Std	42.17	230.30	827	77667	436	21384	50.87	331.13
916	P+WQ/P(1.0,1.0)	Std	36.14	136.17	737	50113	393	15047	58.26	470.54
916	P+WQ/P(1.0,0.1)	Std	23.88	49.55	557	22170	305	7610	63.50	745.67
917	P+XWQ(0.94)	Std	23.26	46.01	552	22039	303	7812	57.89	513.25
917	P+XWQ(0.96)	Std	22.67	39.01	536	18212	293	6439	58.40	513.94
917	P+XWQ(0.98)	Std	22.74	38.18	541	18837	295	6598	59.96	561.95
945	P+S/OPN(0,1.0)	Std	51.65	71.31			538	33562	56.80	88.41

Table 7a -- Continued

Rule	Conditions	Jobs in Q		Total Work		Work Rem.		I-O WINQ	
		Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
)45 P+S/OPN(0.3,1.0)	Std	47.82	)7.1)	1023	55510	445	25134	57.)8	202.)4
)45 P+S/OPN(0.5,1.0)	Std	44.36	110.83	)66	45194	376	16233	58.86	31).04
)45 P+S/OPN(0.75,1.0)	Std	3).60	IOI.48	875	44552	314	)335	61.24	474.12
)45 P+S/OPN(0,0.5)	Std	50.47	103.)2	1088	27139	745	35163	55.38	149.44
)45 P+S/OPN(0,0.8)	Std	51.06	58.28	1085	18463	615	36242	56.41	73.53
)45 P+S/OPN(0,1.2)	Std	51.86	)6.46	1067	35189	474	31242	56.76	112.)6
)45 P+S/OPN(0.5,0.8)	Std	43.79	82.73	)80	31566	443	21177	57.48	215.59
)45f P+S/OPNf(0,4.0)	Std	44.22	173.17	877	70038	278	7125	64.43	617.89
)45f P+S/OPNf(0,10.0)	Std	48.38	161.63	)83	64713	322	11435	61.45	459.28
)45f P+S/OPNf(0.3,10.0)	Std	45.89	165.83	932	69498	291	8024	64.14	578.10
2-) FCFS*S(5)	Std	2).49	51.75			342	7521	61.59	689.63
2-) FCFS*())	Std	38.67	90.06			407	10476	59.08	583.71
10-) LOPN*S(5)	Std	31.89	47.26	675	23221	360	7655	60.42	630.00
15-) NINQ*S(1)	Std	28.36	59.53			346	8)64	51.)9	451.93
15-) NINQ*S(2)	Std	2).64	72.59			353	)864	49.20	403.36
9.413 P*SLACK	Std	23.26	43.23	547	21102	300	7459	62.54	651.33
)0 SHOPNE(±10%)	Std	23.23	41.49	546	18973	297	6616	62.50	643.98
)0 SHOPNE(±100%)	Std	27.13	59.83	606	25214	327	8596	62.40	674.86
)2 2CLASS(0.98,0.75)	Std	44.))	216.34	873	69619	458	19644	58.98	432.01
)2 2CLASS(0.98,1.00)	Std	35.2)	146.78	725	49179	385	14647	60.)5	534.67
929 2CSHOPN(1)	Std	24.46	44.69	561	21184	306	7179	62.32	657.91
929 2CSHOPN(3)	Std	27.29	52.02	602	21670	326	7394	62.09	622.87
929 2CSHOPN(5)	Std	31.17	71.34	652	28734	352	9081	62.12	652.88
929 2CSHOPN(7)	Std	34.57	103.41	696	31702	375	10629	59.74	613.17
929 2CSHOPN(9)	Std	36.11	148.16	705	34768	384	12453	59.92	525.17
)) SHOPNH(1,0)	Std	23.98	46.44	561	21016	308	7099	69.04	753.48
)) SHOPNH(1,1)	Std	23.47	45.86	550	21061	303	7285	66.92	733.87
)) SHOPNH(100,0)	Std	60.66	242.15	1098	)6730	649	40779	206.23	6349.70
2 FCFS	Rep 2	75.04	361.43	1331	103376	700	40830	75.05	297.73
9 SHOPN	Rep 2	28.46	67.01	640	28327	356	11048	86.24	1442.62
911 P-SP(0.)5)	Rep 2	28.33	62.37	641	27495	358	11214	84.90	1305.18
)12 P+WKR(0.)852)	Rep 2	28.07	62.67	650	28135	372	12082	83.66	1377.47
)16 P+WQ/P(0.97,0)	Rep 2	27.91	68.73	631	28359	351	10527	80.99	1315.20
)45 P+S/OPN(0,1)	Rep 2	61.96	85.42	1218	24947	812	53805	67.49	117.56
2 FCFS	Rep 3	92.16	971.01	1634	262686	854	74602	92.33	1015.09
9 SHOPN	Rep 3	33.08	118.60	754	52336	424	19327	108.90	2609.02
911 P-SP(0.)5)	Rep 3	33.24	122.84	768	56977	435	22007	108.72	2714.15
)12 P+WKR(0.)852)	Rep 3	33.42	121.04	778	53814	450	22535	110.36	2723.71
)16 P+WQ/P(0.97,0)	Rep 3	33.92	137.38	778	61078	437	23098	104.50	2211.75
)45 P+S/OPN(0,1)	Rep 3	75.57	306.35	1469	87137	1064	113830	81.92	308.50
9 SHOPN	Rep 4	26.50	85.58	608	36500	340	14863	77.52	1483.99
)12 P+WKR(0.)852)	Rep 4	26.61	86.06	626	39985	358	17347	77.76	1494.92
)16 P+WQ/P(0.97,0)	Rep 4	26.54	)0.57	605	37184	337	14803	74.84	1373.55
2 FCFS	DD II	58.87	335.87	1078	101639	559	27104	58.29	325.70
4 DDATE	DD II	55.54	204.24	1135	45621	868	37322	55.99	229.31
9 SHOPN	DD II	23.25	41.45	545	19129	297	6730	62.54	635.52
10 LOPN	DD II	167.44	1021.87	2709	292333	1369	72087	59.08	79.86
)45 P+S/OPN(0,1)	DD II	57.52	109.38	1097	31702	653	63808	56.85	127.65
2 FCFS	DD III	58.87	335.87	1078	101639	559	27104	58.29	325.70
4 DDATE	DD III	57.51	259.15			563	23557	57.44	276.61
9 SHOPN	DD III	23.25	41.45	545	19129	297	6730	62.54	635.52
)45 P+S/OPN(0,1)	DD III	58.44	171.29			467	30134	57.52	167.83
2 FCFS	DD IV	58.87	335.87	1078	101639	559	27104	58.29	325.70
4 DDATE	DD IV	57.94	268.48	798	35932	569	24938	57.88	257.83
9 SHOPN	DD IV	23.25	41.45	545	19129	297	6730	62.54	635.52
)45 P+S/OPN(0,1)	DD IV	58.86	210.08	759	37742	485	35520	58.01	202.76
2 FCFS	Scale 0.877	23.58	42.03			220	2)06	20.40	32.65
9 SHOPN	Scale 0.877	12.01	5.98			147	786	21.35	38.45
2 FCFS	Rect PTM	40.20	183.71					39.)1	180.50
9 SHOPN	Rect PTM	24.87	72.67					41.09	247.57
90 SHOPNE(±10%)	Rect PTM	25.19	78.86					41.25	260.82
90 SHOPNE(±100%)	Rect PTM	27.79	92.15					41.47	251.66
2 FCFS	Cons PTM	30.74	133.76	622	41832	322	11506	30.65	133.00
9 SHOPN	3 Mach	8.40	11.)5					24.55	241.61
9 SHOPN	5 Mach	13.13	19.13					36.44	324.01
9 SHOPN	7 Mach	18.15	26.41					49.)8	424.85

Table 7b

Experimental Results -- Job Statistics

Rule		Conditions	Shop Time		Lateness		NJL
			Mean	Variance	Mean	Variance	
1	RANDOM	Std (Seed=1)	71.49	9611.4	- 7.43	5990.2	2616
1	RANDOM	Std (Seed=1357)	74.70	10821.7	- 4.22	6913.5	2675
2	FCFS	Std	74.43	5739.2	- 4.49	1686.2	3897
3	FASFS	Std	72.52	1565.2	- 6.41	3358.6	4599
4	DDATE	Std	63.72	6780.3	-15.53	431.5	1544
5	FOPNR	Std	67.07	23925.5	-11.85	11510.8	1094
6	MOPNR	Std	86.90	5815.1	7.98	7855.2	4300
7	LHALF	Std	74.10	5438.7	- 4.82	1861.1	3856
8	FHALF	Std	77.87	7577.8	- 1.05	2701.0	3567
9	SHOPN	Std	34.02	2318.0	-44.90	2878.2	437
10	LOPN	Std	196.43	55980.9	117.51	37919.3	5957
11	LSOPN	Std	68.76	9287.5	-10.16	5878.0	2368
12	LWKR	Std	61.75	20746.3	-17.17	8948.9	834
13	MWKR	Std	125.09	21928.0	46.17	24001.9	4663
14	TWORK	Std	98.55	15223.4	19.64	25059.9	4171
15	NINQ	Std	43.09	3076.9	-35.83	2405.8	984
16	WINQ	Std	53.28	4154.5	-25.64	2473.7	1778
17	XWINQ	Std	46.30	3380.7	-32.62	2447.1	1220
405	OPNDD	Std	68.99	28820.3	- 9.90	14560.2	901
4059	OPNDDP	Std	74.12	5657.8	- 4.80	1667.9	3884
413	SLACK	Std	65.79	6524.1	-13.13	432.6	1916
415	DININQ(1)	Std	53.53	5541.7	-25.39	867.9	926
902	TSHOPN(4)	Std	70.58	5254.4	- 8.34	1670.8	3455
902	TSHOPN(8)	Std	68.24	5056.2	-10.68	1667.6	3205
902	TSHOPN(16)	Std	57.95	4088.3	-20.97	1791.3	2163
902	TSHOPN(32)	Std	45.27	2740.6	-33.65	1920.5	1127
906	P/OPNR(0.5)	Std	40.33	2852.1	-38.59	3469.0	1163
906	P/OPNR(1.0)	Std	47.11	3407.8	-31.81	5324.4	1906
911	P-SP(0.5)	Std	37.32	2629.6	-41.60	2640.0	552
911	P-SP(0.9)	Std	34.58	2600.6	-44.35	2835.8	415
911	P-SP(0.95)	Std	33.95	2382.4	-44.97	2806.4	422
911	P-SP(0.97)	Std	34.11	2411.7	-44.81	2862.8	412
912	P+WKR(0.8264)	Std	41.66	7817.5	-37.26	2871.9	444
912	P+WKR(0.9091)	Std	36.31	3898.3	-42.61	1961.1	344
912	P+WKR(0.9524)	Std	34.54	2976.8	-44.38	2335.9	351
912	P+WKR(0.9756)	Std	33.96	2551.6	-44.96	2579.4	394
912	P+WKR(0.9852)	Std	33.81	2378.2	-45.11	2653.8	395
913	P/WKR(0.05)	Std	34.32	2336.1	-44.61	2970.9	474
913	P/WKR(0.1)	Std	34.10	2220.5	-44.82	3019.2	486
913	P/WKR(0.25)	Std	35.73	2267.0	-43.19	3434.4	687
913	P/WKR(0.5)	Std	40.87	2663.9	38.05	4319.8	1384
913	P/WKR(0.75)	Std	49.98	3434.5	-28.94	5443.1	2308
913	P/WKR(1.0)	Std	76.41	8980.1	- 2.50	10902.6	3404
913	P/WKR(1.2)	Std	96.96	14418.5	17.77	17395.1	3839
913	P/WKR(2.0)	Std	104.32	16096.3	25.40	19048.3	4081
914	P/TWK	Std	54.31	4224.1	-24.61	10082.0	2681
915	P+NINQ(0.5)	Std	39.63	2654.1	-39.29	2561.7	756
915	P+NINQ(0.87)	Std	33.61	2229.4	-45.31	2704.3	413
915	P+NINQ(0.90)	Std	33.53	2262.3	-45.39	2701.4	367
915	P+NINQ(0.9232)	Std	33.52	2284.2	-45.40	2783.7	400
915	P+NINQ(0.97)	Std	34.01	2354.3	-44.91	2835.5	435
916	P+WQ/P(0.5,0)	Std	41.99	2983.9	-36.93	2539.3	915
916	P+WQ/P(0.667,0)	Std	40.14	2749.6	-38.78	2501.0	814
916	P+WQ/P(0.9,0)	Std	34.68	2207.5	-44.24	2615.8	454
916	P+WQ/P(0.95,0)	Std	33.80	2291.6	-45.13	2716.4	403
916	P+WQ/P(0.96,0)	Std	34.02	2443.8	-44.90	2798.9	398
916	P+WQ/P(0.97,0)	Std	33.63	2223.6	-45.29	2746.1	394
916	P+WQ/P(0.5,1.0)	Std	55.27	6184.0	-23.15	4304.7	1689
916	P+WQ/P(1.0,1.0)	Std	48.48	5184.2	-30.44	4074.9	1198
916	P+WQ/P(1.0,0.1)	Std	34.77	2578.2	-44.15	2916.7	424
917	P+XWQ(0.94)	Std	34.07	2265.0	-44.85	2594.2	387
917	P+XWQ(0.96)	Std	33.35	2169.0	-45.57	2733.0	374
917	P+XWQ(0.96)	Std	33.54	2247.0	-45.38	2702.3	390
945	P+X/OPN(0,1.0)	Std	66.13	5459.6	-12.79	226.2	323

Table 7b -- Continued

Rule		Conditions	Shop Time		Lateness		NJL
			Mean	Variance	Mean	Variance	
945	P+S/OPN(0.3,1.0)	Std	61.67	5126.8	-17.25	368.3	158
945	P+S/OPN(0.5,1.0)	Std	57.62	4758.1	-21.30	518.4	132
945	P+S/OPN(0.75,1.0)	Std	52.20	4291.1	-26.72	695.9	172
945	P+S/OPN(0,0.5)	Std	64.96	5871.2	-13.96	232.3	1175
945	P+S/OPN(0,0.8)	Std	65.53	5613.6	-13.39	155.5	496
945	P+S/OPN(0,1.2)	Std	66.10	5287.4	-12.82	350.4	335
945	P+S/OPN(0.5,0.8)	Std	57.24	4953.0	-21.68	340.9	137
945f	P+S/OPNf(0,4.0)	Std	57.06	3908.0	-21.86	1482.5	525
945f	P+S/OPNf(0,10.0)	Std	61.67	4656.9	-17.25	854.2	329
945	P+X/OPNf(0.3,10.0)	Std	58.78		-20.14	1102.5	376
2-9	FCFS*S(5)	Std	41.04	2543.2	-37.88	2203.1	686
2-9	FCFS*S(9)	Std	51.27	2983.7	-27.65	1576.8	1566
10-9	LOPN*S(5)	Std	43.75	2895.4	-35.17	2258.9	1008
15-9	NINQ*S(1)	Std	39.83	2740.6	-39.02	2320.7	642
15-9	NINQ*S(2)	Std	41.21	2907.5	-37.71	2372.0	750
9-413	P*SLACK	Std	34.03	2356.1	-44.89	2781.4	364
90	SHOPNE(+10%)	Std	34.00	2399.5	-44.92	2844.3	439
90	SHOPNE(±100%)	Std	38.32	3018.2	-40.60	2963.1	666
92	2CLASS(0.98,0.75)	Std	58.40	4176.9	-20.52	1867.0	2221
92	2CLASS(0.98,1.00)	Std	47.77	2785.9	-31.15	1743.3	1139
929	2CSHOPN(1)	Std	35.45	2505.2	-43.47	2854.3	525
929	2CSHOPN(3)	Std	38.68	3069.3	-40.24	3132.9	840
929	2CSHOPN(5)	Std	42.84	4162.3	-36.08	3908.5	1331
929	2CSHOPN(7)	Std	46.63	6476.7	-32.29	5918.9	1570
929	2CSHOPN(9)	Std	48.25	12130.4	-30.67	11641.0	904
99	SHOPNH(1,0)	Std	34.87	2666.6	-44.05	2952.4	455
99	SHOPNH(1,1)	Std	34.34	2532.4	-44.58	2925.5	434
99	SHOPNH(100,0)	Std	64.45	30225.0	-14.45	24412.7	1092
2	FCFS	Rep 2	90.64	8164.5	11.27	2121.8	5017
9	SHOPN	Rep 2	39.80	5124.1	-39.57	4648.5	530
911	P-SP(0.95)	Rep 2	30.70	5014.9	-39.67	4464.6	515
912	P+WKR(0.9852)	Rep 2	39.37	5228.8	-40.00	4462.4	507
916	P+WQ/P(0.97,0)	Rep 2	39.24	4821.1	-40.13	4424.3	512
945	P+S/OPN(0,1)	Rep 2	76.23	6274.0	- 3.14	204.7	2691
2	FCFS	Rep 3	112.94	13258.6	31.39	4542.9	5858
9	SHOPN	Rep 3	46.27	13893.2	-35.29	12186.7	632
911	P-SP(0.95)	Rep 3	46.54	14096.7	-35.01	12115.1	619
912	P+WKR(0.9852)	Rep 3	46.75	14643.1	-34.80	12505.3	613
916	P+WQ/P(0.97,0)	Rep 3	47.25	12980.5	-34.30	10934.7	666
945	P+S/OPN(0,1)	Rep 3	93.50	9238.1	11.94	1029.2	4763
9	SHOPN	Rep 4	37.06	3982.5	-41.39	3862.7	514
912	P+WKR(0.9852)	Rep 4	37.35	4187.4	-41.10	3704.8	493
916	P+WQ/P(0.97,0)	Rep 4	37.24	3780.9	-41.21	3667.0	484
2	FCFS	DD II	74.43	5739.2	- 3.91	1125.6	3474
4	DDATE	DD II	70.62	6838.3	- 7.72	529.1	2322
9	SHOPN	DD II	34.02	2318.0	-44.32	2930.3	537
10	LOPN	DD II	196.43	55980.9	118.09	25073.9	5960
945	P+S/OPN(0,1)	DD II	72.94	5548.0	- 5.41	225.6	1878
2	FCFS	DD III	74.43	5739.2	- 4.37	5739.2	2937
4	DDATE	DD III	72.52	1565.2	- 6.28	1565.2	3819
9	SHOPN	DD III	34.02	2318.0	-44.78	2318.0	957
945	P+S/OPN(0,1)	DD III	73.66	1118.1	- 5.14	1118.1	4183
2	FCFS	DD IV	74.43	5739.2	- 4.87	7728.6	3585
4	DDATE	DD IV	72.89	2902.5	- 6.41	2100.8	4244
9	SHOPN	DD IV	34.02	2318.0	-45.28	4416.1	1726
945	P+S/OPN(0,1)	DD IV	73.98	2589.7	- 5.32	1718.1	4573
2	FCFS	Scale 0.877	33.40	1137.4	-35.82	1695.1	947
9	SHOPN	Scale 0.877	20.73	552.6	-48.48	2310.0	163
2	FCFS	Rect PTM	52.75	2849.8			
9	SHOPN	Rect PTM	35.62	2794.0			
90	SHOPNE(+10%)	Rect PTM	35.81	2757.6			
90	SHOPNE(±100%)	Rect PTM	38.73	3054.0			
2	FCFS	Cons PTM	42.63	1921.8	-36.54	1506.9	476
9	SHOPN	3 Mach	14.22	918.7			
9	SHOPN	5 Mach	20.04	1123.6			
9	SHOPN	7 Mach	27.05	1768.4			

Table 8

SUMMARY: EXTREMES OF PERFORMANCE
(Std Conditions)

Criterion Minimum	Item	Best	2nd Best	Best Perf for Rule w/o P_1	Worst	Ratio: Worst & Best
Mean Jobs in Q	Value	22.67	22.77	34.03	167.44	7.39
	Rule	917 P+XWQ(0.96)	915 P+NINQ(0.90)	17 XWINQ	10 LOPN	
Mean Total Work	Value	435	524	587	2709	6.23
	Rule	914 P/TWK	913 P/WKR(0.25)	14 TWORK	10 LOPN	
Mean Work Rem	Value	220	227	252	1369	6.22
	Rule	913 P/WKR(1.0)	906 P/OPNR(1.0)	6 MOPNR	10 LOPN	
Mean Work Completed	Value	190	213	213	1340	7.04
	Rule	914 P/TWK	3 FASFS	3 FASFS	10 LOPN	
Mean I-O WINQ	Value	39.16	41.26	39.16	206	5.27
	Rule	16 WINQ	17 XWINQ	16 WINQ	99 SHOPNH(100,0)	
Variance Shop Time	Value	1565	2169	1565	59980	35.77
	Rule	3 FASFS	917 P+XWQ(0.96)	3 FASFS	10 LOPN	
Variance Lateness	Value	155.5	431.5	155.5	37919	24.39
	Rule	945 P+S/OPN(0,0.8)	4 DDATE	945 P+S/OPN(0,0.8)	10 LOPN	
% No. Jobs Late	Value	1.52%	3.95%	3.71%	68.47%	45.05%
	Rule	945 P+S/OPN(0.5,1)	912 P+WKR(0.9091)	945 P+S/OPN(0,1.0)	10 LOPN	

Table 9

SUMMARY: BEST PERFORMANCE OF RULES OF DIFFERENT INFORMATION CLASSIFICATIONS
(Std Conditions)

Criterion Minimum	Item	Information Classification			
		Local Operation	Local Job	Global Current	Global Prediction
Mean Jobs in Q	Value	23.25	22.98	22.77	22.67
	Rule	9 SHOPN	912 P+WKR(0.9852)	915 P+NINQ(0.90)	917 P+XWQ(0.96)
Mean Total Work	Value	545	435	435	435
	Rule	9 SHOPN	914 P/TWK	914 P/TWK	914 P/TWK
Mean Work Rem	Value	297	220	220	220
	Rule	9 SHOPN	913 P/WKR(1.0)	913 P/WKR(1.0)	913 P/WKR(1.0)
Mean I-O WinQ	Value	57.62	51.81	39.16	39.16
	Rule	405 OPNDD	12 LWKR	16 WINQ	16 WINQ
Variance Shop Time	Value	2318	1565	1565	1565
	Rule	9 SHOPN	3 FASFS	3 FASFS	3 FASFS
Variance Lateness	Value	1577	156	156	156
	Rule	2-9 FCFS*S(9)	945 P+S/OPN(0,0.8)	945 P+S/OPN(0,0.8)	945 P+S/OPN(0,0.8)
% No Jobs Late	Value	5.02%	1.52%	1.52%	1.52%
	Rule	9 SHOPN	945 P+S/OPN(0.5,1)	945 P+S/OPN(0.5,1)	945 P+S/OPN(0.5,1)

Table 10

CLASSIFICATION OF PRIORITY RULES BY INFORMATION REQUIREMENTS

Rule		Information Required
	A. LOCAL, OPERATION[a]	
1	RANDOM	None.
2	FCFS	Time job arrived in this queue.
9	SHOPN	Processing time of the imminent operation.
10	LOPN	Processing time of the imminent operation.
405	OPNDD	Due-date of the imminent operation (Job due-date, number of operations on the job, and time of job arrival at shop are required to calculate the operation due-date, but it may be considered an attribute of the operation which is determined at time job enters shop.)
4059	OPNDDP(a)	Time job arrived in this queue. Processing time of the imminent operation. Due-date of the imminent operation.
902	TSHOPN(a)	Time job arrived in this queue. Processing time of the imminent operation.
2-9	FCFS*S(a)	Time job arrived in this queue. Processing time of the imminent operation. Number of jobs in this queue.
10-9	LOPN*S(a)	Time job arrived in this queue. Processing time of the imminent operation. Number of jobs in this queue.
90	SHOPNE(+a%)	An estimate of the processing time of the imminent operation.
92	2CSHOPN(a,p)	An estimate of the processing time of the imminent operation.
	B. LOCAL, JOB[b]	
3	FASFS	Time job arrived in the shop.
4	DDATE	Due-date of the job.
5	FOPNR	Number of operations remaining on the job.
6	MOPNR	Number of operations remaining on the job.
7	LHALF	Total number of operations on the job. Number of operations remaining on the job.
8	FHALF	Total number of operations on the job. Number of operations remaining on the job.
11	LSOPN	Processing time of the next-after-imminent operation.
12	LWKR	Sum of the processing times for all operations remaining on the job.
13	MWKR	Sum of the processing times for all operations remaining on the job.
14	TWORK	Sum of the processing times for all operations on the job.
413	SLACK	Due-date of the job. Sum of the processing times for all operations remaining on the job.
906	P/OPNR(a)	Processing time of the imminent operation. Number of operations remaining on the job.
911	P-SP(a)	Processing time of the imminent operation. Processing time of the next-after-imminent operation.
912	P+WKR(a)	Processing time of the imminent operation. Sum of the processing times for all operations remaining on the job.
913	P/WKR(a)	Processing time of the imminent operation. Sum of the processing times for all operations remaining on the job.
914	P/TWK	Processing time of the imminent operation. Sum of the processing times for all operations on the job.
945	P+S/OPN(A,b)	Processing time of the imminent operation. Sum of the processing times for all operations remaining on the job. Number of operations remaining on the job. Due-date of the job.

Table 10 -- Continued

	Rule	Information Required
		LOCAL, JOB (Continued)
945f	P+S/OPNf(a,b)	Processing time of the imminent operation. Sum of the processing times for all operations remaining on the job. Number of operations remaining on the job. Due-date of the job.
9-413	P*SLACK	Processing time of the imminent operation. Identification of jobs waiting for their last operation. Due-date of jobs waiting for their last operation.
		C. GLOBAL, CURRENT STATUS[c]
15	NINQ	Number of jobs in every queue which will receive (after the imminent operation) a job which is in the present queue. Processing time of the imminent operation (to resolve ties).
16	WINQ	Sum of the imminent operation processing times of the jobs in every queue which will receive (after the imminent operation) a job which is in the present queue.
415	DDNINQ(a)	Number of jobs in every queue which will receive (after the imminent operation) a job which is in the present queue. Due-date of the job.
915	P+NINQ(a)	Processing time of the imminent operation. Number of jobs in every queue which will receive (after the imminent operation) a job which is in the present queue.
916	P+WQ/P(a,b)	Processing time of the imminent operation. Sum of the imminent operation processing times of the jobs in every queue which will receive (after the imminent operation) a job which is in the present queue.
15-9	NINQ*S9(a)	Processing time of the imminent operation. Number of jobs in every queue which will receive (after the imminent operation) a job which is in the present queue.
		D. GLOBAL, PREDICTED STATUS[d]
17	XWINQ	Sum of the imminent operation processing times of the jobs in every queue which will receive (after the imminent operation) a job which is in the present queue. Processing time of the next-after-current operation of jobs which are now on machines and which, after completing the current operation, will move to a queue which will receive a job (after the imminent operation) which is in the queue from which selection is being made.
917	P+XWQ(a)	Processing time of the imminent operation. Sum of the imminent operation processing times of the jobs in every queue which will receive (after the imminent operation) a job which is in the present queue. Processing time of the next-after-current operation of jobs which are now on machines and which, after completing the current operation, will move to a queue which will receive a job (after the imminent operation) which is in the queue from which selection is being made.
99	SHOPNH(a,b)	Processing time of the imminent operation. Number of jobs in this queue. Processing time of the next-after-current operation of jobs which are now on machines and which, after completing the current operation, will move to the queue from which selection is being made; also the time at which these jobs will arrive.

Note: A, B, C, and D are information classifications.

[a]Priority based only on attributes of the particular queue from which selection is to be made and attributes of the imminent operation of jobs in that queue.

[b]Priority based only on attributes of the particular queue from which selection is to be made and attributes of the jobs in that queue.

[c]Priority based on current value of attributes of any queues or machines in the shop and attributes of any jobs in the shop.

[d]Priority based on the current or predicted values of attributes of any queues or machines in the shop and attributes of any jobs in the shop.

Table 11

SUMMARY: REPLICATION - SHOP STATISTICS

Measure	Conditions	Priority Rule: FCFS 2	SHOPN 9	P-SP (0.95) 911	P+WKR (0.9852) 912	P+WQ/P (0.97,0) 916	P+S/OPN (0,1) 945
Mean Jobs in Q	STD	58.87	23.25	23.15	22.98	22.83	51.65
	Rep 2	75.04	28.46	28.33	28.07	27.91	61.96
	Rep 3	92.16	33.08	33.24	33.42	33.92	75.57
	Rep 4		26.50		26.61	26.54	
	Mean	75.36	28.27	28.24	28.16	28.22	63.06
	S23						
	Mean S234		27.82		27.77	27.80	
Mean Total Work	Std	1078	545	547	552	539	
	Rep 2	1331	640	641	650	631	1218
	Rep 3	1634	754	768	778	778	1469
	Rep 4		608		626	605	
	Mean	1348	646	652	660	649	
	S23						
	Mean S234		637		652	638	
Mean Work Rem	Std	559	297	302	309	294	538
	Rep 2	700	356	358	372	351	812
	Rep 3	854	424	435	450	437	1064
	Rep 4		340		358	337	
	Mean	704	359	365	377	361	805
	S23						
	Mean S234		354		372	355	
Mean I-O WinQ	Std	58	63	62	61	60	57
	Rep 2	75	86	85	84	81	67
	Rep 3	92	109	109	110	105	82
	Rep 4		78		78	78	
	Mean	75	86	85	85	82	69
	S23						
	Mean S234		84		83	81	

Table 12

COMPARISON OF SHOPN PERFORMANCE IN REPLICATE RUNS

Interval	Mean Number of Jobs in Queue for Interval											
	Std			Rep 2			Rep 3			Rep 4		
	All Q's	Max Q		All Q's	Max Q		All Q's	Max Q		All Q's	Max Q	
0- 400	29.17	3	5.53	17.80	4	2.99	28.55	3	4.81	35.16	3	5.98
400- 800	29.44	3	7.48	18.70	6	5.02	27.12	2	5.22	41.28	3	7.74
800- 1200	22.61	3	5.33	21.90	4	6.96	33.25	3	8.28	26.52	3	5.21
1200- 1600	25.23	7	4.04	27.93	5	6.40	27.98	3	8.62	10.40	9	1.69
1600- 2000	30.70	5	5.06	31.76	5	11.42	41.47	1	8.35	20.63	8	4.21
2000- 2400	21.70	6	3.77	53.24	5	14.40	41.99	2	12.64	19.81	2	3.24
2400- 2800	18.87	9	3.11	46.77	7	11.44	44.28	2	16.77	11.69	4	2.57
2800- 3200	19.15	6	3.47	20.12	5	6.70	40.14	2	16.55	22.68	8	4.71
3200- 3600	27.22	6	4.01	26.54	3	5.82	45.19	2	20.11	19.88	7	4.36
3600- 4000	33.47	8	5.84	29.49	6	6.43	49.35	2	20.69	26.35	6	3.58
4000- 4400	35.30	3	8.70	33.76	9	6.19	44.85	2	18.53	19.44	3	5.06
4400- 4800	28.48	5	4.71	25.78	9	4.81	50.07	2	20.14	29.44	9	6.29
4800- 5200	25.31	2	4.51	28.01	3	3.84	31.00	2	15.61	17.90	9	3.19
5200- 5600	13.36	7	2.91	35.99	2	7.13	26.54	4	4.86	29.44	7	6.54
5600- 6000	15.03	4	2.12	30.64	5	5.65	16.88	7	3.46	24.53	5	5.20
6000- 6400	14.93	4	2.51	27.64	8	9.29	21.14	3	5.55	15.48	1	2.81
6400- 6800	25.11	2	4.61	28.76	9	7.51	16.59	9	3.30	36.95	2	11.41
6800- 7200	24.81	7	3.43	27.16	9	6.09	21.37	5	3.57	47.78	2	16.21
7200- 7600	18.07	2	4.08	25.12	7	4.69	50.93	7	9.50	40.36	2	10.36
7600- 8000	30.37	6	8.66	16.76	5	2.26	41.84	6	8.55	26.60	3	6.93
8000- 8400	32.61	9	7.78	20.93	1	4.89	37.15	4	6.00	32.94	6	8.97
8400- 8800	18.56	7	3.94	21.14	1	3.54	34.69	4	6.38	38.44	6	14.27
8800- 9200	14.34	6	2.76	16.60	5	2.49	21.58	3	3.66	23.59	6	4.98
9200- 9600	14.05	1	2.42	33.47	1	11.47	16.95	7	4.36	30.90	5	9.93
9600-10000	16.68	2	2.50	34.27	9	5.65	30.86	7	8.82	16.85	5	4.39
10000-10400	24.40	5	5.25	31.20	9	7.43	23.85	1	5.56	24.68	6	4.75
10400-10800	26.71	5	8.07	26.33	1	3.83	23.06	5	5.43	34.37	6	9.76
0-10800	23.47	1	2.43	28.07	1	2.77	32.91	1	3.09	26.82	1	2.49
		2	2.75		2	2.91		2	7.55		2	3.49
		3	2.69		3	2.60		3	3.49		3	3.12
		4	2.25		4	2.97		4	3.03		4	3.39
		5	2.81		5	4.16		5	3.23		5	2.83
		6	2.78		6	2.94		6	2.99		6	3.80
		7	2.75		7	3.01		7	3.76		7	2.47
		8	2.45		8	3.13		8	3.07		8	2.62
		9	2.57		9	3.57		9	2.71		9	2.62

Table 13

COMPARISON OF P+WQ/P(0.97,0) PERFORMANCE IN REPLICATE RUNS

Interval	Mean Number of Jobs in Queue for Interval							
	Std		Rep 2		Rep 3		Rep 4	
	All Q's	Max Q	All Q's	Max Q	All Q's	Max Q	All Q's	Max Q
0- 400	27.97	3 5.33	17.83	4 2.88	29.26	3 5.63	34.55	3 5.72
400- 800	30.00	3 7.08	18.36	6 4.50	26.44	2 4.76	38.99	3 7.97
800- 1200	21.85	3 5.05	21.52	4 6.67	34.38	3 7.72	25.53	3 5.70
1200- 1600	27.47	7 4.55	26.77	5 5.93	29.80	3 8.60	10.25	9 1.54
1600- 2000	29.12	5 4.75	30.85	5 9.74	40.11	2 7.33	20.39	8 4.26
2000- 2400	21.43	7 4.16	54.75	5 12.83	42.68	2 12.01	20.09	2 3.44
2400- 2800	18.98	8 3.30	47.17	7 10.83	43.14	2 15.52	11.62	2 2.46
2800- 3200	19.61	6 3.51	20.02	5 5.41	42.22	2 15.09	23.26	8 4.76
3200- 3600	26.35	6 3.51	25.47	3 5.67	46.98	2 16.78	18.71	7 4.17
3600- 4000	32.72	8 5.96	28.99	6 5.81	52.97	2 16.99	24.97	5 3.59
4000- 4400	34.59	3 8.41	32.13	9 5.72	47.33	2 16.29	20.39	3 5.27
4400- 4800	28.87	5 5.28	26.29	9 5.07	57.39	2 19.23	30.81	9 6.15
4800- 5200	22.45	2 4.11	27.55	6 5.90	37.84	2 15.03	18.83	9 3.19
5200- 5600	12.12	7 2.68	34.06	2 6.75	29.16	4 6.31	29.46	7 5.90
5600- 6000	14.25	4 2.05	29.45	5 6.09	16.57	4 3.11	24.11	7 5.24
6000- 6400	14.68	4 2.59	27.46	8 9.57	21.39	3 5.34	15.44	8 2.34
6400- 6800	24.69	2 5.44	27.19	8 7.15	17.04	9 4.07	36.92	2 10.56
6800- 7200	24.85	2 3.60	25.37	8 5.83	21.45	5 3.32	50.28	2 14.59
7200- 7600	17.94	2 3.60	25.16	7 4.35	49.28	7 9.67	42.13	2 8.75
7600- 8000	30.22	6 9.94	17.03	4 2.26	41.36	6 8.77	27.63	3 7.06
8000- 8400	29.86	9 6.12	20.97	1 5.05	37.01	4 6.13	33.59	6 7.55
8400- 8800	18.72	7 2.87	21.26	1 3.80	34.43	4 6.24	38.88	6 13.16
8800- 9200	14.20	6 2.65	16.64	3 2.79	21.01	7 3.55	23.77	6 5.45
9200- 9600	14.91	1 2.81	32.16	1 10.21	16.33	7 3.44	29.74	5 8.91
9600-10000	15.45	2 2.57	33.58	9 5.62	29.18	7 8.19	15.71	5 3.71
10000-10400	23.48	5 5.22	30.88	9 7.02	22.74	1 4.28	24.92	6 4.90
10400-10800	24.86	5 2.84	24.62	1 3.80	23.73	5 5.60	33.51	6 9.39
0-10800	23.02	1 2.41	27.53	1 2.81	33.75	1 3.08	26.83	1 2.59
		2 2.66		2 2.80		2 6.98		2 3.51
		3 2.63		3 2.61		3 3.69		3 3.11
		4 2.28		4 2.98		4 3.41		4 3.37
		5 2.76		5 3.95		5 3.40		5 2.94
		6 2.68		6 2.80		6 3.31		6 3.68
		7 2.76		7 2.97		7 3.83		7 2.42
		8 2.44		8 3.09		8 3.22		8 2.59
		9 2.40		9 3.53		9 2.85		9 2.82

Table 14

FCFS AND SHOPN PERFORMANCE AT DIFFERENT LEVELS OF UTILIZATION

Rule	Conditions	Utilization 400-10,800	Mean Jobs in Q	Variance Shop Time	Variance Lateness
2 FCFS	Scale 0.877	0.77540	23.58	1137.4	1695.1
9 SHOPN	Scale 0.877	0.77571	12.01	552.6	2310.0
2 FCFS	Std	0.88321	58.87	5739.2	1686.2
9 SHOPN	Std	0.88378	23.25	2318.0	2878.2
2 FCFS	Rep 2	0.90083	75.04	8164.5	2121.8
9 SHOPN	Rep 2	0.90372	28.46	5124.1	4648.5
2 FCFS	Rep 3	0.91653	92.16	13258.6	4542.9
9 SHOPN	Rep 3	0.91850	33.08	13893.2	12186.7
9 SHOPN	Rep 4	0.89289	26.50	3982.5	3862.7
2 FCFS	Theoretical	0.77540	24.09		
2 FCFS	Theoretical	0.88321	60.11		
2 FCFS	Theoretical	0.90083	73.65		
2 FCFS	Theoretical	0.91653	90.57		

Table 15

EFFECT OF SHOP SIZE ON SHORTEST OPERATION RULE PERFORMANCE

No. of Machines (3)	SHOPN			FCFS			$\frac{\text{SHOPN}}{\text{FCFS}}$
	Mean Jobs in Q (2)	Source & Utilization	Jobs/Q (2)/(1)	Mean Jobs in Q (3)	Source & Utilization	Jobs/Q (3)/(1)	(2)/(3)
1	2.54	Theoretical 0.884	2.54	6.72	Theoretical 0.884	6.72	0.38
3	8.40	Experimental 0.883	2.80	20.16	Theoretical 0.884	6.72	0.42
5	13.13	Experimental 0.879	2.63	33.60	Theoretical 0.884	6.72	0.39
7	18.15	Experimental 0.882	2.59	47.04	Theoretical 0.884	6.72	0.39
9	23.25	Experimental 0.884	2.58	60.49	Theoretical 0.884	6.72	0.38

Table 16

EFFECT OF PROCESSING TIME DISTRIBUTION

Processing Time Distribution	Mean	Variance	Mean Jobs in Q	
			SHOPN	FCFS
Exponential	1.00	1.00	23.25	58.87
Rectangular	1.00	0.33	24.87	40.20
Constant	1.00	0	30.74	30.74

Table 17

EFFECT OF ERRORS IN ESTIMATING PROCESSING TIME OF MEAN NUMBER OF JOBS IN QUEUE

	Exponential Processing Times		Rectangular Processing Times	
Statistic	Mean Jobs in Q	% inc.	Mean Jobs in Q	% inc.
Percentage errors of estimate uniformly distributed between				
± 0%	23.25	-	24.87	-
± 10%	23.23	-	25.19	1.3
± 100%	27.13	16.7	27.79	11.7
± ∞	58.87	153.2	40.20	61.6
Times classified as "short" or "long"; below or above the mean. Probability of correct classification.				
1.00	35.29	-		
0.75	44.99	27.5		
0.50	58.87	66.8		

Table 18

SUMMARY: COMBINATIONS OF SHORTEST OPERATION AND FIRST COME, FIRST SERVED RULES

Truncation of SHOPN (TSHOPN(a)			Shift from FCFS to SHOPN if Queue becomes Large (FCFS*S(a))		
"a" Max Time in Queue	Mean Jobs in Q	Ratio to SHOPN	Shift Point SHOPN if N ≥	Mean Jobs in Q	Ratio to SHOPN
0	58.78	2.53	∞	58.87	2.53
4	55.67	2.39	9	38.67	1.66
8	53.50	2.30	5	29.49	1.27
16	44.20	1.90	0	23.25	1.00
32	32.85	1.41			
∞	23.25	1.00			

Table 19

EFFECT OF SUPERIMPOSING CLASS PRECEDENCE UPON SHOPN PERFORMANCE (STD Conditions)

Percentage of Jobs in Preferred Class	Mean Jobs in Q	Mean Total Work	Mean Work Rem	Mean Work Completed	Lateness Variance	No. Jobs Late
0 (SHOPN)	23.25	545	297	248	2878	437
10	24.46	561	306	256	2854	525
30	27.29	602	326	276	3133	840
50	31.17	652	352	300	3909	1331
70	34.57	696	375	321	5919	1570
90	36.11	706	384	323	11641	904
100 (SHOPN)	23.25	545	297	248	2878	437

Table 20

SUMMARY: WORK-IN-PROCESS INVENTORY
(STD Conditions)

	Rule	Mean No. Jobs in Shop	Mean Total Work on Jobs in Shop	Mean Work Remaining on Jobs in Shop	Mean Work Completed or in Progress on Jobs in Shop
1	RANDOM (Seed=1357)	67.38	1080	554	526
2	FCFS	66.83	1078	559	519
3	FASF	65.46	777	563	214
9	SHOPN	31.21	545	297	248
10	LOPN	175.40	2709	1369	1340
11	LSOPN	62.25	1007	527	480
14	TWORK	90.87	587	323	264
16	WINQ	48.38	801	421	380
17	XWINQ	41.98	709	379	330
913	P/WKR(0.05)	31.44	539	290	249
913	P/WKR(0.75)	45.95	597	224	373
913	P/WKR(2.0)	98.55	1011	229	782
914	P/TWK	50.35	435	244	191
945	P+S/OPN(0.5,1)	52.32	968	376	592

Table 21

SUMMARY: PROCESSING TIME RATIO RULES
(STD Conditions)

	913 P/WKR(a)			906 P/OPNR(a)			914 P/TWK(a)		
a	Jobs in Q	Total Work	Work Rem	Jobs in Q	Total Work	Work Rem	Jobs in Q	Total Work	Work Rem
0.00	23.25	545	297	23.25	545	297	23.25	545	297
0.05	23.48	539	90						
0.10	23.37	530	279						
0.25	24.85	524	260						
0.50	29.63	540	235	29.17		238			
0.75	37.99	597	224						
1.00	62.94		220	35.42		227	42.39	435	244
1.20	83.23	941	222						
2.00	90.59	1011	229						
∞	109.97		276	72.30		252	82.91	587	323

Table 22

SUMMARY: LATENESS PERFORMANCE OF RULE 945 P+S/OPN(a,b)

a	Statistic	b = 0.5	b = 0.8	b = 1.0	b = 1.2
	Mean lateness	-13.96	-13.39	-12.79	-12.82
	Standard deviation of lateness	15.20	12.50	15.00	18.70
	Number of jobs late	1175	496	323	335
	Percentage of jobs late	13.51	5.70	3.71	3.82
0.3	Mean lateness			-17.25	
	Standard deviation of lateness			19.20	
	Number of jobs late			158	
	Percentage of jobs late			1.82	
0.5	Mean lateness		-21.68	-21.30	
	Standard deviation of lateness		18.5	22.8	
	Number of jobs late		137	132	
	Percentage of jobs late		1.57	1.52	
0.75	Mean lateness			-26.72	
	Standard deviation of lateness			26.40	
	Number of jobs late			172	
	Percentage of jobs late			1.98	
1.00	Mean lateness	-44.90	-44.90	-44.90	-44.90
	Standard deviation of lateness	53.70	53.70	53.70	53.70
	Number of jobs late	437	437	437	437
	Percentage of jobs late	5.02	5.02	5.02	5.02

$$V_i(t) = a.P_{i,J} + (1-a).\frac{D_i - t - \sum_{j=J}^{M_i} P_{i,j}}{(M_i - J + 1)^b}$$

Table 23

SUMMARY: REPLICATION - JOB STATISTICS

Measure	Conditions	Priority Rule: FCFS 2	SHOPN 9	P-SP (0.95) 911	P+WKR (0.9852) 912	P+WQ/P (0.97,0) 916	P+S/OPN (0,1) 945
Mean Shop Time	STD	74.43	34.02	33.95	33.81	33.63	66.13
	Rep 2	90.64	39.80	39.70	39.37	39.24	76.23
	Rep 3	112.94	46.27	46.54	46.75	47.25	93.50
	Rep 4		37.06		37.35	37.24	
	MEAN	92.67	40.03	40.07	39.98	40.04	78.62
	MEAN S23		39.29		39.32	39.34	
	MEAN S234						
Std Dev Shop Time	STD	75.8	48.1	48.8	48.8	47.2	73.9
	Rep 2	90.3	71.6	70.8	72.3	69.5	79.2
	Rep 3	115.1	117.9	118.7	121.0	113.9	96.1
	Rep 4		63.1		64.7	61.5	
	MEAN	93.7	79.2	79.4	80.7	76.9	83.1
	MEAN S23		75.2		76.7	73.0	
	MEAN S234						
Mean Lateness	STD	-4.49	-44.90	-44.97	-45.11	-45.29	-12.79
	Rep 2	11.27	-39.57	-39.67	-40.00	-40.13	- 3.14
	Rep 3	31.39	-35.29	-35.01	-34.80	-34.30	11.94
	Rep 4		-41.39		-41.10	-41.21	
	MEAN	-12.72	-39.92	-39.88	-39.97	-39.91	- 1.33
	MEAN S23		-40.29		-40.26	-40.23	
	MEAN S234						
Std Dev Lateness	STD	41.1	53.6	53.0	51.5	52.4	15.0
	Rep 2	46.1	68.2	66.8	66.8	66.5	14.3
	Rep 3	67.4	110.4	110.1	111.8	104.6	32.1
	Rep 4		62.1		60.9	60.5	
	MEAN	51.5	77.4	76.6	76.7	74.4	20.5
	MEAN S23		76.1		72.8	71.0	
	MEAN S234						
% Jobs Late	STD	44.79	5.02	4.85	4.59	4.53	3.71
	Rep 2	57.67	6.09	5.92	5.83	5.88	30.93
	Rep 3	67.33	7.26	7.11	7.05	7.66	54.75
	Rep 4		5.91		5.67	5.56	
	MEAN	56.60%	6.12%	5.96%	5.82%	6.02%	29.80%
	MEAN S23		6.07%		5.79%	5.91%	
	MEAN S234						

Table 24

SUMMARY OF LATENESS STATISTICS
STD, DD II, DD III, AND DD IV Conditions

Rule	Statistic	Type of Due-Date I (~ Work)	II (~ No. Opns)	III (Constant)	IV (Random)
2 FCFS	Mean	-4.49	-3.91	-4.37	-4.87
	Variance	1686	1125	5739	7728
	√Var	41.1	33.6	75.8	87.9
	NJL	3897	3474	2937	3585
	% Late	44.8	39.9	33.8	41.2
4 DDATE	Mean	-15.53	-7.72	-6.28	-6.41
	Variance	431	529	1565	2100
	√Var	20.8	23.0	39.6	45.8
	NJL	1544	2322	3819	4244
	% Late	17.8	26.7	43.9	48.8
9 SHOPN	Mean	-44.90	-44.32	-44.78	-45.28
	Variance	2878	2930	2318	4416
	√Var	53.7	54.1	48.1	66.5
	NJL	437	537	957	1726
	% Late	5.0	6.2	11.0	19.8
945 P+S/OPN(0,1)	Mean	-12.79	-5.41	-5.14	-5.32
	Variance	226	225	1118	1718
	√Var	15.0	15.0	33.4	41.5
	NJL	323	1878	4183	4573
	% Late	3.7	21.6	48.1	52.6
10 LOPN	Mean	117.50	118.09		
	Variance	37919	25074		
	√Var	194.7	158.3		
	NJL	5957	5960		
	% Late	68.5	68.5		

Table 25

NUMBER OF JOBS LATE

Job Numbers	2 FCFS				4 DDATE				9 SHOPN				945 P+S/OPN(0,1)			
	I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
401- 500	42	30	43	46	10	8	48	48	7	7	22	28	1	0	69	70
501- 600	54	47	39	35	8	6	63	47	8	10	12	18	1	0	71	68
601- 700	54	53	33	48	5	13	60	65	5	4	10	22	2	0	73	73
701- 800	39	24	26	38	7	13	32	51	6	4	10	19	1	0	55	65
801- 900	31	32	28	39	5	4	35	39	6	7	9	17	1	0	67	57
901-1000	35	18	25	39	8	9	40	31	2	4	9	13	1	0	56	49
1001-1100	37	25	35	47	3	4	36	47	4	11	14	25	1	2	68	61
1101-1200	51	53	33	46	5	9	47	64	3	6	12	21	1	1	69	69
1201-1300	58	60	39	44	8	15	60	64	5	6	12	19	4	2	82	75
1301-1400	49	60	41	53	9	9	61	69	5	7	11	25	1	1	68	81
1401-1500	66	68	44	50	12	19	75	73	6	8	17	25	0	18	74	85
1501-1600	69	59	39	43	15	21	77	72	6	11	10	18	4	20	80	79
1601-1700	73	69	49	51	25	44	75	58	7	8	18	21	1	43	81	79
1701-1800	77	76	50	45	45	71	72	63	10	12	14	12	4	66	79	80
1801-1900	69	79	44	51	42	63	56	64	3	5	11	19	1	54	75	81
1901-2000	62	63	46	60	43	60	59	72	6	6	13	22	5	61	70	78
2001-2100	45	46	32	38	13	53	56	48	2	2	3	14	0	31	61	67
2101-2200	46	46	46	58	19	30	50	59	3	6	11	24	0	27	77	78
2201-2300	64	76	43	52	32	51	64	56	6	9	10	16	3	61	82	60
2301-2400	47	53	42	43	51	75	55	56	5	5	10	16	1	72	69	61
2401-2500	51	44	30	43	44	73	34	50	9	8	10	14	5	70	50	52
2501-2600	38	32	29	31	17	42	35	48	4	3	7	11	1	42	52	40
2601-2700	29	18	33	44	18	34	26	48	4	2	10	20	2	26	18	28
2701-2800	20	6	13	27	7	10	7	18	0	0	4	12	0	6	0	18
2801-2900	36	19	32	37	6	10	26	50	3	4	16	29	0	3	39	58
2901-3000	38	30	40	38	10	11	56	59	9	9	18	19	0	3	59	67
3001-3100	42	31	30	35	6	15	31	43	2	6	9	16	1	6	32	51
3101-3200	46	48	37	48	11	29	30	47	7	11	14	29	1	4	32	59
3201-3300	69	61	52	53	26	38	66	72	15	15	21	34	3	16	72	73
3301-3400	70	68	44	52	32	55	66	75	6	8	15	26	3	37	69	73
3401-3500	62	66	45	49	35	51	70	69	14	11	14	23	3	63	68	76
3501-3600	64	69	47	50	43	66	55	69	9	9	13	19	1	70	71	77
3601-3700	68	78	48	52	37	59	71	76	8	13	20	28	4	69	78	79
3701-3800	85	81	54	54	65	73	78	74	15	17	22	26	1	77	83	81
3801-3900	79	83	55	57	55	77	85	78	10	14	21	28	13	77	85	79
3901-4000	61	60	49	53	61	74	71	76	4	7	12	21	22	68	73	81
4001-4100	78	78	55	57	63	73	81	82	5	2	12	25	35	79	83	86
4101-4200	66	80	39	45	48	72	57	71	10	8	13	22	24	78	64	79
4201-4300	68	66	48	53	36	66	62	68	6	10	13	23	9	82	64	62
4301-4400	71	79	56	55	36	66	81	72	8	12	17	25	19	75	91	78
4401-4500	78	77	43	45	50	73	70	68	3	1	8	15	21	85	81	77
4501-4600	59	67	47	51	34	64	69	66	5	6	10	20	5	82	77	71
4601-4700	69	74	45	54	39	56	69	69	6	5	20	29	6	74	79	78
4701-4800	49	43	30	28	20	46	54	48	4	4	4	14	6	71	67	57
4801-4900	35	15	26	42	7	26	42	40	2	3	6	18	2	18	50	41
4901-5000	25	8	16	26	1	7	10	17	1	1	0	11	1	0	1	17

Table 25 -- Continued

Job	2 FCFS				4 DDATE				9 SHOPN				945 P+S/OPN(0,1)			
Numbers	I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
5001-5100	25	6	20	34	1	1	15	23	3	2	8	21	1	0	32	31
5101-5200	16	13	18	32	1	0	24	27	4	7	9	21	4	0	27	23
5201-5300	18	5	18	35	1	3	14	36	0	0	5	19	1	0	2	31
5301-5400	14	7	17	28	1	1	16	22	0	3	8	18	1	0	4	18
5401-5500	11	7	13	18	0	0	12	10	0	1	6	6	0	0	0	6
5501-5600	16	2	15	16	1	2	11	11	2	1	4	10	3	0	0	9
5601-5700	21	4	20	34	2	2	14	26	1	3	6	22	2	0	0	15
5701-5800	15	13	19	23	2	3	11	19	1	6	4	13	0	0	0	12
5801-5900	20	11	20	37	1	1	6	32	1	3	5	21	0	0	0	17
5901-6000	37	29	37	43	6	5	35	36	8	13	19	29	2	0	33	44
6001-6100	33	22	26	34	5	7	28	43	7	8	15	25	1	0	20	49
6101-6200	42	33	37	38	8	12	53	50	4	6	10	23	2	0	56	50
6201-6300	32	28	27	35	5	8	50	39	6	7	8	19	2	0	48	50
6301-6400	32	26	30	41	5	6	34	47	10	6	19	24	0	0	44	45
6401-6500	44	32	38	46	8	14	51	43	6	3	9	23	2	0	56	43
6501-6600	47	37	29	40	7	11	36	40	4	3	10	19	2	0	40	31
6601-6700	38	25	23	40	9	24	23	28	2	3	4	16	2	0	6	12
6701-6800	30	17	34	40	5	7	24	37	3	4	9	20	0	0	19	33
6801-6900	34	28	28	42	4	8	27	40	3	3	9	22	0	0	31	46
6901-7000	43	32	36	45	15	12	48	57	8	11	19	28	1	0	63	76
7001-7100	58	51	37	44	24	19	68	66	5	6	16	22	0	0	62	73
7101-7200	61	59	50	50	24	27	68	77	12	15	13	26	5	2	64	82
7201-7300	56	61	40	49	35	44	67	66	3	8	16	20	11	10	65	76
7301-7400	65	69	40	48	57	53	70	71	7	10	11	15	26	22	79	72
7401-7500	64	68	50	51	50	64	69	70	7	12	27	30	8	28	85	76
7501-7600	65	68	46	49	23	37	83	68	6	9	16	21	0	23	85	73
7601-7700	56	61	36	46	24	37	55	65	7	4	4	21	1	45	69	64
7701-7800	49	46	36	42	13	19	36	52	3	4	6	16	3	4	56	57
7801-7900	42	48	38	40	5	18	54	56	6	7	18	27	3	0	76	67
7901-8000	46	36	25	36	7	13	43	52	2	1	5	16	1	1	41	55
8001-8100	35	24	25	31	2	7	23	35	3	4	6	14	2	1	24	32
8101-8200	37	11	28	37	4	1	25	25	1	2	5	15	0	1	10	16
8201-8300	22	14	28	29	1	7	20	24	3	6	8	18	3	1	10	21
8301-8400	25	17	23	29	3	2	8	18	2	5	7	16	0	0	1	12
8401-8500	21	14	18	28	2	2	17	14	6	5	9	16	2	1	0	12
8501-8600	21	8	15	22	2	0	6	10	2	1	4	7	1	0	0	4
8601-8700	15	5	14	28	1	0	11	31	0	4	7	18	1	0	0	22
8701-8800	15	5	18	29	2	0	9	22	4	5	8	16	0	0	2	14
8801-8900	16	4	16	26	2	0	13	16	4	2	5	9	3	0	6	10
8901-9000	25	9	18	29	3	1	11	23	3	4	9	19	4	0	3	23
9001-9100	13	1	11	26	1	1	8	18	3	4	3	14	2	0	0	17
Total NJL	3897	3474	2937	3585	1544	2322	3819	4244	437	537	957	1726	323	1878	4183	4573
Percentage	44.8	39.9	33.8	41.2	17.8	26.7	43.9	48.8	5.0	6.2	11.0	19.8	3.7	21.6	48.1	52.6

Table 26

COMPARISON OF FCFS AND SHOPN IN A ONE-MACHINE SHOP

Utilization λ/μ	Mean Number of Jobs in Queue		N_s/N_f
	SHOPN N_s	FCFS N_f	
0.10	0.0105	0.0111	0.949
0.20	0.0448	0.0500	0.895
0.30	0.1078	0.1286	0.838
0.40	0.2074	0.2667	0.778
0.50	0.3563	0.5000	0.713
0.55	0.4558	0.6722	0.678
0.60	0.5775	0.9000	0.642
0.65	0.7282	1.2071	0.603
0.70	0.9185	1.6333	0.562
0.72	1.0094	1.8514	0.545
0.74	1.1110	2.1062	0.528
0.76	1.2256	2.4067	0.509
0.78	1.3558	2.7655	0.490
0.80	1.5058	3.2000	0.471
0.82	1.6809	3.7356	0.450
0.84	1.8891	4.4100	0.428
0.86	2.1426	5.2829	0.406
0.88	2.4607	6.4533	0.381
0.90	2.8772	8.1000	0.355
0.91	3.1404	9.2011	0.341
0.92	3.4565	10.5800	0.327
0.93	3.8459	12.3557	0.311
0.94	4.3415	14.7267	0.295
0.95	5.0009	18.0500	0.277
0.96	5.9349	23.0400	0.258
0.97	7.3920	31.3633	0.236
0.98	10.0798	48.0200	0.210

NOTE: For exponentially distributed processing times, mean = $1/\mu$; for exponentially distributed interarrival intervals, mean = $1/\lambda$.

$$N_f = \frac{\lambda/\mu}{1-\lambda/\mu} - \lambda/\mu$$

$$N_s = \frac{\lambda^2}{\mu} \Bigg/ \int_0^\infty \frac{e^{-\mu t} dt}{\left\{1 - (\lambda/\mu\,[1 - e^{-\mu t}(1 + \mu t)]\right\}^2}$$

The foregoing equation, solving for N_s, was taken from Thomas E. Phipps, Jr., "Machine Repair as a Priority Waiting-Line Problem," Operations Research, February 1956, p. 85.

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