NASr-21(02)

MEMORANDUM
RM-4007-NASA
APRIL 1964

SATELLITE LIFETIME PROGRAM

S. J. Belcher, L. N. Rowell and M. C. Smith

PREPARED FOR:
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

The RAND Corporation
SANTA MONICA • CALIFORNIA

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This research is sponsored by the National Aeronautics and Space Administration under Contract No. NASr-21. This report does not necessarily represent the views of the National Aeronautics and Space Administration.

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PREFACE

The formal work statement for NASA Task Order NASr-21(02) covering communications satellite studies to be carried out by The RAND Corporation includes a specific requirement for the review of the potentialities of communication systems using passive satellites. In the study carried out to fulfill that requirement, it was necessary to consider the orbital motions of passive communication satellites. To facilitate the study, the digital computer program described here was obtained from the Massachusetts Institute of Technology, Lincoln Laboratories, and was adapted to the needs of the study.

SUMMARY

This Memorandum describes a digital computer program, written in FORTRAN II, which simulates mathematically the motions of an earth satellite. The program permits cheap and accurate predictions of orbital changes of close in (2000 n mi or less) satellites for months or years ahead. It is thus useful in assessing the worth of systems in which satellite lifetimes and satellite orbital stability are important. For example, the program has already been used to study a proposed system of passive communications satellites, in which the effectiveness of the system depends on several satellites staying in orbit in strict configuration with one another.

The program could also be used to estimate the drag characteristics of an unknown satellite, or to improve or refine a new model atmosphere. Improvements in the model atmosphere, in turn, will increase the accuracy of prediction of trajectories of rockets and satellites.

This Memorandum is intended to acquaint the prospective user with the basic theory and possible functions of the program, and to serve as a reference manual.

ACKNOWLEDGMENT

The authors wish to thank I. I. Shapiro and H. M. Jones, of the Lincoln Laboratory, Massachusetts Institute of Technology, for the use of their program and for their time and effort in making the program understandable.

The authors are also grateful to W. L. Sibley for his interest and efforts in getting programs to run on the RAND computer.

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LIST OF SYMBOLS

Symbol	Program Symbol	
A		aspect averaged cross-sectional area of the satellite in cm^2
A_D		acceleration due to atmospheric drag
A_{RP}		perturbing acceleration caused by solar radiation pressure on the satellite
$A_{RP_x}, A_{RP_y}, A_{RP_z}$		components of the perturbing acceleration along the x, y, and z axes caused by the sun
$\bar{A}_{EM}$		acceleration of the earth caused by the moon
$\bar{A}_{PM}$		acceleration of the satellite caused by the moon
$\bar{A}_R$		relative acceleration of the satellite and earth caused by the moon
a	A	semimajor axis in ERU (earth radius units)
a_E		semimajor axis of the earth's orbit around the sun
a_M, e_M	C(50),C(51)	semimajor axis and eccentricity of the moon's orbit, respectively, where a_M is in ERU
a_S		semi-major axis of the shadow ellipse
c		speed of light
C_D		atmospheric drag coefficient
c_n		constants of integration as defined by Eqs. (91) to (96), where n = 1 - 6

DAY	DAY	DAY is JD - 2436203.5 days
E		eccentric anomaly of the satellite
E_M	ANEM	eccentric anomaly of the moon's position in radians
E_{oM}	ANEM1	initial eccentric anomaly of the moon's position in radians
e	E	eccentricity
G		gravitational constant
$h(v)$	HFT	altitude of the satellite above the oblate earth
I, I_o	FLUX	solar flux near the earth and at the earth's surface in dynes/cm sec
$I_k(n,m)$		defined by Eq. (52)
i	OINC	inclination of the satellite's orbit plane to the plane of the equator in radians
i_1		angle between the earth-sun line and the direction of W (or ζ-axis)
i_M	OIM	inclination of the moon's orbit plane to the equator in radians
i_{ME}	OIM1	inclination of the moon's orbit to the ecliptic in radians
JD		Julian date
J_n	GJAY,C(46), C(47),C(48)	earth oblateness constants
K		a constant between 0 and 2 which depends on the reflection characteristics of the satellite
M		mass of the satellite

M_E	M	mass of the Earth
MJD	DMJ	modified Julian date in days (JD - 2430000.5)
M_M	ANMM	mean anomaly of the moon's position in radians
m_M		mass of the moon
n		mean orbital angular rate
P_n		nth order Legendre polynomial
p	P	semilatus rectum in ERU
Q_n		defined by Eqs. (85) to (90)
R,S,W		R = components of the disturbing acceleration in the radial direction; S = components in the orbit plane perpendicular to the radial direction and in the direction of motion; W = components perpendicular to the orbit plane with positive direction defined by the cross product of unit vectors in the R and S directions
R_D, S_D, W_D		components of the drag acceleration along the R, S, and W directions
R_E	RE	mean equatorial radius of the earth in cm
R_M, S_M, W_M	RM,SM,WM	components of the perturbing acceleration caused by the moon and with the same directions as R, S, and W, respectively
R_{RP}, S_{RP}, W_{RP}		components of the perturbing acceleration along the R, S, and W directions caused by solar radiation pressure
R(U)		defined by Eq. (42)

$R_1(\omega), S_1(\omega), W_1(\omega)$		direction cosines of the angles between earth radii to the moon and to perigee position; between an earth radius to the moon and the semi-latus rectum of the satellite's orbit; and between earth radii to the moon and normal to the satellite orbit plane, respectively
$R_2(\omega), S_2(\omega), W_2(\omega)$		definitions are similar to those of $R_1(\omega)$, $S_1(\omega)$ and $W_1(\omega)$ with moon replaced by sun
r		distance from the center of the earth to the satellite
r_E	$(Q(30,5))^{-\frac{1}{2}}$	instantaneous distance of the earth from the sun in astronomical units
r_{EM}		distance between the earth and the moon
$r_{EMR}, r_{EMS}, r_{EMW}$		components of $\bar{r}_{EM}$ along the R, S, and W directions, respectively
$r_{EMx}, r_{EMy}, r_{EMz}$		components of $\bar{r}_{EM}$ along the X, Y, and Z axes, respectively
r_{PM}		distance between the satellite and the moon
U		earth's gravitational potential
U_1		higher harmonics of the earth's gravitational potential
u		argument of the latitude of the satellite's position in radians
u_M	UM	argument of the latitude of the moon's position
V		satellite velocity
V_a		velocity of the atmosphere at the satellite's position relative to a fixed coordinate system

V_r		satellite velocity relative to the atmosphere
v	V	true anomaly in radians
v_M	VM	true anomaly of the moon's position in radians
v_1, v_2	V_1, V_2	true anomalies of the shadow boundaries of the earth's shadow on the satellite's orbit plane in radians
x, y, z		rectangular coordinate system --with the x-axis in the direction of the vernal equinox; the z-axis in the direction of the north pole; and the y-axis in the equator plane and directed so as to form a right-handed coordinate system
α_M		angle between earth radii to the satellite and to the moon
β	OST	angular position of the earth-sun line relative to the vernal equinox direction in radians
γ		as defined by Eq. (15a)
$\Delta(\ )$	D(21,N),D(22,N) D(23,N),D(24,N) D(25,N) N = 5,...,13	change in an orbital element during the computation interval
$\Delta(\ )_D$	D(21,6),D(22,6) D(23,6),D(24,6) D(25,6)	change in an orbital element during the computation interval caused by atmospheric drag
$\Delta(\ )_M$	D(21,11),D(22,11) D(23,11),D(24,11) D(25,11)	change in an orbital element during the computation interval caused by the moon's attraction
$\Delta(\ )_{RP}$	D(21,5),D(22,5) D(23,5),D(24,5) D(25,5)	change in an orbital element during the computation interval caused by solar radiation pressure
δ		geocentric colatitude

θ		elevation angle of the satellite's velocity vector above the perpendicular to the radius to the satellite
λ		as defined by equation after Eq. (76)
μ	GM	product of the gravitational constant and the mass of the earth in cm^3/sec^2
μ_M		product of the gravitational constant and the mass of the moon
ξ, η		ξ-axis defined by the projection of the earth-sun line onto the satellite orbit plane, and the η-axis 90^o removed in the direction of motion and in the orbit plane
ρ	DENS	atmospheric density in $grams/cm^3$
σ		epoch (time of perigee passage)
ϕ	PHI	angle between an earth radius to perigee and the ξ-axis
Ψ	PSI	angle between the ecliptic plane and the earth's equatorial plane in radians
Ω	ASCN	right ascension of the ascending node of the satellite orbit and measured positive eastward from the vernal equinox direction in radians
Ω_M	ASM	right ascension of the ascending node of the moon's orbit plane
Ω_{ME}	ASM1	ecliptic longitude of the ascending node of the moon in radians
ω	OMEGA	argument of perigee in radians
ω_E	Ω_1	angular rate of rotation of the earth and atmosphere

ω_M	OMM	argument of the moon's perigee from the equator plane in radians
ω_{ME}	OMM1	argument of moon's perigee from the ecliptic in radians
$1_R, 1_S, 1_W$		unit vectors along the R, S, and W directions
1_{zE}		unit vector along the earth's polar axis in the direction of the north pole
2h,3h,4h,5h		subscripts corresponding to the second, third, fourth, and fifth harmonic, respectively

PROGRAM CONSTANTS

Constant	Value	Location	Remarks
R_E	6378.388 km	C(546), Setup, KK = 1	Equatorial radius of the earth
$A_2 (= \frac{3}{2} J_2)$	1.623270×10^{-3}	C(544), GJAY, Setup, KK = 1, 7	Coefficient of the second harmonic of the earth's potential
$A_3 (- J_3)$	2.27×10^{-6}	C(46), Setup, KK = 1, 7	Coefficient of the third harmonic of the earth's potential
$A_4 (- \frac{35}{8} J_4)$	9.2×10^{-6}	C(47), Setup, KK = 1, 7	Coefficient of the fourth harmonic of the earth's potential
$A_5 (= -J_5)$	0.26×10^{-6}	C(48), Setup, KK = 1, 7	Coefficient of the fifth harmonic of the earth's potential
m_M	$1.23 \times 10^{-2}\ M_E$	C(49), Setup, KK = 1, 11	Moon mass in units of earth mass
a_M	$60.27\ R_E$	C(50), Setup, KK = 1, 11	Semi major axis of the moon's orbit in units of R_E
e_M	0.0549	C(51), Setup, KK = 1, 11	Eccentricity of the moon's orbit
i_{ME}	0.089802 rad	C(52), Setup, KK = 1, 11	Inclination of the moon's orbit plane to the ecliptic
$\bar{r}_{EM}$	$60.5\ R_E$	C(53), Setup, KK = 1, 11	Mean distance from the earth to the moon in units of R_E
M_s	$3.33432 \times 10^5\ M_E$	C(54), Setup, KK = 1, 12	Mass of sun in units of earth mass
$\bar{r}_E$	$2.344 \times 10^5\ R_E$	C(55), Setup, KK = 1, 12	Mean distance from the earth to the sun in units of R_E
Ψ	23.44441 deg	C(504), C(543), Setup, KK = 1	Angle between ecliptic plane and equator plane
$\mu (= GM_E)$	$3.9863 \times 10^{20}\ cm^3/sec^2$	C(545), Setup, KK = 1	Earth's gravitational constant
$\dot{\beta}$	1.72029×10^{-2} rad/day	C(547), Setup, KK = 1	Angular velocity of the earth around the sun
I_o/C	$4.5 \times 10^{-5}\ dynes/cm^2$	C(548), Setup, KK = 1, 5	Solar radiation pressure at earth's surface
$\omega_E (= \Omega_1)$	0.72722×10^{-4} rad/sec	KK = 6	Angular rate of rotation of the earth

I. INTRODUCTION

This computer program was designed to study the orbital motion of balloon or dipole satellites above the appreciable atmosphere.[1] The original version of the program was developed at the Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts, by H. M. Jones and I. I. Shapiro. We obtained the program from them for a study of the stability of the Echo I satellite orbit. When that study was completed we became interested in a network based on three medium-altitude passive communication satellites, orbiting in a fixed relationship with one another to provide continuous coverage. To determine the value of such a net, it was necessary to know how long the satellites would maintain their relative positions, and also how long they would stay up. To improve the accuracy of this information as provided by the MIT program, it was necessary to modify the program to include an oblate, rotating atmosphere model.

The modified program is extremely useful for making long-term predictions of the orbital motion of close-in satellites (2000 n mi or less). The program can, in effect, look ahead for months or years --for thousands of revolutions--and predict the approximate time the satellite orbit will decay. The program described here is the only tool, so far as is known, that will make such long-term predictions cheaply and yet sufficiently accurately to be useful.

The MIT program has certain other potentially valuable uses. It could be used, for example, to assess the drag characteristics of an unknown satellite, to get some indication of its size and shape.

The program should also be useful in helping to derive an accurate model of the upper atmosphere. At present, as is well known to workers in this field, no such accurate model exists, and so discrepancies always appear when the actual motions of a satellite are compared with the motions that are predicted for it, based on the values from any of the existing model atmospheres. By making such comparisons, and correcting for them in the light of our currently increasing knowledge of the actual density and solar pressure effects in the upper atmosphere, the program described here can be used to help prepare and refine a new

working model atmosphere. Among other benefits, such improvements in the model atmosphere will increase the accuracy of prediction of trajectories of rockets and satellites.

Since the interest in satellite lifetime studies is in the long-term orbital behavior, the short-period perturbations are neglected. The program computes the changes in orbital elements by assuming constant orbital elements during an integral number of orbital periods in which the short-period perturbations are "averaged out." The orbital elements used in the next computation period are obtained by adding the computed changes to the present set of elements. This procedure of considering "average" elements is usually called the Kryloff-Bogoliuboff method.

II. THEORY

METHOD OF ORBIT COMPUTATION FOR CLOSE-IN EARTH SATELLITES

The disturbing accelerations considered are small compared with that of the central force field. Because of this the orbit is described in terms of the osculating parameters. The time variations of these parameters are related to the perturbing accelerations by the following set of nonlinear first-order differential equations:[2]

$$\frac{da}{dt} = \frac{2e \sin v}{n\sqrt{1 - e^2}} R + \frac{2a\sqrt{1 - e^2}}{nr} S \tag{1}$$

$$\frac{de}{dt} = \frac{\sqrt{1 - e^2} \sin v}{na} R + \frac{\sqrt{1 - e^2}}{na^2 e} \left[\frac{a^2(1 - e^2)}{r} - r\right] S \tag{2}$$

$$\frac{d\omega}{dt} = - \frac{\sqrt{1 - e^2} \cos v}{nae} R + \frac{\sqrt{1 - e^2}}{nae} \left[1 + \frac{r}{p}\right] S \sin v - \cos i \frac{d\Omega}{dt} \tag{3}$$

$$\frac{d\Omega}{dt} = \frac{r \sin u}{na^2 \sqrt{1 - e^2} \sin i} W \tag{4}$$

$$\frac{di}{dt} = \frac{r \cos u}{na^2\sqrt{1 - e^2}} W \tag{5}$$

$$\frac{d\sigma}{dt} = - \frac{1}{na} \left[\frac{2r}{a} - \frac{1 - e^2}{e} \cos v\right] R - \frac{(1 - e^2)}{nae} \left[1 + \frac{r}{p}\right] S \sin v \tag{6}$$

where a, e, i, Ω, ω, and σ are the orbital elements defined in the usual way, R is the component of disturbing acceleration in the radial direction, S is the component in the orbital plane perpendicular to the radial direction (making an acute angle with the satellite's velocity vector), and W is the component perpendicular to the orbit plane whose positive direction is defined by that of the cross product of the unit vectors in the directions of increasing R and S, respectively, illustrated in Fig. 1. As for the other symbols, the semilatus rectum p is

$$p = a(1 - e^2) \tag{7}$$

v is the true anomaly of the satellite's position, and u is the argument of latitude of the satellite's position and is given by

$$u = v + \omega \tag{8}$$

The mean orbital angular rate n is given by

$$n = \sqrt{\frac{\mu}{a^3}} \tag{9}$$

with μ the product of the gravitational constant and the mass of the earth,

$$GM_E = \mu \cong 3.98 \times 10^{20} \text{ cm}^3/\text{sec}^2$$

In satellite lifetime studies only the long-term changes in orbital elements are important; therefore the short-period oscillations of the orbital elements that occur during each orbit of the satellite can be left out. Considering only the long-period effects greatly reduces the computation time and the accumulation of certain types of computational errors. Based on this assumption the analysis will not include

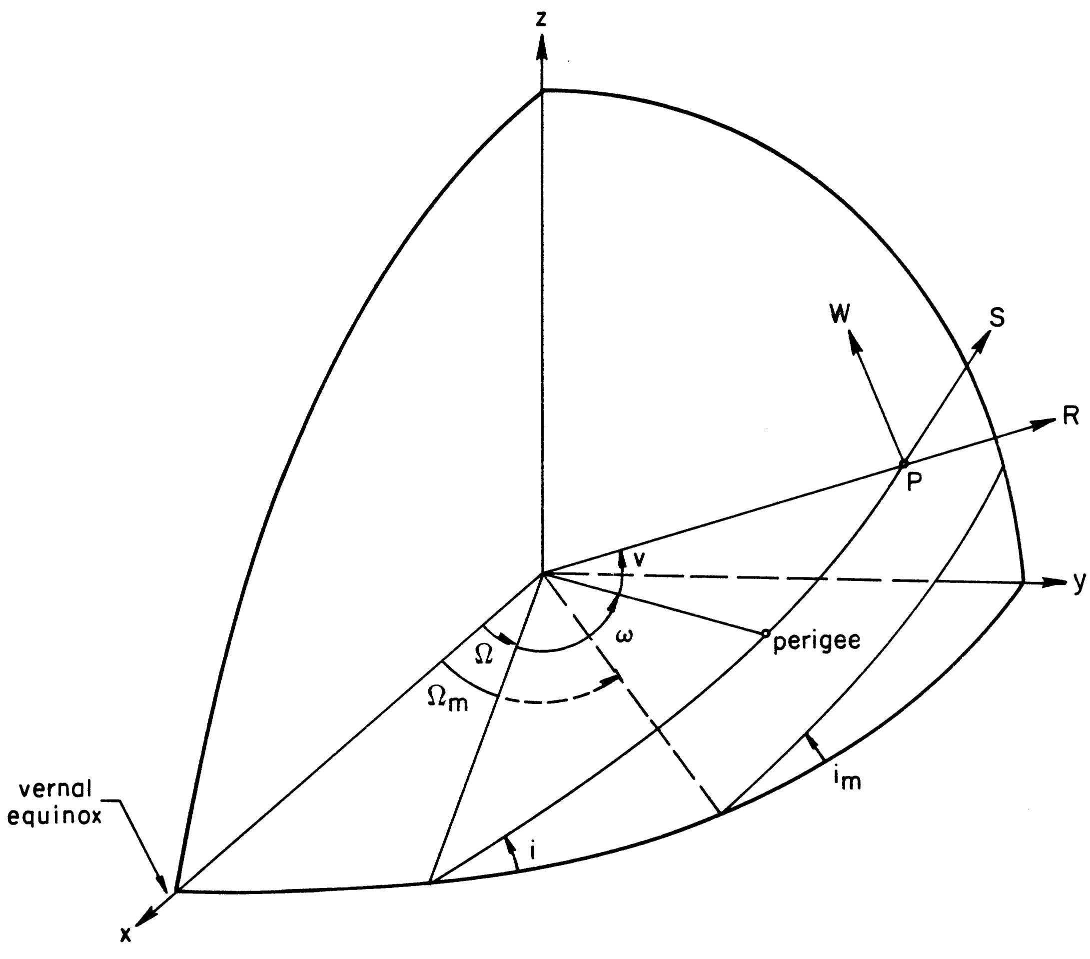

Fig. 1 — Coordinate systems and projection of orbits on a unit sphere

the instantaneous location of the satellite along its orbit. Therefore Eq. (6), which measures the change in time of perigee passage, can be eliminated. The integration of Eqs. (1) through (5) over one period neglects the short-period oscillations of the osculating elements. As a consequence the orbital elements are "average," and the elements on the right side of Eqs. (1) through (5) can be kept constant during this integration. In this way the approximate values of the changes in the osculating elements are obtained during one revolution. The mean values of the elements of the second revolution are found by adding the changes in the elements obtained from the first orbital integration to the orbital elements of the first revolution. These new orbital elements are kept constant during the next integration. This process is repeated for the number of revolutions desired.

It is convenient to change the independent variable from t to the true anomaly v. (One reason for this change of independent variable is to avoid some of the difficulties associated with the solution of Kepler's equation.) This change of variables gives:[1]

$$\frac{dv}{dt} = \frac{\sqrt{\mu p}}{r^2}\left\{1 + \frac{r^2}{\mu e}\left[R \cos v - \left(1 + \frac{r}{p}\right) S \sin v\right]\right\} \tag{10}$$

In a dissipative medium such as the atmosphere, p will decrease less rapidly than a, thus a is replaced by p using Eq. (7).[3] Then, using Eq. (10), Eqs. (1) through (5) have the following form:

$$\frac{dp}{dv} = \frac{2r^3\gamma}{\mu} S \tag{11}$$

$$\frac{de}{dv} = \frac{r^2\gamma}{\mu}\left\{R \sin v + S \frac{r}{p}\left[2 \cos v + e(1 + \cos^2 v)\right]\right\} \tag{12}$$

$$\frac{d\omega}{dv} = \frac{r^2\gamma}{\mu e}\left\{-R \cos v + S\left(1 + \frac{r}{p}\right)\sin v\right\} - \frac{d\Omega}{dv}\cos i \tag{13}$$

$$\frac{d\Omega}{dv} = W \frac{r^3\gamma}{\mu p} \frac{\sin u}{\sin i} \tag{14}$$

$$\frac{di}{dv} = W \frac{r^3\gamma}{\mu p} \cos u \tag{15}$$

where

$$\gamma = \left\{ 1 + \frac{r^2}{\mu e}\left[R \cos v - S\left(1 + \frac{r}{p}\right)\sin v\right]\right\}^{-1} \tag{15a}$$

The introduction of the perturbing accelerations R, S, and W into Eqs. (10) through (15) give the set of first-order nonlinear differential equations that are to be solved.

FIRST-ORDER PERTURBING FORCES

Of the many perturbing forces that exist, the following are included:

1. Higher harmonics of the earth's gravitational field.
2. Lunar and solar gravitational fields.
3. Direct solar radiation pressure.
4. Atmosphere drag of an oblate, neutral and rotating atmosphere.

These particular perturbations were found to be necessary for balloon satellite orbit studies. Other forces can be included by finding the R, S, and W acceleration components and integrating Eqs. (11) through (15).

Higher Harmonics of the Earth's Gravitational Field

The earth's gravitational field is simplified to the extent that the asymmetries in the mass distribution are neglected. Also, the asym-

metries in the satellite mass distribution are neglected. Then, the gravitational potential energy of the earth can be expanded in spherical harmonics as follows:

$$U(r,\delta) = \frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty} J_n\left(\frac{1}{r}\right)^n P_n(\cos\delta)\right] \tag{16}$$

where δ, the geocentric colatitude, is given by $\cos\delta = \sin i \sin u$. P_n is the nth order Legendre Polynomial, and J_n is constant, determined empirically to be

$$\begin{aligned} J_2 \times 10^6 &= (1082.18 \pm 0.03)\ R_E^2 \\ J_3 \times 10^6 &= (-2.27 \pm 0.2)\ R_E^3 \\ J_4 \times 10^6 &= (-2.12 \pm 0.05)\ R_E^4 \\ J_5 \times 10^6 &= (-0.26 \pm 0.2)\ R_E^5 \end{aligned} \tag{17}$$

and $R_E \cong 6378.388$ km is the mean equatorial radius of the earth.

The R, S, and W components of the perturbing acceleration for each of the higher harmonics are given by

$$R = -\frac{\partial U_1}{\partial r}, \quad S = -\frac{1}{r}\frac{\partial U_1}{\partial u}, \quad W = -\frac{1}{r \sin u}\frac{\partial U_1}{\partial i} \tag{18}$$

where the higher harmonics represent the perturbations in the gravitational field and

$$U_1 = -\frac{\mu}{r}\sum_{2}^{\infty} J_n \left(\frac{1}{r}\right)^n P_n(\cos\delta)$$

The nonvanishing mean changes in the orbital elements caused by these perturbations after one revolution, with the assumption that $\gamma = 1$ are:

For the second harmonic,

$$\Delta\omega_{2h} = \frac{3\pi}{2} \frac{J_2}{p^2} (5 \cos^2 i - 1)$$

$$\Delta\Omega_{2h} = - \frac{3\pi J_2}{p^2} \cos i \tag{19}$$

For the third harmonic,

$$\Delta p_{3h} = 2p \tan i \, \Delta i_{3h}$$

$$\Delta e_{3h} = - \frac{3\pi}{4} \frac{J_3}{p^3} (1 - e^2) \cos \omega \sin i \, (5 \cos^2 i - 1)$$

$$\Delta\omega_{3h} = \frac{3\pi}{4} \frac{J_3}{p^3} \frac{(1 + 4e^2)}{e} \sin \omega \sin i \, (5 \cos^2 i - 1) - \Delta\Omega_{3h} \cos i$$

$$\Delta\Omega_{3h} = \frac{3\pi}{4} \frac{J_3}{p^3} e \sin \omega \cot i \, (15 \cos^2 i - 11) \tag{20}$$

$$\Delta i_{3h} = \frac{3\pi}{4} \frac{J_3}{p^3} e \cos \omega \cos i \, (5 \cos^2 i - 1)$$

For the fourth harmonic,

$$\Delta p_{4h} = 2p \tan i \, \Delta i_{4h}$$

$$\Delta e_{4h} = - \frac{15\pi}{16} \frac{J_4}{p^4} e(1 - e^2) \sin 2\omega \sin^2 i \left[7 \cos^2 i - 1 \right]$$

$$\Delta\omega_{4h} = -\frac{15\pi}{8}\frac{J_4}{p^4}\Big\{8 - 28\sin^2 i + 21\sin^4 i$$

$$-\sin^2\omega\sin^2 i\,(7\cos^2 i - 1) + e^2\Big[6 - 14\sin^2 i + \frac{63}{8}\sin^4 i$$

$$+ \sin^6\omega\,(6 - 35\sin^2 i + \frac{63}{2}\sin^4 i\Big]\Big\}$$

$$\Delta\Omega_{4h} = \frac{15\pi}{16}\frac{J_4}{p^4}\cos i\,\Big\{2\,(7\cos^2 i - 3) + e^2\Big[7\cos^2 i - 1$$

$$+ 4\sin^2\omega\,(7\cos^2 i - 4)\Big]\Big\}$$

$$\Delta i_{4h} = \frac{15\pi}{32}\frac{J_4}{p^4}e^2\sin 2\omega\sin 2i\Big[7\cos^2 i - 1\Big] \qquad (21)$$

For the fifth harmonic,

$$\Delta P_{5h} = 2p\tan i\,\Delta i_{5h}$$

$$\Delta e_{5h} = \frac{15\pi}{16}\frac{J_5}{p^5}\cos\omega\sin i\,\Big\{8 - 28\sin^2 i + 21\sin^4 i + 0(e^2)\Big\}$$

$$\Delta\omega_{5h} = -\frac{15\pi}{16}\frac{J_5}{p^5}\frac{\sin\omega\sin i}{e}\Big\{8 - 28\sin^2 i + 21\sin^4 i + 0(e^2)\Big\}$$

$$- \cos i\,\Delta\Omega_{5h}$$

$$\Delta\Omega_{5h} = -\frac{15\pi}{16}\frac{J_5}{p^5}e\sin\omega\cot i\,\Big\{8 - 84\sin^2 i + 105\sin^4 i + 0(e^2)\Big\}$$

$$\Delta i_{5h} = -\frac{15\pi}{16}\frac{J_5}{p^5}e\cos\omega\cos i\,\Big\{8 - 28\sin^2 i + 21\sin^4 i + 0(e^2)\Big\}$$

(22)

Considering the relative sizes of the coefficients of Eq. (19), terms including J_2^2 should be included since they are as important as the linear terms in J_3, J_4, and J_5. This would require that γ no longer be taken as one and also that the secular perturbations of second order, and long-periodic perturbations of the first order be included.[4] If these terms were added in the program the accuracy of the results would be improved slightly.

Lunar and Solar Gravitational Fields

Consider now the perturbations of a close-in earth satellite orbit caused by the presence of the sun and moon.[1] Because the algebraic form of the formulae associated with the moon-induced perturbations are of the same form as the sun-induced perturbations, only one set of equations appropriate to either perturbing source will be developed.

Assume that the lunar orbit is known, then the relative acceleration of the satellite and earth due to the moon is (see Fig. 2)

$$\bar{A}_R = \bar{A}_{PM} - \bar{A}_{EM} \tag{23}$$

where $\bar{A}_{PM}$ is the acceleration of the satellite caused by the moon, $\bar{A}_{EM}$ is the corresponding acceleration of the earth. The acceleration of the satellite caused by the moon is

$$\bar{A}_{PM} = \frac{Gm_M}{r_{PM}^3} \bar{r}_{PM} \tag{24}$$

where r_{PM} is the distance between the satellite and the moon, G is the gravitational constant and m_M is the mass of the moon. From the law of cosines

$$r_{PM}^2 = r^2 + r_{EM}^2 - 2r\, r_{EM} \cos \alpha_M \tag{25}$$

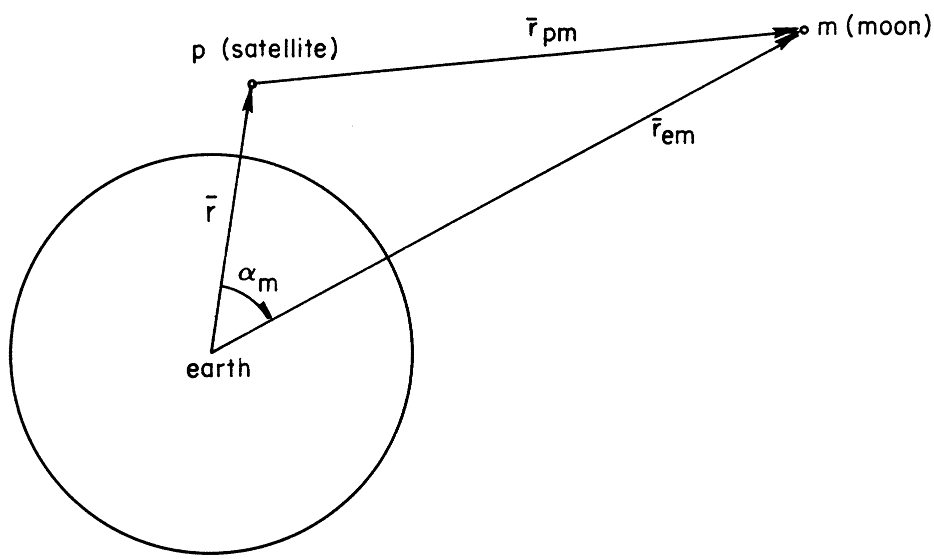

Fig. 2 — Typical orientation of earth, satellite and moon

where the quantities $\bar{r}$, $\bar{r}_{EM}$, $\bar{r}_{PM}$, α_M are illustrated in Fig. 2. The distance from the center of earth to the satellite, r, is computed from

$$r = \frac{a(1 - e^2)}{1 + e \cos v}$$

and the distance from the earth to the moon, r_{EM}, is computed from

$$r_{EM} = a_M \left[1 - e_M \cos E_M\right]$$

which is described in Appendix D.
From Eq. (25), r_{PM}^{-3} can be approximated by

$$r_{PM}^{-3} = r_{EM}^{-3} \left\{1 + 3\left(\frac{r}{r_{EM}}\right)\cos \alpha_M - \frac{3}{2}\left(\frac{r}{r_{EM}}\right)^2 \left[1 - 5 \cos^2\alpha_M\right] + 0\left[\left(\frac{r}{r_{EM}}\right)^3\right]\right\} \tag{26}$$

Then from Eqs. (23) and (24) we get

$$\bar{A}_R = \frac{Gm_M}{r_{EM}^3} \left\{\bar{r}_{PM}\left[1 + 3\left(\frac{r}{r_{EM}}\right)\cos \alpha_M + \ldots\ldots\right] - \bar{r}_{EM}\right\} \tag{27}$$

Since $\bar{r}_{PM} = \bar{r}_{EM} - \bar{r}$, Eq. (27) becomes

$$\bar{A}_R = \frac{\mu}{r_{EM}^3} \left\{ \left[3\left(\frac{r}{r_{EM}}\right)\cos \alpha_M + \frac{3}{2}\left(\frac{r}{r_{EM}}\right)^2 (5 \cos^2\alpha_M - 1)\right] \bar{r}_{EM} - \left[1 + 3\left(\frac{r}{r_{EM}}\right)\cos \alpha_M + \ldots\ldots\right]\bar{r} \right\} \tag{28}$$

The components of perturbing acceleration are along the radius to the satellite position R_M, perpendicular to the radius and in the direction of motion of the satellite S_M, and perpendicular to the orbit plane with its positive direction defined by that of the cross product of the unit vectors in the directions of increasing R_M and S_M, respectively (see Fig. 1). These are the same directions as were defined for the earth's higher harmonics but are now related to the moon and satellite interaction.

The acceleration components are determined by using Fig. 1 to relate the x,y,z axes to the R,S,W axes, respectively. This is done by first rotating the x,y,z axes through the angle Ω about the z-axis, then by rotating through the angle i about the new x-axis, and finally by rotating through the angle u about the new z-axis. After these rotations the final x,y,z axes will be aligned along the R,S,W axes, respectively. The resulting transformation in terms of the x,y,z components of r_{EM} is

$$r_{EMR} = (\cos u \cos \Omega - \sin u \sin \Omega \cos i)\, r_{EMx} + (\cos u \sin \Omega + \sin u \cos \Omega \cos i)\, r_{EMy} + \sin u \sin i\, r_{EMz} \tag{29}$$

$$r_{EMS} = -(\sin u \cos \Omega + \cos u \sin \Omega \cos i)\, r_{EMx} - (\sin u \sin \Omega - \cos u \cos \Omega \cos i)\, r_{EMy} + \cos u \sin i\, r_{EMz} \tag{30}$$

$$r_{EMW} = \sin \Omega \sin i\, r_{EMx} - \cos \Omega \sin i\, r_{EMy} + \cos i\, r_{EMz} \tag{31}$$

From Eq. (24) it is evident that the transformation for acceleration components is the same as for displacement. If i_M denotes the angle of inclination of the moon's orbit plane to the earth's equatorial plane, Ω_M denotes the angular position of the earth-moon line of nodes (measured from the vernal equinox), and u_M the position of the moon with respect to the nodal crossing, then the rectangular components of r_{EM} can be written as

$$r_{EMx} = r_{EM} (\cos u_M \cos \Omega_M - \sin u_M \sin \Omega_M \cos i_M) \tag{32}$$

$$r_{EMy} = r_{EM} (\cos u_M \sin \Omega_M + \sin u_M \cos \Omega_M \cos i_M) \tag{33}$$

$$r_{EMz} = r_{EM} \sin u_M \sin i_M \tag{34}$$

Finally, the components of r_{EM} along the R,S,W, directions are obtained from Eqs. (29) through (31), and Eqs. (32) through (34) and are

$$\begin{aligned} r_{EMR} = r_{EM} \Big\{ &\left[\cos u \cos(\Omega - \Omega_M) - \sin u \sin(\Omega - \Omega_M) \cos i\right] \cos u_M \\ &+ \left[\cos u \sin(\Omega - \Omega_M) + \sin u \cos (\Omega - \Omega_M) \cos i\right] \sin u_M \cos i_M \\ &+ \sin u \sin i \sin u_M \sin i_M \Big\} \end{aligned} \tag{35}$$

$$\begin{aligned} r_{EMS} = r_{EM} \Big\{ &\left[-\sin u \cos(\Omega - \Omega_M) - \cos u \sin(\Omega - \Omega_M) \cos i\right] \cos u_M \\ &+ \left[-\sin u \sin(\Omega - \Omega_M) + \cos u \cos(\Omega - \Omega_M) \cos i \right] \sin u_M \cos i_M \\ &+ \cos u \sin i \sin u_M \sin i_M \Big\} \end{aligned} \tag{36}$$

$$r_{EMW} = r_{EM}\Big[\sin(\Omega - \Omega_M)\sin i \cos u_M$$

$$- \cos(\Omega - \Omega_M)\sin i \sin u_M \cos i_M + \cos i \sin u_M \sin i_M\Big] \quad (37)$$

Now, using Eqs. (28) and (35) through (37) with $\mu_M = GM_m$ we can write

$$R_M = \frac{\mu_M}{r_{EM}^3}\left[3\frac{r}{r_{EM}} r_{EMR} \cos \alpha_M - r\right] \quad (38)$$

$$S_M = \frac{\mu_M}{r_{EM}^3}\left[3\frac{r}{r_{EM}} r_{EMS} \cos \alpha_M\right] \quad (39)$$

$$W_M = \frac{\mu_M}{r_{EM}^3}\left[3\frac{r}{r_{EM}} r_{EMW} \cos \alpha_M\right] \quad (40)$$

where higher-order terms in (r/r_{EM}) have been omitted. Also from Eq. (35) the cosine of the angle between $\bar{r}$ and $\bar{r}_{EM}$ can be found, since

$$\cos \alpha_M = \frac{r_{EMR}}{r_{EM}} \quad (41)$$

Now it is possible to calculate the changes in the satellite orbital elements during one revolution by using the perturbation Eqs. (11) through (15) and the perturbing acceleration components given by Eqs. (38) through (40). Because the integrations are with respect to v, the true anomaly, it is convenient to replace u, using Eq. (8), in Eqs. (35) through (37). This is accomplished by defining terms in Eqs. (38) through (40) as follows:

$$\frac{r_{EMR}}{r_{EM}} \equiv R(u) = R_1(\omega)\cos v + S_1(\omega)\sin v \quad (42)$$

$$\frac{r_{EMS}}{r_{EM}} \equiv S(u) = S_1(\omega) \cos v - R_1(\omega) \sin v \tag{43}$$

$$\frac{r_{EMW}}{r_{EM}} = W_1(\omega) \tag{44}$$

The quantities $R_1(\omega)$, $S_1(\omega)$ and $W_1(\omega)$ are, respectively, the cosine of the angle between the earth-moon line and the line from earth's center to the perigee point of the satellite's orbit, the cosine of the angle between the earth-moon line and the semilatus rectum of the satellite's orbit, and the cosine of the angle between the earth-moon line and the line normal to the orbit plane. By replacing u with $(v + \omega)$ in Eqs. (35) through (37) and rearranging, it is not difficult to show that

$$\begin{aligned} R_1(\omega) = & \left[\cos \omega \cos (\Omega - \Omega_M) - \sin \omega \sin(\Omega - \Omega_M) \cos i\right] \cos u_M \\ & + \left[\cos \omega \sin (\Omega - \Omega_M) + \sin \omega \cos (\Omega - \Omega_M) \cos i\right] \sin u_M \cos i_M \\ & + \sin \omega \sin i \sin u_M \sin i_M \end{aligned} \tag{45}$$

$$\begin{aligned} S_1(\omega) = & \left[- \sin \omega \cos (\Omega - \Omega_M) - \cos \omega \sin (\Omega - \Omega_M) \cos i\right] \cos u_M \\ & + \left[- \sin \omega \sin (\Omega - \Omega_M) + \cos \omega \cos (\Omega - \Omega_M) \cos i\right] \sin u_M \cos i_M \\ & + \cos \omega \sin i \sin u_M \sin i_M \end{aligned} \tag{46}$$

and

$$\begin{aligned} W_1(\omega) = & \sin (\Omega - \Omega_M) \sin i \cos u_M - \cos(\Omega - \Omega_M) \sin i \sin u_M \cos i_M \\ & + \cos i \sin u_M \sin i_M \end{aligned} \tag{47}$$

The components of the perturbing acceleration can now be written as

$$R_M = 3\frac{\mu_M r}{r_{EM}^3}\left\{R_1^2 \cos^2 v + 2R_1S_1 \sin v \cos v + S_1^2 \sin^2 v - \frac{1}{3}\right\} \quad (48)$$

$$S_M = 3\frac{\mu_M r}{r_{EM}^3}\left\{R_1S_1 \cos^2 v - (R_1^2 - S_1^2) \sin v \cos v - R_1S_1 \sin^2 v\right\} \quad (49)$$

$$W_M = 3\frac{\mu_M r}{r_{EM}^3}\left\{W_1(R_1 \cos v + S_1 \sin v)\right\} \quad (50)$$

Using Eqs. (48) through (50) it is possible to evaluate the changes in the orbital elements over one period. Consider, for example, Eqs. (11) and (49). For $\gamma = 1$ we get

$$\Delta p_M = 6p\frac{\mu_M}{\mu}\left(\frac{p}{r_{EM}}\right)^3$$

$$\cdot \int_0^{2\pi} \frac{R_1S_1 \cos^2 v - (R_1^2 - S_1^2) \sin v \cos v - R_1S_1 \sin^2 v}{(1 + e \cos v)^4}\, dv \quad (51)$$

where integrals of the type

$$I_k(n,m) = \int_0^{2\pi} \frac{\cos^n v \sin^m v}{(1 + e \cos v)^k}\, dv \quad (52)$$

are present. These are evaluated in Appendix A. By selecting the appropriate integrals from this Appendix, Eq. (51) reduces to

$$\Delta p_M = 6p\left(\frac{\mu_M}{\mu}\right)\left(\frac{p}{r_{EM}}\right)^3 \left[R_1 S_1\left(I_4(2,0) - I_4(0,2)\right) - (R_1^2 - S_1^2)\ I_4(1,1)\right]$$

where

$$I_4(2,0) - I_4(0,2) = \pi(1 - e^2)^{-7/2}\ 5e^2$$

and

$$I_4(1,1) = 0$$

hence

$$\Delta p_M = L\ 10pe^2(1 - e^2)^{-1}\ R_1 S_1 \qquad (53)$$

where

$$L = 3\left(\frac{\mu_M}{\mu}\right)\left(\frac{p}{r_{EM}}\right)^3 \pi(1 - e^2)^{-5/2} \simeq 10^{-7}$$

In a similar way the changes in the other elements are found to be

$$\Delta e_M = -\ L\ 5e\ R_1 S_1 \qquad (54)$$

$$\Delta \omega_M = L(4R_1^2 - S_1^2 - 1) - \cos i\ \Delta\Omega_M \qquad (55)$$

$$\Delta\Omega_M = L\ \frac{W_1}{\sin i}\left[R_1(1 - e^2)^{-1}\ (1 + 4e^2)\ \sin\omega + S_1\ \cos\omega\right] \qquad (56)$$

$$\Delta i_M = LW_1\left[R_1(1 - e^2)^{-1}\ (1 + 4e^2)\ \cos\omega - S_1\ \sin\omega\right] \qquad (57)$$

and R_1, S_1, and W_1 are given by Eqs. (45) through (47), respectively.

The solar gravitational field perturbations are analogous to those just found for the moon. If the subscripts M are replaced by S, then Eqs. (53) through (57) give the appropriate equations for the interaction between the sun and satellite.

Direct Solar Radiation Pressure

The force experienced by an object subjected to solar radiation is proportional to the product of the solar energy flux and the cross-sectional area of the object perpendicular to the direction of the flux and also the object's reflective properties.[(1)] The acceleration caused by radiation pressure in a radial direction away from the sun is

$$A_{RP} = K\left(\frac{I}{C}\right)\left(\frac{A}{M}\right) \tag{58}$$

where I is the solar flux near the earth, C the speed of light, $\frac{A}{M}$ the aspect averaged cross-sectional area-to-mass ratio of the satellite, and K is a constant $(0 \leq K \leq 4/3)$ which depends on the reflection characteristics of the satellite. For a perfectly absorbing satellite $K = 1$ and for a flat, specularly reflecting satellite $K = 4/3$, and for a completely transparent satellite $K = 0$.

Because the earth's orbit is elliptical the solar flux will vary during the year according to

$$I = I_o\left(\frac{a_E}{r_E}\right)^2 \tag{59}$$

where a_E is the semimajor axis of the earth's orbit, r_E is the instantaneous distance of the earth from the sun, and I_o is the value of the solar constant at the distance a_E from the sun.* The variation in

*At the surface of the earth it has been observed that $I_o/C \approx 4.5 \times 10^{-5}$ dynes/cm^2 and that it is almost a constant except for short periods of time when it may vary a few per cent.

distance between the sun and the satellite caused by the satellite's motion is ignored.

Let Ψ be the angle between the ecliptic plane and the earth's equatorial plane, and let β be the angular position of the earth-sun line relative to the vernal equinox. Then, the rectangular components of acceleration caused by solar radiation pressure on the satellite are

$$A_{RPx} = - A_{RP} \cos \beta \tag{60}$$

$$A_{RPy} = -A_{RP} \sin \beta \cos \Psi \tag{61}$$

$$A_{RPz} = - A_{RP} \sin \beta \sin \Psi \tag{62}$$

where the minus signs denote that the acceleration is in a direction opposite to that of the sun. The R, S, W components are found by substituting Eqs. (60) through (62) into equations of the same form as Eqs. (29) through (31).

$$\begin{aligned} R_{RP} = - A_{RP} \Big[&(\cos u \cos \Omega - \sin u \sin \Omega \cos i) \cos \beta \\ &+ (\cos u \sin \Omega + \sin u \cos \Omega \cos i) \sin \beta \cos \Psi \\ &+ \sin u \sin i \sin \beta \sin \Psi \Big] \end{aligned} \tag{63}$$

$$\begin{aligned} S_{RP} = - A_{RP} \Big[&- (\sin u \cos \Omega + \cos u \sin \Omega \cos i) \cos \beta \\ &- (\sin u \sin \Omega - \cos u \cos \Omega \cos i) \sin \beta \cos \Psi \\ &+ \cos u \sin i \sin \beta \sin \Psi \Big] \end{aligned} \tag{64}$$

$$W_{RP} = -A_{RP}\Big[\sin\Omega\sin i\cos\beta - \cos\Omega\sin i\sin\beta\cos\Psi$$

$$+ \cos i\sin\beta\sin\Psi\Big] \quad (65)$$

As before, the integration is with respect to the true anomaly, v, hence it is convenient to separate variables by replacing u with $(\omega + v)$ as was done in Eqs. (42) through (44).

$$R_{RP} = -A_{RP}\left[R_2(\omega)\cos v + S_2(\omega)\sin v\right] \quad (66)$$

$$S_{RP} = -A_{RP}\left[S_2(\omega)\cos v - R_2(\omega)\sin v\right] \quad (67)$$

$$W_{RP} = -A_{RP}\,W_2(\omega) \quad (68)$$

where $R_2(\omega)$, $S_2(\omega)$, and $W_2(\omega)$ are similar to $R_1(\omega)$, $S_1(\omega)$, and $W_1(\omega)$ of Eqs. (45) through (47) but now with respect to the earth-sun line. Thus

$$R_2(\omega) = (\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i)\cos\beta$$

$$+ (\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i)\sin\beta\cos\Psi$$

$$+ \sin\omega\sin i\sin\beta\sin\Psi \quad (69)$$

$$S_2(\omega) = -(\sin\omega\cos\Omega + \cos\omega\sin\Omega\cos i)\cos\beta$$

$$+ (-\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i)\sin\beta\cos\Psi$$

$$+ \cos\omega\sin i\sin\beta\sin\Psi \quad (70)$$

$$W_2(\omega) = \sin\Omega\sin i\cos\beta - \cos\Omega\sin i\sin\beta\cos\Psi$$

$$+ \cos i\sin\beta\sin\Psi \quad (71)$$

If the satellite remains in the sun during the entire orbit the changes in the elements can be found in the same way as changes caused by the earth-sun gravitational interaction.

If we use Eq. (58) to replace A_{RP} in Eqs. (66) through (68) and then substitute into Eqs. (11) through (15) we get integrals of the type given by Eq. (51). These are integrated to give

$$\Delta p_{RP} = \frac{6\pi\lambda e}{(1 - e^2)^{5/2}} pS_2 \tag{72}$$

$$\Delta e_{RP} = - \frac{3\pi\lambda}{(1 - e^2)^{3/2}} S_2 \tag{73}$$

$$\Delta\omega_{RP} = \frac{3\pi\lambda}{e(1 - e^2)^{3/2}} R_2 + \Delta\Omega_{RP} \cos i \tag{74}$$

$$\Delta\Omega_{RP} = - \frac{3\pi\lambda e}{(1 - e^2)^{5/2}} W_2 \frac{\sin \omega}{\sin i} \tag{75}$$

$$\Delta i_{RP} = \frac{3\pi\lambda e}{(1 - e^2)^{5/2}} W_2 \cos \omega \tag{76}$$

where

$$\lambda = \frac{KA}{M} \left(\frac{I}{C}\right)\left(\frac{p^2}{\mu}\right)$$

Influence of the Earth's Shadow[1]

If the satellite passes into the earth's shadow during some portion of its orbit, the determination of the changes in the orbital elements during a period is much more difficult.

In order to simplify the problem a bit assume the shadow region cast by the earth to be bounded by a circular cylinder. The axis of the cylindrical shadow is the same as the earth-sun line, its radius is assumed to be equal to the earth's mean equatorial radius and its end is found by a plane which passes through the earth's center and is perpendicular to the earth-sun line. This is an approximation for the following three reasons:

1. The sun is an extended source which will produce an earth shadow consisting of a penumbra and an umbra region. These will vary during the year due to the ellipticity of the earth's orbit.
2. The atmosphere of the earth causes refraction of the sunlight.
3. The cross-section of the earth in the plane perpendicular to the earth-sun line varies throughout the year.

The boundary of the orbital plane intersection with the cylindrical shadow is in general an ellipse, see Fig. 3. Hence, from Fig. 4

$$\left(\frac{\xi \cos i_1}{R_E}\right)^2 + \left(\frac{\eta}{R_E}\right)^2 = 1 \tag{77}$$

The components of satellite position are $\xi = r \cos(v - \phi)$ and $\eta = r \sin(v - \phi)$.

If these components are substituted into Eq. (77) we get

$$R_E^2 - r^2 \left[1 - \sin^2 i_1 \cos^2(v - \phi)\right] = 0 \tag{78}$$

when the satellite is on the boundary of the shadow. From

$$R_2^2(\omega) + S_2^2(\omega) + W_2^2(\omega) = 1$$

and

$$\sin^2 i_1 = 1 - W_2^2(\omega)$$

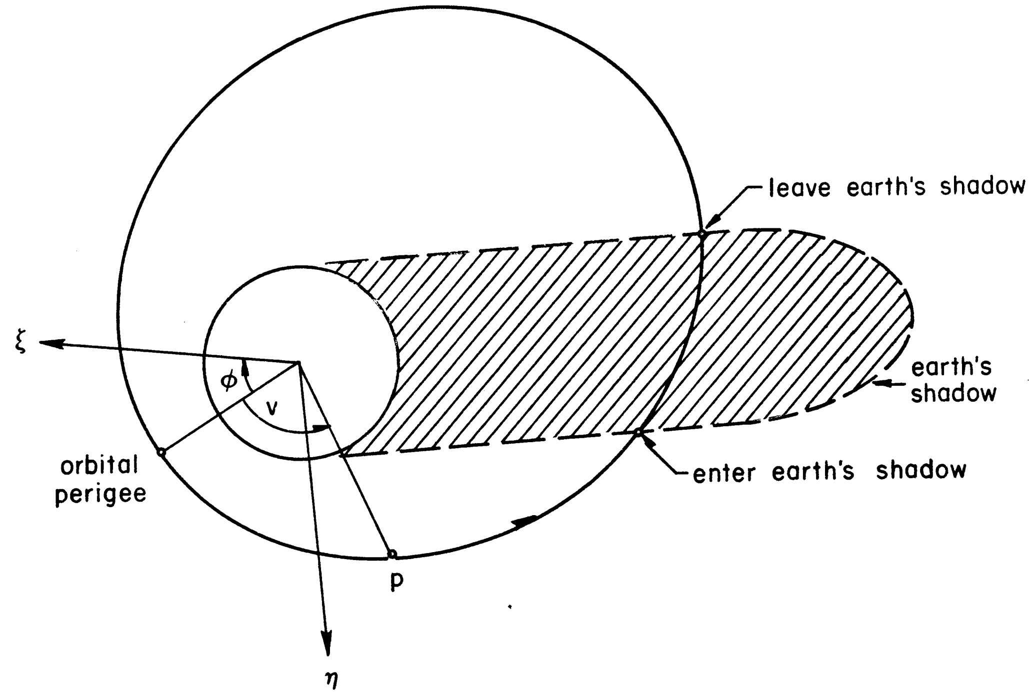

Fig. 3 — Satellite passage through earth's shadow, projected onto the satellite orbit plane

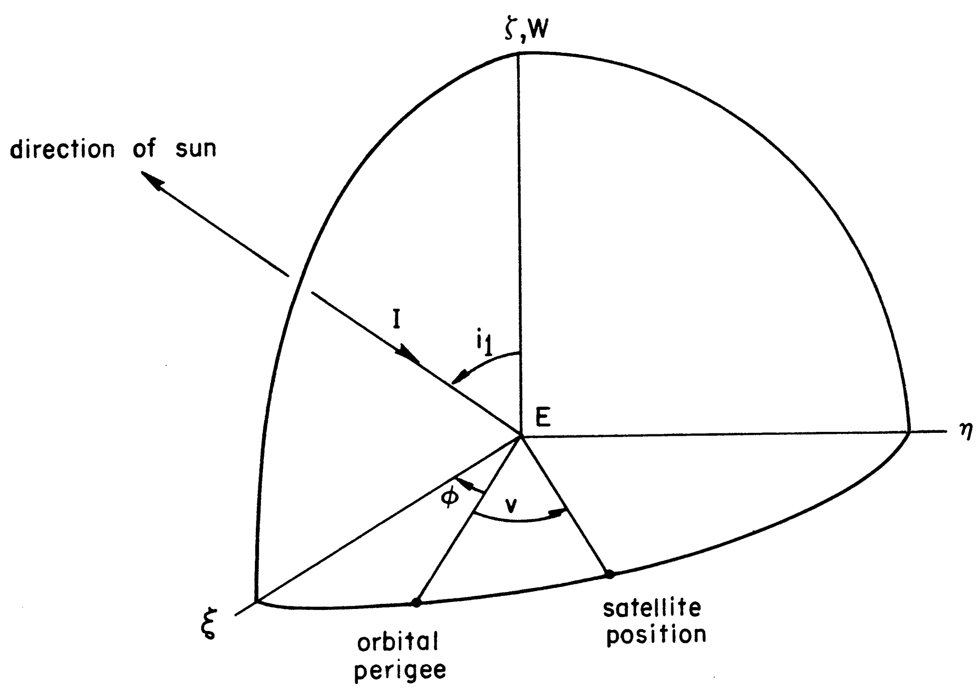

Fig. 4 — Relative orientation of sun and satellite

and Eqs. (69) and (70), Eq. (78) becomes

$$\cos(v - \phi) + \left[\frac{p^2 - R_E^2 (1 + e \cos v)^2}{p^2\left(R_2^2(\omega) + S_2^2(\omega)\right)}\right]^{1/2} = 0 \qquad (79)$$

where p is the semilatus rectum of the satellite orbit.

The boundaries of the portion of the satellite orbit that is in the earth's shadow are found in the program by evaluating the left side of Eq. (79). If it is positive the satellite is in the shadow. When the left side of Eq. (79) changes sign the incremental value of the true anomaly v is reduced and the left side of Eq. (79) is again evaluated in order to determine the boundaries of the shadow more precisely. The search for the boundaries can be simplified by certain preliminary tests. If the perigee distance of the orbit is greater than the semimajor axis of the shadow ellipse, then the satellite is always in the sunshine, i.e., if $a(1 - e) > a_S$, where a_S is the semimajor axis of the shadow ellipse. If, on the other hand, the apogee distance of the satellite orbit $a(1 + e) < a_S$ then the satellite must pass through the shadow. Once the shadow boundary has been determined, the new boundaries for the next orbit are based on the previous boundaries as a first approximation.

For calculation of the changes in orbital elements caused by the solar radiation pressure, A_{RP} can now be redefined as

$$A_{RP} = \begin{cases} K \dfrac{I}{C} \dfrac{A}{M} & \text{for satellite in the sunshine} \\ 0 & \text{for satellite in the shadow} \end{cases}$$

Now Eqs. (11) through (15) with $\gamma = 1$ can be rewritten in terms of indefinite integrals as follows:

$$\Delta p_{RP} = 2\lambda p \left[R_2 Q_1 - S_2 Q_2\right] \tag{80}$$

$$\Delta e_{RP} = \lambda\left[R_2 Q_3 - S_2 Q_4\right] \tag{81}$$

$$\Delta\omega_{RP} = + \Delta\Omega_{RP} \cos i + \frac{\lambda}{e}\left[R_2 Q_5 - S_2 Q_6\right] \tag{82}$$

$$\Delta\Omega_{RP} = - \frac{\lambda}{\sin i} W_2 \left[Q_1 \cos \omega + Q_2 \sin \omega\right] \tag{83}$$

$$\Delta i_{RP} = - \lambda W_2 \left[Q_2 \cos \omega - Q_1 \sin \omega\right] \tag{84}$$

where

$$\lambda = K \left(\frac{I}{C}\right)\left(\frac{A}{M}\right)\left(\frac{p^2}{\mu}\right)$$

and

$$Q_1 = \int \frac{\sin v}{(1 + e \cos v)^3} dv \tag{85}$$

$$Q_2 = \int \frac{\cos v}{(1 + e \cos v)^3} dv \tag{86}$$

$$Q_3 = \int \frac{\sin v \,(e + \cos v)}{(1 + e \cos v)^3} dv \tag{87}$$

$$Q_4 = \int \frac{(1 + 2e \cos v + \cos^2 v)}{(1 + e \cos v)^3} dv \tag{88}$$

$$Q_5 = \int \frac{(2 + e \cos v - \cos^2 v)}{(1 + e \cos v)^3} dv \tag{89}$$

$$Q_6 = \int \frac{\sin v \cos v}{(1 + e \cos v)^3} dv \tag{90}$$

The limits of integration depend on the position of the shadow relative to the orbit. If the orbit is entirely in the sunshine the integration is from 0 to 2π, in this case for $Q_i(v)$ the quantity $Q_i(2\pi) - Q_i(0)$ is substituted. If the satellite enters the shadow during a portion of its orbit, say at v_1, $(0 \le v_1 \le 2\pi)$ and leaves it at v_2, $(0 \le v_2 \le 2\pi)$ then the region of integration will be from either 0 to v_1 and v_2 to 2π, or from v_2 to v_1 depending on whether the perigee point $(v = 0)$ is in the sunshine or shadow. In these cases for $Q_i(v)$ the quantity

$$Q_i(v_1) - Q_i(v_2) + \sigma(v_2 - v_1)\left[Q_i(2\pi) - Q_i(0)\right]$$

where

$$\sigma(v_2 - v_1) = \begin{cases} 0 \text{ if } (v_2 - v_1) \le 0 \\ \\ 1 \text{ if } (v_2 - v_1) \ge 0 \end{cases}$$

is substituted. When $v_2 - v_1 \ge 0$ the perigee point lies in the sunlight, and when $v_2 - v_1 \le 0$ the perigee point is in the shadow.

To evaluate the integrals Eqs. (85) through (90) when the limits are 0 to 2π it is useful to note that they are a combination of the form Eq. (52) which are evaluated in Appendix A. For intermediate limits of integration, quantities with factors $(1 + e \cos v)$ in the denominator are simplified if the eccentric anomaly is used as the independent variable. In any case Eqs. (85) through (90) integrate to the following:

$$Q_1 = - \frac{\cos v\ (2 + e \cos v)}{(1 + e \cos v)^2} + c_1 \tag{91}$$

$$Q_2 = -\frac{3eE}{2(1-e^2)^{5/2}} + \frac{\sin v\left[2 + e^2 + e(1+2e^2)\cos v\right]}{2(1-e^2)^2(1+e\cos v)^2} + c_2 \tag{92}$$

$$Q_3 = \frac{\sin^2 v}{2(1+e\cos v)^2} + c_3 \tag{93}$$

$$Q_4 = \frac{3E}{2(1-e^2)^{3/2}} - \frac{\sin v\left[3e - (1-4e^2)\cos v\right]}{2(1-e^2)(1+e\cos v)^2} + c_4 \tag{94}$$

$$Q_5 = \frac{3E}{2(1-e^2)^{3/2}} - \frac{\sin v\left[3e + (1+2e^2)\cos v\right]}{2(1-e^2)(1+e\cos v)^2} + c_5 \tag{95}$$

$$Q_6 = -\frac{\cos^2 v}{2(1+e\cos v)^2} + c_6 \tag{96}$$

where the eccentric anomaly E is related to the true anomaly by

$$\tan\frac{v}{2} = \left(\frac{1+e}{1-e}\right)^{1/2} \tan\frac{E}{2}$$

Atmospheric Drag of an Oblate, Neutral, and Rotating Atmosphere[5]

An accurate theoretical prediction of the effects of the atmosphere on satellite orbits is not possible at present. Air densities are known to vary with time and longitude as well as with altitude but in not an entirely predictable manner. In addition, the effects of neutral and charge drag are not fully understood. Therefore this discussion is limited to simplified models that yield reasonably accurate orbital elements when compared to the orbital elements of a known satellite.

The general differential equations for determining the changes in the orbital elements, Eqs. (11) through (15), are again applied

after obtaining the components of the drag acceleration R_D, S_D, and W_D.

Atmospheric drag force enters through two mechanisms, that of neutral drag and charge drag. Fortunately, for ordinary satellites (excluding needles) charge drag can usually be neglected. Therefore the following discussion pertains to the effects of neutral drag on the orbital elements. We assume that neutral drag force is caused by the satellite's absorbing (and isotropically re-emitting) all molecules (and ions) in its path. If we assume the speed of the molecule is negligible compared to that of the satellite, air is absorbed at the rate of $\rho A \overline{V}_r$, where ρ is the air density, $\overline{V}_r$ is the satellite velocity relative to the rotating atmosphere and A is the area of the satellite when projected on the plane perpendicular to $\overline{V}_r$. The rate of momentum transfer is $\rho A V_r \overline{V}_r$, which leads to a deceleration (an acceleration directed oppositely to the direction of $\overline{V}_r$) of

$$A_D = \frac{1}{2} C_D \frac{A}{M} \rho V_r^2 \tag{97}$$

where C_D is the drag coefficient and A/M is the aspect averaged cross-sectional area-to-mass ratio of the satellite.

The velocity vector of the satellite relative to the atmosphere is

$$\overline{V}_r = \overline{V} - \overline{V}_a \tag{98}$$

where $\overline{V}$ is the velocity vector of the satellite in inertial space (the orbital velocity) and $\overline{V}_a$ is the velocity vector of the atmosphere in inertial space at the satellite's position. Their magnitudes are given by

$$V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

and

$$V_a = \omega_E r \sin \delta$$

where ω_E is the angular rate of rotation of the earth and atmosphere, r is the distance of the satellite from the center of the earth, and δ is the geocentric colatitude of the satellite.

The components of the drag acceleration are obtained by expressing the drag acceleration vector as

$$\bar{A}_D = -\frac{1}{2} C_D \frac{A}{M} \rho V_r^2 1_{V_r} \tag{99}$$

where 1_{V_r} is a unit vector in the direction of $\bar{V}_r$, and then projecting $\bar{V}_r$ onto the R, S, and W directions. The inertial velocity vector of the satellite is

$$\bar{V} = V \sin \theta \, 1_R + V \cos \theta \, 1_S \tag{100}$$

where θ is the elevation angle of $\bar{V}$ above a perpendicular to the radius vector in orbital plane. The velocity vector of the atmosphere is

$$\begin{aligned}
\bar{V}_a &= \omega_E 1_{zE} \times \bar{r} = \omega_E 1_{zE} \times r \, 1_R \\
&= \omega_E (\sin u \sin i \, 1_R + \cos u \sin i \, 1_S + \cos i \, 1_W) \times r \, 1_R \\
&= \omega_E r \cos i \, 1_S - \omega_E r \cos u \sin i \, 1_W \qquad (101)
\end{aligned}$$

where 1_R, 1_S, and 1_W are unit vectors in the R, S, and W directions and 1_{zE} is a unit vector along the earth's polar axis.

Using Eqs. (100) and (101) the relative velocity of the satellite is

$$\bar{V}_r = V \sin \theta \, 1_R + (V \cos \theta - \omega_E r \cos i) \, 1_S + \omega_E r \cos u \sin i \, 1_W \tag{102}$$

By rewriting Eq. (99) as

$$\overline{A}_D = -\frac{1}{2} C_D \frac{A}{M} \rho V_r \overline{V}_r$$

and substituting for $\overline{V}_r$ we get

$$\overline{A}_D = -\frac{1}{2} C_D \frac{A}{M} \rho V_r \Big[V \sin\theta \, 1_R + (V \cos\theta - \omega_E r \cos i) \, 1_S + \omega_E r \cos u \sin i \, 1_W \Big] \tag{103}$$

Thus, the components of acceleration are

$$\begin{aligned} R_D &= -\frac{1}{2} C_D \frac{A}{M} \rho V_r V \sin\theta \\ S_D &= -\frac{1}{2} C_D \frac{A}{M} \rho V_r (V \cos\theta - \omega_E r \cos i) \\ W_D &= -\frac{1}{2} C_D \frac{A}{M} \rho V_r \omega_E r \cos u \sin i \end{aligned} \tag{104}$$

The components of $\overline{V}$ as functions of the orbital elements and the true anomaly v are

$$\begin{aligned} V \sin\theta &= \sqrt{\frac{\mu}{p}} \, e \sin v \\ V \cos\theta &= \sqrt{\frac{\mu}{p}} \, (1 + e \cos v) \end{aligned} \tag{105}$$

The final forms of the acceleration components are

$$R_D = -\frac{1}{2} C_D \frac{A}{M} \rho V_r \sqrt{\frac{\mu}{p}} e \sin v$$

$$S_D = -\frac{1}{2} C_D \frac{A}{M} \rho V_r \left[\sqrt{\frac{\mu}{p}}(1 + e \cos v) - \omega_E r \cos i\right]$$

$$W_D = -\frac{1}{2} C_D \frac{A}{M} \rho V_r \omega_E r \cos (\omega + v) \sin i \qquad (106)$$

where

$$V_r = \sqrt{V^2 \sin^2\theta + (V \cos \theta - \omega_E r \cos i)^2 + \omega_E^2 r^2 \sin^2 i \cos^2 (\omega + v)} \qquad (107)$$

By substituting Eq. (105) into Eq. (107) and simplifying we get V_r in the desired form, i.e.,

$$V_r = \left(\frac{\mu}{p}\right)^{1/2} \left[(1 + e^2 + 2e \cos v) - 2\frac{p^{3/2}}{\sqrt{\mu}} \omega_E \cos i\right.$$

$$\left. + \frac{p^3 \omega_E^2}{\mu(1 + e \cos v)^2} \left\{1 - \frac{1}{2} (1 - \cos^2 i) \left[1 - \cos (2\omega + 2v)\right]\right\}\right]^{1/2}$$

$$= \left(\frac{\mu}{p}\right)^{1/2} (X + Y)^{1/2} \qquad (108)$$

where

$$X = 1 + e^2 + 2e \cos v$$

and

$$Y = -2 \frac{p^{3/2}}{\sqrt{\mu}} \omega_E \cos i$$

$$+ \frac{p^3 \omega_E^2}{\mu(1 + e \cos v)^2} \left\{1 - \frac{1}{2} (1 - \cos^2 i) \left[1 - \cos (2\omega + 2v)\right] \right\}$$

Clearly, X and Y are associated with the nonrotating and rotating atmosphere, respectively.

In Appendix B it is shown that

$$\sqrt{X + Y} \approx \sqrt{X} + \frac{Y}{2\sqrt{X}} \approx \sqrt{X} + \frac{Y_1}{2\sqrt{X}} \tag{109}$$

where

$$Y_1 = - \frac{2p^{3/2}}{\sqrt{\mu}} \omega_E \cos i$$

is only the first term of the equation for Y. Thus, the final form of V_r used in the program is

$$V_r \approx \sqrt{\frac{\mu}{p}} \sqrt{1 + e^2 + 2e \cos v} - \frac{p \omega_E \cos i}{\sqrt{1 + e^2 + 2e \cos v}} \tag{110}$$

The approximate equations for the changes in the orbital elements for one revolution of the satellite are found by substituting Eqs. (104), (105), and (110) into Eqs. (11) through (15) and integrating. This gives

$$\Delta p_D = - C_d \frac{A}{M} \gamma p^2 \int_o^{2\pi} \rho \frac{\sqrt{1 + e^2 + 2e \cos v}}{(1 + e \cos v)^2} dv$$

$$+ C_d \frac{A}{M} \gamma \frac{p^{\frac{7}{2}}}{\sqrt{\mu}} \omega_E \cos i \int_o^{2\pi} \rho \frac{1}{(1 + e \cos v)^2 \sqrt{1 + e^2 + 2e \cos v}} dv$$

$$+ C_d \frac{A}{M} \gamma \frac{p^{\frac{7}{2}}}{\sqrt{\mu}} \omega_E \cos i \int_o^{2\pi} \rho \frac{1 + e^2 + 2e \cos v}{(1 + e \cos v)^4} dv$$

$$- C_d \frac{A}{M} \gamma \frac{p^5}{\mu} \omega_E^2 \cos^2 i \int_o^{2\pi} \rho \frac{1}{(1 + e \cos v)^4 \sqrt{1 + e^2 + 2e \cos v}} dv \qquad (111)$$

$$\Delta e_D = - C_d \frac{A}{M} \gamma p \int_o^{2\pi} \rho \frac{(e + \cos v) \sqrt{1 + e^2 + 2e \cos v}}{(1 + e \cos v)^2} dv$$

$$+ \frac{1}{2} C_d \frac{A}{M} \gamma \frac{p^{\frac{5}{2}}}{\sqrt{\mu}} \omega_E \cos i \int_o^{2\pi} \rho \frac{\left[2 \cos v + e(1 + \cos^2 v)\right] \sqrt{1 + e^2 + 2e \cos v}}{(1 + e \cos v)^4} dv$$

$$+ 2 C_d \frac{A}{M} \gamma \frac{p^{\frac{5}{2}}}{\sqrt{\mu}} \omega_E \cos i \int_o^{2\pi} \rho \frac{e + \cos v}{(1 + e \cos v)^2} dv$$

$$- C_d \frac{A}{M} \gamma \frac{p^4}{\mu} \omega_E^2 \cos^2 i \int_o^{2\pi} \rho \frac{2 \cos v + e(1 + \cos^2 v)}{(1 + e \cos v)^4} dv \qquad (112)$$

$$\Delta\omega_D = - C_d \frac{A}{M} \gamma \frac{p}{e} \int_o^{2\pi} \rho \frac{\sin v \sqrt{1 + e^2 + 2e \cos v}}{(1 + e \cos v)^2} dv$$

$$+\frac{1}{2} C_d \frac{A}{M} \gamma \frac{p^{\frac{5}{2}}}{e\sqrt{\mu}} \omega_E \cos i \int_o^{2\pi} \rho \frac{(2 + e \cos v) \sin v \sqrt{1 + e^2 + 2e \cos v}}{(1 + e \cos v)^4} dv$$

$$+\frac{1}{4} C_d \frac{A}{M} \gamma \frac{p^{\frac{5}{2}}}{\sqrt{\mu}} \omega_E \cos i \int_o^{2\pi} \rho \frac{\sin (2\omega + 2v) \sqrt{1 + e^2 + 2e \cos v}}{(1 + e \cos v)^4} dv$$

$$+ C_d \frac{A}{M} \gamma \frac{p^{\frac{5}{2}}}{e\sqrt{\mu}} \omega_E \cos i \int_o^{2\pi} \rho \frac{\sin v}{\sqrt{1 + e^2 + 2e \cos v}} dv$$

$$- \frac{1}{2} C_d \frac{A}{M} \gamma \frac{p^4}{\mu e} \omega_E^2 \cos^2 i \int_o^{2\pi} \rho \frac{(2 + e \cos v) \sin v}{(1 + e \cos v)^4 \sqrt{1 + e^2 + 2e \cos v}} dv$$

$$- \frac{1}{4} C_d \frac{A}{M} \gamma \frac{p^4}{\mu} \omega_E^2 \cos^2 i \int_o^{2\pi} \rho \frac{\sin (2\omega + 2v)}{(1 + e \cos v)^4 \sqrt{1 + e^2 + 2e \cos v}} dv \qquad (113)$$

$$\Delta\Omega_D = - \frac{1}{4} C_d \frac{A}{M} \gamma \frac{p^{\frac{5}{2}}}{\sqrt{\mu}} \omega_E \int_o^{2\pi} \rho \frac{\sin (2\omega + 2v) \sqrt{1 + e^2 + 2e \cos v}}{(1 + e \cos v)^4} dv$$

$$+ \frac{1}{4} C_d \frac{A}{M} \gamma \frac{p^4}{\mu} \omega_E^2 \cos i \int_o^{2\pi} \rho \frac{\sin (2\omega + 2v)}{(1 + e \cos v)^4 \sqrt{1 + e^2 + 2e \cos v}} dv \qquad (114)$$

$$\Delta i_D = -\frac{1}{2} C_d \frac{A}{M} \gamma \frac{p^{\frac{5}{2}}}{\sqrt{\mu}} \omega_E \int_0^{2\pi} \rho \frac{\cos(\omega + v)\sqrt{1 + e + 2e\cos v}}{(1 + e\cos v)^4} dv$$

$$+ \frac{1}{2} C_d \frac{A}{M} \gamma \frac{p^4}{\mu} \omega_E^2 \cos i \int_0^{2\pi} \rho \frac{\cos^2(\omega + v)}{(1 + e\cos v)^4 \sqrt{1 + e^2 + 2e\cos v}} dv \tag{115}$$

The atmospheric density ρ appears in the integrand of Eqs. (111) through (115) as

$$\rho = f\left[h(v)\right]^*$$

which is a function of v only for a given satellite orbit because the orbital elements are constant during the integration period. The altitude above the geoid, h(v) can be written as

$$h(v) = r(v) - r_E(v)$$

$$= \frac{p}{1 + e\cos v} - \frac{R_E^2 (1 - e_E^2)}{1 - e_E^2 \left[1 - \sin^2 i \sin^2(\omega + v)\right]} \tag{116}$$

The γ in Eqs. (111) through (115) is given by Eq. (16) and is assumed to be unity. For all except low, nearly circular orbits γ is approximately equal to one.

After evaluating the coefficients of the integrals in Eqs. (111) through (115) for several different orbits, Eq. (113) was modified by omitting the third, fifth, and sixth terms which are small compared

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*The dependency of ρ on h varies with the atmospheric model used. Two of the current atmospheres are discussed in Appendix C.

to the remaining terms. Thus, the final form of Eqs. (111) through (115) and the ones which are in the program are Eqs. (111), (112), (114), (115), and Eq. (117), which is

$$\Delta\omega_D = - C_D \frac{A}{M} \gamma \frac{p}{e} \int_o^{2\pi} \rho \frac{\sin v \sqrt{1 + e^2 + 2e \cos v}}{(1 + e \cos v)^2} dv$$

$$+ \frac{1}{2} C_D \frac{A}{M} \frac{p^{\frac{5}{2}}}{e\sqrt{\mu}} \omega_E \cos i \int_o^{2\pi} \rho \frac{(2 + e \cos v) \sin v \sqrt{1 + e^2 + 2e \cos v}}{(1 + e \cos v)^4} dv$$

$$+ C_D \frac{A}{M} \frac{p^{\frac{5}{2}}}{e\sqrt{\mu}} \omega_E \cos i \int_o^{2\pi} \rho \frac{\sin v}{\sqrt{1 + e^2 + 2e \cos v}\ (1 + e \cos v)^2} dv \tag{117}$$

III. COMPUTER PROGRAM

GENERAL DESCRIPTION

The program consists of a main routine with several subroutines to take care of the perturbations on the satellite, together with a subroutine to collect these changes and give new values upon completion of an orbit. Some of these routines are denoted by a number which is the value of KK in the code. This is used by the subroutine ODCHEK for printout of overflows and divide checks so that the routine in which they occur will be known.

The main flow of logic can be seen from Fig. 5. The dotted lines signify alternate paths to the solid lines, depending on the values of certain constants at the time.

Many of the routines listed are vacuous at the moment but the description of their original purpose is included in most cases for reference purposes.

Most of the quantities calculated are placed in COMMON storage so that they may be used from subroutine to subroutine. A list of these variables, their description, where they are used, and where they are calculated is given in Table 1. The routine number, KK, and the letters in the last two columns of Table 1, are used to denote where they are computed and used, an M is used to denote the MAIN routine and an S is used to denote the SETUP routine.

A case is terminated when the satellite is down or the maximum number of revolutions has been reached (specified by K(1)). A blank card must follow the end of all cases to be run to cut off the run neatly. This blank card causes an end of file to be written on tape 13. The program calls for standard output tape printout plus a printout of one file of tape 13 regardless of how many cases have been run.

MAIN FLOW OF LOGIC

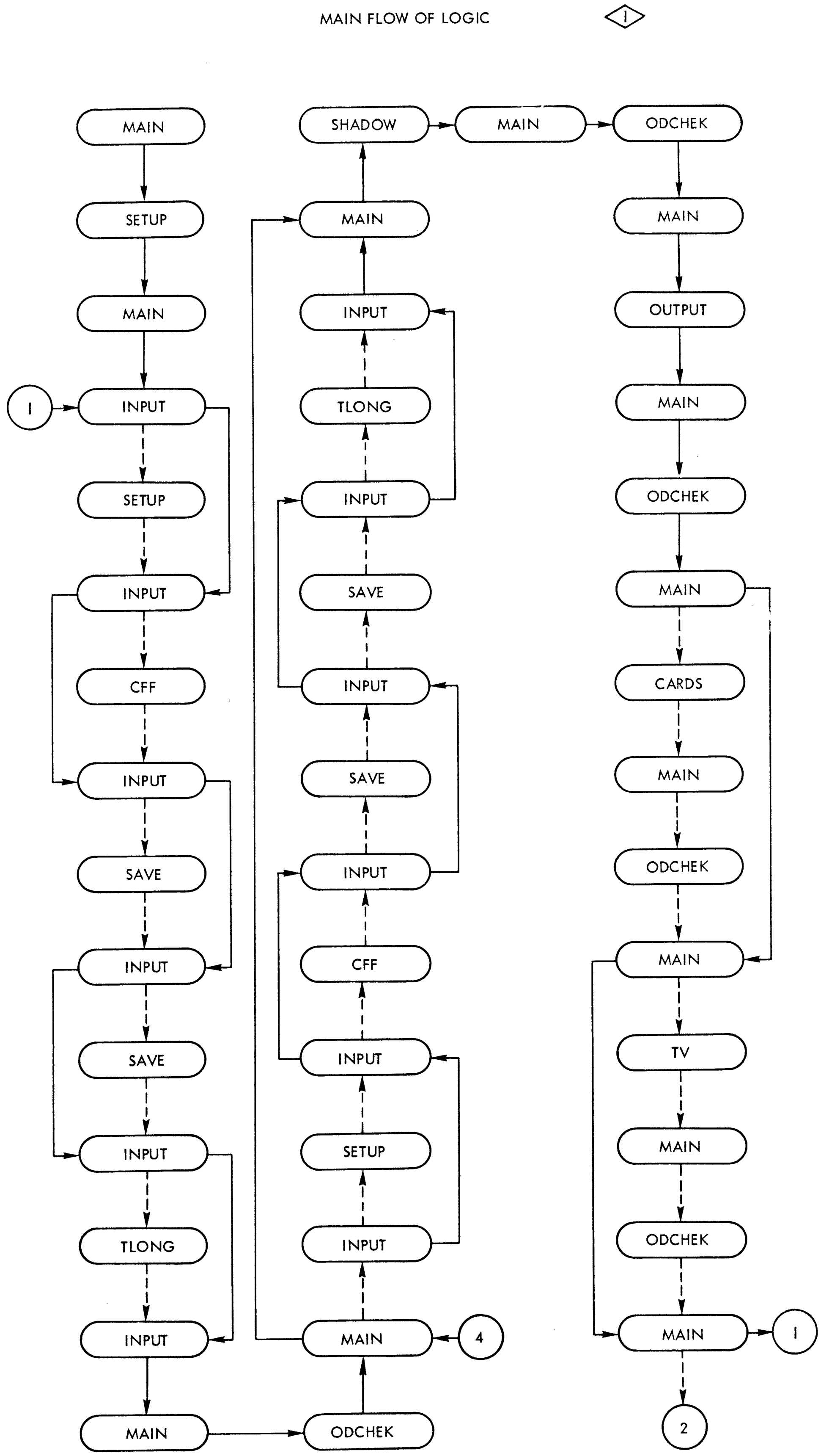


Fig. 5 — Main flow of logic

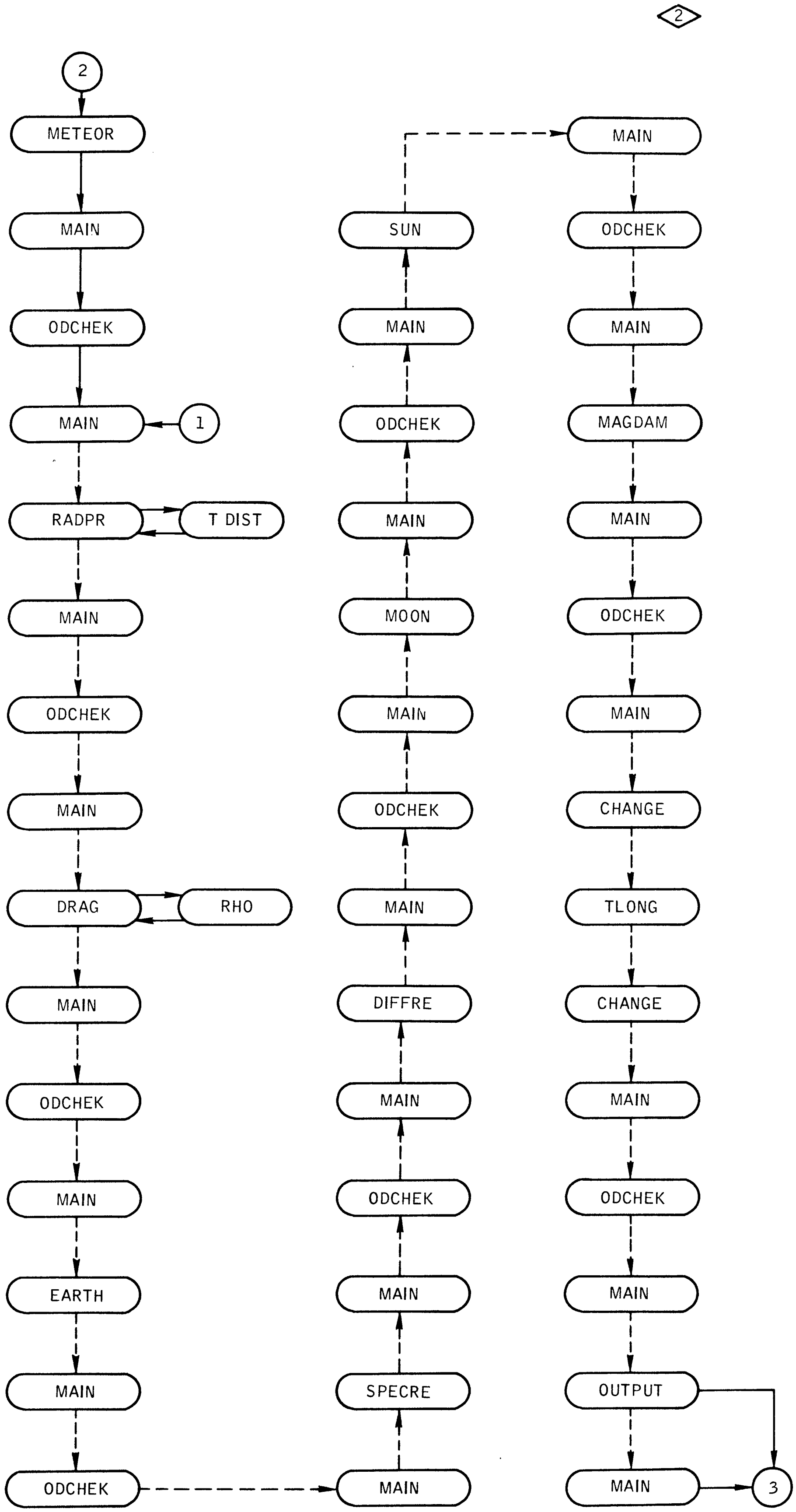

2
2
METEOR
MAIN
ODCHEK
MAIN
1
RADPR
T DIST
MAIN
ODCHEK
MAIN
DRAG
RHO
MAIN
ODCHEK
MAIN
EARTH
MAIN
ODCHEK
MAIN
SPECRE
MAIN
ODCHEK
MAIN
DIFFRE
MAIN
ODCHEK
MAIN
MOON
MAIN
ODCHEK
MAIN
SUN
MAIN
ODCHEK
MAIN
MAGDAM
MAIN
ODCHEK
MAIN
CHANGE
TLONG
CHANGE
MAIN
ODCHEK
MAIN
OUTPUT
MAIN
3

Fig. 5 — cont'd

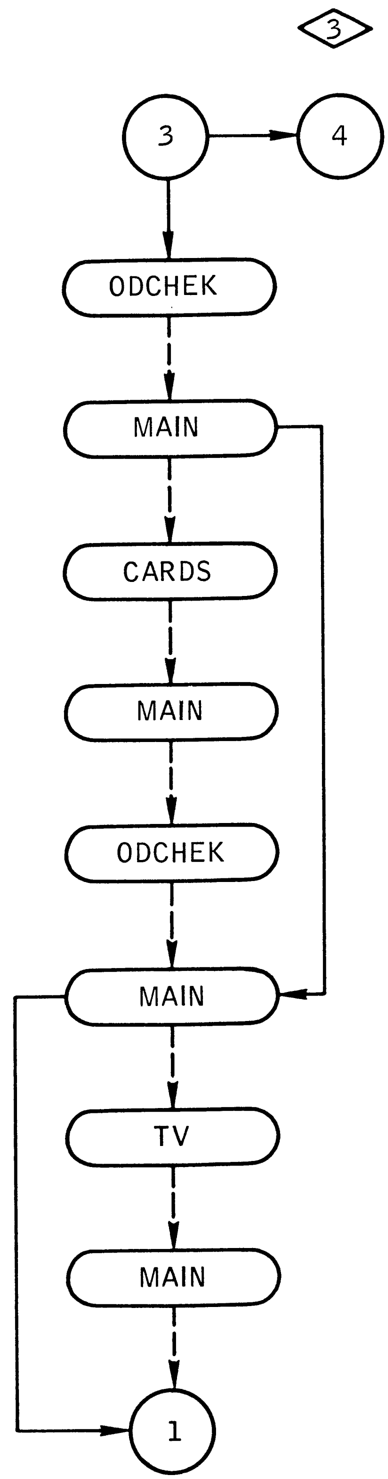


Fig. 5 — cont'd

MAIN

The MAIN routine calls most of the other subroutines directly, thus controlling the main logic flow. It tests to see if the maximum number of revolutions has been reached. If not, it proceeds with the computation as before. If so, the case is terminated.

When either of the two previously mentioned halts occurs, the logic control proceeds to the input routine to start another case. When a blank card is encountered the run is terminated. Thus the program will continue reading cases and processing them until no more data follow or a blank card is input. If no blank card is present the run will be terminated but an end of file will not be written on tape 13. As a result tape 13 is printed with your run plus whatever other miscellaneous information succeeds it until an end of file is reached.

SETUP

The subroutine SETUP first sets the K and C regions to zero, then initializes the following: 1) a standard test case, 2) values for the constants in the Gaussian quadrature to be used in DRAG, 3) certain other parameters, 4) and a solar flux table beginning at day 731 (which corresponds to 0^h GCT on 31 Dec., 1957). All values in SETUP may be changed by inputting new values in the INPUT routine. Thus only those numbers which are different from the standard case need to be input.

INPUT (KK=1)

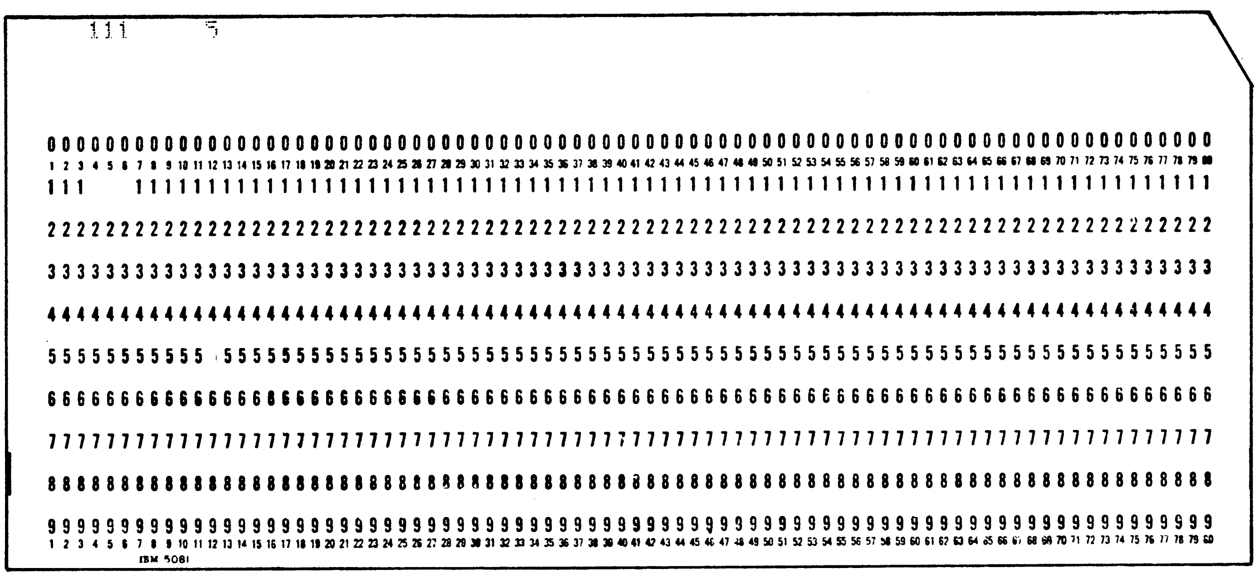
Number of Cards	Format		Description
1	12I6	NUR	number of run
		N1K	number of records (cards) of sequentially ordered integer data to be read in
		N1C	number of records (cards) of sequentially ordered floating point numbers to be read in

Number of Cards	Format	Description
		NM maximum value of NUR for the same initial conditions
		NK number of records (cards) of randomly ordered integers to be read in
		NC number of records (cards) of randomly ordered floating point numbers to be read in
		NO number of records (cards) of randomly ordered octal numbers to be read in
		NS if +, INPUT calls SETUP for reinitialization
		NCV if > 0, number of records (cards) of CV vector (flux table) to be read in, if = 0, flux table in SETUP is to be used
		NH number of records (cards) of Hollerith information to be read in (maximum = 2)
NH	72H	Hollerith Information - will be printed as output heading
N1K	12I6	INTEGERS for K region starting at K(1) and ending at K(12N1K)
NK	12I6	randomly ordered integers in the K region II, IK(II) where II is the index and IK(II) the value. There are 6 pairs per card
N1C	1P6E12.5	sequentially ordered C region numbers starting with C(531) and ending with C(min of 590 and 530 + 6N1C) if N1C > 10 the following N1C-10 cards will be put into sequential locations starting at C(1) and ending at C(min of 200 and 6N1C-60)
NC	4(I4, 1PE14.7)	randomly ordered C region numbers II, VI(II), where II is the index and VI(II) is the value
NCV	18F4.2	flux table values - if new flux table input a starting day C(561) will need to be input. The maximum length is 1000 entries.
		start of new case (i.e., Card 1 again) -- or blank if execution terminates with this case

The appropriate indexes and values for input to the K and C regions can be deduced from the description of variables in Table 1. Figure 6 shows a usual input case.

Any one of the numbers on the first card may be zero, thus eliminating that part of the input. The input routine outputs onto all output tapes all the quantities which have been input, as indicated by the switches in the K region (list of variables in COMMON storage). Then it clears out the D and Q regions, computes some initial quantities and returns to the main routine.

The effect of any routine can be deleted by input of the KK number into the K region beginning with K(111); thus reading the following card would delete solar radiation pressure:



These cards are input as part of the randomly ordered integers.

OUTPUT (KK=2)

The OUTPUT routine outputs the titles and input quantities for the case. This routine also outputs the semilatus rectum, eccentricity, omega, ascending node, inclination, modified Julian day, semimajor axis,

FIG. 6 SAMPLE INPUT

0000000001111111111222222222233333333334444444444555555555566666666667777777778
1234567890123456789012345678901234567890123456789012345678901234567890123456789 0

RUN #	# SEQ INT.	#SEQ F.P.	MAX RUN #	# RAND INT.	# RAND F.P.	# RAND OCT.	± SETUP	CVVECTOR	# HOLLER.	

HOLLERITH

1	TOTAL # REV. IN RUN	2	# /CALC.	10	#/TAPE 13	11	99999	20	#/TAPE 11	21	1st TAPE 11	

531	XRAMO		532	XDAMO		537	XP		538	XE	
539	XØMEGAD		540	XASCND		541	XØINCD		542	XDAY	
564	ROTATION										

1234567890123456789012345678901234567890123456789012345678901234567890123456789 0

height of perigee above the sphere, direction cosine of sunlight, angle between perigee and the projection of the sun line on the orbit plane, and the true anomalies of the shadow boundaries for each computation interval, K(10). This routine also exits neatly after normal halts like "Satellite is Down" or when the last revolution has been attained. It prints on three output tapes: the normal output tape, tape 11, and tape 13, depending on the values of K(10), K(11) and other selected values in the K-region set up by SETUP or INPUT. Tape 11 contains debugging output and tape 13 contains formal output for the customer. All angles are output in degrees, p and a are in earth radii, MJD (MJD = JD-2430000.5) is in days. On the standard output tape subroutine RHO outputs the table of height vs density. More will be said about the atmosphere later.

CARDS (KK=3)

This routine writes on tape 11 and also writes an end of file on tape 11 when finished. In the normal course of logic flow this subroutine is by-passed. To get the output here the value of K(12) must be made greater than zero.

SHADOW (KK=4)

This routine finds the shadow boundaries to use for computation of the radiation pressure. This is done by an iteration procedure and a shadow function given by Eq. (79).

$$\mathrm{SHADF}(v) = \cos(v-\varphi) + \left(\frac{p^2 - (1 + e\cos v)^2}{p^2\,(R^2 + S^2)}\right)^{1/2}$$

where

R = the direction cosine between perigee and the sun

S = the direction cosine between the semilatus rectum at $v = 90^{\circ}$ and the sun

p = the semilatus rectum of the orbit

e = the eccentricity of the orbit

v = the true anomaly

ϕ = the angle between perigee and the projection of the sun line on the orbit plane

The true anomalies v_1 and v_2 of the shadow boundaries are determined by this routine.

RADPR (KK=5)

This routine computes the effect of radiation pressure on the satellite. The equations have been analytically integrated so that the result is obtained by straight computation using the shadow boundaries v_1 and v_2 as the limits of integration (see Eqs. (91) through (96)). The effect of this routine can be deleted in two ways--as previously mentioned, by setting the proper K region constants, and also by setting XRAM less than 10^{-8}. XRAM is the effective area to mass ratio and $K \frac{I}{C} \frac{A}{M}$ is defined as the acceleration caused by solar radiation pressure in CGS units. The effective area includes the factor K which depends on the distribution of scattered light.

DRAG (KK=6)

DRAG computes the air drag on the satellite due to a given atmosphere (see Eqs. (111) through (115)). It consists of a numerical integration which calls upon routine RHO to supply it with a density for a given height. There are two separate integrations, one for a symmetric atmosphere and one for an asymmetric atmosphere. The one that will be used is determined by the value of IM. IM is set by a preliminary pass through RHO. The value of IM in RHO depends on the values of K(9), B1A, and B1B. Gaussian quadrature is used for the integration procedure in both cases. The symmetric case integrates from zero to π and breaks up this interval into π/KA subintervals. On each of these subintervals it performs a nine-point gaussian quad-

rature. The asymmetric case integrates from minus π to π, breaking it up into 2π/KA subintervals upon which to perform nine-point gaussian quadratures. The option of having a rotating atmosphere is provided. It will be included unless C(564) is input as zero. As in RADPR, if XDAM is set less than 10^{-8} the effect of the routine will be cancelled. XDAM is the effective area to mass ratio and is equal to $\frac{1}{2} C_D \frac{A}{M}$. It is defined for a model where air molecules are at rest when swept up by the satellite and are essentially absorbed by the satellite (or re-emitted at low velocities in the satellite's reference system).

RHO

In RHO the density for a given height is calculated. A table may be used or the density may be calculated using formulas such as those of Jacchia which are presently used:[6] HT1 is the height above which the Jacchia atmosphere is used and HT2 is the height above which the formula for the Jacchia atmosphere changes. At present HT1 is 200 km and HT2 is 600 km.

The table of heights and densities is programmed into the routine directly in the form of FORTRAN source cards with the limit of 500 values for height and the corresponding 500 for density. The height is called ARDCH and the density is called ARDCR. ARDCH is in km and ARDCR is in G/CM^3. There are several DO loops and an output statement with an index which must correspond to the largest value of the index in the table. The height for which to interpolate is calculated from the true anomaly, v, eccentricity, e, the radius of the earth, RE, and the semilatus rectum, p, by the following formula:

$$H = \frac{p\ (RE)}{1 + e \cos v} - RE$$

where p is in earth radius units and RE is the radius of the earth in cm. In the case of table look up it is modified to include the oblateness of the earth. This modification is not made for Jacchia calculations. Interpolation is made linearly on the logarithms of the density.

If the largest height in the table is exceeded, one must put in a value of RHO after statement 72 for use at this point. The value of HT1 may be controlled by input of B1C and B1B or may be programmed as a constant directly into RHO. HT1 may be set to zero by setting B1C less than 10^{-3}. The atmosphere table included in the deck is 1962 ARDC.

There are two other options for calculating the density. These are a power law and a parabolic log model. Their use is governed by the value of K(9). If K(9) is > 0 the routine calculates RHO by the use of Jacchia formulas or table interpolation as previously discussed. If K(9) = 0 the power law formulas are used as follows:

$$\mathrm{RHO} = \mathrm{RHO1}\left[\frac{\mathrm{H1} - \mathrm{DH}}{\mathrm{H} - \mathrm{DH}}\right]^{\mathrm{DEXP}}$$

where RHO1, H1, DH, DEXP are air density constants as found defined in SETUP. If K(9) < 0 the parabolic log model is used as follows:

$$\mathrm{RHO} = e^{\mathrm{RHO1} + \mathrm{DH}(\mathrm{H}-\mathrm{H1}) + \mathrm{DEXP}(\mathrm{H}-\mathrm{H1})^2}$$

where the variables are defined as before. K(9) is set > 0 by SETUP.

The flux table, CV, is used by RHO in the computation of the Jacchia atmosphere. The table presently included starts on day 731 and has 730 entries. If the DAY calculated for use in interpolating is greater than 1000, the flux is set equal to CD(15). If the entries in the table are exhausted but DAY is not greater than 1000, the flux is also set equal to CD(15) which is equivalent to C(563) and can be input as any number. SETUP currently assigns 1 to CD(15).

EARTH (KK=7)

The EARTH routine computes the first-order oblateness terms of the earth through the fifth harmonic (see Eqs. (19) through (22)). It also includes second-order corrections. By setting K(71) less than zero a printout of the second-order corrections can be obtained. These come out on the tape unit specified by K(35) which must be input.

MOON (KK=1)

MOON considers the changes in the orbital elements due to the moon (see Eqs. (53) through (57) and (D-1) through (D-11)). Powers, higher than the first, of the ratio of the distance from the earth to the satellite to the distance from the earth to the moon are ignored. TDISTM is the distance from the earth to the moon in earth radii. In getting these changes calculations are made of the ecliptic longitude of the ascending node of the moon (ASM1), the orbital inclination of the moon to the ecliptic (OIM1), the orbital inclination of the moon to the equator (OIM), the equatorial longitude of the ascending node of the moon (ASM), the argument of perigee from the ecliptic of the moon (OMM1), the argument of perigee from the equator of the moon (OMM), the mean anomaly of the moon (ANMM) and the true anomaly of the moon (VM) (see Appendix D). The distance from the earth to the moon (TDISTM), and the angular quantities RM,SM,WM, along with ELM are also calculated (see passages on lunar and solar gravitational fields).

SUN (KK=12)

The SUN routine computes the attraction of the sun and can only be run in conjunction with the MOON routine.

Routine DIFFRE,KK=13, is vacuous at the moment but is meant to contain the calculation for diffuse reflection.

CHANGE (KK=14)

This routine accumulates the incremental changes of the orbital elements. It also recalculates part of COMMON storage. It checks to see if the next interval will produce a "Satellite is Down" halt, and if so, changes the computation interval to one revolution and proceeds. If the computation interval is already one revolution, nothing is done and the computation proceeds. It also brings DAY up to date. The changes are collected in the following way:

$$\text{Element} = \begin{matrix}\text{previous value}\\ \text{of the element}\end{matrix} + \begin{pmatrix}\text{computation}\\ \text{interval}\end{pmatrix} \sum \begin{matrix}\text{incremental changes}\\ \text{of this element from}\\ \text{all subroutines}\end{matrix}$$

VACUOUS ROUTINES

The routines SPECRE,KK=8, METEOR,KK=9, and MAGDAM,KK=10, are vacuous at the moment but are provided for possible additions later. SPECRE would consider specular reflection, METEOR would consider meteor collisions, and MAGDAM would consider magnetic damping. SAVE and CFF are vacuous at the moment also. TV, KK=15, is a vacuous subroutine and was intended to be used for displaying results on an oscilloscope.

TLONG

TLONG computes the true longitude of the sun.

TDIST

TDIST computes the square of the earth to sun distance in astronomical units.

ODCHEK

ODCHEK is called after each routine to make sure that accumulator overflows, and divide checks, do not go unnoticed. The occurrence of these two items is output to tape 13 and also to the regular output tape. A PDUMP of core can be obtained by setting K(33) equal to the revolution at which you wish to print out. Also see the list of integers in COMMON storage for values of K(31) and K(32) which are necessary.

Copies of the regular output and tape 13 output are included with this write-up (Figs. 7 and 8). Flow charts of the various non-vacuous routines are given in Part B of this Section.

An arccos routine, ACOS, is written in FAP and used by several subroutines.

```
LINCOLN LABORATORIES EARTH SATELLITE PROGRAM
   10/23/63  LINCOLN LAB ROCKET CHECK
NUR=     1 N1K=     0 N1C=     0  NM=     1  NK=     1  NC=     3  NO=     0  NS=    -1 NCV=     0  NH=     2
K 1=    10 K 2=     1 K 3=     0 K 4=    13 K 5=     6 K 6=     2 K 7=     0 K 8=     6 K 9=     6 K10=     1 K11=99999 K12=     0
K20=     1 K21=     1 K

XRAM=   -0.          XDAM=    0.100000    SC1=    0.             SC2=    0.             SC3=    0.             SC4=    0.
  XP=  1.35091399      XE=  0.30000000 XOMEGD=   45.000000 XASCND=    0.          XOINCD=   45.000000   XDAY= 1827.000000
PSID=  2.344441E 01 GJAY=  1.623270E-03    GM=  3.986300E 20    RE=  6.378388E 08    OS=  1.720290E-02 RADP=  4.500000E-05
RHO1=  1.602100E 01   H1=  1.985000E-08    DH=  6.363000E 00 DEXP=  2.600000E-08 CD(5=  0.                CD(6=  0.
 C(555)=  6.0000000E 07  C(556)=  1.9000000E-01  C(557)=  5.4999999E-08  C(558)=  1.9000000E 00
 C(559)=  5.7300000E-08  C(560)=  1.0000000E 00  C(561)=  7.3099999E 02  C(562)=  3.0000000E 01
 C(563)=  1.0000000E 00  C(564)=  1.0000000E 00  C(
 C( 49)=  1.2300000E-02  C( 50)=  6.0270000E 01  C( 51)=  5.4899999E-02  C( 52)=  8.9801999E-02
 C( 53)=  6.0500000E 01  C( 54)=  3.3343200E 05  C( 55)=  2.3440000E 04  C(
   J      P          E         OMEGAD    ASCND     OINCD     M.J.D.       A         HTPER       W          PHID      V1D       V2D
  -0 1.3509140 0.3000000  45.0000   0.        45.0000 8030.0000 1.4845209 0.0391646 0.3619839 235.5784 344.3289 456.8643
```

Atmosphere, h (km)	ρ (gm/cm^3)
0.	0.12249999E-02
0.20000000E 01	0.10659999E-02
0.40000000E 01	0.81934999E-03
0.59999999E 01	0.66010999E-03
0.80000000E 01	0.52578999E-03
0.09999999E 02	0.41350999E-03
0.12000000E 02	0.31193999E-03
0.13999999E 02	0.22786000E-03
0.16000000E 02	0.16647000E-03
0.18000000E 02	0.12165000E-03
0.20000000E 02	0.88909999E-04
0.22000000E 02	0.64510000E-04
0.23999999E 02	0.46937999E-04
0.26000000E 02	0.34256999E-04
0.27999999E 02	0.25076000E-04
0.30000000E 02	0.18409999E-04
0.31999999E 02	0.13554999E-04
⋮	⋮
0.68399999E 03	0.18250000E-15
0.68600000E 03	0.17860000E-15
0.68799999E 03	0.17479999E-15
0.69000000E 03	0.17110000E-15
0.69199999E 03	0.16750000E-15
0.69400000E 03	0.16390000E-15
0.69599999E 03	0.16040000E-15
0.69800000E 03	0.15700000E-15
0.70000000E 03	0.15369999E-15

```
10   END OF RUN NUMBER      1
```

Fig. 7—Standard output tape

```
LINCOLN LABORATORIES EARTH SATELLITE PROGRAM
   10/23/63  LINCOLN LAB ROCKET CHECK
NUR=     1 N1K=     0 N1C=     0  NM=     1  NK=     1  NC=     3  NO=     0  NS=    -1 NCV=     0  NH=     2
K 1=    10 K 2=     1 K 3=     0 K 4=    13 K 5=     6 K 6=     2 K 7=     0 K 8=     6 K 9=     6 K10=     1 K11=99999 K12=     0
K20=     1 K21=     1 K

XRAM=   -0.          XDAM=    0.100000    SC1=    0.           SC2=    0.           SC3=    0.          SC4=    0.
  XP=  1.35091399     XE=  0.30000000 XOMEGD=   45.000000 XASCND=    0.         XOINCD=   45.000000  XDAY= 1827.000000
PSID=  2.344441E 01 GJAY=  1.623270E-03   GM=  3.986300E 20   RE=  6.378388E 08   OS=  1.720290E-02 RADP=  4.500000E-05
RHO1=  1.602100E 01   H1=  1.985000E-08   DH=  6.363000E 00 DEXP=  2.600000E-08 CD(5=  0.                CD(6=  0.
 C(555)=  6.0000000E 07  C(556)=  1.9000000E-01  C(557)=  5.4999999E-08  C(558)=  1.9000000E 00
 C(559)=  5.7300000E-08  C(560)=  1.0000000E 00  C(561)=  7.3099999E 02  C(562)=  3.0000000E 01
 C(563)=  1.0000000E 00  C(564)=  1.0000000E 00  C(
 C( 49)=  1.2300000E-02  C( 50)=  6.0270000E 01  C( 51)=  5.4899999E-02  C( 52)=  8.9801999E-02
 C( 53)=  6.0500000E 01  C( 54)=  3.3343200E 05  C( 55)=  2.3440000E 04  C(
   J       P        E       OMEGAD    ASCND    OINCD     M.J.D.      A      HTPER       W       PHID     V1D      V2D
  -0 1.3509140 0.3000000  45.0000   0.        45.0000 8030.0000 1.4845209 0.0391646 0.3619839 235.5784 344.3289 456.8643

   1 1.3509087 0.2999963  45.2400 359.7736  45.0000 8030.1061 1.4845114 0.0391634 0.3613763 235.6753 344.4044 456.9476
     6.6281875-8.245E-060.1061263  -0.        0.1000  279.9618   45.0136  314.5335  125.0518  34.5717 234.4953 324.9754
 RADPR  0.            -0.            0.            -0.            -0.            -0.            -0.            -0.
 DRAG  -4.7631571E-06-4.2666737E-06 6.1912641E-06-2.6919693E-06-2.9371649E-06-8.0473764E-06-8.1313380E-06 7.0073304E-07
 EARTH -0.             0.            2.4015916E-01-2.2642421E-01-0.            0.            0.            0.
 SPECRE 0.             0.            0.            0.            0.            0.            0.            0.
 METEOR 0.             0.            0.            0.            0.            0.            0.            0.
 MAGDAM 0.             0.            0.            0.            0.            0.            0.            0.
 MOON  -1.0303245E-06 1.1567419E-06-1.0300102E-04-1.2572523E-06 6.8068483E-06-3.6954613E-07-3.7340176E-07-1.9758803E-06
 SUN    4.7985056E-07-5.3872666E-07-1.4412654E-05-1.4135965E-05 9.9956366E-06 1.7210783E-07 1.7390350E-07 9.2022200E-07
 DIFFRE 0.             0.            0.            0.            0.            0.            0.            0.

   2 1.3509033 0.2999927  45.4801 359.5471  45.0000 8030.2122 1.4845020 0.0391622 0.3607470 235.7719 344.4796 457.0312

   3 1.3508979 0.2999892  45.7202 359.3207  45.0000 8030.3184 1.4844926 0.0391609 0.3600962 235.8682 344.5544 457.1149

   4 1.3508925 0.2999856  45.9603 359.0942  45.0000 8030.4244 1.4844832 0.0391596 0.3594236 235.9643 344.6290 457.1990

   5 1.3508870 0.2999821  46.2004 358.8678  45.0001 8030.5306 1.4844737 0.0391582 0.3587295 236.0600 344.7032 457.2832

   6 1.3508815 0.2999786  46.4405 358.6413  45.0001 8030.6367 1.4844643 0.0391567 0.3580139 236.1554 344.7771 457.3676

   7 1.3508760 0.2999752  46.6806 358.4149  45.0001 8030.7428 1.4844549 0.0391553 0.3572766 236.2506 344.8507 457.4523

   8 1.3508705 0.2999717  46.9207 358.1884  45.0001 8030.8489 1.4844454 0.0391537 0.3565179 236.3454 344.9240 457.5371

   9 1.3508650 0.2999683  47.1609 357.9619  45.0001 8030.9550 1.4844360 0.0391522 0.3557375 236.4400 344.9969 457.6221

  10 1.3508594 0.2999649  47.4010 357.7355  45.0001 8031.0611 1.4844265 0.0391507 0.3549358 236.5342 345.0695 457.7073

  10   END OF RUN NUMBER      1
```

Fig. 8—Tape 13 output

TABLE 1 LIST OF VARIABLES IN COMMON STORAGE
MANY OF THESE VARIABLES ARE DEFINED BUT NOT CURRENTLY USED

LIST OF INTEGERS IN COMMON STORAGE

K-VECT LOC.	MNEM. NAME	OCTAL LOC.	DEFINITION	CALC. IN	USED IN
K(1)	=N	77461	TOTAL NO. OF PERIODS IN RUN	S,1	M,2
K(2)	=L	77460	NO. OF PERIODS PER CALCULATION	S,1	M,2,14
K(3)	=KD	77457	INITIAL VALUE OF J **(J = Period number).**	S,1	M
K(4)	=KE	77456	CONTROLS NUMBER OF PERTURBATIONS INCLUDED	S,1	2,14
K(5)	=KA	77455	NUMBER OF GAUSS QUADRATURES IN DRAG	S,1	6
K(6)		77454	2.**K(6)=NO. OF DEGREES IN SHADOW SEARCH INTERVAL,DELV. IF K(6) OVER 99,NEGLECT SHADOW (V1=V2=0).	S,1	4
K(7)		77453	2.**K(7)=NO. OF DEGREES IN SHADOW INTERPOLATION INTERVAL,EPSV.	S,1	4
K(8)		77452	NO. OF GAUSS QUADRATURES IN RADPR (MAG. NEEDLE)	S,1	5
K(9)		77451	CONTROLS RHO. IF +,JACCHIA MODEL OR TABLE. IF 0,POWERLAW. IF -,PARABOLIC LOG MODEL.	S,1	RHO
K(10)		77450	NO. OF PERIODS PER TAPE 13 OUTPUT. IF 0 OR -, NO TAPE 13	S,1	2
K(11)		77447	NO. OF PERIODS PER STANDARD OUTPUT TAPE PRINT. IF 0 OR -,NO STANDARD OUTPUT TAPE.	S,1	2
K(12)		77446	NO. OF PERIODS PER CARDS OUTPUT. IF 0 OR -, NO CARDS	S,1	M
K(13)		77445	NO. OF PERIODS PER TV OUTPUT. IF 0 OR -, NO TV	S,1	M
K(14)		77444	THRESH. TAPE 13. IF 0 OR -, K(10) ALL THE WAY	S,1	2
K(15)		77443	NO. OF PERIODS PER TAPE 13 AFTER JKD=K(14). IF 0 OR -, NO TAPE 13 AFTER JKD=K(14).	S,1	2
K(16)		77442	THRESH. STANDARD OUTPUT TAPE. IF 0 OR -, K(11) ALL THE WAY	S,1	2
K(17)		77441	NO. OF PERIODS PER STANDARD OUTPUT TAPE AFTER JKD=K(16). IF 0 OR -, NO STANDARD OUTPUT TAPE AFTER JKD=K(16).	S,1	2
K(18)		77440	NO. OF LINES OF SHORT OUTPUT TAPE 13. IF 0, ONE LINE. IF +, GET D(1,K)-D(8,K) BEGINNING THIRD LINE. IF -, GET D(1,K)-D(5,K),2 VALUES OF K PER LINE BEGINNING THIRD LINE	S,1	2
K(19)		77437	NO. OF LINES PER STANDARD OUTPUT TAPE. SIMILAR TO K(18)	S,1	2
K(20)		77436	NO. OF PERIODS PER LONG OUTPUT TAPE 13. IF 0, NO LONG OUTPUT	S,1	2
K(21)		77435	FIRST VALUE OF JKD GIVING LONG OUTPUT TAPE 13	S,1	2
K(22)		77434	MAX. VALUE OF JKD FOR LONG OUTPUT TAPE 13	S,1	2
K(23)		77433	(NOT USED)		
K(24)		77432	NO. OF LINES FOR D MATRIX ON LONG OUTPUT	S,1	2
K(25)		77431	ROW OF Q MATRIX TO GO ON LONG OUTPUT	S,1	2
K(26)		77430	ROW OF Q MATRIX TO GO ON LONG OUTPUT		
K(27)		77427	ROW OF Q MATRIX TO GO ON LONG OUTPUT		
K(28)		77426	ROW OF Q MATRIX TO GO ON LONG OUTPUT		
K(29)		77425	ROW OF Q MATRIX TO GO ON LONG OUTPUT		
K(30)		77424	(NOT USED)		
K(31)		77423	CORE DUMP VALUE OF KK. IF +, DUMP COMMON ONLY. IF -, DUMP ALL CORE WHEN JKD=K(33).	S,1	ODCHEK
K(32)		77422	CORE DUMP VALUE OF KK. SIMILAR TO K(31)	S,1	ODCHEK
K(33)		77421	FINAL VALUE OF JKD FOR CORE DUMP	S,1	ODCHEK
K(34)	=M2	77420	INPUT TAPE (NOT USED IN CURRENT INPUTS)	S,1	1

TABLE 1 CONTINUED

K-VECT LOC.	MNEM. NAME	OCTAL LOC.	DEFINITION	CALC. IN	USED IN
K(35)	=M3	77417	OUTPUT TAPE FOR OFF LINE PRINTING	S,1	1,2,7
K(36)	=M4	77416	OUTPUT TAPE FOR OFF LINE CARD PUNCHING (K(36) NOT USED IN CURRENT OUTPUTS. K(35) USED ONLY IN EARTH CURRENTLY)	S,1	1,3
K(37)		77415	CONTROLS EXIT IN CASE OF BAD DATA INDEX (IF +, CALL EXIT)	S,1	1
K(38)-K(100)			(NOT USED)		
K(101)	=NUR	77315	RUN NUMBER	1	1
K(102)	=N1K	77314	NO. OF RECORDS OF SEQUENTIAL INTEGERS READ IN.	1	1
K(103)	=N1C	77313	NO. OF RECORDS OF SEQUENTIAL F.P.NOS. READ IN. (IF N1C=1 OR 2,ADD A BLANK RECORD)	1	1
K(104)	=NM	77312	MAX. VALUE OF NUR FOR SAME INITIAL CONDITIONS.	1	1
K(105)	=NK	77311	NO. OF RECORDS OF RANDOM INTEGERS READ IN.	1	1
K(106)	=NC	77310	NO. OF RECORDS OF RANDOM F.P.NOS. READ IN.	1	1
K(107)	=NO	77307	NO. OF RECORDS OF RANDOM OCTAL NOS. READ IN.	1	1
K(108)	=NS	77306	IF +, INPUT CALLS SETUP	1	1
K(109)	=NCV	77305	NO. OF RECORDS OF CV-VECTOR READ IN.	1	1
K(110)	=NH	77304	NO. OF RECORDS OF HOLLERITH READ IN. (MAX=2)	1	1
K(111)-K(118)			USED TO DELETE INFLUENCE OF SPECIFIED ROUTINES	1	14
K(119)-K(196)			(NOT USED)		
K(197)	=IM	77155	IF 1, ATMOSPHERE ASSYMETRIC. IF 0, SYMMETRIC	RHO	6,RHO
K(198)	=KK	77154	INDEX TO IDENTIFY LOCATION OF STATEMENT CALL ODCHEK	MAIN	ODCHEK
K(199)	=JKD	77153	NO. OF PERIODS SINCE START OF RUN. JKD=J-KD	MAIN	(MANY)
K(200)	=J	77152	PERIOD NUMBER	MAIN	2,3

TABLE 1 CONTINUED

ABBREVIATIONS FOR UNITS IN COMMON STORAGE LIST (ALPHABETICAL)

CM. =CENTIMETERS	DAY. =DAYS	DEG. =DEGREES
DIML. =DIMENSIONLESS	DYNE. =DYNES	G. =GRAMS
RAD. =RADIANS	RE. =EARTH RADII	REV. =REVOLUTIONS
SEC. =SECONDS		

LIST OF FLOATING POINT NUMBERS IN COMMON STORAGE

C-VECT LOC.	MNEM. NAME	OCTAL LOC.	DEFINITION	DIMENSIONS	CALC. IN	USED IN
C(1)		77151	DAYS AFTER LAUNCH WHEN YRAM= C(2)	DAYS	S,1	14
C(2)		77150	VALUE OF YRAM C(1) DAYS AFTER LAUNCH	CM**2/G	S,1	14
C(3)		77147	DAYS AFTER LAUNCH WHEN YRAM= C(4)	DAYS	S,1	14
C(4)		77146	VALUE OF YRAM C(3) DAYS AFTER LAUNCH	CM**2/G	S,1	14
C(5)		77145	DAYS AFTER LAUNCH WHEN YRAM= C(6)	DAYS	S,1	14
C(6)		77144	VALUE OF YRAM C(5) DAYS AFTER LAUNCH	CM**2/G	S,1	14
C(7)		77143	DAYS AFTER LAUNCH WHEN YRAM= C(8)	DAYS	S,1	14
C(8)		77142	VALUE OF YRAM C(7) DAYS AFTER LAUNCH	CM**2/G	S,1	14
C(9)		77141	DAYS AFTER LAUNCH WHEN YRAM= C(10)	DAYS	S,1	14
C(10)		77140	VALUE OF YRAM C(9) DAYS AFTER LAUNCH	CM**2/G	S,1	14
C(11)		77137	DAYS AFTER LAUNCH WHEN YRAM= C(12)	DAYS	S,1	14
C(12)		77136	VALUE OF YRAM C(11) DAYS AFTER LAUNCH	CM**2/G	S,1	14
C(13)		77135	(NOT USED)			
C(14)		77134	(NOT USED)			
C(15)	=SMASS	77133	MASS OF SATELLITE	G.	S,1	9
C(16)	=SLENGT	77132	LENGTH OF SATELLITE	CM.	S,1	9
C(17)	=SEVMOM	77131	METEOROID MOMENTUM WHICH SEVERS	G*CM/SEC	S,1	9
C(18)	=PMIN	77130	MINIMUM MOMENTUM OF METEOROID	G*CM/SEC	S,1	9
C(19)	=PRATE	77127	AVERAGE NO. OF COLLISIONS PER DAY	1/DAY	S,1	9
C(20)	=PEXP	77126	EXPONENT IN METEOR COLLISION LAW	DIML.	S,1	9
C(21)	=DINCD	77125	INCLINATION OF DISPENSER SPIN AXIS	DEG.	S,1	9
C(22)	=DLAMD	77124	R.D. OF DISPENSER SPIN AXIS	DEG.	S,1	9
C(23)	=DLIFE	77123	DISPENSER LIFETIME	DAYS	S,1	9
C(24)	=DRPS	77122	DISPENSER ROTATION RATE	REV/SEC	S,1	9
C(25)	=DR1	77121	DISPENSER RADIUS, INITIAL	CM.	S,1	9
C(26)	=DR2	77120	DISPENSER RADIUS, FINAL	CM.	S,1	9
C(27)	=DUD	77117	ARGUMENT OF LATITUDE,U, AT WHICH	DEG.	S,1	9
			DISPENSING STARTS		S,1	9
C(28)		77116	VM=(C(28)+C(29)*RANF(0))*TANGENTIAL	DIML.	S,1	9
C(29)		77115	VELOCITY OF DISPENSER	DIML.	S,1	9
C(30)		77114	COSTH=C(30)*(2.*RANF(0)-1.)	DIML.	S,1	9
C(31)		77113	AMOMD=C(31)+C(32)*RANF(0))*AMOM	DIML.	S,1	9
C(32)		77112	AMOMD=C(31)+C(32)*RANF(0))*AMOM	DIML.	S,1	9
C(33)		77111	AMOMV=C(33)*(2.*RANF(0)-1.)*AMOM	DIML.	S,1	9
C(34)		77110	AMOMN=C(34)*(2.*RANF(0)-1.)*AMOM	DIML.	S,1	9
C(35)		77107	(NOT USED)			
C(36)		77106	(NOT USED)			
C(37)		77105	(NOT USED)			
C(38)		77104	RADIUS TO EARTH MAGNETIC DIPOLE	RE	S,1	5
C(39)		77103	COLATITUDE TO MAGNETIC DIPOLE	DEG.	S,1	5
C(40)		77102	EAST LONGITUDE TO MAGNETIC DIPOLE	DEG.	S,1	5
C(41)		77101	COLATITUDE OF DIPOLE VECTOR	DEG.	S,1	5
C(42)		77100	EAST LONGITUDE OF DIPOLE VECTOR	DEG.	S,1	5
C(43)		77077	DIPOLE MOMENT MAGNITUDE		S,1	
C(44)		77076	WEST LONGITUDE OF VERN. EQ. AT DAY=0.	DEG.	S,1	5

TABLE 1 CONTINUED

C-VECT LOC.	MNEM. NAME	OCTAL LOC.	DEFINITION	DIMENSIONS	CALC. IN	USED IN
C(45)		77075	(NOT USED)			
C(46)		77074	COEFFICIENT OF THIRD HARMONIC	DIML.	S,1	7
C(47)		77073	COEFFICIENT OF FOURTH HARMONIC	DIML.	S,1	7
C(48)		77072	COEFFICIENT OF FIFTH HARMONIC	DIML.	S,1	7
C(49)		77071	MOON MASS DIVIDED BY EARTH MASS	DIML.	S,1	11
C(50)		77070	SEMIMAJOR AXIS OF MOON'S ORBIT	RE.	S,1	11
C(51)		77067	ECCENTRICITY OF MOON'S ORBIT	DIML.	S,1	11
C(52)	=OIM1	77066	INCLINATION OF MOON'S ORBIT TO ECLIPTIC	RAD.	S,1	11
C(53)		77065	MEAN DISTANCE FROM EARTH TO MOON	RE.	S,1	11
C(54)		77064	SUN MASS DIVIDED BY EARTH MASS	DIML.	S,1	12
C(55)		77063	MEAN DISTANCE FROM EARTH TO SUN	RE.	S,1	12
C(70)		77044	FRACTION OF REFLECTED RADIATION	DIML.	S,1	5

TABLE 1 CONTINUED

C-VECT. LOC.	MATRIX NAME	MNEM. NAME	OCTAL LOC.	DEFINITION		UNITS	CALC. IN	USED IN
C(501)	=Q(1,1)	=PI	76165	3.1415926536		RAD.	S,1	(MANY)
C(502)	=Q(2,1)	=CIRC	76164	PI*2.		RAD.	S,1	(MANY)
C(503)	=Q(3,1)	=CONV	76163	PI/180		RAD/DEG	S,1	(MANY)
C(504)	=Q(4,1)	=PSI	76162	ANGLE BET. ECLIPTIC AND EQUATOR		RAD.	1	(MANY)
C(505)	=Q(5,1)	=CPS	76161	COSF(PSI)		DIML.	1	
C(506)	=Q(6,1)	=SPS	76160	SINF(PSI)		DIML.	1	
C(507)	=Q(7,1)	=TIME	76157	CIRC/86400.*RE*SQRTF(RE/GM) PERIOD OF EARTH SKIMMING SAT.		DAYS	1	
C(529)	=Q(29,1)	=YRAM	76131	CURRENT VALUE OF MAX. RAM		CM**2/G	14	9
C(530)	=Q(30,1)	=YDAM	76130	CURRENT VALUE OF MAX. DAM		CM**2/G	14	9
C(531)	=Q(1,2)	=XRAM	76127	INITIAL VALUE OF MAX. RAM		CM**2/G	S,1	9
C(532)	=Q(2,2)	=XDAM	76126	INITIAL VALUE OF MAX. DAM		CM**2/G	S,1	9
C(533)	=Q(3,2)	=SC1	76125	(NOT USED)				
C(534)	=Q(4,2)	=SC2	76124	(NOT USED)				
C(535)	=Q(5,2)	=SC3	76123	(NOT USED)				
C(536)	=Q(6,2)	=SC4	76122	(NOT USED)				
C(537)	=Q(7,2)	=XP	76121	INITIAL VALUE OF P		RE	S,1	1
C(538)	=Q(8,2)	=XE	76120	INITIAL VALUE OF E		DIML.	S,1	1
C(539)	=Q(9,2)	=XOMEGD	76117	INITIAL VALUE OF OMEGAD		DEG.	S,1	1
C(540)	=Q(10,2)	=XASCND	76116	INITIAL VALUE OF ASCND		DEG.	S,1	1
C(541)	=Q(11,2)	=XOINCD	76115	INITIAL VALUE OF OINCD		DEG.	S,1	1
C(542)	=Q(12,2)	=XDAY	76114	INITIAL VALUE OF DAY		DAYS	S,1	1
C(543)	=Q(13,2)	=PSID	76113	ANGLE BET ECLIPTIC AND EQUATOR		DEG.	S,1	
C(544)	=Q(14,2)	=GJAY	76112	SECOND HARM. COEFF. A2		DIML.	S,1	7
C(545)	=Q(15,2)	=GM	76111	GRAV.CONST*EARTH MASS		CM**3/SEC**2	S,1	(MANY)
C(546)	=Q(16,2)	=RE	76110	RADIUS OF EARTH		CM.	S,1	(MANY)
C(547)	=Q(17,2)	=OS	76107	ANG. VEL. OF EARTH ABOUT SUN		RAD/DAY	S,1	
C(548)	=Q(18,2)	=RADP	76106	SOLAR RAD. PRESSURE		DYNE/CM**2	S,1	5
C(549)	=Q(19,2)	=RHO1	76105	AIR DENSITY CONSTANT	CD(1)		S,1	RHO
C(550)	=Q(20,2)	=H1	76104	AIR DENSITY CONSTANT	CD(2)		S,1	RHO
C(551)	=Q(21,2)	=DH	76103	AIR DENSITY CONSTANT	CD(3)		S,1	RHO
C(552)	=Q(22,2)	=DEXP	76102	AIR DENSITY CONSTANT	CD(4)		S,1	RHO
C(553)	=Q(23,2)		76101	COEFF. OF H**2	CD(5)		S,1	RHO
C(554)	=Q(24,2)		76100	COEFF. OF H**3	CD(6)		S,1	RHO
C(555)	=Q(25,2)	=HT2	76077	THRESHHOLD FOR SECOND BULGE COEFF. FORMULA	CD(7)	CM	S,1	RHO
C(556)	=Q(26,2)	=B1A	76076	8.*TA IN FIRST BULGE COEFF. FORMULA	CD(8)		S,1	RHO
C(557)	=Q(27,2)	=B1B	76075	COEFF. OF H IN FIRST BULGE COEFF. FORMULA	CD(9)		S,1	RHO
C(558)	=Q(28,2)	=B1C	76074	CONST. WRT H IN FIRST BULGE COEFF. FORMULA	CD(10)		S,1	RHO
C(559)	=Q(29,2)	=B2B	76073	COEFF. IN SECOND BULGE COEFF. FORMULA	CD(11)		S,1	RHO
C(560)	=Q(30,2)	=B2E	76072	EXPONENT IN SECOND BULGE COEFF. FORMULA	CD(12)		S,1	RHO
C(561)	=Q(1,3)		76071	DAY CORRESPONDING TO CV(0) IN FLUX TABLE	CD(13)	DAY	S,1	RHO
C(562)	=Q(2,3)	=BLAG	76070	LAG ANGLE BETWEEN SUN LINE AND BULGE MAX.	CD(14)		S,1	RHO
C(563)	=Q(3,3)		76067	VALUX OF FLUX WHEN FLUX TABLE EXHAUSTED	CD(15)		S,1	RHO

TABLE 1 CONTINUED

C-VECT. LOC.	MATRIX NAME	MNEM. NAME	OCTAL LOC.	DEFINITION	UNITS	CALC. IN	USED IN
C(564)	=Q(4,3)		76066	=1,ROTATING ATMOSPHERE CD(16) =0,NONROTATING.		S,1	RHO
C(591)	=Q(1,4)	=COM	76033	COSF(OMEGA)	DIML.	4	
C(592)	=Q(2,4)	=SOM	76032	SINF(OMEGA)	DIML.	4	
C(593)	=Q(3,4)	=CAS	76031	COSF(ASCN)	DIML.	4	
C(594)	=Q(4,4)	=SAS	76030	SINF(ASCN)	DIML.	4	
C(595)	=Q(5,4)	=COI	76027	COSF(OINC)	DIML.	4	
C(596)	=Q(6,4)	=SOI	76026	SINF(OINC)	DIML.	4	
C(597)	=Q(7,4)	=COT	76025	COSF(OST)	DIML.	4	
C(598)	=Q(8,4)	=SOT	76024	SINF(OST)	DIML.	4	
C(599)	=Q(9,4)	=R	76023	DIR. COS. BET. PERIGEE AND SUN	DIML.	4	
C(600)	=Q(10,4)	=S	76022	DIR. COS. BET. SEMI-LATUS RECTUM AT V=90. DEG. AND SUN	DIML.	4	
C(601)	=Q(11,4)		76021	(NOT USED)			
C(602)	=Q(12,4)	=RS	76020	R**2 + S**2	DIML.	4	
C(603)	=Q(13,4)	=VA	76017	LOWER BOUND OF POSSIBLE SHADOW	RAD.	4	
C(604)	=Q(14,4)	=VB	76016	UPPER BOUND OF POSSIBLE SHADOW	RAD.	4	
C(605)	=Q(15,4)	=SAE	76015	P*ABSF(W)-1.	RE	4	
C(606)	=Q(16,4)	=SA	76014	SAE/E	RE	4	
C(607)	=Q(17,4)	=DELV	76013	INTERVAL FOR SHADOW SEARCH	RAD.	4	
C(608)	=Q(18,4)	=EPSV	76012	INTERVAL FOR SHADOW INTERPOL.	RAD.	4	
C(621)	=Q(1,5)		75775	INTEGRAL USED IN RADPR	DIML.	5	
C(622)	=Q(2,5)		75774	INTEGRAL USED IN RADPR	DIML.	5	
C(623)	=Q(3,5)		75773	INTEGRAL USED IN RADPR	DIML.	5	
C(624)	=Q(4,5)		75772	INTEGRAL USED IN RADPR	DIML.	5	
C(625)	=Q(5,5)		75771	INTEGRAL USED IN RADPR	DIML.	5	
C(626)	=Q(6,5)		75770	INTEGRAL USED IN RADPR	DIML.	5	
C(627)	=Q(7,5)		75767	INTEGRAL USED IN RADPR	DIML.	5	
C(628)	=Q(8,5)		75766	INTEGRAL USED IN RADPR	DIML.	5	
C(629)	=Q(9,5)		75765	INTEGRAL USED IN RADPR	DIML.	5	
C(630) - C(640)				(NOT USED)			
C(641)	=Q(21,5)	=EA1	75751	ECCENTRIC ANOM. GOING INTO SHAD.	RAD.	5	
C(642)	=Q(22,5)	=EA2	75750	ECCENTRIC ANOM. COMING OUT SHAD.	RAD.	5	
C(643)	=Q(23,5)	=PCR	75747	P**2*CR (ACCEL. DUE TO RADP) / (ACCEL. DUE TO GRAVITY AT SEMI-LATUS RECTUM)	DIML.	5	
C(644)	=Q(24,5)	=CR	75746	RE**2*RAM*RADP/GM*Q(30,5) (ACCEL. DUE TO RADP) / (ACCEL. DUE TO GRAVITY AT EARTH SURFACE)	DIML.	5	
C(645)	=Q(25,5)	=ES	75745	E**2	DIML.	5	
C(646)	=Q(26,5)	=RA	75744	1.-ES	DIML.	5	
C(647)	=Q(27,5)	=RB	75743	SQRTF((1.-E)/(1.+E))	DIML.	5	
C(648) - C(649)				(NOT USED)			
C(650)	=Q(30,5)		75740	TDIST(0)=1/(SQUARE OF EARTH-SUN DISTANCE IN ASTON. UNITS)	DIML.	5	
C(651)	=Q(1,6)	=SUM1	75737	INTEGRAL , DRAG CHANGE IN P	G/CM**3	6	
C(652)	=Q(2,6)	=SUM2	75736	INTEGRAL , DRAG CHANGE IN E	G/CM**3	6	
C(653)	=Q(3,6)	=SUM3	75735	INTEGRAL , DRAG CHANGE IN OMEGA	G/CM**3	6	
C(654)	=Q(4,6)	=DV	75734	INTERVAL COVERED BY ONE QUAD.	RAD.	6	
C(655)	=Q(5,6)	=BIGA	75733	2.*DAM*RE*P (COEFFICIENT)	CM**3/G		
C(656)	=Q(6,6)	=V	75732	TRUE ANOMALY	RAD.	6	
C(657)	=Q(7,6)	=X	75731	COSF(V)	DIML.	6	

TABLE 1 CONTINUED

C-VECT. LOC.	MATRIX NAME	MNEM. NAME	OCTAL LOC.	DEFINITION	UNITS	CALC. IN	USED IN
C(658)	=Q(8,6)	=Y	75730	SINF(V)	DIML.	6	
C(659)	=Q(9,6)	=DA	75727	1.+E*COSF(V)	DIML.	6	
C(660)	=Q(10,6)	=DENS	75726	AIR DENSITY AT V	G/CM**3	6	
C(661)	=Q(11,6)	=DUMX	75725	INTEGRAND FOR CHANGE IN P	G/CM**3	6	
C(662)	=Q(12,6)	=DUMY	75724	DUMX*C(IA+10) WHERE C(IA+10) IS A GAUSSIAN QUAD. COEFF.	G/CM**3	6	
C(663) - C(665)				(NOT USED)			
C(666)	=Q(16,6)	=RHO	75720	DENSITY CALCULATED BY RHO	G/CM**3	RHO	
C(667)	=Q(17,6)	=H	75717	ALTITUDE ABOVE SPHERE ,RE	CM.	RHO	
C(668)	=Q(18,6)	=HT1	75716	ALT. BELOW WHICH BULGE=0.	CM.	RHO	
C(669)	=Q(19,6)	=TA	75715	FACTOR IN BULGE BELOW HT2	DIML.	RHO	
C(670)	=Q(20,6)	=TB	75714	FACTOR IN BULGE ABOVE HT2	DIML.	RHO	
C(671)	=Q(21,6)	=TC	75713	CONST. IN BULGE ABOVE HT2	1./CM.	RHO	
C(672)	=Q(22,6)	=DAYREF	75712	DAY-XDAY	DAY	RHO	
C(673)	=Q(23,6)	=FLUX	75711	SOLAR MICROWAVE FLUX	DYNES/CM./SEC.	RHO	
C(674)	=Q(24,6)	=BETA	75710	ASCN-BLAGD*CONV	RAD.	RHO	
C(675)	=Q(25,6)	=RL	75707	DIR. COS BET. PERIGEE AND BULGE	DIML.	RHO	
C(676)	=Q(26,6)	=SL	75706	DIR. COS BET.V=90 DEG AND BULGE	DIML.	RHO	
C(677)	=Q(27,6)	=ZT	75705	DIR. COS BET V=V AND BULGE	DIML.	RHO	
C(678) - C(680)				(NOT USED)			
C(1101)	=D(1,1)	=P	75035	SEMI-LATUS RECTUM OF ORBIT	RE.	14	MANY
C(1102)	=D(2,1)	=E	75034	ECCENTRICITY	DIML.	14	MANY
C(1103)	=D(3,1)	=OMEGAD	75033	ARGUMENT OF PERIGEE	DEG.	2	2,3
C(1104)	=D(4,1)	=ASCND	75032	R.A. OF ASCENDING NODE	DEG.	2	2,3
C(1105)	=D(5,1)	=OINCD	75031	INCLINATION OF ORBIT	DEG.	2	2,3
C(1106)	=D(6,1)	=DMJ	75030	MODIFIED JULIAN DAY-30,000. DMJ=DAY+6203.	DAYS	2	2
C(1107)	=D(7,1)	=DAY	75027	DAYS SINCE MIDNIGHT U.T.12/30/57	DAYS	14	MANY
C(1108) - C(1122)				(NOT USED)			
C(1123)	=D(23,1)	=OMEGA	75007	ARGUMENT OF PERIGEE	RAD.	14	MANY
C(1124)	=D(24,1)	=ASCN	75006	R.A. OF ASCENDING NODE	RAD.	14	MANY
C(1125)	=D(25,1)	=OINC	75005	INCLINATION OF ORBIT	RAD.	14	MANY
C(1126) - C(1130)				(NOT USED)			
C(1131)	=D(1,2)	=A	74777	SEMI-MAJOR AXIS OF ORBIT	RE.	14	MANY
C(1132)	=D(2,2)	=HTPER	74776	HEIGHT OF PERIGEE ABOVE SPHERE	RE.	14	MANY
C(1133)	=D(3,2)	=W	74775	DIRECTION COSINE OF SUNLIGHT WITH THE NORMAL TO THE ORBIT	DIML.	4	MANY
C(1134)	=D(4,2)	=PHID	74774	ANGLE BETWEEN PERIGEE AND PROJECTION OF SUN LINE ON ORBIT PLANE--MEASURED POSITIVELY IN THE DIRECTION OF MOTION FROM PERIGEE TO THE SUN LINE	DEG.	2	2,3
C(1135)	=D(5,2)	=V1D	74773	TRUE ANOMALY ON ENTERING SHADOW	DEG.	2	2,3
C(1136)	=D(6,2)	=V2D	74772	TRUE ANOMALY ON LEAVING SHADOW	DEG.	2	2,3
C(1137) - C(1153)				(NOT USED)			
C(1154)	=D(24,2)	=PHI	74750	PHID IN RADIANS	RAD.	4	MANY
C(1155)	=D(25,2)	=V1	74747	TRUE ANOMALY ON ENTERING SHADOW	RAD.	4	MANY
C(1156)	=D(26,2)	=V2	74746	TRUE ANOMALY ON LEAVING SHADOW	RAD.	4	MANY
C(1157) - C(1160)				(NOT USED)			
C(1161)	=D(1,3)	=QMM	74741	PERIGEE DISTANCE A*(1.-E)*RE/10.**8	10**8*CM.	2	2
C(1162)	=D(2,3)	=DLNTAU	74740	TOTAL CHANGE IN ONE PERIOD OF LOG OF PERIOD	DIML.	2	2

TABLE 1 CONTINUED

C-VECT. LOC.	MATRIX NAME	MNEM. NAME	OCTAL LOC.	DEFINITION	UNITS	CALC. IN	USED IN
C(1163)	=D(3,3)	=PERIOD	74737	SATELLITE PERIOD, TIME*A**1.5	DAYS	14,2	MANY
C(1164)	=D(4,3)	=RAM	74736	EFFECTIVE AREA TO MASS RATIO FOR RADPR	CM**2/G.	9	MANY
C(1165)	=D(5,3)	=DAM	74735	EFFECTIVE AREA TO MASS RATIO FOR DRAG	CM**2/G.	9	MANY
C(1166)	=D(6,3)	=OSTD	74734	TRUE LONGITUDE OF SUN	DEG.	2	2
C(1167) - C(1185)				(NOT USED)			
C(1186)	=D(26,3)	=OST	74710	TRUE LONGITUDE OF SUN	RAD.	14	MANY
C(1187) - C(1190)				(NOT USED)			
C(1191)	=D(1,4)	=OMBARD	74703	OMEGAD + ASCND	DEG.	2	2
C(1192)	=D(2,4)	=OMTILD	74702	OMEGAD - ASCND	DEG.	2	2
C(1193)	=D(3,4)	=OM1D	74701	OMEGAD + ASCND - OSTD	DEG.	2	2
C(1194)	=D(4,4)	=OM2D	74700	OMEGAD - ASCND - OSTD	DEG.	2	2
C(1195)	=D(5,4)	=OM3D	74677	OMEGAD - ASCND + OSTD	DEG.	2	2
C(1196)	=D(6,4)	=OM4D	74676	OMEGAD + ASCND + OSTD	DEG.	2	2
C(1197) - C(1220)				(NOT USED)			

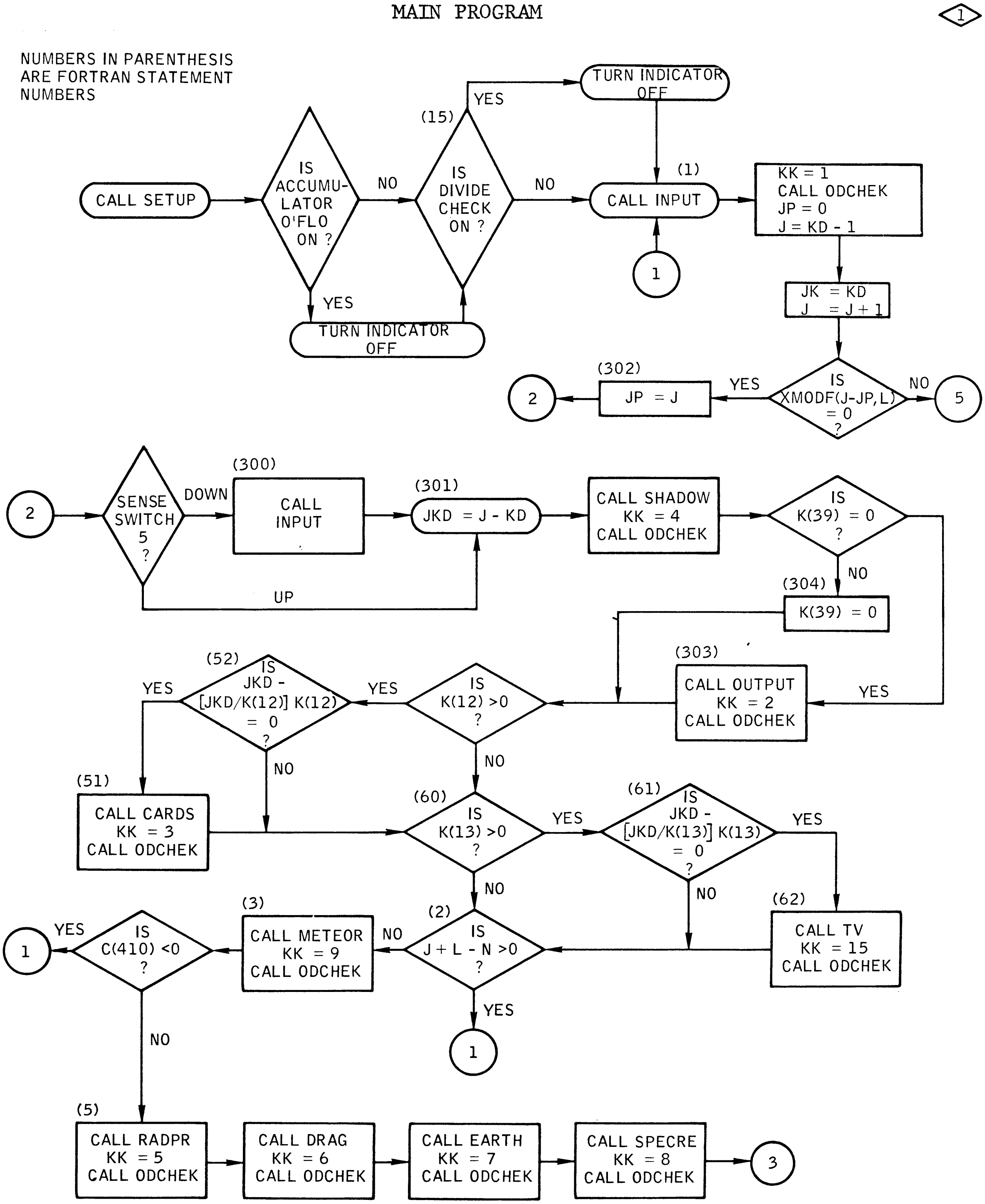
TABLE 1 CONTINUED

THE REMAINDER OF THE D-MATRIX CONTAINS INCREMENTS TO ORBITAL ELEMENTS
DURING ONE PERIOD DUE TO THE VARIOUS PERTURBING FORCES

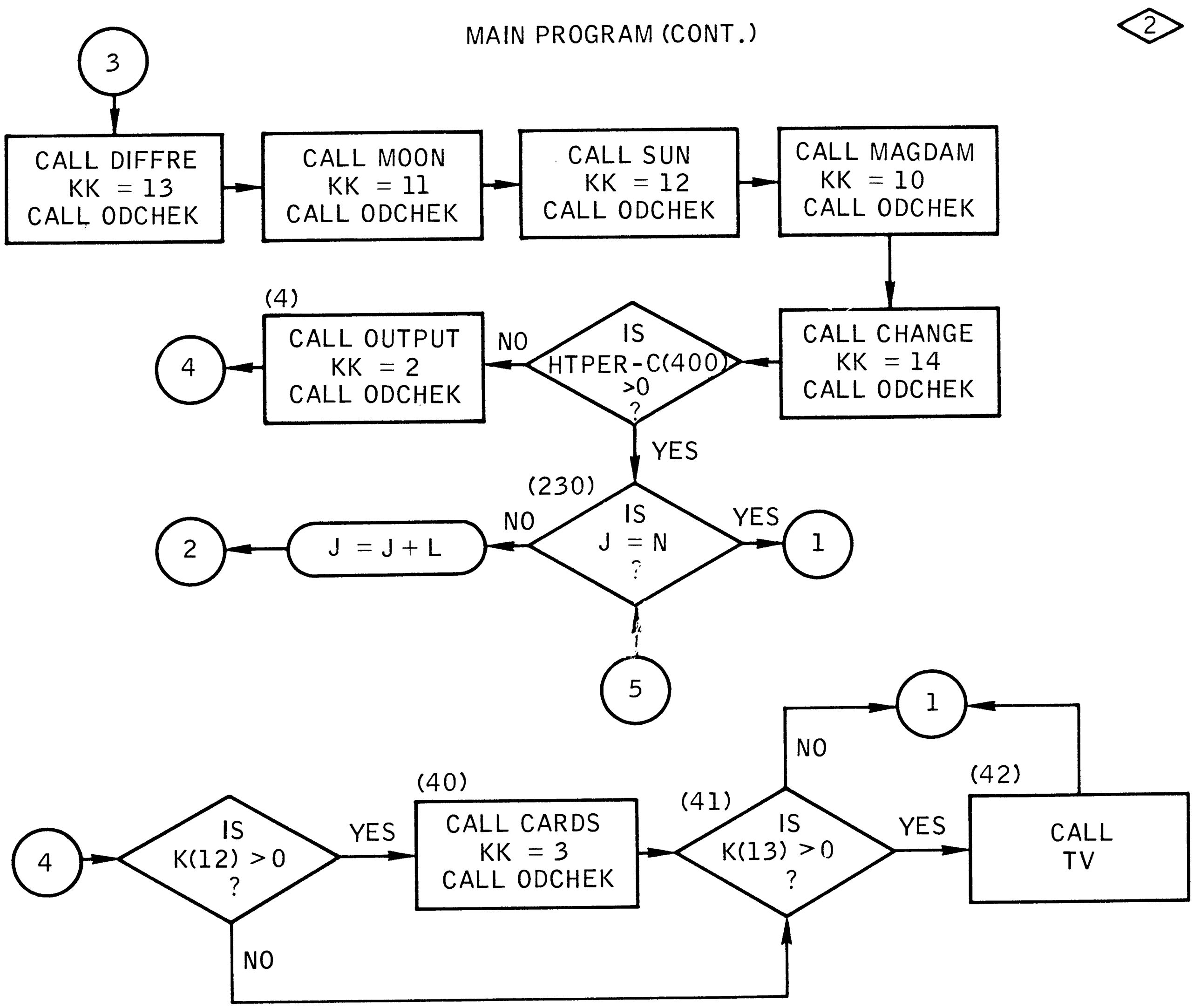
D(1,I)	=CHANGE IN P	DUE TO I-TH PERT. FORCE	2	2
D(2,I)	=CHANGE IN E	DUE TO I-TH PERT. FORCE	2	2
D(3,I)	=CHANGE IN OMEGAD	DUE TO I-TH PERT. FORCE	2	2
D(4,I)	=CHANGE IN ASCND	DUE TO I-TH PERT. FORCE	2	2
D(5,I)	=CHANGE IN OINCD	DUE TO I-TH PERT. FORCE	2	2
D(6,I)	=CHANGE IN A	DUE TO I-TH PERT. FORCE	2	2
D(7,I)	=CHANGE IN LOG OF PERIOD	DUE TO I-TH PERT. FORCE	2	2
D(8,I)	=CHANGE IN HTPER	DUE TO I-TH PERT. FORCE	2	2
D(21,I)	=CHANGE IN P	DUE TO I-TH PERT. FORCE	I	14,2
D(22,I)	=CHANGE IN E	DUE TO I-TH PERT. FORCE	I	14,2
D(23,I)	=CHANGE IN OMEGA	DUE TO I-TH PERT. FORCE	I	14,2
D(24,I)	=CHANGE IN ASCN	DUE TO I-TH PERT. FORCE	I	14,2
D(25,I)	=CHANGE IN OINC	DUE TO I-TH PERT. FORCE	I	14,2

FLOW CHARTS

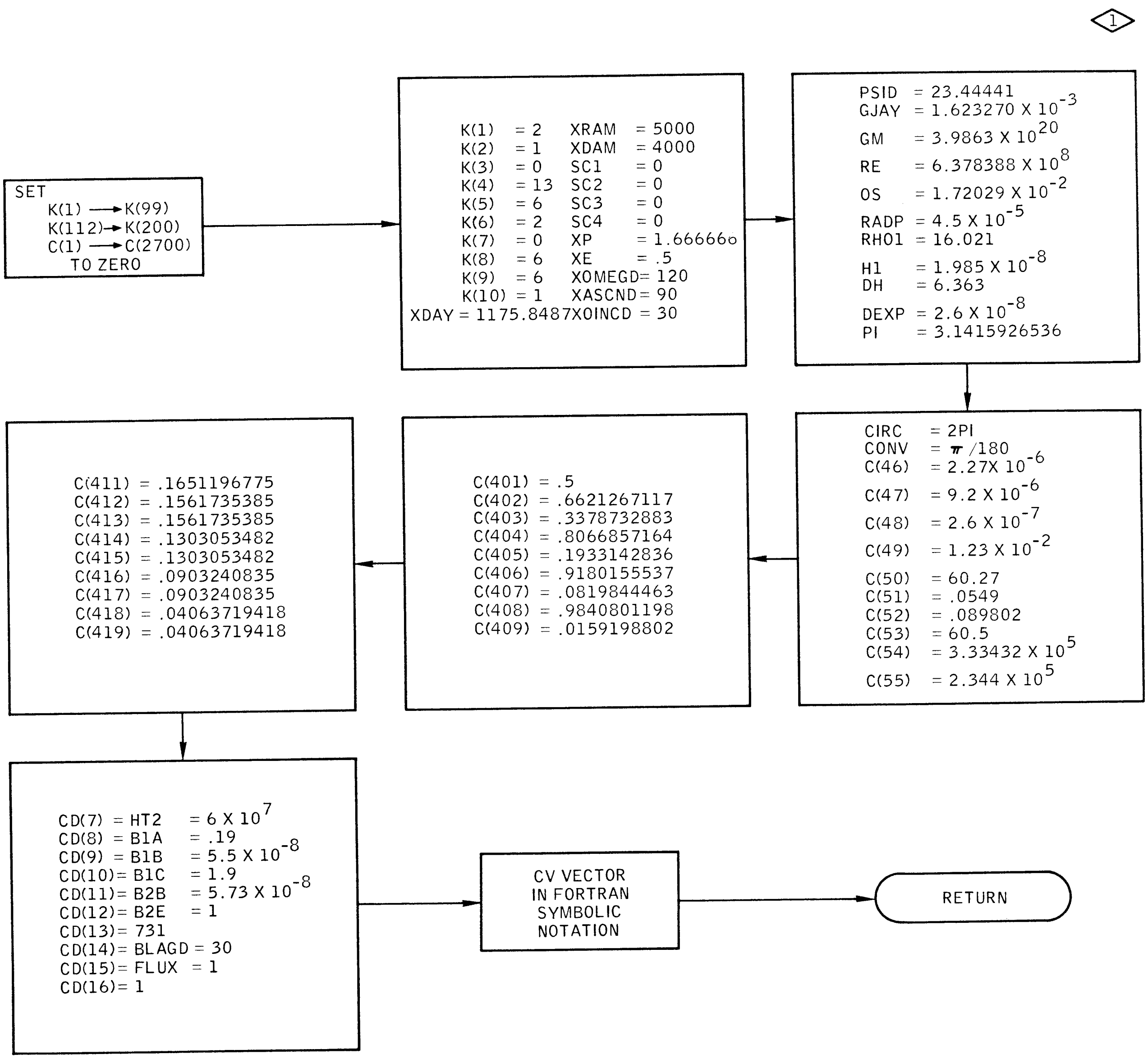
MAIN PROGRAM



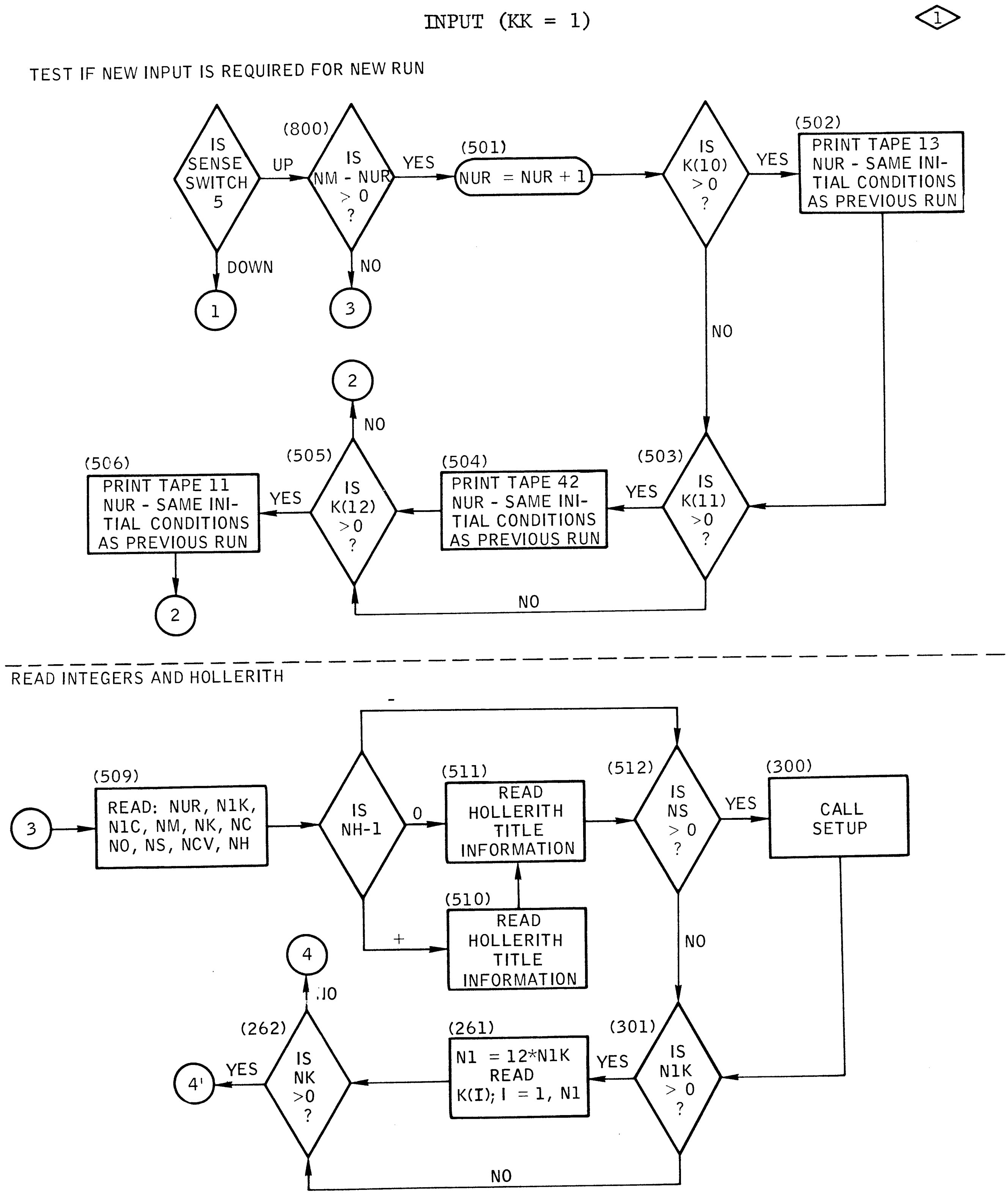
MAIN PROGRAM (CONT.)



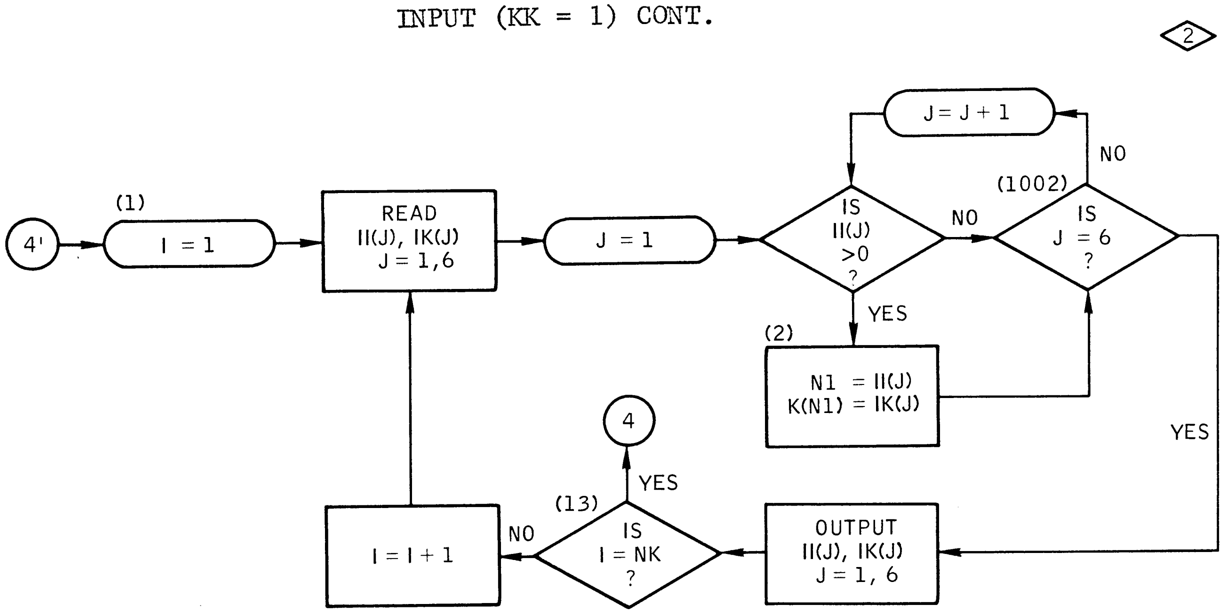
SETUP

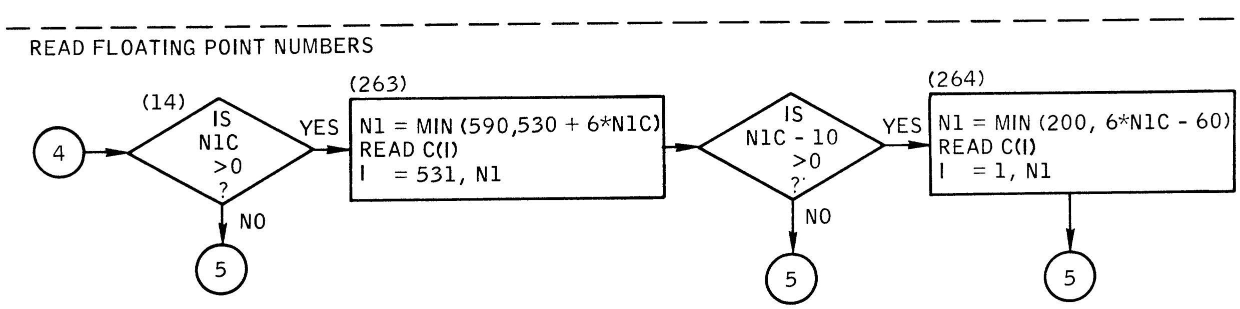

1
SET
K(1) → K(99)
K(112) → K(200)
C(1) → C(2700)
TO ZERO
K(1) = 2 XRAM = 5000
K(2) = 1 XDAM = 4000
K(3) = 0 SC1 = 0
K(4) = 13 SC2 = 0
K(5) = 6 SC3 = 0
K(6) = 2 SC4 = 0
K(7) = 0 XP = 1.666660
K(8) = 6 XE = .5
K(9) = 6 XOMEGD = 120
K(10) = 1 XASCND = 90
XDAY = 1175.8487 XOINCD = 30
PSID = 23.44441
GJAY = 1.623270 X 10^{-3}
GM = 3.9863 X 10^{20}
RE = 6.378388 X 10^{8}
OS = 1.72029 X 10^{-2}
RADP = 4.5 X 10^{-5}
RHO1 = 16.021
H1 = 1.985 X 10^{-8}
DH = 6.363
DEXP = 2.6 X 10^{-8}
PI = 3.1415926536
CIRC = 2PI
CONV = π/180
C(46) = 2.27X 10^{-6}
C(47) = 9.2 X 10^{-6}
C(48) = 2.6 X 10^{-7}
C(49) = 1.23 X 10^{-2}
C(50) = 60.27
C(51) = .0549
C(52) = .089802
C(53) = 60.5
C(54) = 3.33432 X 10^{5}
C(55) = 2.344 X 10^{5}
C(401) = .5
C(402) = .6621267117
C(403) = .3378732883
C(404) = .8066857164
C(405) = .1933142836
C(406) = .9180155537
C(407) = .0819844463
C(408) = .9840801198
C(409) = .0159198802
C(411) = .1651196775
C(412) = .1561735385
C(413) = .1561735385
C(414) = .1303053482
C(415) = .1303053482
C(416) = .0903240835
C(417) = .0903240835
C(418) = .04063719418
C(419) = .04063719418
CD(7) = HT2 = 6 X 10^{7}
CD(8) = B1A = .19
CD(9) = B1B = 5.5 X 10^{-8}
CD(10) = B1C = 1.9
CD(11) = B2B = 5.73 X 10^{-8}
CD(12) = B2E = 1
CD(13) = 731
CD(14) = BLAGD = 30
CD(15) = FLUX = 1
CD(16) = 1
CV VECTOR IN FORTRAN SYMBOLIC NOTATION
RETURN

INPUT (KK = 1)

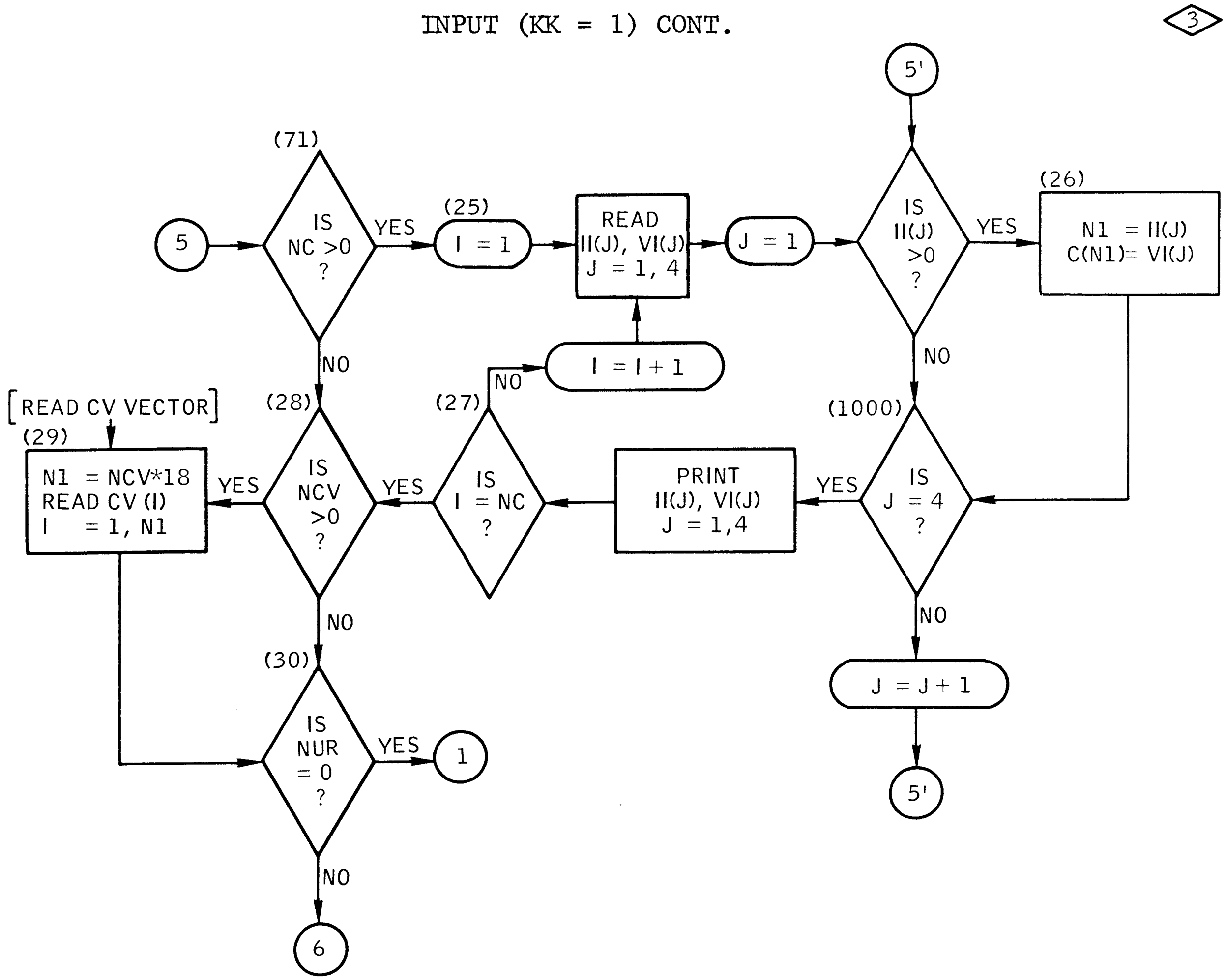

1
TEST IF NEW INPUT IS REQUIRED FOR NEW RUN
IS SENSE SWITCH 5
UP
DOWN
1
(800)
IS NM - NUR > 0 ?
YES
NO
3
(501)
NUR = NUR + 1
IS K(10) > 0 ?
YES
NO
(502)
PRINT TAPE 13 NUR - SAME INITIAL CONDITIONS AS PREVIOUS RUN
(503)
IS K(11) > 0 ?
YES
NO
(504)
PRINT TAPE 42 NUR - SAME INITIAL CONDITIONS AS PREVIOUS RUN
(505)
IS K(12) > 0 ?
YES
NO
2
(506)
PRINT TAPE 11 NUR - SAME INITIAL CONDITIONS AS PREVIOUS RUN
2
READ INTEGERS AND HOLLERITH
3
(509)
READ: NUR, N1K, N1C, NM, NK, NC NO, NS, NCV, NH
IS NH-1
-
0
+
(511)
READ HOLLERITH TITLE INFORMATION
(510)
READ HOLLERITH TITLE INFORMATION
(512)
IS NS > 0 ?
YES
NO
(300)
CALL SETUP
(301)
IS N1K > 0 ?
YES
NO
(261)
N1 = 12*N1K READ K(I); I = 1, N1
(262)
IS NK >0 ?
YES
NO
4
4'

INPUT (KK = 1) CONT.

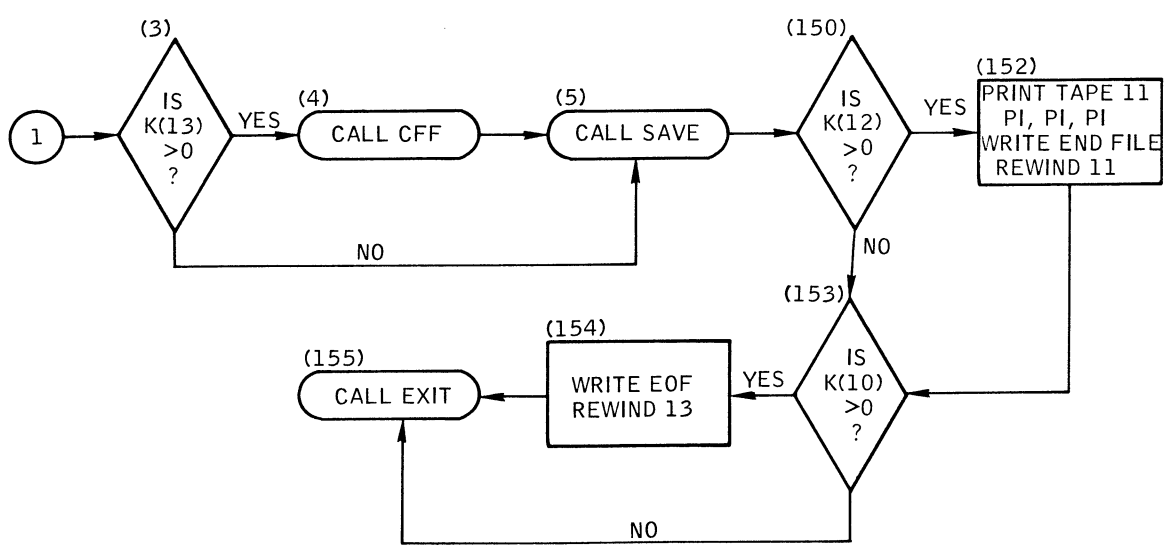




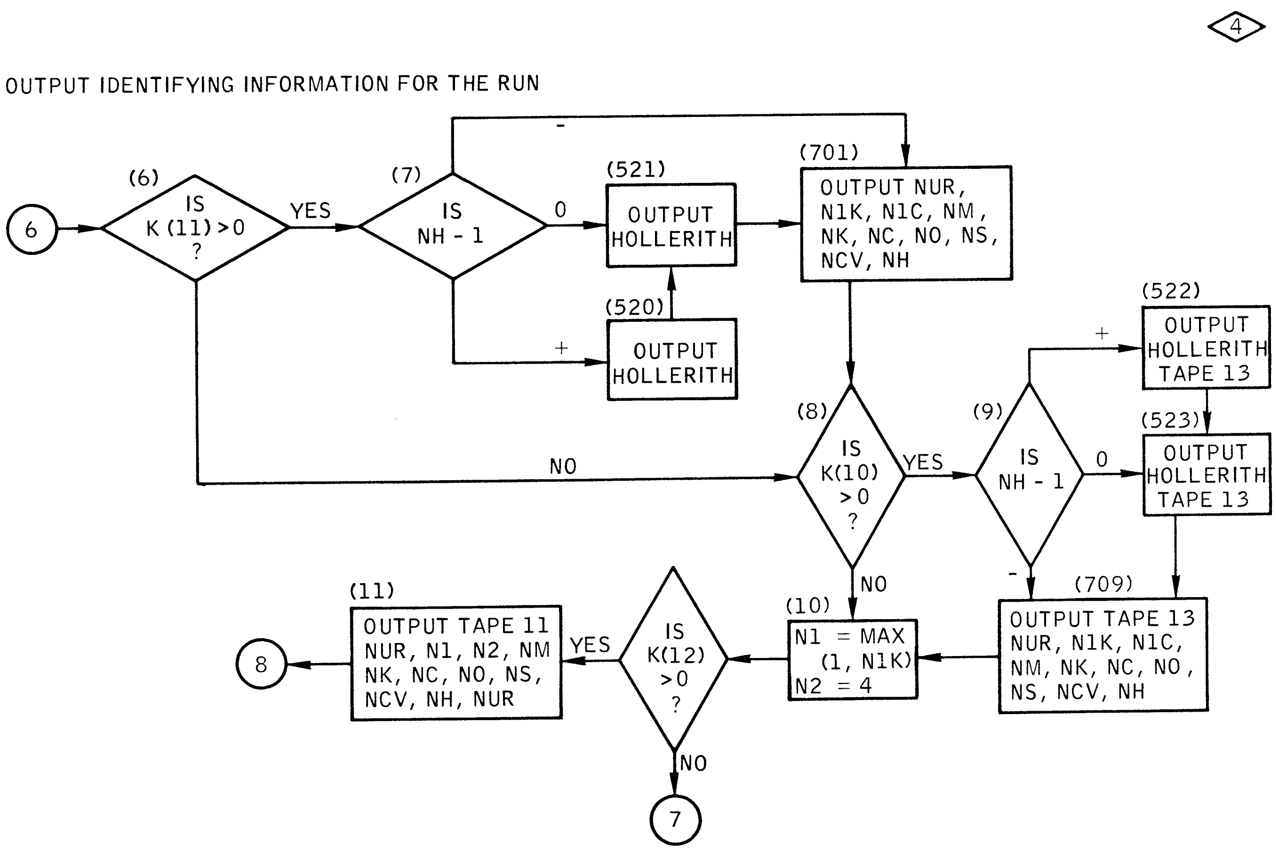
INPUT (KK = 1) CONT.



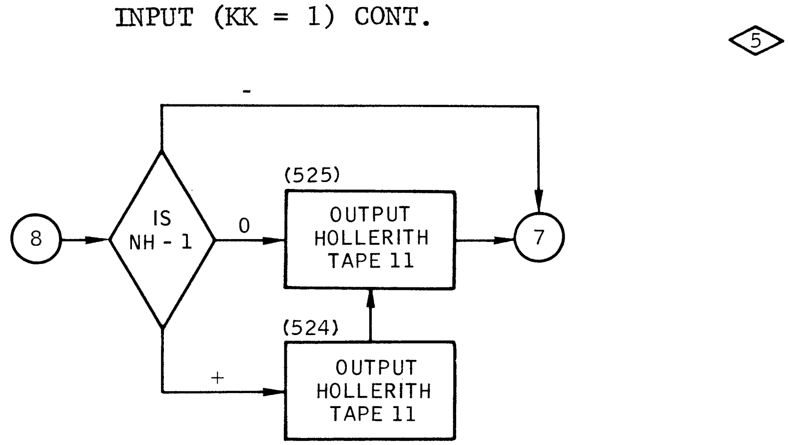
TERMINATE A SET OF RUNS IF NUR = 0



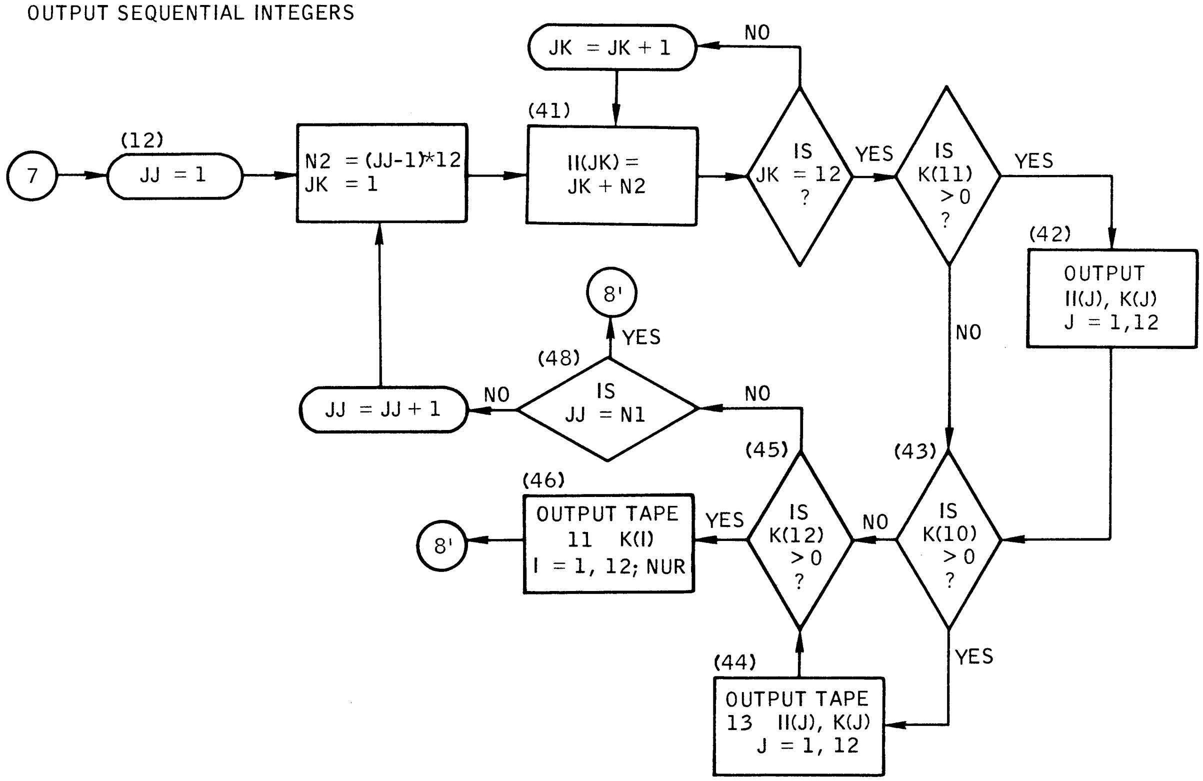
INPUT (KK = 1) CONT.



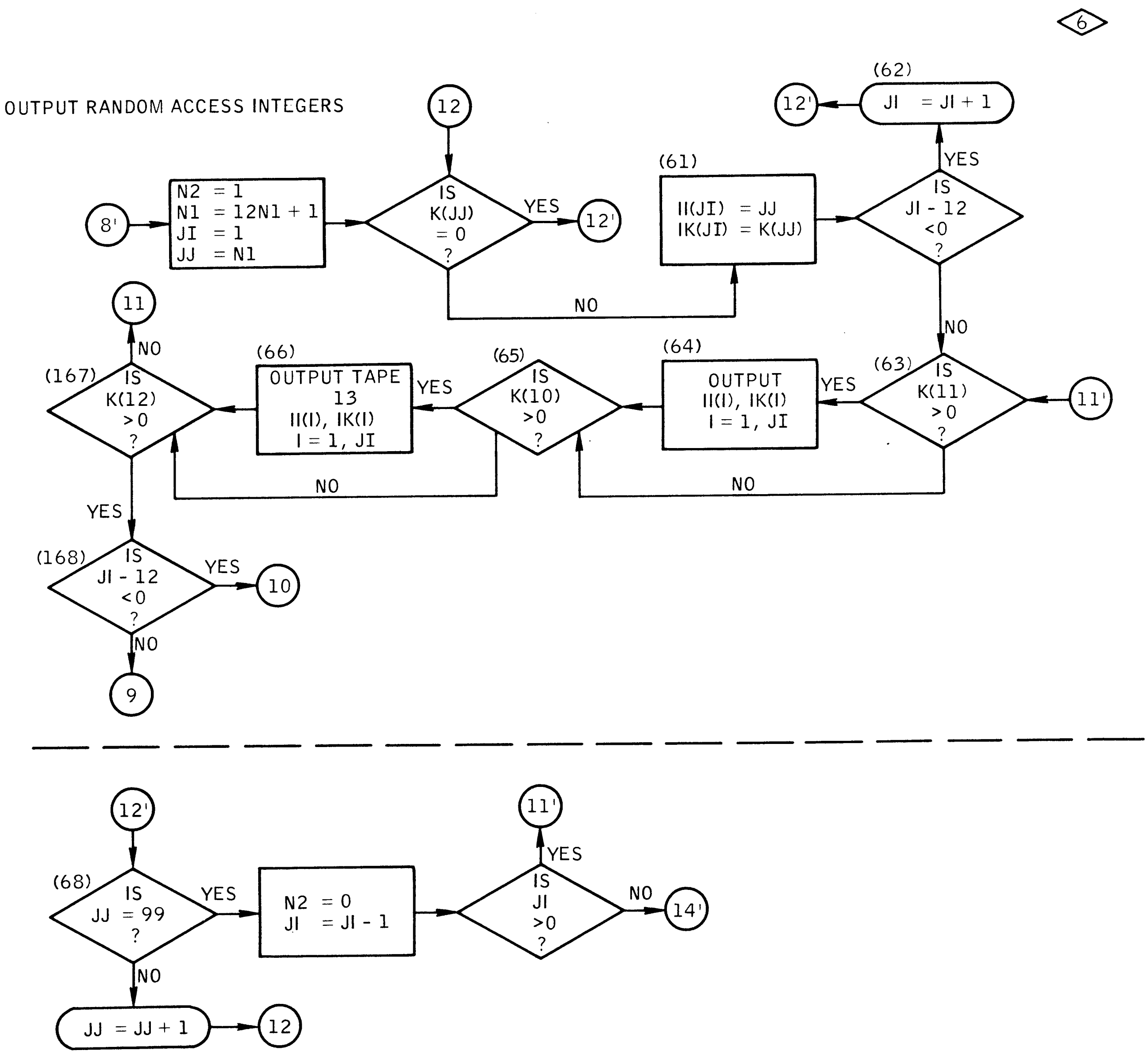
INPUT (KK = 1) CONT.



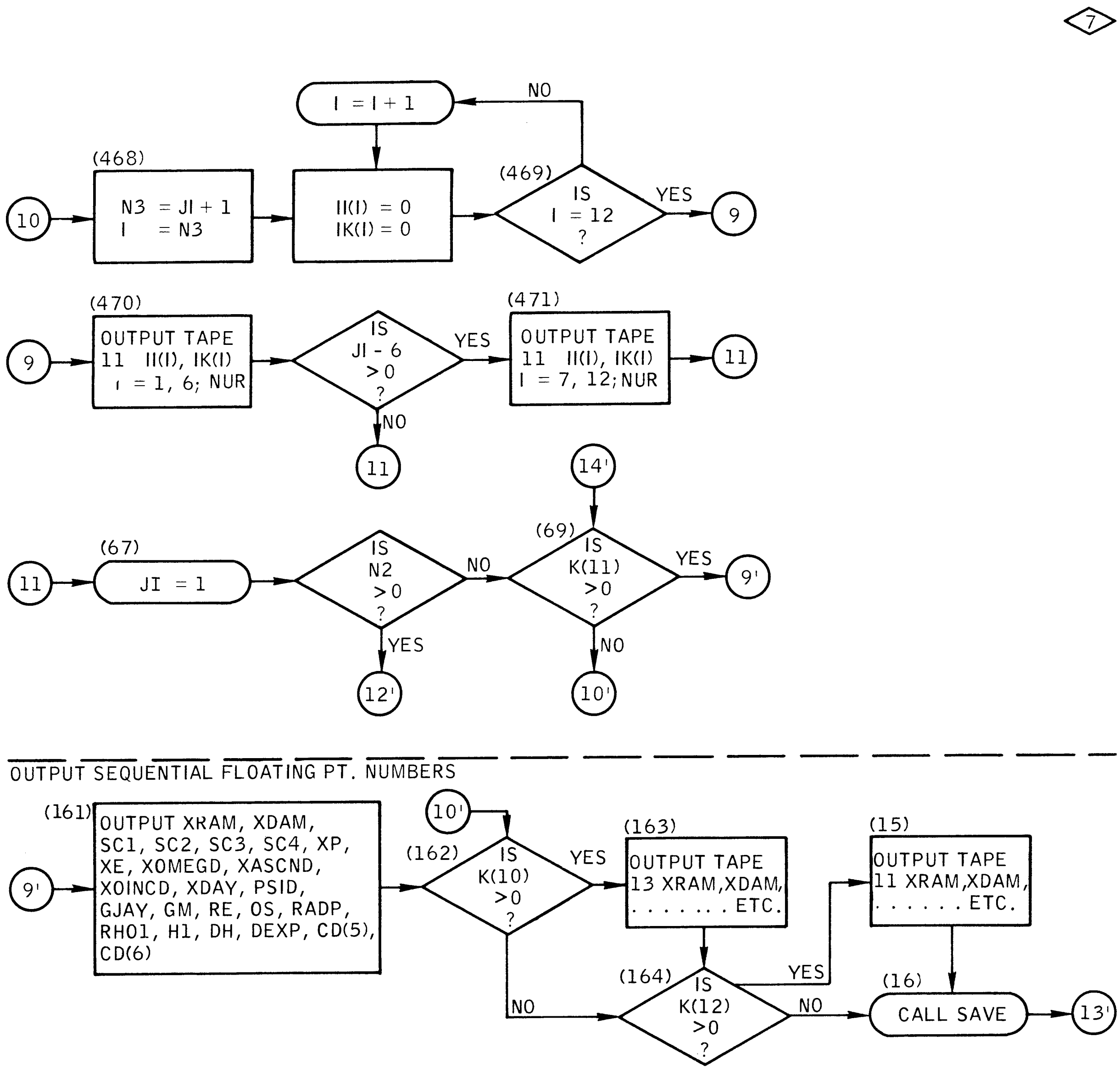
OUTPUT SEQUENTIAL INTEGERS



INPUT (KK = 1) CONT.

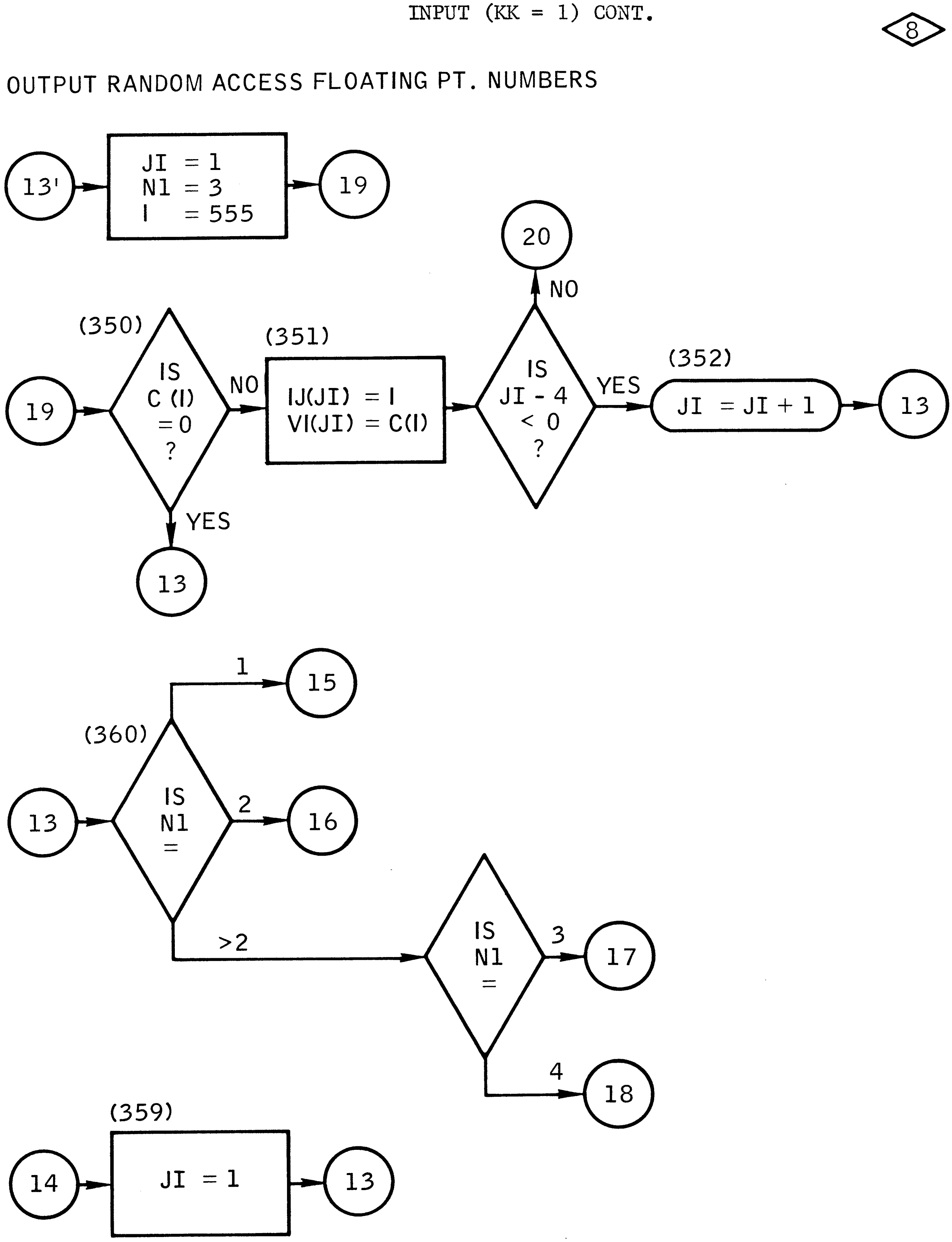


INPUT (KK = 1) CONT.

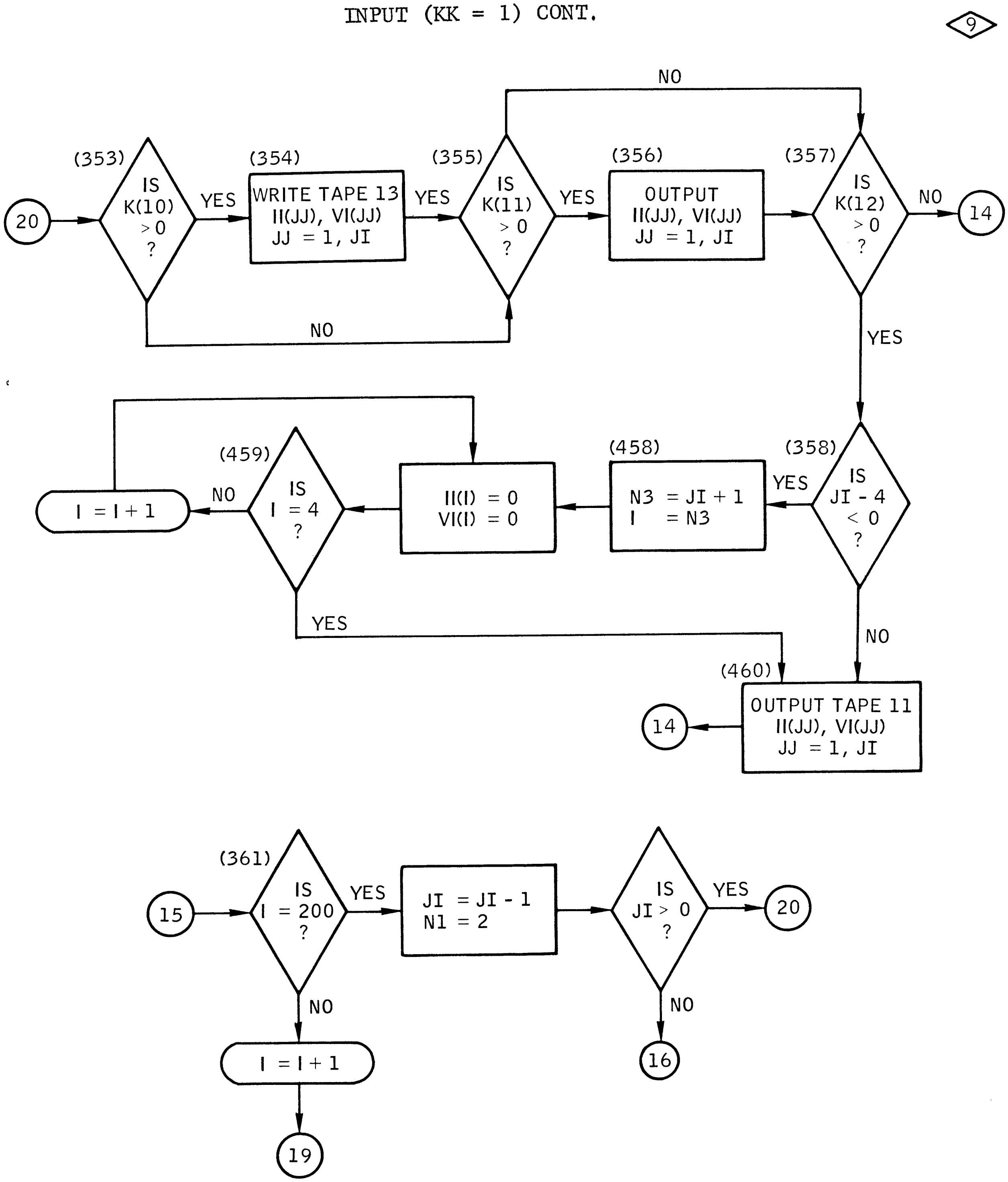


INPUT (KK = 1) CONT.

OUTPUT RANDOM ACCESS FLOATING PT. NUMBERS

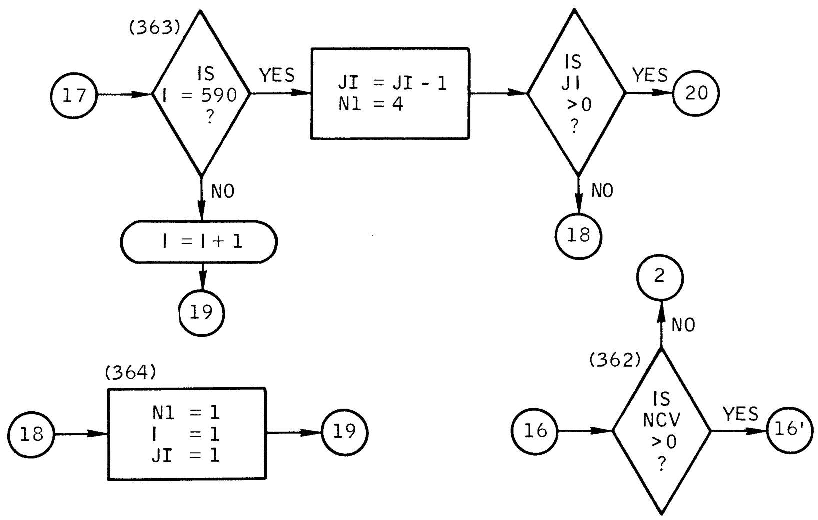


INPUT (KK = 1) CONT.

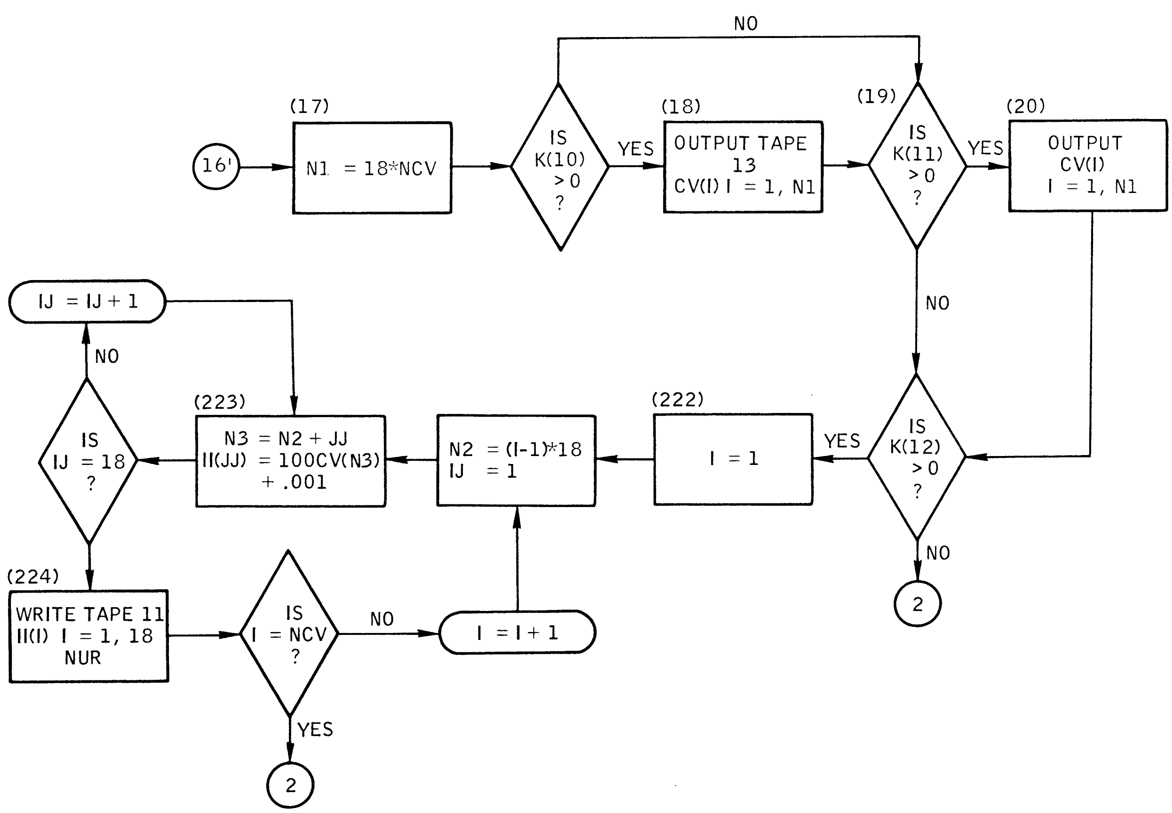


INPUT (KK = 1) CONT.

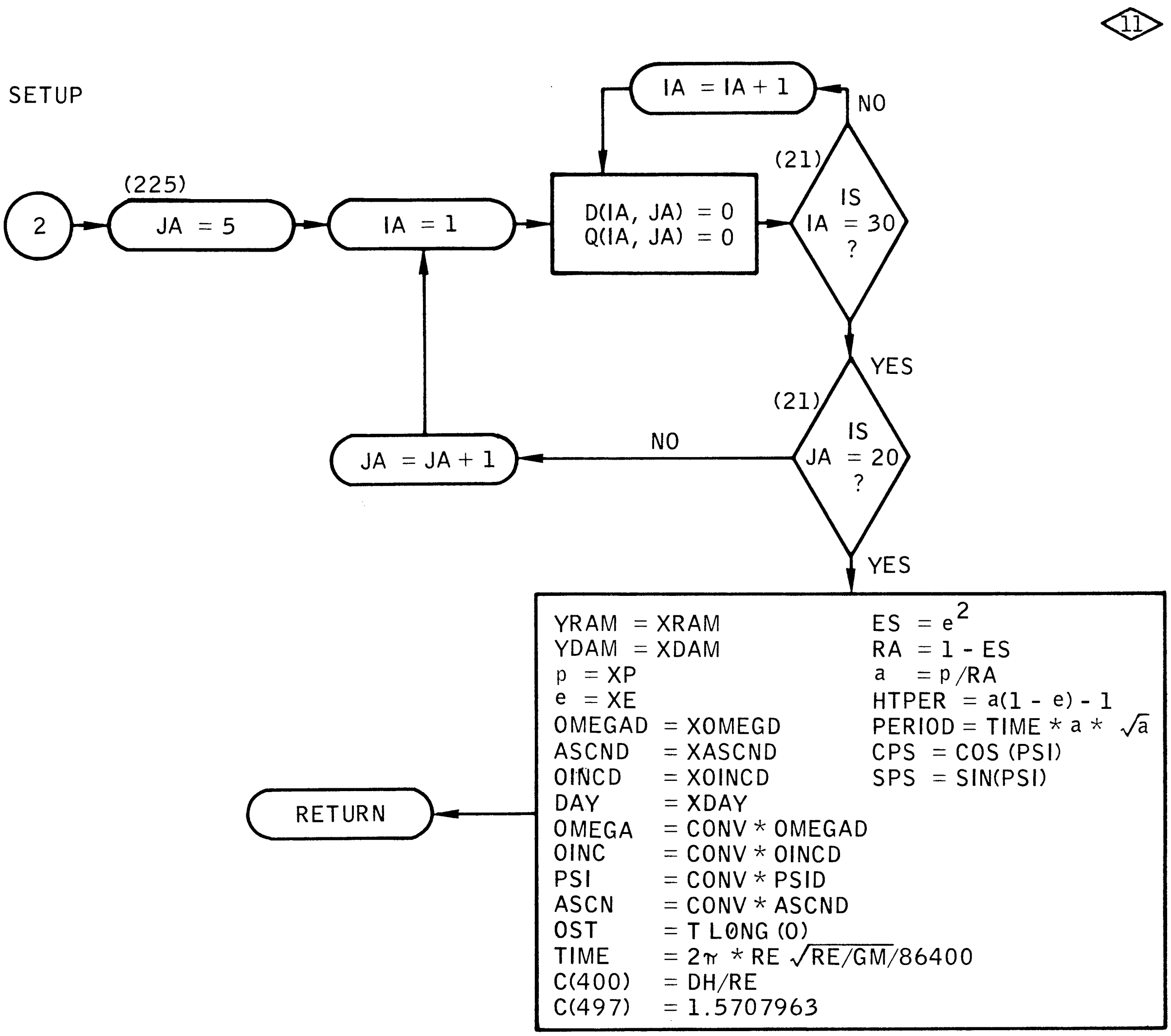
10



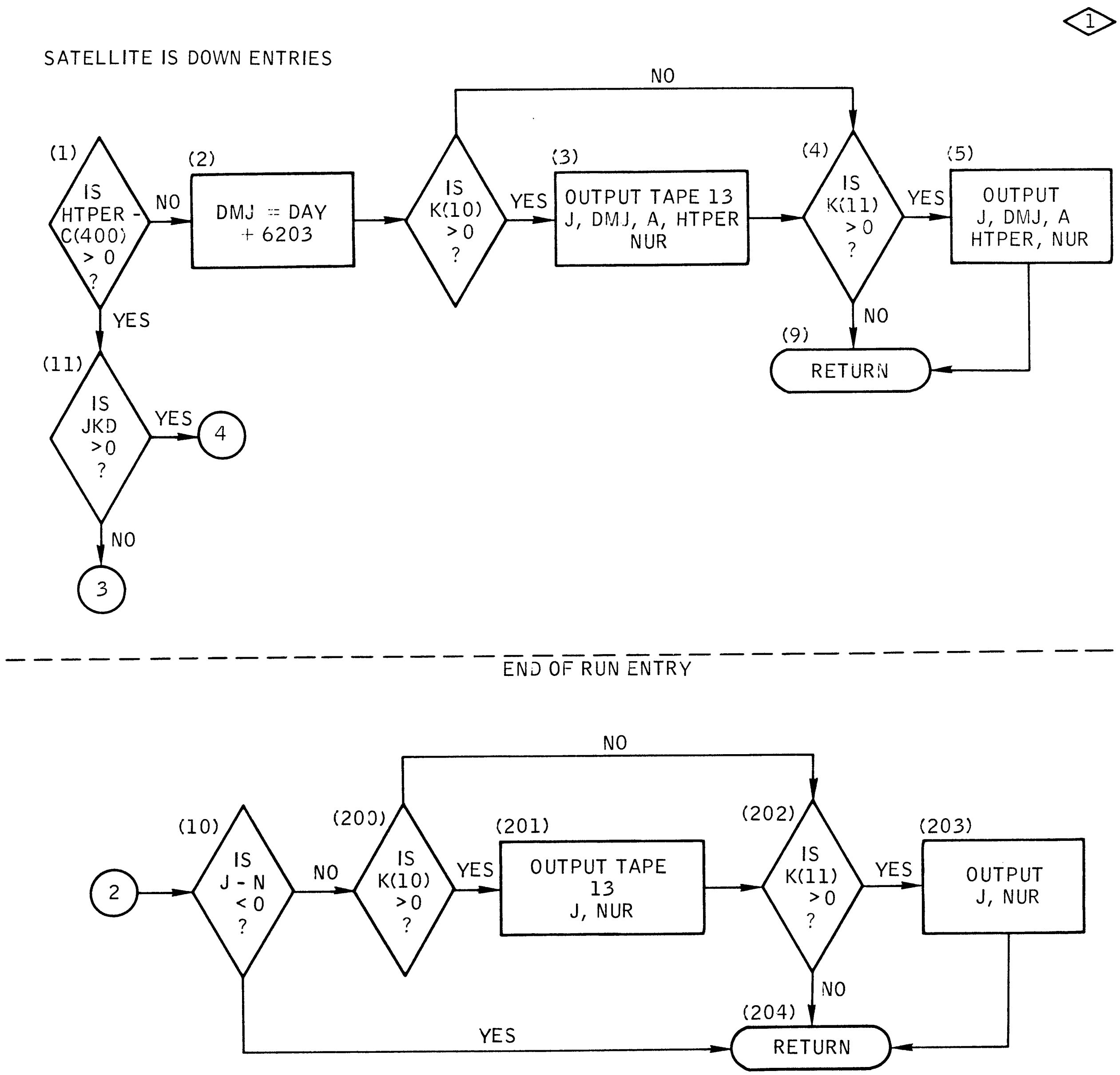
OUTPUT CV-VECTOR



INPUT (KK = 1) CONT.

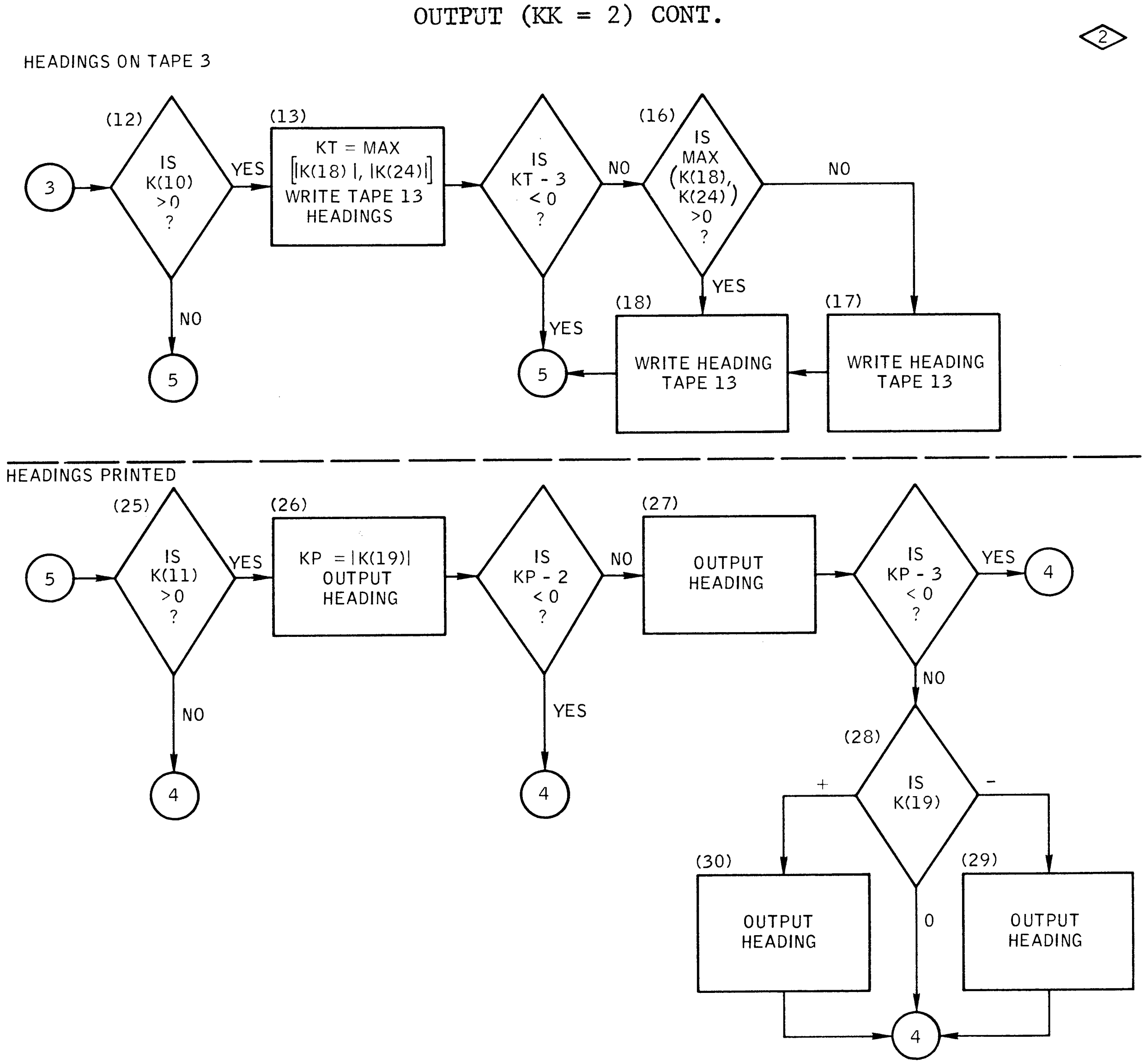


OUTPUT (KK = 2)

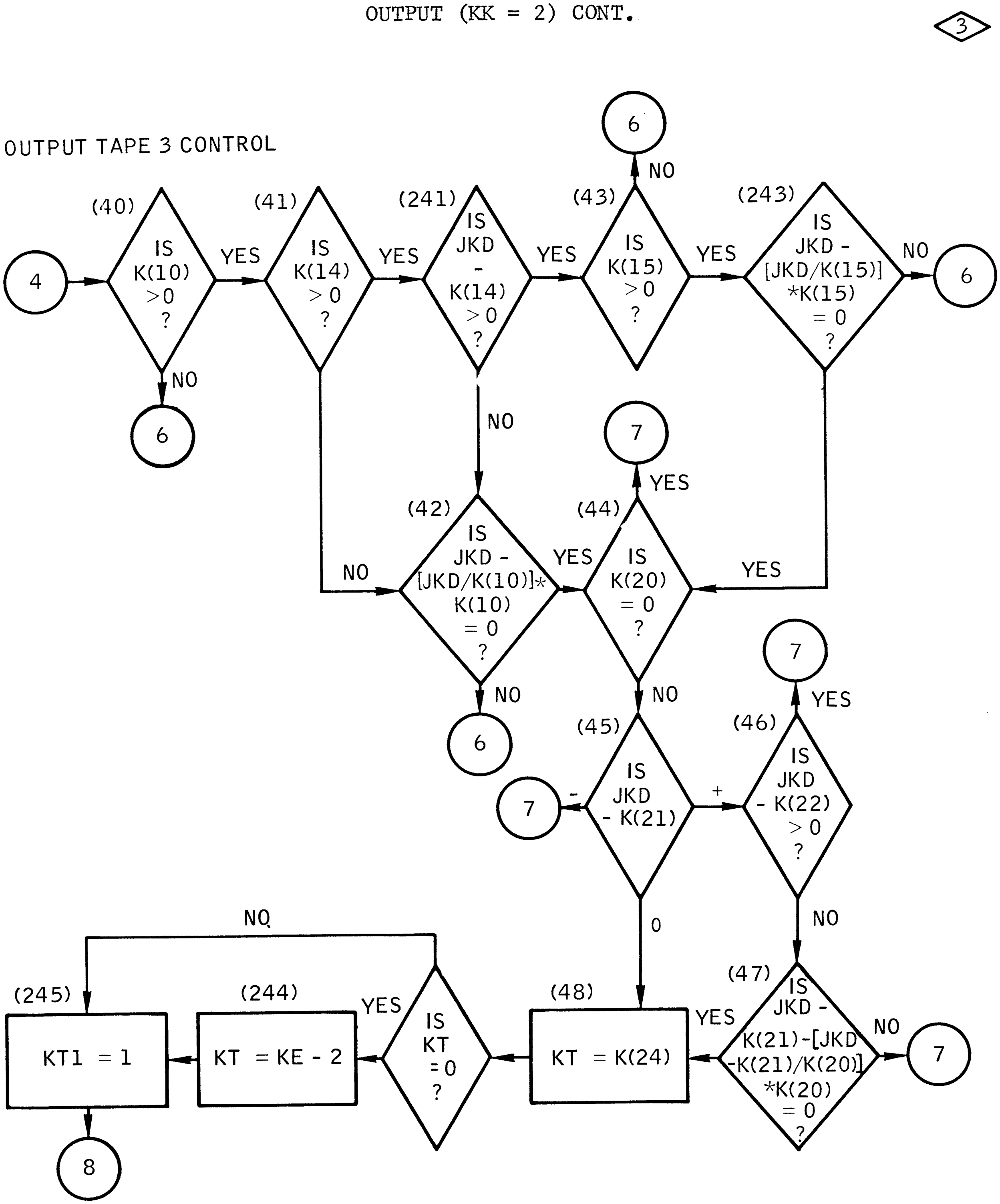


1
SATELLITE IS DOWN ENTRIES
(1)
IS HTPER - C(400) > 0 ?
NO
(2)
DMJ = DAY + 6203
IS K(10) > 0 ?
YES
(3)
OUTPUT TAPE 13 J, DMJ, A, HTPER NUR
NO
(4)
IS K(11) > 0 ?
YES
(5)
OUTPUT J, DMJ, A HTPER, NUR
NO
(9)
RETURN
YES
(11)
IS JKD >0 ?
YES
4
NO
3
END OF RUN ENTRY
NO
(10)
IS J - N < 0 ?
2
NO
(200)
IS K(10) >0 ?
YES
(201)
OUTPUT TAPE 13 J, NUR
(202)
IS K(11) >0 ?
YES
(203)
OUTPUT J, NUR
NO
(204)
RETURN
YES

OUTPUT (KK = 2) CONT.

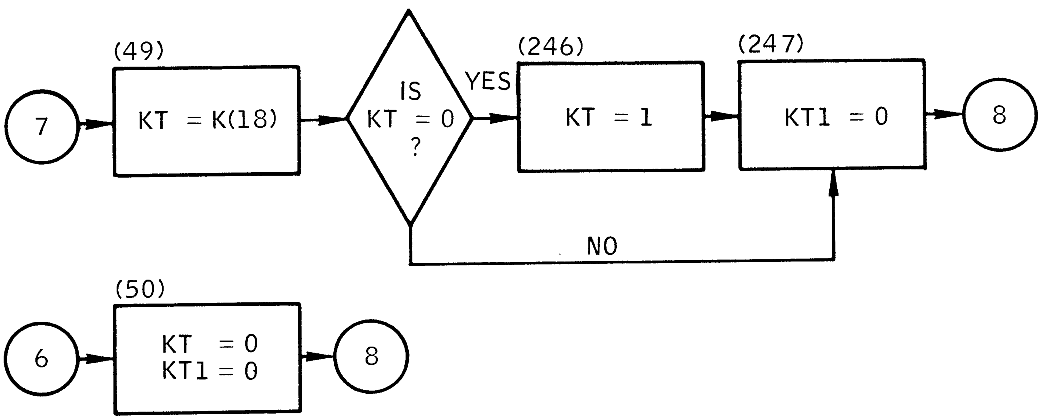


OUTPUT (KK = 2) CONT.

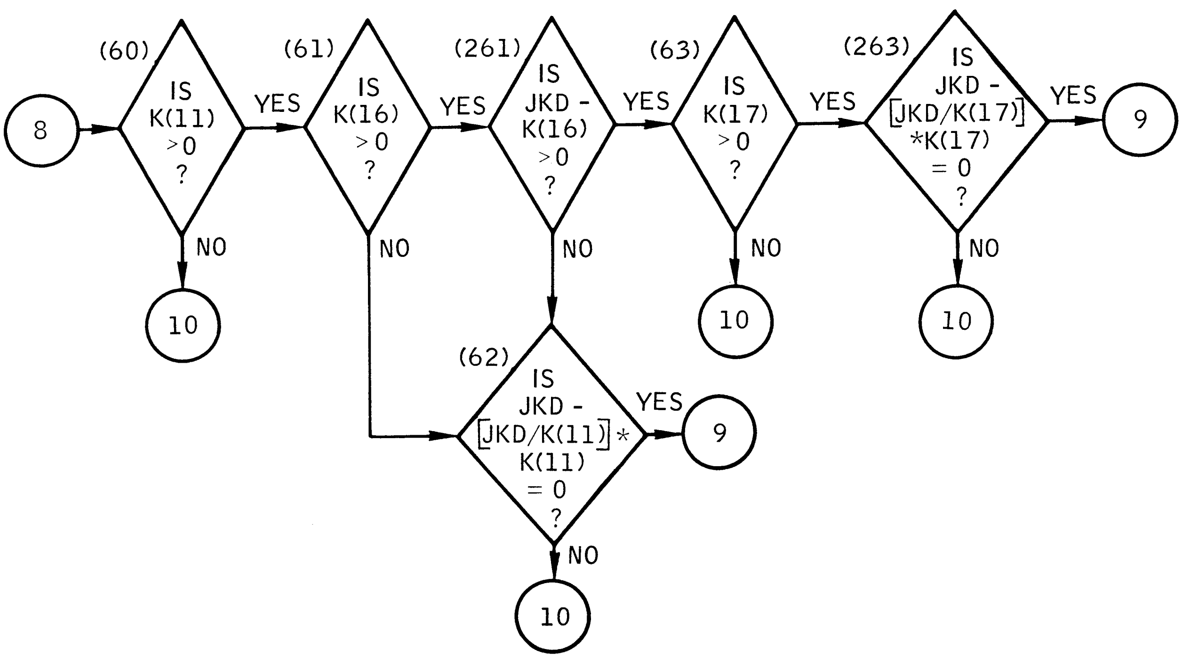


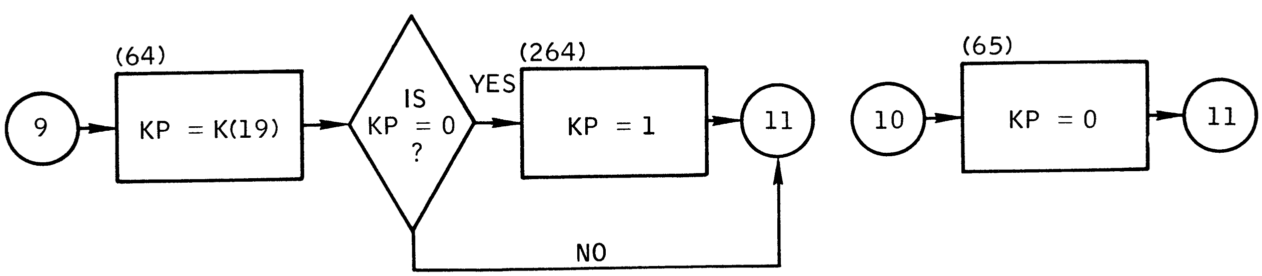
OUTPUT (KK = 2) CONT.

4

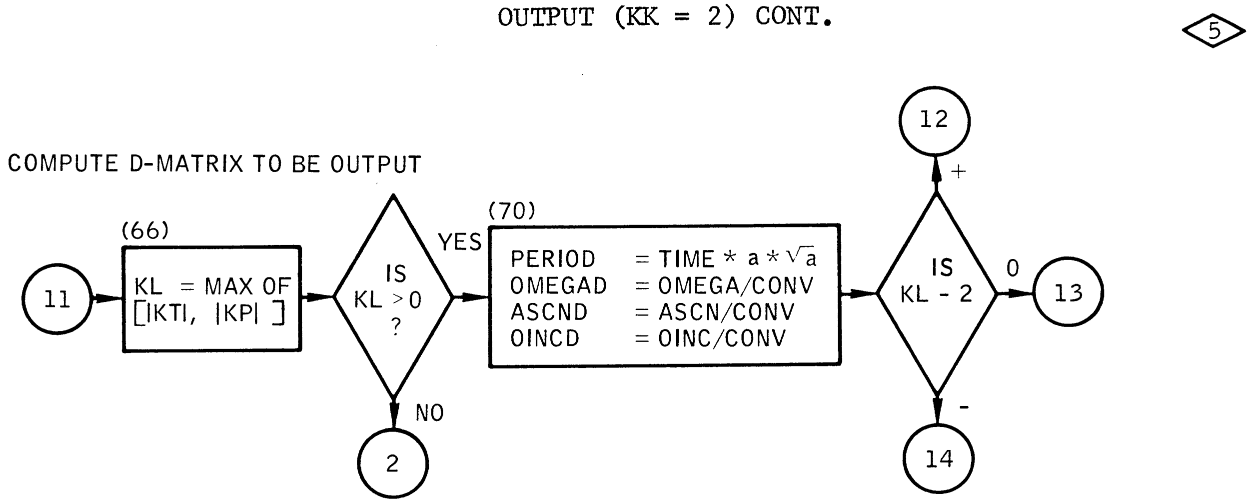


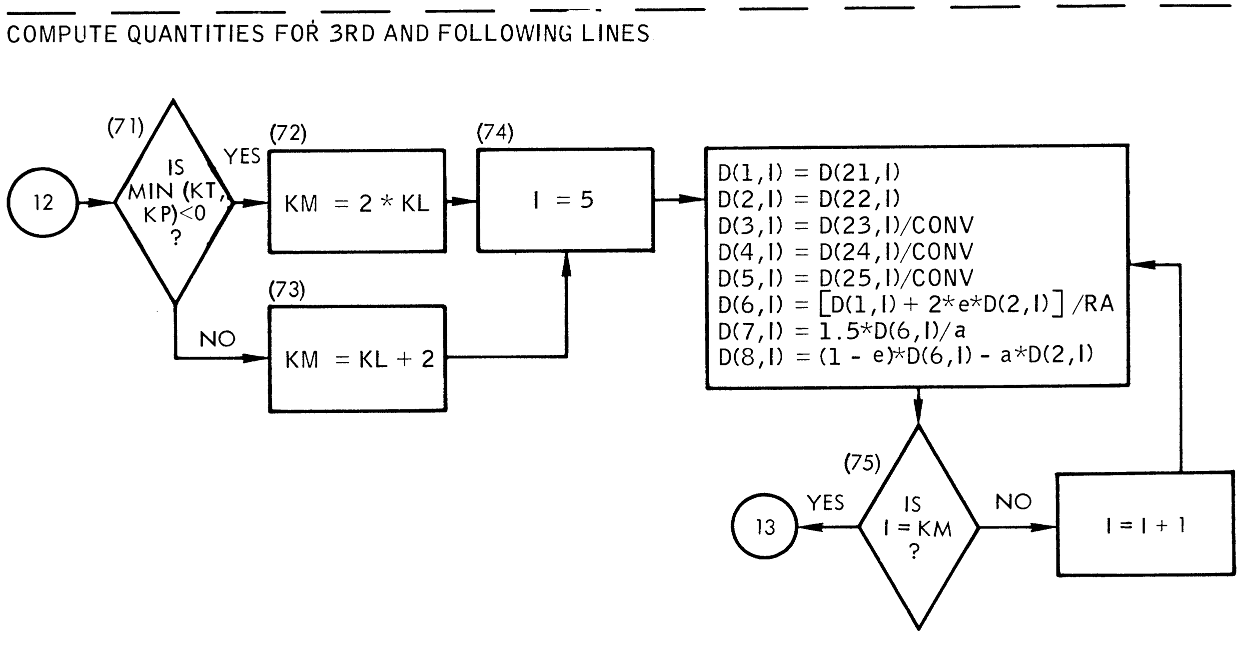
PRINT CONTROL



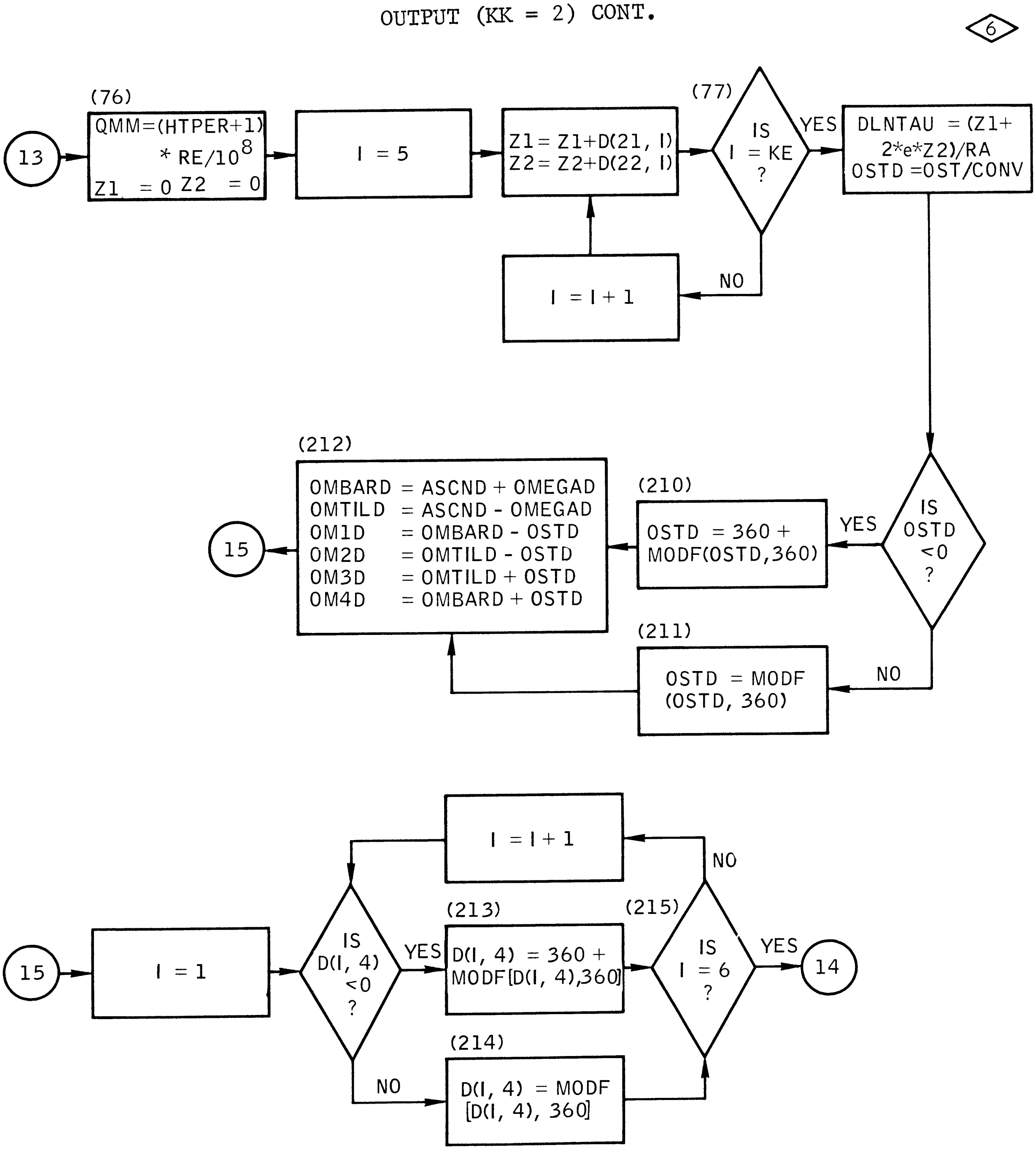


OUTPUT (KK = 2) CONT.

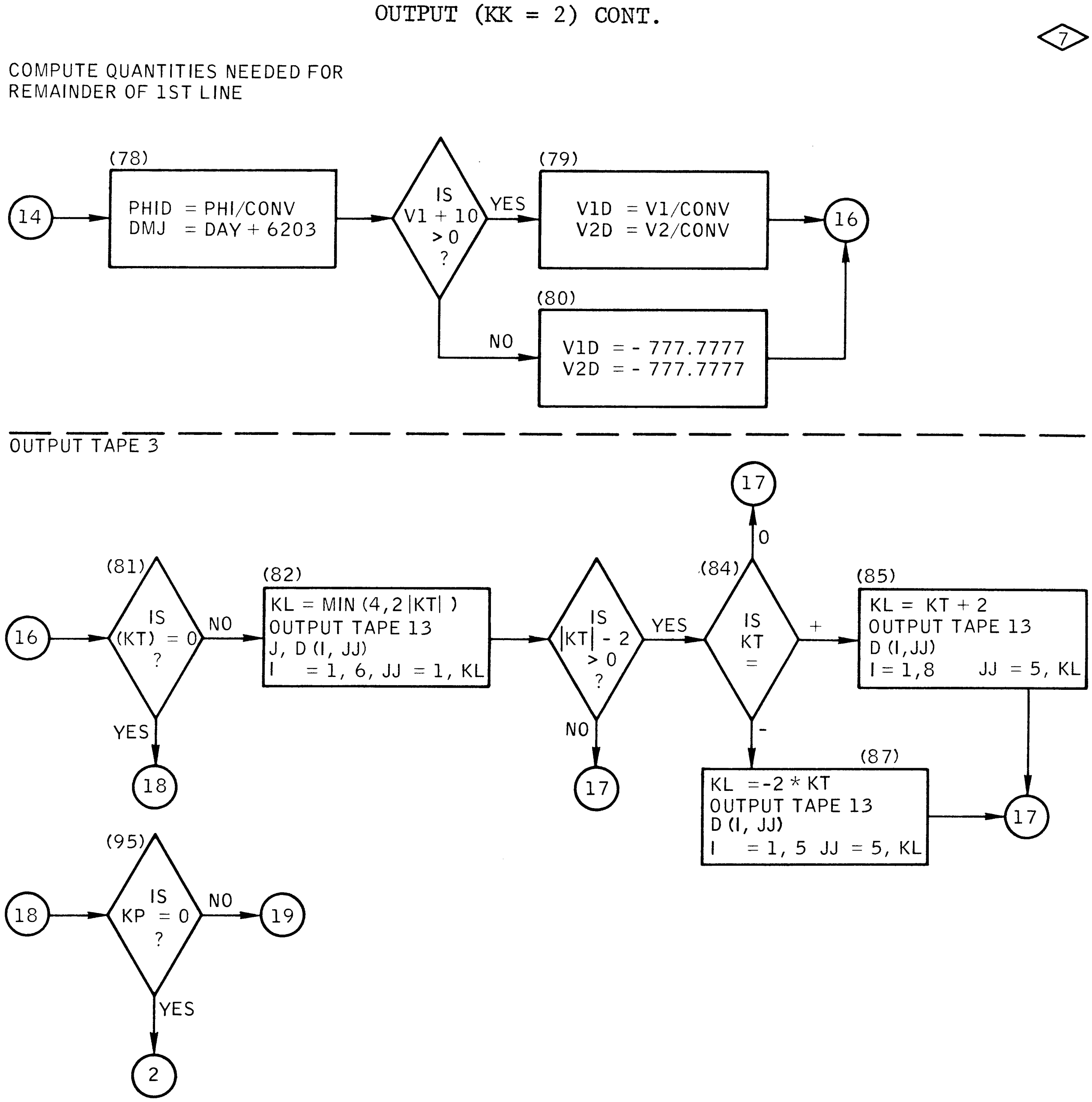




OUTPUT (KK = 2) CONT.

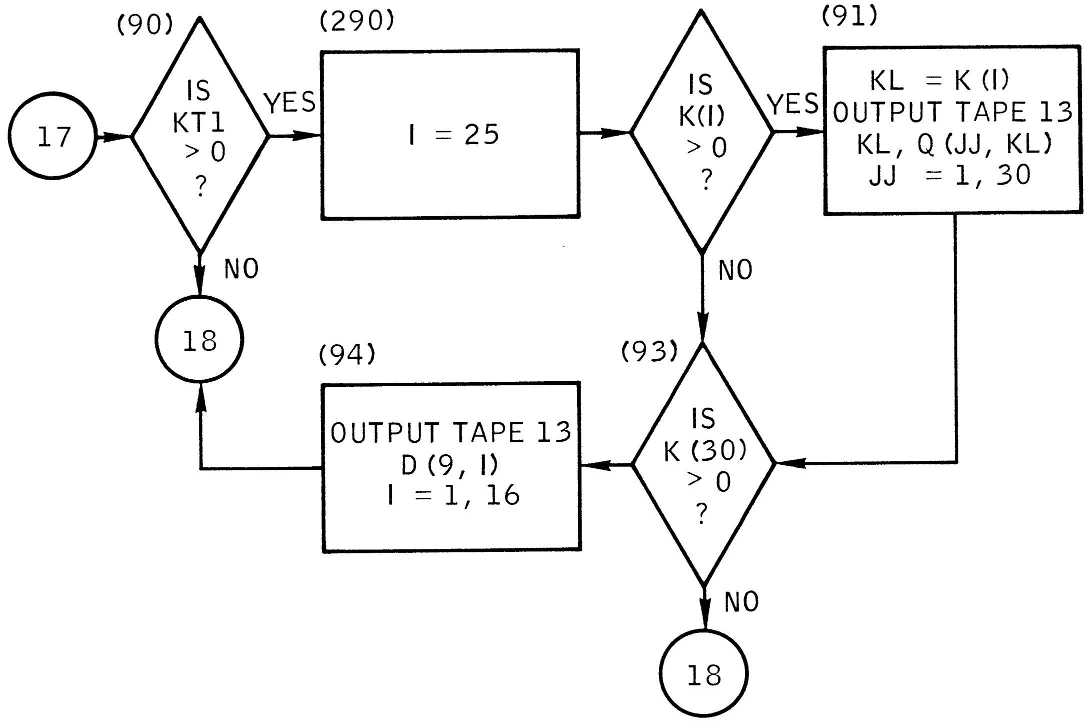


OUTPUT (KK = 2) CONT.

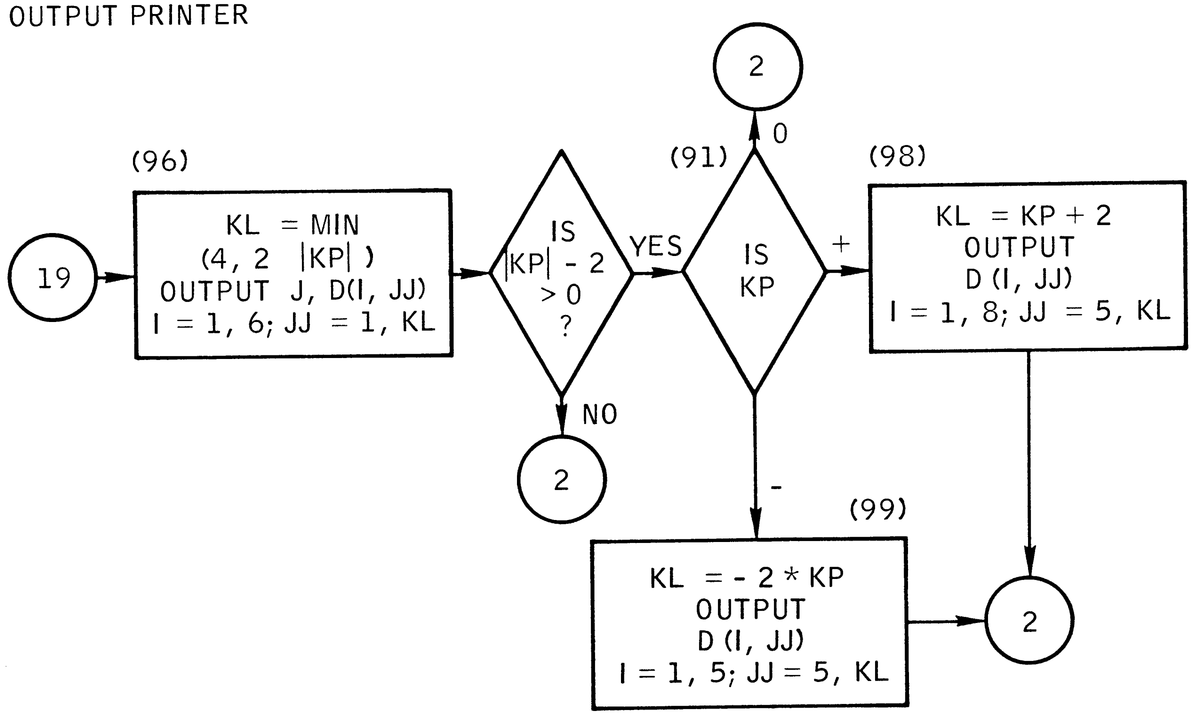


OUTPUT (KK = 2) CONT.

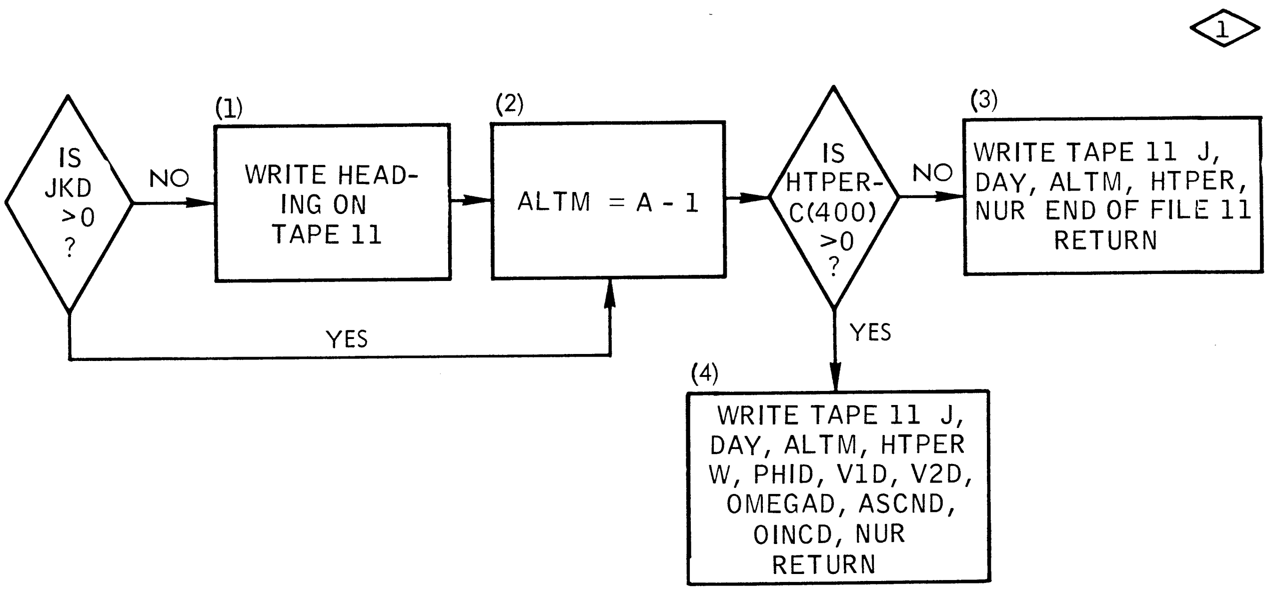
8



OUTPUT PRINTER

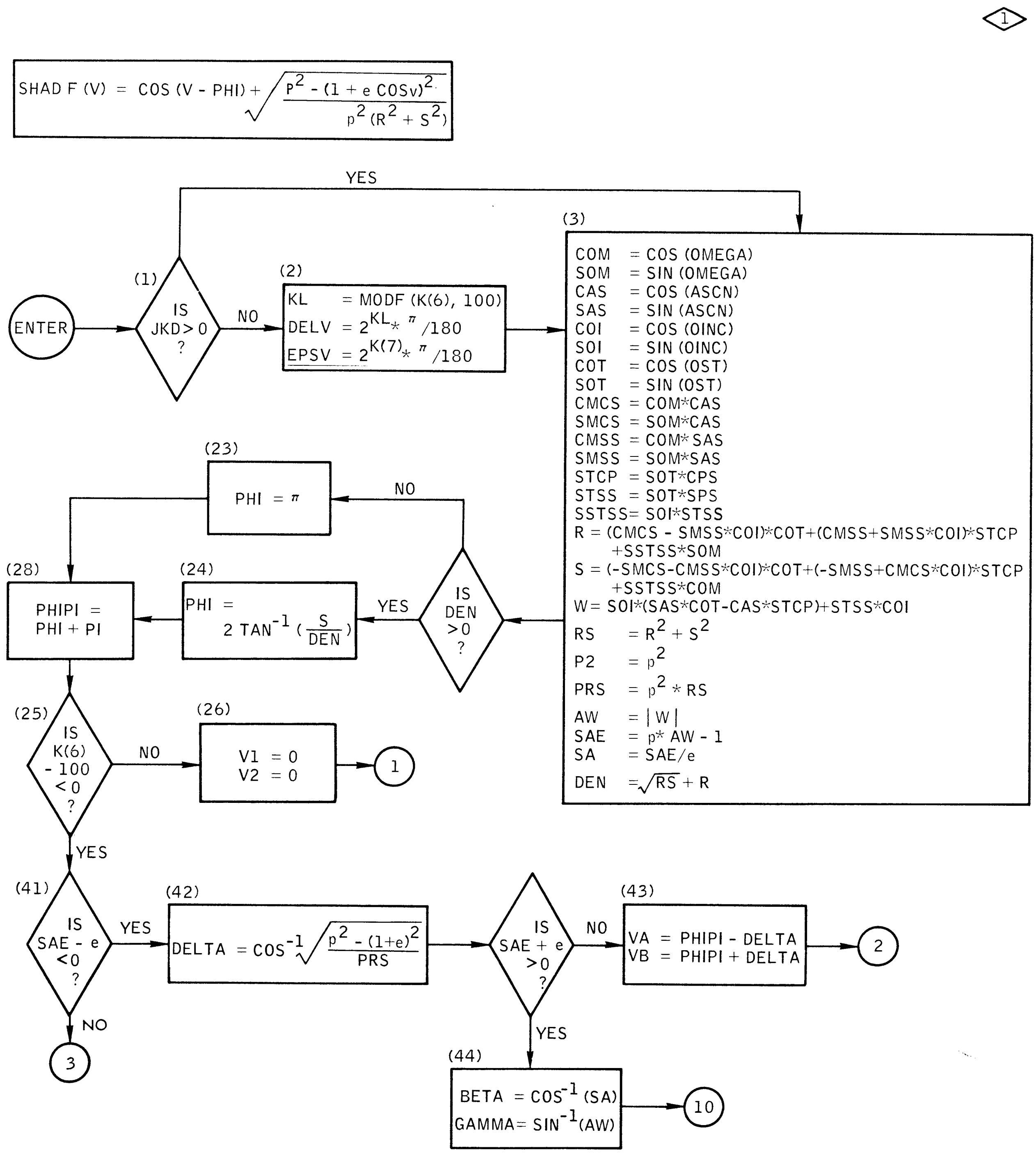


CARDS (KK = 3)

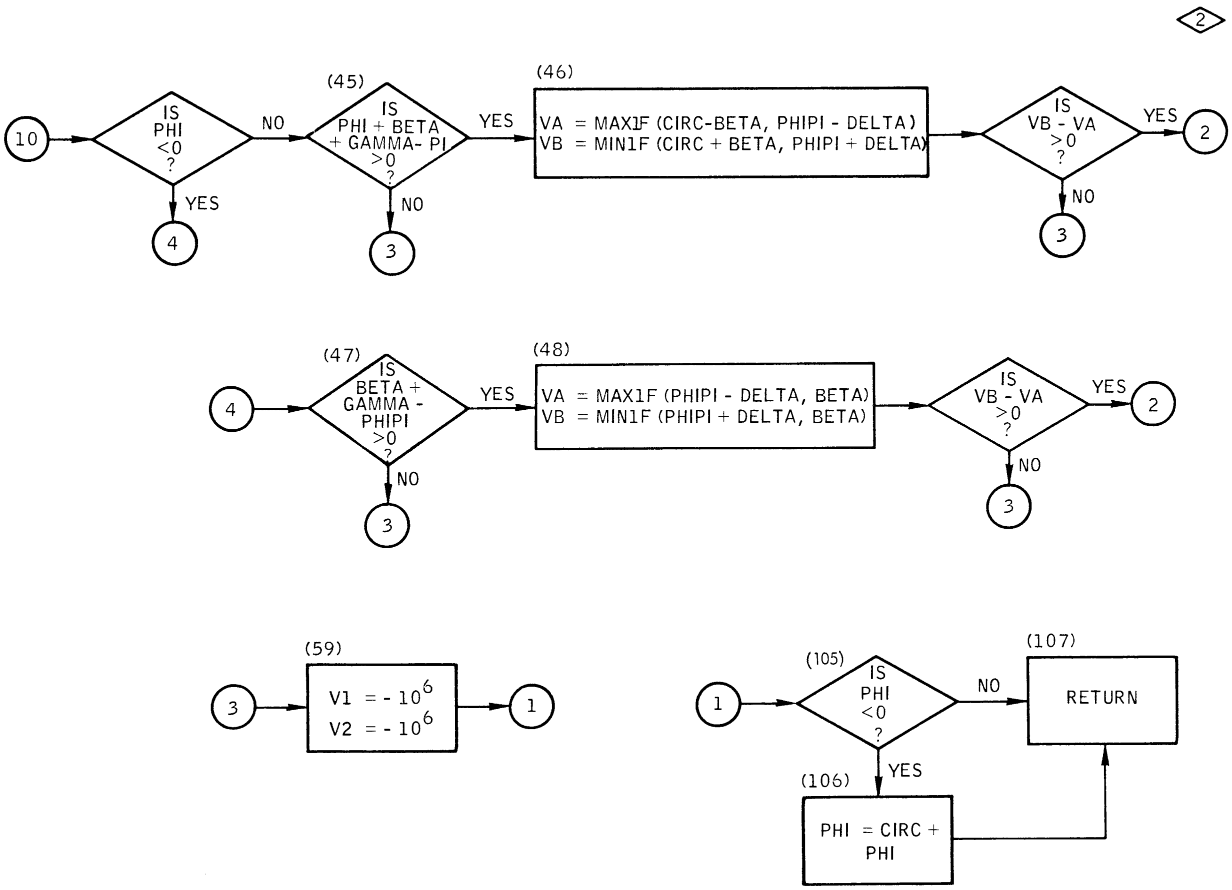


NOTE: NO REWIND OF TAPE 11 HERE IF END FILE MADE

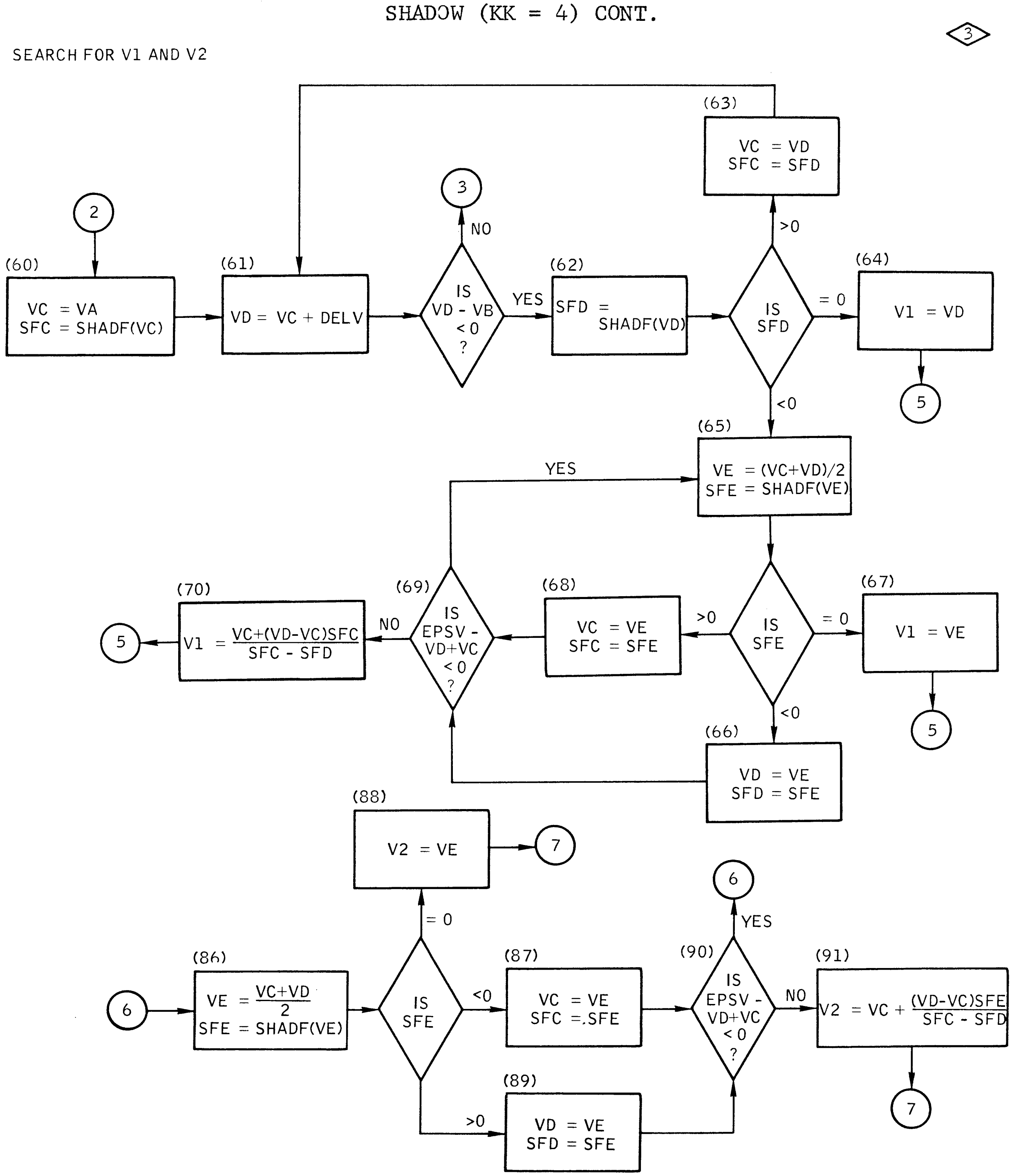
SHADOW (KK = 4)


1
SHAD F (V) = COS (V - PHI) + √((P² - (1 + e COSv)²) / (p² (R² + S²)))
ENTER
(1)
IS JKD > 0 ?
YES
NO
(2)
KL = MODF (K(6), 100)
DELV = 2^KL * π /180
EPSV = 2^K(7) * π /180
(3)
COM = COS (OMEGA)
SOM = SIN (OMEGA)
CAS = COS (ASCN)
SAS = SIN (ASCN)
COI = COS (OINC)
SOI = SIN (OINC)
COT = COS (OST)
SOT = SIN (OST)
CMCS = COM*CAS
SMCS = SOM*CAS
CMSS = COM*SAS
SMSS = SOM*SAS
STCP = SOT*CPS
STSS = SOT*SPS
SSTSS= SOI*STSS
R = (CMCS - SMSS*COI)*COT+(CMSS+SMSS*COI)*STCP +SSTSS*SOM
S = (-SMCS-CMSS*COI)*COT+(-SMSS+CMCS*COI)*STCP +SSTSS*COM
W= SOI*(SAS*COT-CAS*STCP)+STSS*COI
RS = R² + S²
P2 = p²
PRS = p² * RS
AW = |W|
SAE = p* AW - 1
SA = SAE/e
DEN = √RS + R
(23)
PHI = π
NO
IS DEN > 0 ?
YES
(24)
PHI = 2 TAN⁻¹ (S / DEN)
(28)
PHIPI = PHI + PI
(25)
IS K(6) - 100 < 0 ?
NO
(26)
V1 = 0
V2 = 0
1
YES
(41)
IS SAE - e < 0 ?
YES
NO
3
(42)
DELTA = COS⁻¹ √((p² - (1+e)²) / PRS)
IS SAE + e > 0 ?
NO
(43)
VA = PHIPI - DELTA
VB = PHIPI + DELTA
2
YES
(44)
BETA = COS⁻¹ (SA)
GAMMA= SIN⁻¹(AW)
10

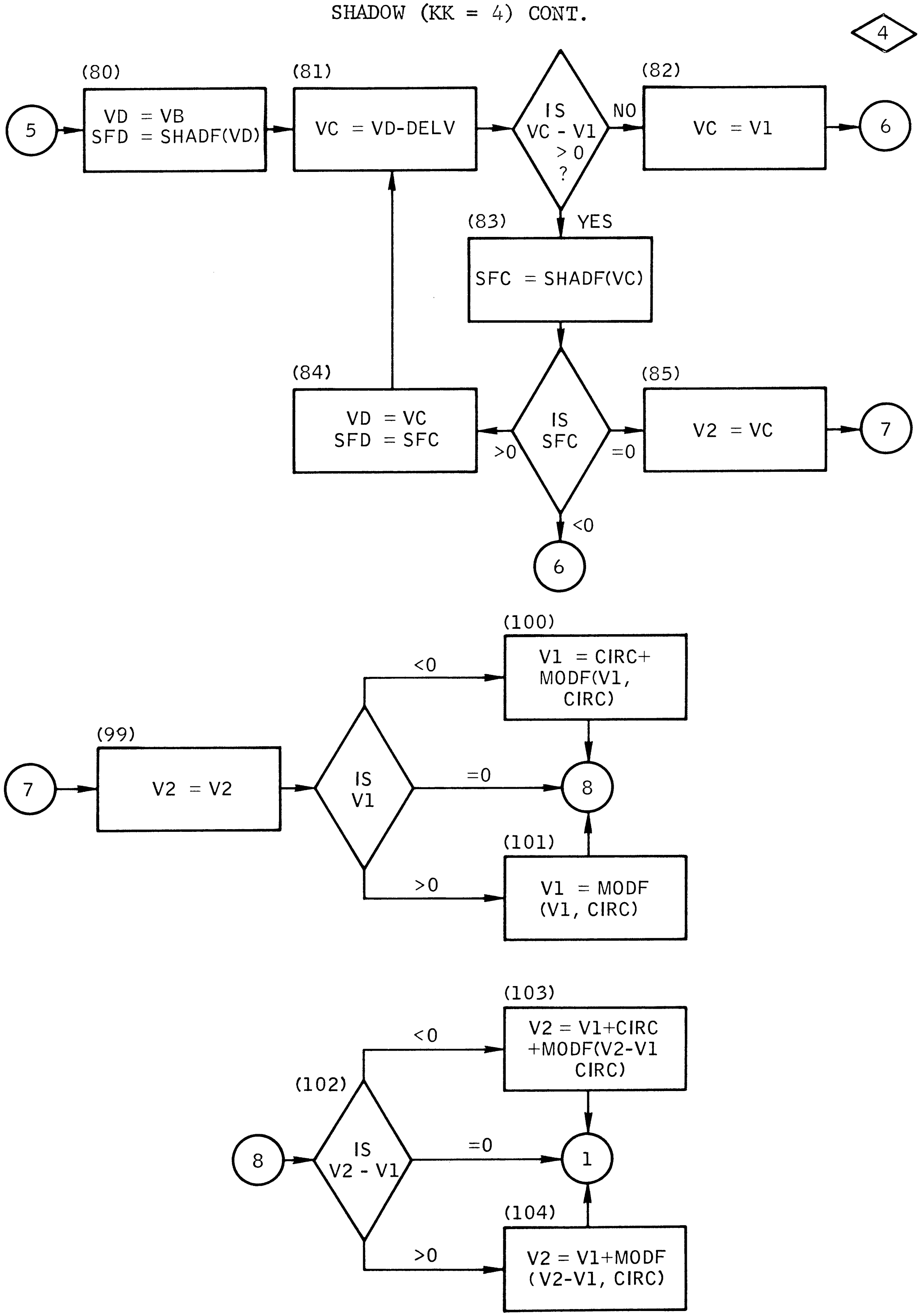
SHADOW (KK = 4) CONT.



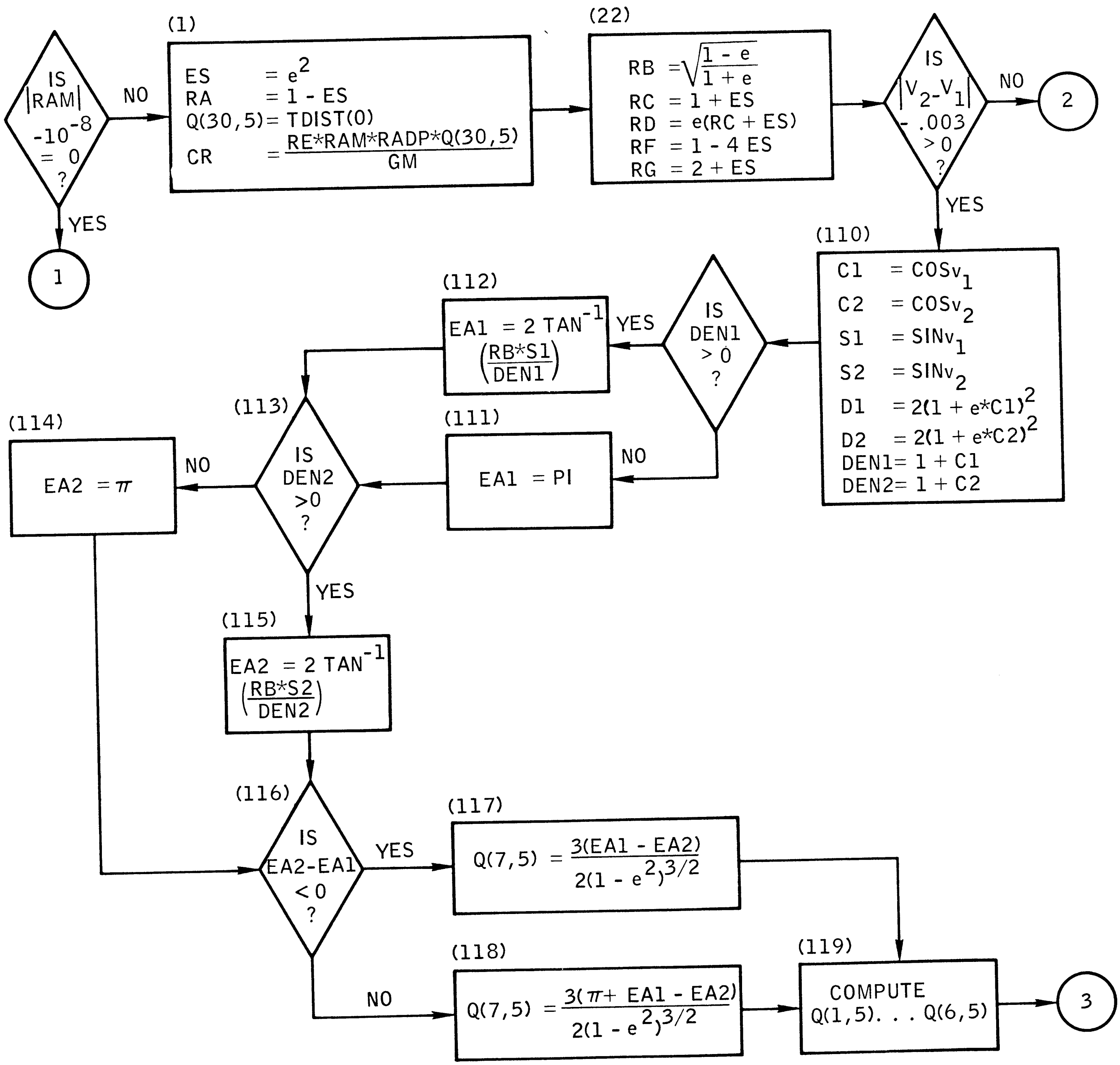
SHADOW (KK = 4) CONT.



SHADOW (KK = 4) CONT.



RADPR (KK = 5)


IS |RAM| -10^-8 = 0 ?
NO
YES
1
(1)
ES = e^2
RA = 1 - ES
Q(30,5) = TDIST(0)
CR = RE*RAM*RADP*Q(30,5) / GM
(22)
RB = √((1 - e)/(1 + e))
RC = 1 + ES
RD = e(RC + ES)
RF = 1 - 4 ES
RG = 2 + ES
IS |V2-V1| - .003 > 0 ?
NO
2
YES
(110)
C1 = COSv1
C2 = COSv2
S1 = SINv1
S2 = SINv2
D1 = 2(1 + e*C1)^2
D2 = 2(1 + e*C2)^2
DEN1 = 1 + C1
DEN2 = 1 + C2
IS DEN1 > 0 ?
YES
NO
(112)
EA1 = 2 TAN^-1 (RB*S1 / DEN1)
(111)
EA1 = PI
(113)
IS DEN2 > 0 ?
NO
YES
(114)
EA2 = π
(115)
EA2 = 2 TAN^-1 (RB*S2 / DEN2)
(116)
IS EA2-EA1 < 0 ?
YES
NO
(117)
Q(7,5) = 3(EA1 - EA2) / 2(1 - e^2)^3/2
(118)
Q(7,5) = 3(π + EA1 - EA2) / 2(1 - e^2)^3/2
(119)
COMPUTE Q(1,5). . . Q(6,5)
3

RADPR (KK = 5) CONT.

2

(150)

$$Q(5,5) = \frac{3\pi}{(1-e^2)^{3/2}} = Q(4,5)$$

$$Q(2,5) = \frac{-3\pi e}{(1-e^2)^{5/2}}$$

$$Q(1,5) = Q(3,5) = Q(6,5) = 0$$

2 → (150) → 3

(161)

Q(8,5) = COS(OMEGA)*Q(1,5)+SIN(OMEGA)*Q(2,5)
Q(9,5) = COS(OMEGA)*Q(2,5)-SIN(OMEGA)*Q(1,5)
PCR = p^2*CR
D(21,5) = 2p*PCR*(R*Q(1,5)-S*Q(2,5))
D(22,5) = PCR*(R*Q(3,5)-S*Q(4,5))
D(23,5) = $\frac{\text{PCR*(R*Q(5,5)-S*Q(6,5))}}{e}$+ $\frac{\text{W*COI}}{\text{SOI}}$ *Q(8,5)
D(24,5) = - PCR*W*Q(8,5)/SOI
D(25,5) = - PCR*W*Q(9,5)

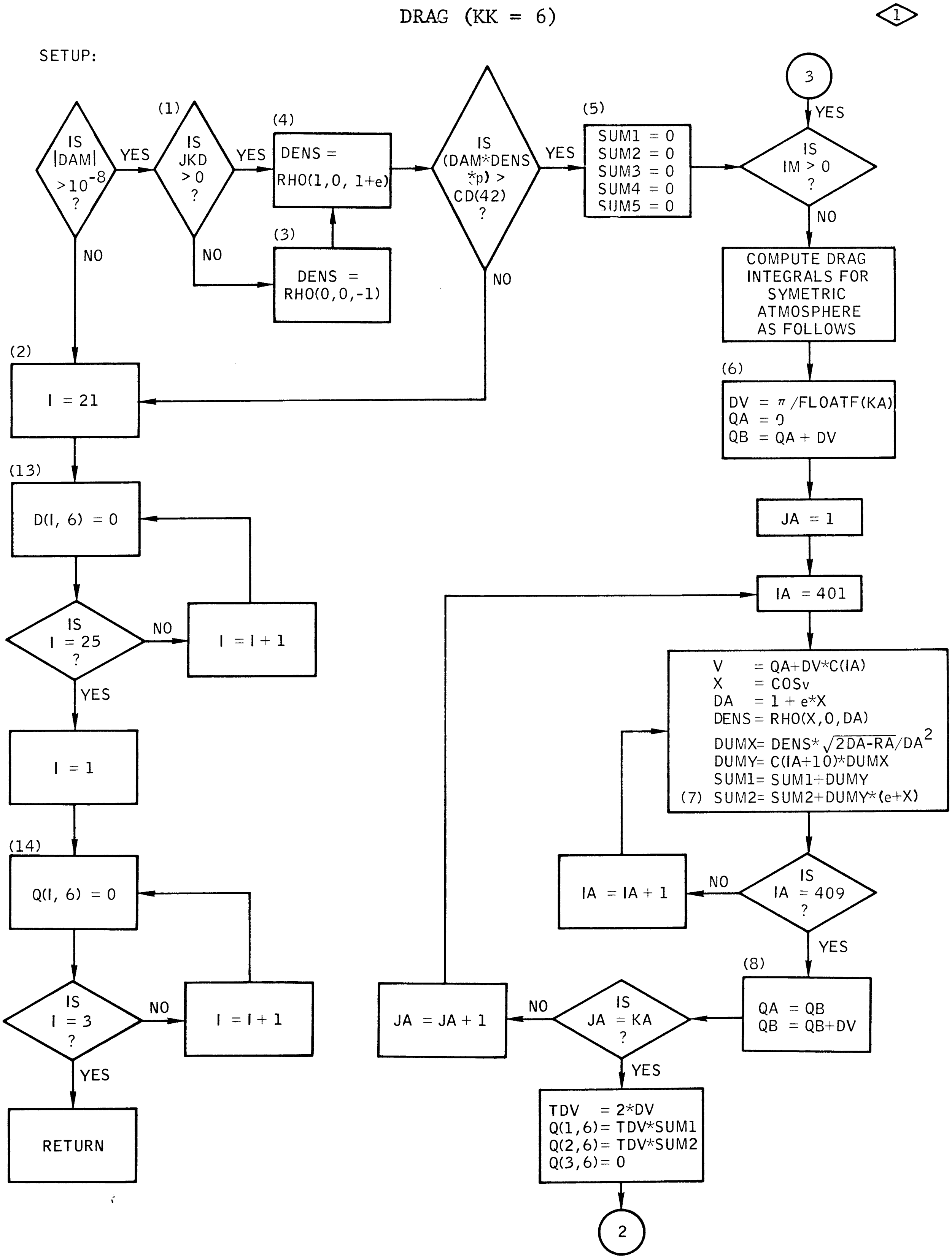
3 → (161) → RETURN

1 → (170) IA = 21 → (171) D(IA,5) = 0 → IS IA = 25 ?

YES → RETURN

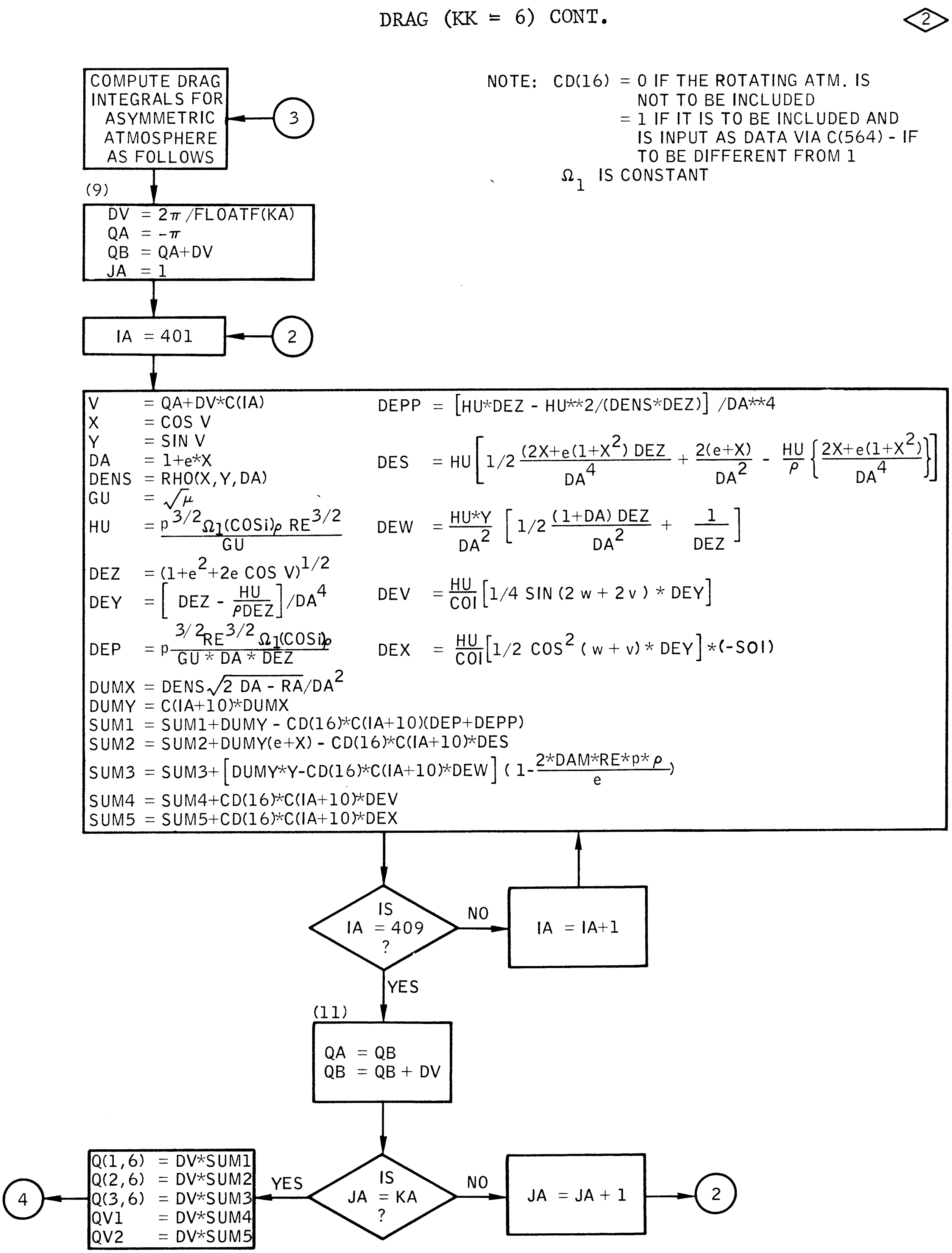
NO → IA = IA + 1 → (171)

DRAG (KK = 6)

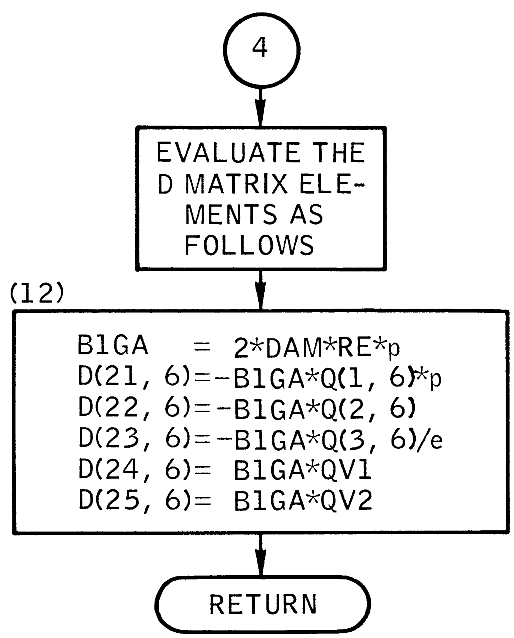

1
SETUP:
IS |DAM| >10^-8 ?
YES
NO
(1)
IS JKD > 0 ?
YES
NO
(4)
DENS = RHO(1,0, 1+e)
(3)
DENS = RHO(0,0,-1)
IS (DAM*DENS *p) > CD(42) ?
YES
NO
(5)
SUM1 = 0
SUM2 = 0
SUM3 = 0
SUM4 = 0
SUM5 = 0
3
YES
IS IM > 0 ?
NO
COMPUTE DRAG INTEGRALS FOR SYMETRIC ATMOSPHERE AS FOLLOWS
(6)
DV = π/FLOATF(KA)
QA = 0
QB = QA + DV
JA = 1
IA = 401
V = QA+DV*C(IA)
X = COSv
DA = 1 + e*X
DENS = RHO(X,0,DA)
DUMX = DENS*√(2DA-RA)/DA^2
DUMY = C(IA+10)*DUMX
SUM1 = SUM1+DUMY
(7) SUM2 = SUM2+DUMY*(e+X)
IA = IA + 1
NO
IS IA = 409 ?
YES
(8)
QA = QB
QB = QB+DV
IS JA = KA ?
NO
JA = JA + 1
YES
TDV = 2*DV
Q(1,6) = TDV*SUM1
Q(2,6) = TDV*SUM2
Q(3,6) = 0
2
(2)
I = 21
(13)
D(I, 6) = 0
IS I = 25 ?
NO
I = I + 1
YES
I = 1
(14)
Q(I, 6) = 0
IS I = 3 ?
NO
I = I + 1
YES
RETURN

DRAG (KK = 6) CONT.

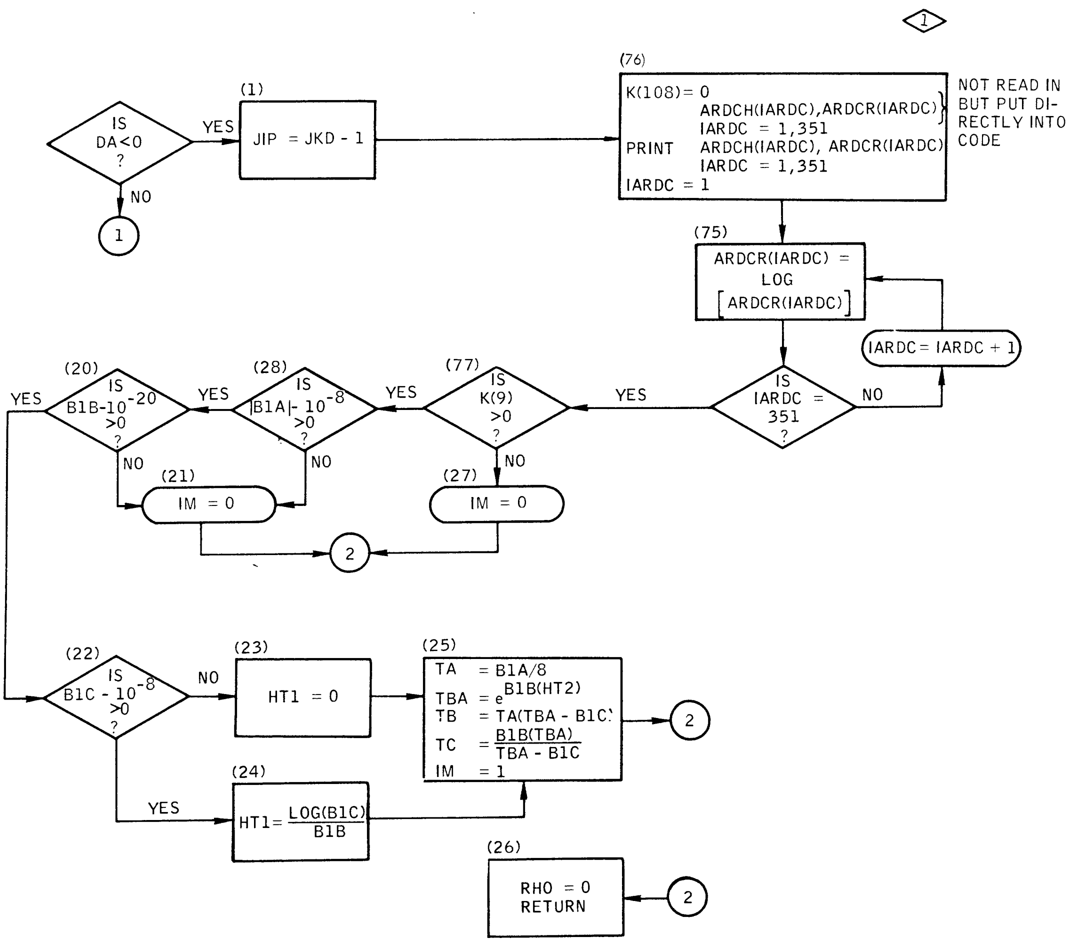
NOTE: CD(16) = 0 IF THE ROTATING ATM. IS NOT TO BE INCLUDED
= 1 IF IT IS TO BE INCLUDED AND IS INPUT AS DATA VIA C(564) - IF TO BE DIFFERENT FROM 1
Ω_1 IS CONSTANT



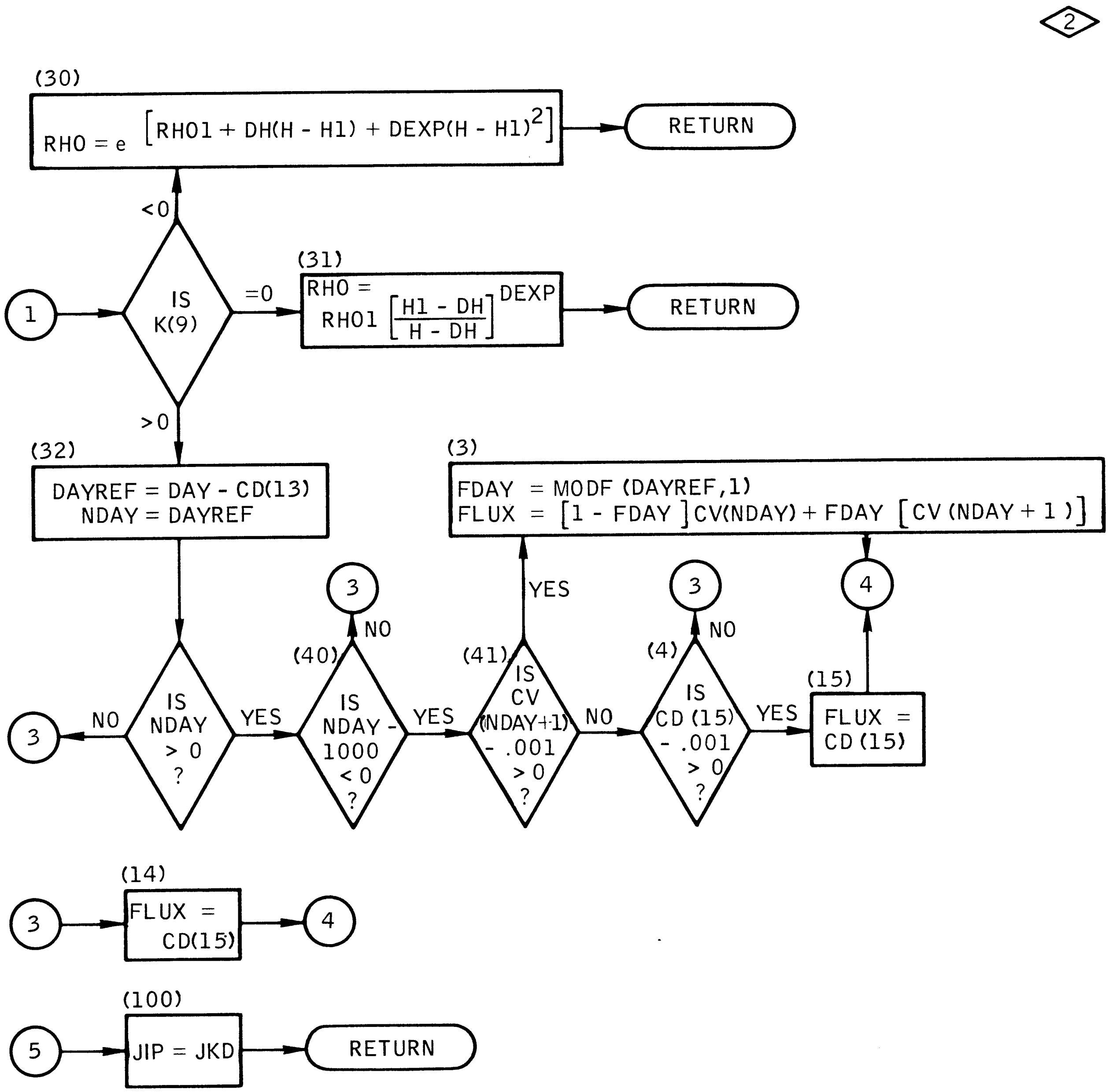
DRAG (KK = 6) CONT. 3



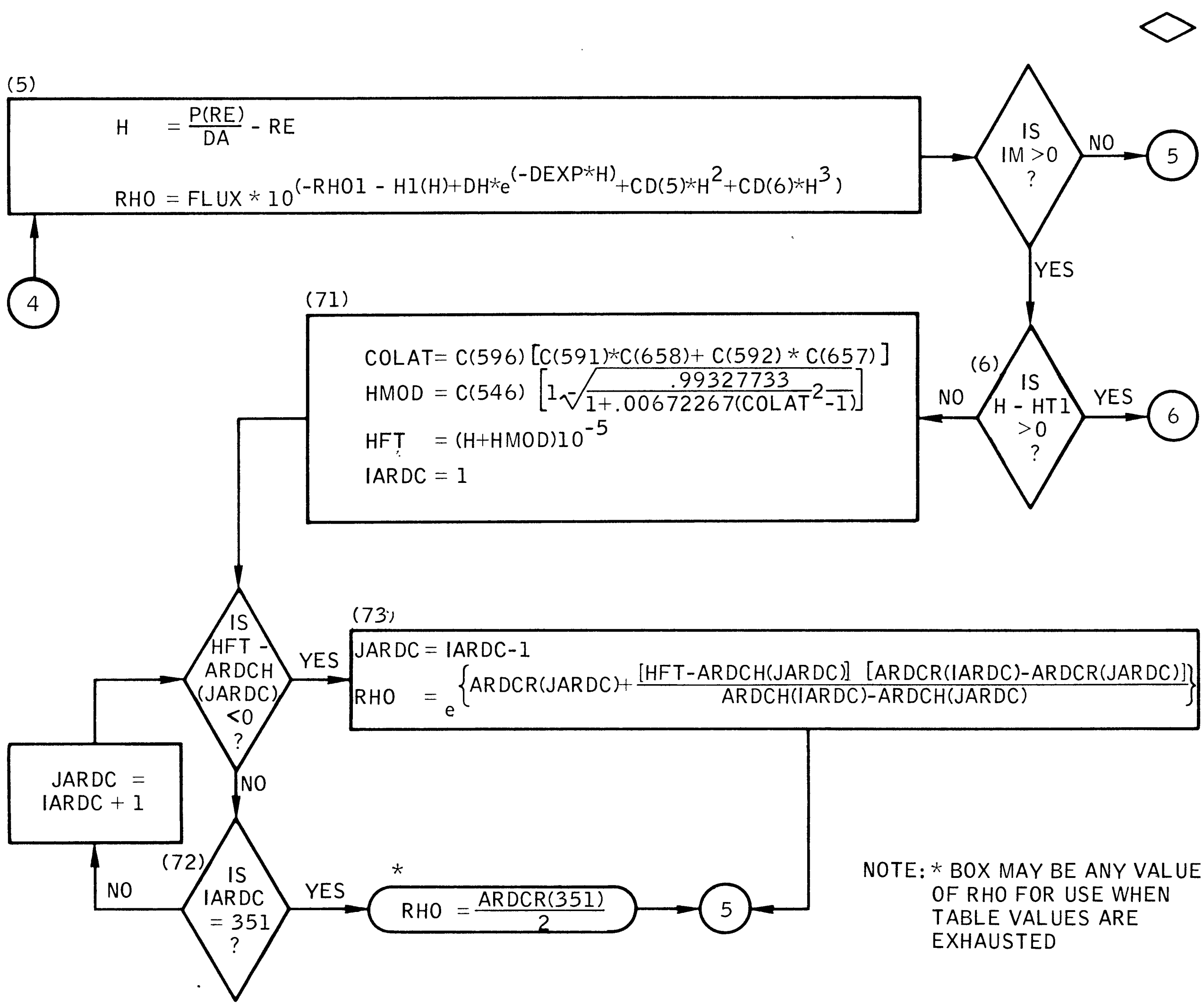
RHO


1
(76)
K(108) = 0
ARDCH(IARDC), ARDCR(IARDC)
IARDC = 1,351
PRINT ARDCH(IARDC), ARDCR(IARDC)
IARDC = 1,351
IARDC = 1
NOT READ IN BUT PUT DIRECTLY INTO CODE
(1)
IS DA<0 ?
YES
NO
JIP = JKD - 1
1
(75)
ARDCR(IARDC) = LOG [ARDCR(IARDC)]
IARDC = IARDC + 1
IS IARDC = 351 ?
NO
YES
(77)
IS K(9) >0 ?
(28)
IS |B1A| - 10^-8 >0 ?
(20)
IS B1B-10^-20 >0 ?
(21)
IM = 0
(27)
IM = 0
2
(22)
IS B1C - 10^-8 >0 ?
(23)
HT1 = 0
(24)
HT1 = LOG(B1C) / B1B
(25)
TA = B1A/8
TBA = e^B1B(HT2)
TB = TA(TBA - B1C)
TC = B1B(TBA) / TBA - B1C
IM = 1
(26)
RHO = 0
RETURN

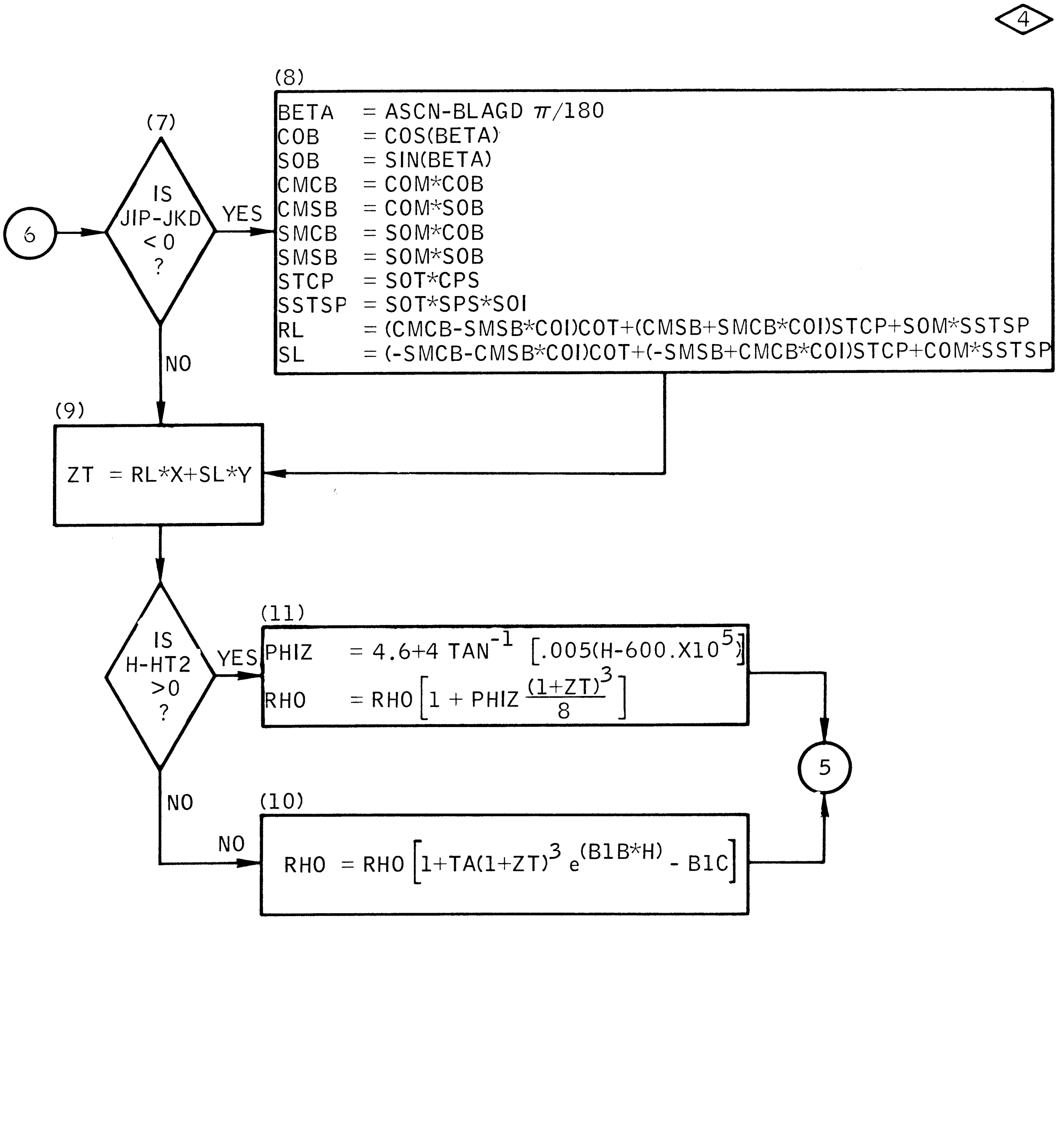
RHO CONT.


2
(30)
RHO = e [RHO1 + DH(H - H1) + DEXP(H - H1)^2]
RETURN
<0
1
IS K(9)
=0
(31)
RHO = RHO1 [H1 - DH / H - DH] DEXP
RETURN
>0
(32)
DAYREF = DAY - CD(13)
NDAY = DAYREF
(3)
FDAY = MODF (DAYREF,1)
FLUX = [1 - FDAY] CV(NDAY) + FDAY [CV (NDAY + 1)]
3
NO
IS NDAY > 0 ?
YES
(40)
IS NDAY - 1000 < 0 ?
3
NO
YES
(41)
IS CV (NDAY+1) - .001 > 0 ?
YES
NO
(4)
IS CD (15) - .001 > 0 ?
3
NO
YES
(15)
FLUX = CD (15)
4
(14)
3
FLUX = CD(15)
4
(100)
5
JIP = JKD
RETURN

RHO CONT.


(5)
H = P(RE)/DA - RE
RHO = FLUX * 10^(-RHO1 - H1(H)+DH*e^(-DEXP*H)+CD(5)*H^2+CD(6)*H^3)
4
IS IM >0 ?
NO
5
YES
(6)
IS H - HT1 >0 ?
YES
6
NO
(71)
COLAT= C(596) [C(591)*C(658)+ C(592) * C(657)]
HMOD = C(546) [1-√(.99327733/(1+.00672267(COLAT^2-1)))]
HFT = (H+HMOD)10^-5
IARDC = 1
IS HFT - ARDCH (JARDC) <0 ?
YES
NO
(73)
JARDC = IARDC-1
RHO = e{ARDCR(JARDC)+ [HFT-ARDCH(JARDC)] [ARDCR(IARDC)-ARDCR(JARDC)] / ARDCH(IARDC)-ARDCH(JARDC)}
JARDC = IARDC + 1
(72)
IS IARDC = 351 ?
NO
YES
*
RHO = ARDCR(351)/2
5
NOTE: * BOX MAY BE ANY VALUE OF RHO FOR USE WHEN TABLE VALUES ARE EXHAUSTED

RHO CONT.


4
(8)
BETA = ASCN-BLAGD π/180
COB = COS(BETA)
SOB = SIN(BETA)
CMCB = COM*COB
CMSB = COM*SOB
SMCB = SOM*COB
SMSB = SOM*SOB
STCP = SOT*CPS
SSTSP = SOT*SPS*SOI
RL = (CMCB-SMSB*COI)COT+(CMSB+SMCB*COI)STCP+SOM*SSTSP
SL = (-SMCB-CMSB*COI)COT+(-SMSB+CMCB*COI)STCP+COM*SSTSP
(7)
IS JIP-JKD < 0 ?
6
YES
NO
(9)
ZT = RL*X+SL*Y
IS H-HT2 >0 ?
YES
(11)
PHIZ = 4.6+4 TAN-1 [.005(H-600.X10^5)]
RHO = RHO [1 + PHIZ (1+ZT)^3 / 8]
5
NO
NO
(10)
RHO = RHO [1+TA(1+ZT)^3 e^(B1B*H) - B1C]

EARTH (KK = 7)

1

$Q(1,7) = p^2 \qquad Q(2,7) = \frac{3\pi}{p^3} \qquad Q(3,7) = SOI^2$

$Q(4,7) = Q(3,7)^2 \qquad Q(5,7) = SOM^2 \qquad Q(6,7) = Q(5,7)^2$

$Q(7,7) = SOM*COM \qquad Q(8,7) = SOI*COM \qquad Q(9,7) = ES^2$

$Q(10,7) = COI/SOI \qquad Q(11,7) = 4-5*Q(3,7)$

$Q(12,7) = 6-7*Q(3,7) \qquad Q(13,7) = 1-3.5*Q(3,7)+2.625*Q(4,7)$

$Q(14,7) = 1.-2*Q(5,7) \qquad Q(15,7) = Q(10,7)/2p$

$$Q(16,7) = \frac{peQ(2,7)*Q(8,7)*Q(11,7)*C(46)}{2}$$

$$Q(17,7) = \frac{-Q(2,7)*C(47)*ES*Q(7,7)*Q(3,7)*Q(12,7)*2}{7}$$

$$Q(18,7) = \frac{5*Q(2,7)*e*Q(8,7)}{p}\left[\left(1-3.5*Q(3,7)+2.625*Q(4,7)\right)+ES\left\{.75+1.75*Q(3,7)*\left(-1-2*Q(5,7)\right) + \frac{63}{64}*Q(4,7)*\left(1+4*Q(5,7)\right)\right\}\right]*C(48)$$

$D(21,7) = Q(16,7) + Q(17,7) + Q(18,7)$

$Q(19,7) = .25*Q(2,7)*C(46)*Q(8,7)*RA*Q(11,7)$

$$Q(20,7) = \frac{Q(2,7)}{7p}*C(47)*e*RA*Q(3,7)*Q(7,7)*Q(12,7)$$

$$Q(21,7) = \frac{Q(2,7)}{3*Q(1,7)}*C(48)*Q(8,7)*\left[-7.5*Q(13,7)+ES*\left\{1.875*\left((1-7*Q(14,7)*Q(3,7)\right) + \frac{315}{32}*\left(1.25-3*Q(5,7)\right)*Q(4,7)\right\} + Q(9,7)*\left\{1.875*\left[3-7\left(1+2*Q(5,7)\right)*Q(3,7)\right] + \frac{945}{128}\left(4*Q(5,7)+1\right)*Q(4,7)\right\}\right]$$

$D(22,7) = Q(19,7)+Q(20,7)+Q(21,7)$

$$Q(22,7) = \frac{-2\pi*GJAY*COI}{Q(1,7)}$$

$$Q(23,7) = \frac{Q(2,7)}{4}*C(46)*e*SOM*Q(10,7)*\left(15*Q(3,7)-4\right)$$

$$Q(24,7) = \frac{-Q(2,7)*C(47)}{14p}*COI*\left[2*\left\{4-7*Q(3,7)\right\}+ES*\left\{Q(12,7)+4*Q(5,7)*\left[3-7*Q(3,7)\right]\right\}\right]$$

1

EARTH (KK = 7) CONT.

2

1

$$
\begin{aligned}
Q(25,7) &= \frac{2.5*Q(2,7)*C(48)*e}{Q(1,7)}*Q(10,7)*SOM*\Big[\Big\{1-10.5*Q(3,7)+\frac{105}{8}*Q(4,7)\Big\}+ \\
&\quad ES*\Big\{.75-1.75*Q(3,7)*\big[3+2*Q(5,7)\big]+\frac{105}{64}*Q(4,7)*\big[3+4*Q(5,7)\big]\Big\}\Big]
\end{aligned}
$$

$$D(24,7) = Q(22,7)+Q(23,7)+Q(24,7)+Q(25,7)$$

$$Q(26,7) = \frac{\pi}{Q(1,7)}*GJAY*Q(11,7)$$

$$
\begin{aligned}
Q(27,7) &= \frac{-Q(2,7)*C(46)}{e}*SOM*SOI\Big[1-1.25*Q(3,7)+ES\big\{4-5*Q(3,7)\big\}\Big] \\
&\quad -COI*Q(23,7)
\end{aligned}
$$

$$
\begin{aligned}
Q(28,7) &= \frac{C(47)*\pi}{Q(1,7)^2}*\Big[\frac{12}{7}-\frac{3}{7}*Q(3,7)*\big\{17+6*Q(5,7)\big\}+3*Q(4,7)*\big\{2+Q(5,7)\big\} \\
&\quad +ES*\Big\{\frac{9}{7}-\frac{45}{7}\big[.5+Q(5,7)\big]*Q(3,7)+\frac{15}{8}*Q(4,7)*\big[1+4*Q(5,7)\big]\Big\}\Big] \\
&\quad -COI*Q(24,7)
\end{aligned}
$$

$$
\begin{aligned}
Q(29,7) &= \frac{Q(2,7)}{4Q(1,7)*e}*5*C(48)*SOI*SOM*\Big[2-7*Q(3,7)+\frac{21}{4}*Q(4,7)+ES*\Big\{\frac{41}{2} \\
&\quad -7*Q(3,7)*\Big[\frac{19}{2}+Q(5,7)\Big]+\frac{63}{8}*Q(4,7)*\Big[\frac{73}{12}+Q(5,7)\Big]\Big]\Big\} \\
&\quad +Q(9,7)*\Big[9-7*Q(3,7)*\big\{3+2*Q(5,7)\big\}+\frac{63}{16}*Q(4,7)*\big\{3+4*Q(5,7)\big\}\Big] \\
&\quad -COI*Q(25,7)
\end{aligned}
$$

$$D(23,7) = Q(26,7)+Q(27,7)+Q(28,7)+Q(29,7)$$

$$Q(30,7) = \frac{Q(10,7)}{2p}$$

$$Q(1,7) = Q(30,7)*Q(16,7)$$

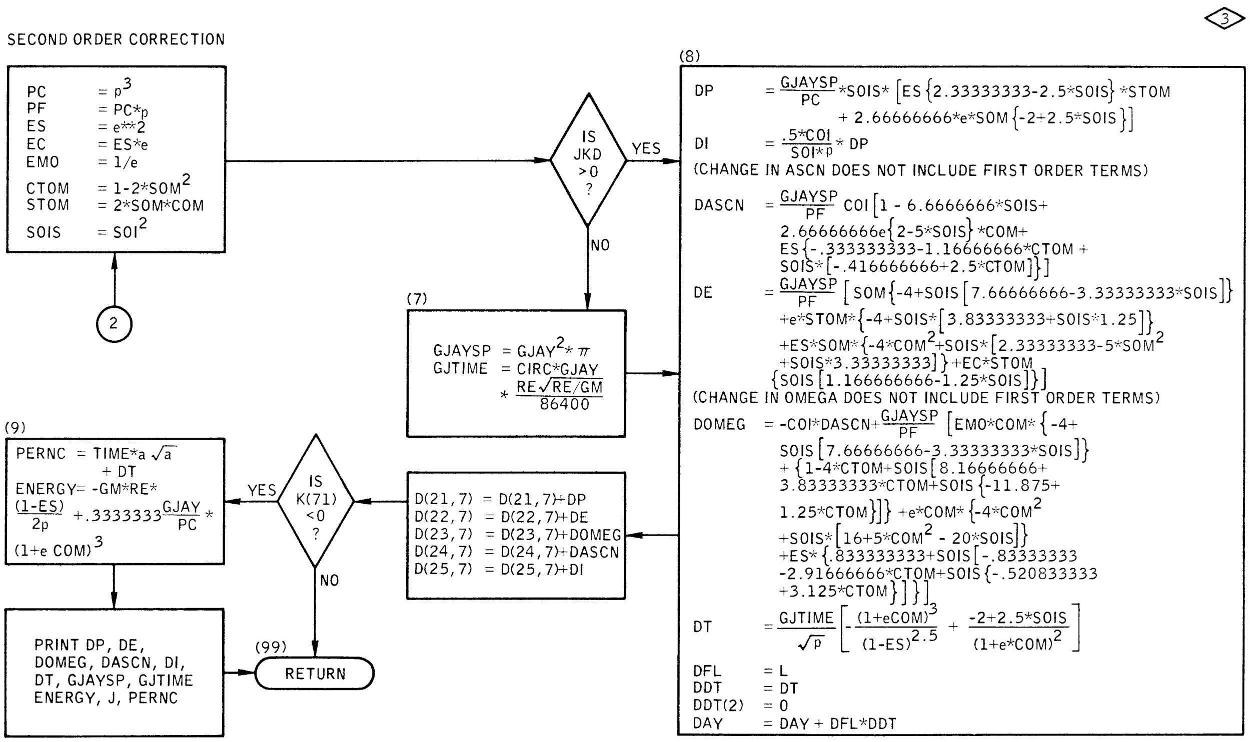
$$Q(2,7) = Q(30,7)*Q(17,7)$$

$$Q(3,7) = Q(30,7)*Q(18,7)$$

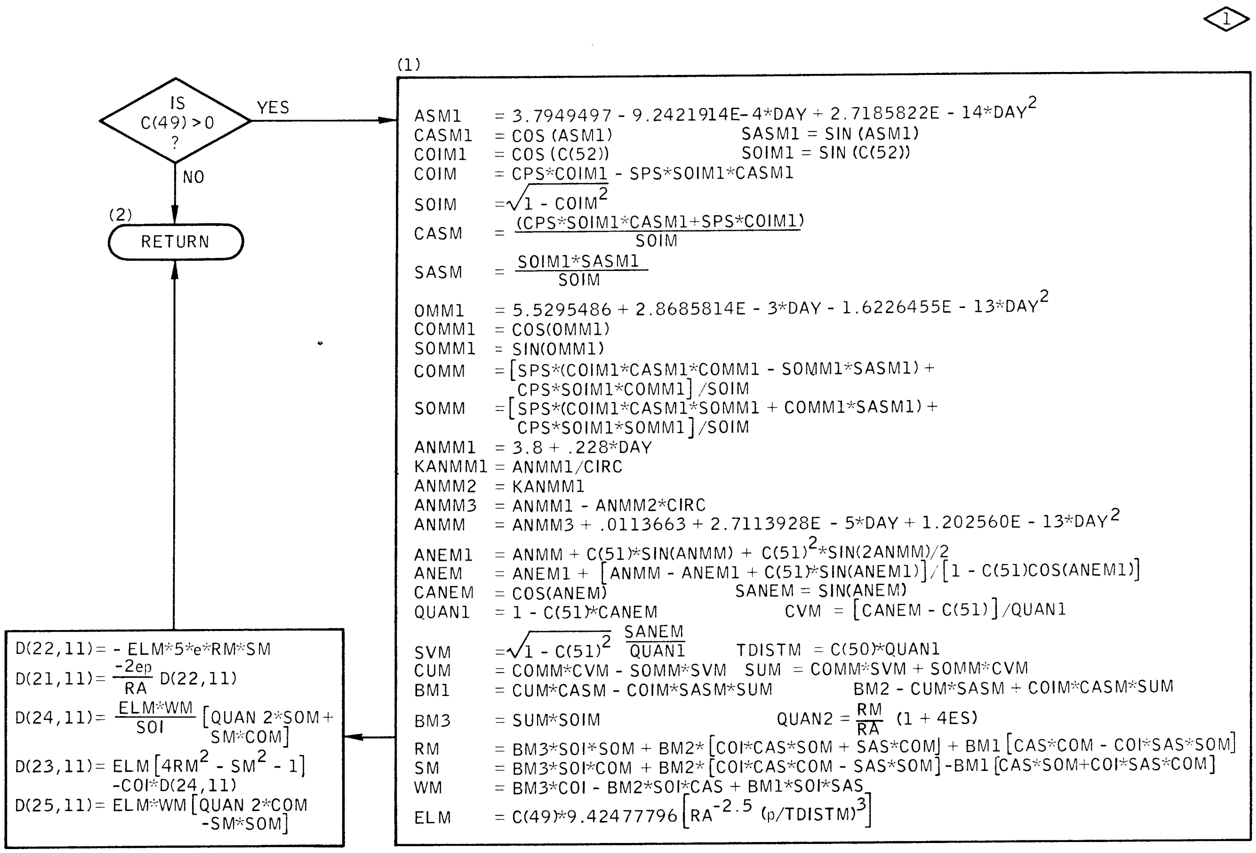
$$D(25,7) = Q(30,7)*D(21,7)$$

2

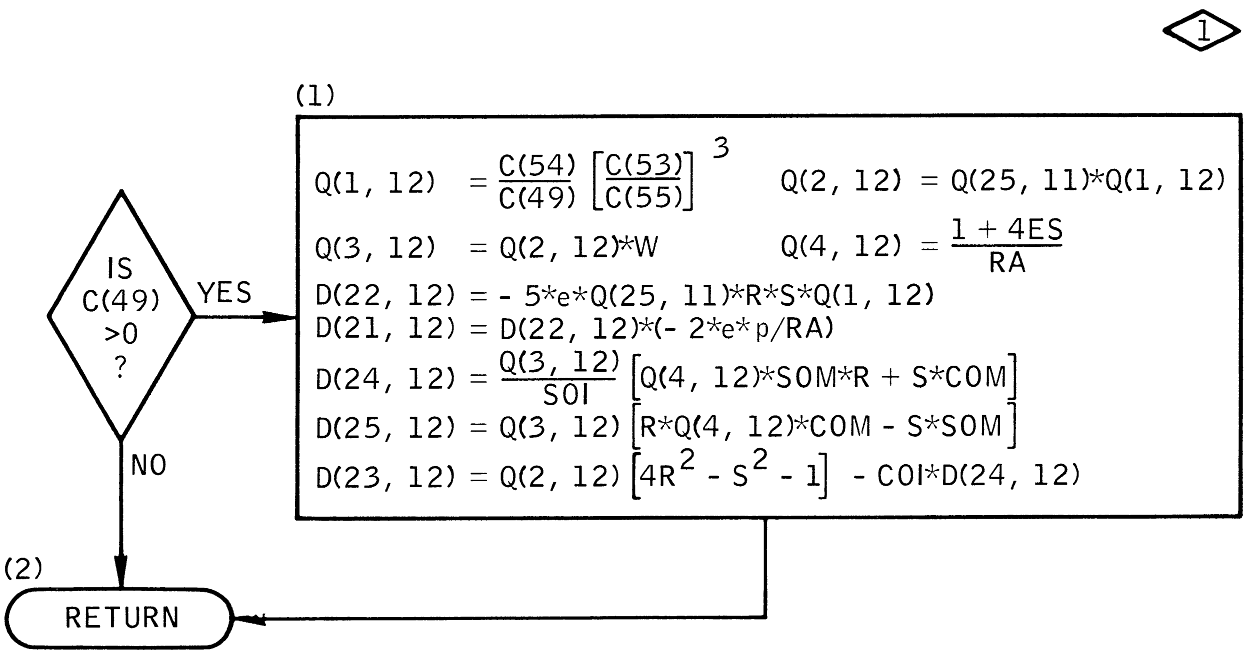
EARTH (KK = 7) CONT.



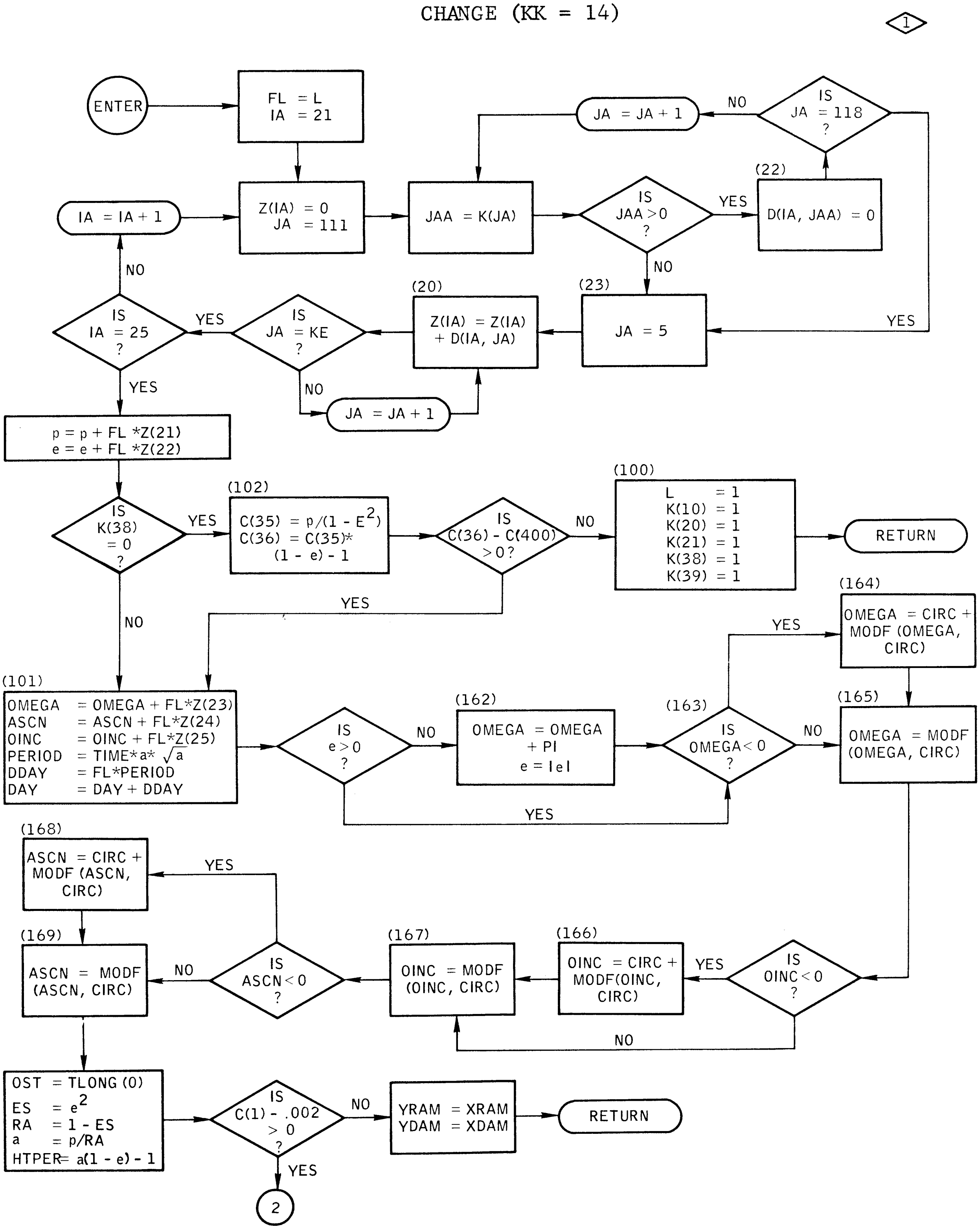
MOON (KK = 11)


1
(1)
IS C(49) > 0 ?
YES
NO
(2)
RETURN
ASM1 = 3.7949497 - 9.2421914E-4*DAY + 2.7185822E - 14*DAY^2
CASM1 = COS (ASM1)
SASM1 = SIN (ASM1)
COIM1 = COS (C(52))
SOIM1 = SIN (C(52))
COIM = CPS*COIM1 - SPS*SOIM1*CASM1
SOIM = √(1 - COIM^2)
CASM = (CPS*SOIM1*CASM1+SPS*COIM1) / SOIM
SASM = SOIM1*SASM1 / SOIM
OMM1 = 5.5295486 + 2.8685814E - 3*DAY - 1.6226455E - 13*DAY^2
COMM1 = COS(OMM1)
SOMM1 = SIN(OMM1)
COMM = [SPS*(COIM1*CASM1*COMM1 - SOMM1*SASM1) + CPS*SOIM1*COMM1]/SOIM
SOMM = [SPS*(COIM1*CASM1*SOMM1 + COMM1*SASM1) + CPS*SOIM1*SOMM1]/SOIM
ANMM1 = 3.8 + .228*DAY
KANMM1 = ANMM1/CIRC
ANMM2 = KANMM1
ANMM3 = ANMM1 - ANMM2*CIRC
ANMM = ANMM3 + .0113663 + 2.7113928E - 5*DAY + 1.202560E - 13*DAY^2
ANEM1 = ANMM + C(51)*SIN(ANMM) + C(51)^2*SIN(2ANMM)/2
ANEM = ANEM1 + [ANMM - ANEM1 + C(51)*SIN(ANEM1)]/[1 - C(51)COS(ANEM1)]
CANEM = COS(ANEM)
SANEM = SIN(ANEM)
QUAN1 = 1 - C(51)*CANEM
CVM = [CANEM - C(51)]/QUAN1
SVM = √(1 - C(51)^2) SANEM/QUAN1
TDISTM = C(50)*QUAN1
CUM = COMM*CVM - SOMM*SVM
SUM = COMM*SVM + SOMM*CVM
BM1 = CUM*CASM - COIM*SASM*SUM
BM2 - CUM*SASM + COIM*CASM*SUM
BM3 = SUM*SOIM
QUAN2 = RM/RA (1 + 4ES)
RM = BM3*SOI*SOM + BM2*[COI*CAS*SOM + SAS*COM] + BM1[CAS*COM - COI*SAS*SOM]
SM = BM3*SOI*COM + BM2*[COI*CAS*COM - SAS*SOM] -BM1[CAS*SOM+COI*SAS*COM]
WM = BM3*COI - BM2*SOI*CAS + BM1*SOI*SAS
ELM = C(49)*9.42477796[RA^-2.5 (ρ/TDISTM)^3]
D(22,11) = - ELM*5*e*RM*SM
D(21,11) = -2eρ/RA D(22,11)
D(24,11) = ELM*WM/SOI [QUAN 2*SOM + SM*COM]
D(23,11) = ELM[4RM^2 - SM^2 - 1] -COI*D(24,11)
D(25,11) = ELM*WM[QUAN 2*COM -SM*SOM]

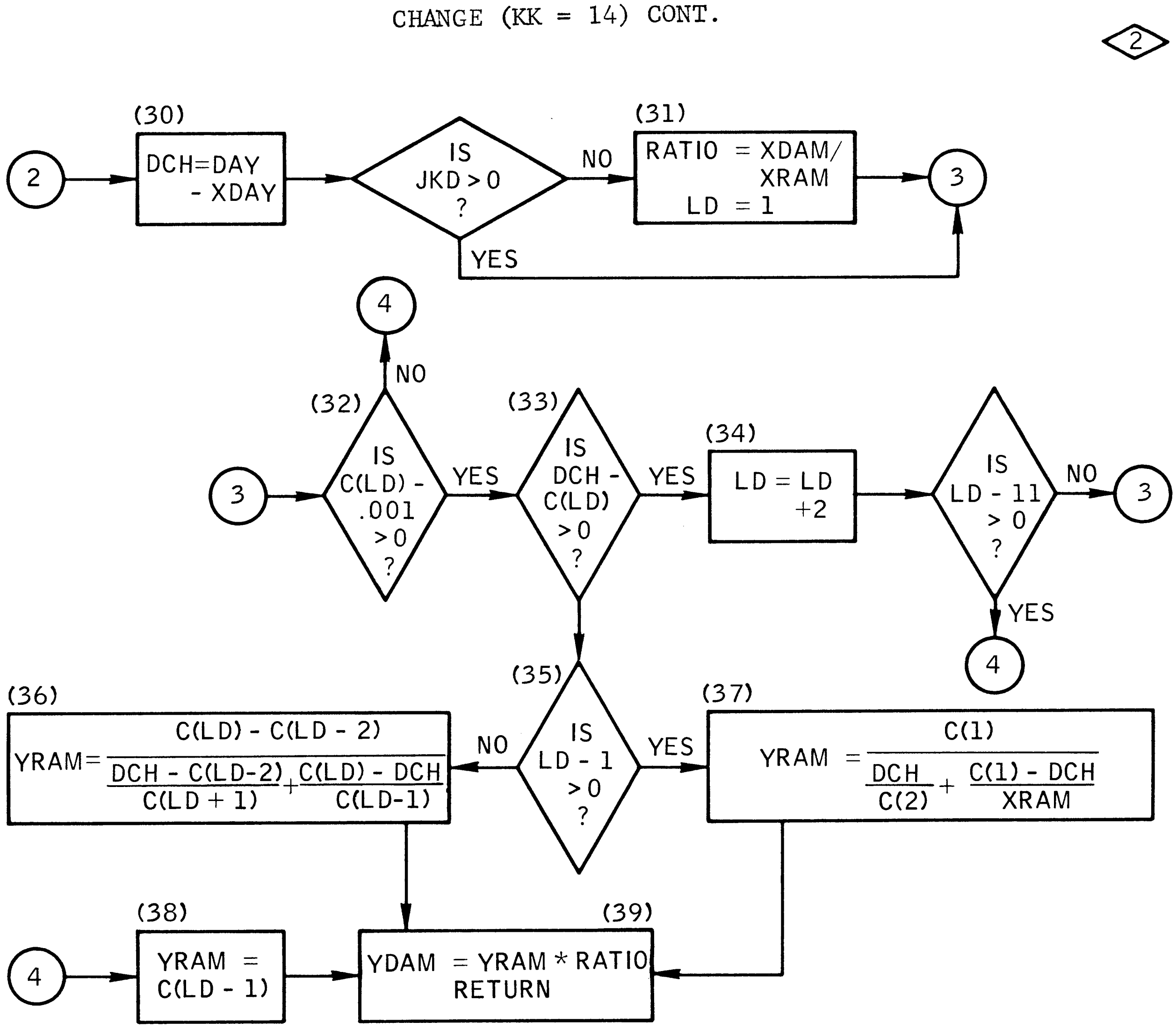
SUN (KK = 12)


1
(1)
$Q(1, 12) = \frac{C(54)}{C(49)} \left[\frac{C(53)}{C(55)}\right]^3$
$Q(2, 12) = Q(25, 11)*Q(1, 12)$
$Q(3, 12) = Q(2, 12)*W$
$Q(4, 12) = \frac{1 + 4ES}{RA}$
$D(22, 12) = - 5*e*Q(25, 11)*R*S*Q(1, 12)$
$D(21, 12) = D(22, 12)*(- 2*e*p/RA)$
$D(24, 12) = \frac{Q(3, 12)}{SOI} \left[Q(4, 12)*SOM*R + S*COM\right]$
$D(25, 12) = Q(3, 12) \left[R*Q(4, 12)*COM - S*SOM\right]$
$D(23, 12) = Q(2, 12) \left[4R^2 - S^2 - 1\right] - COI*D(24, 12)$
IS C(49) >0 ?
YES
NO
(2)
RETURN

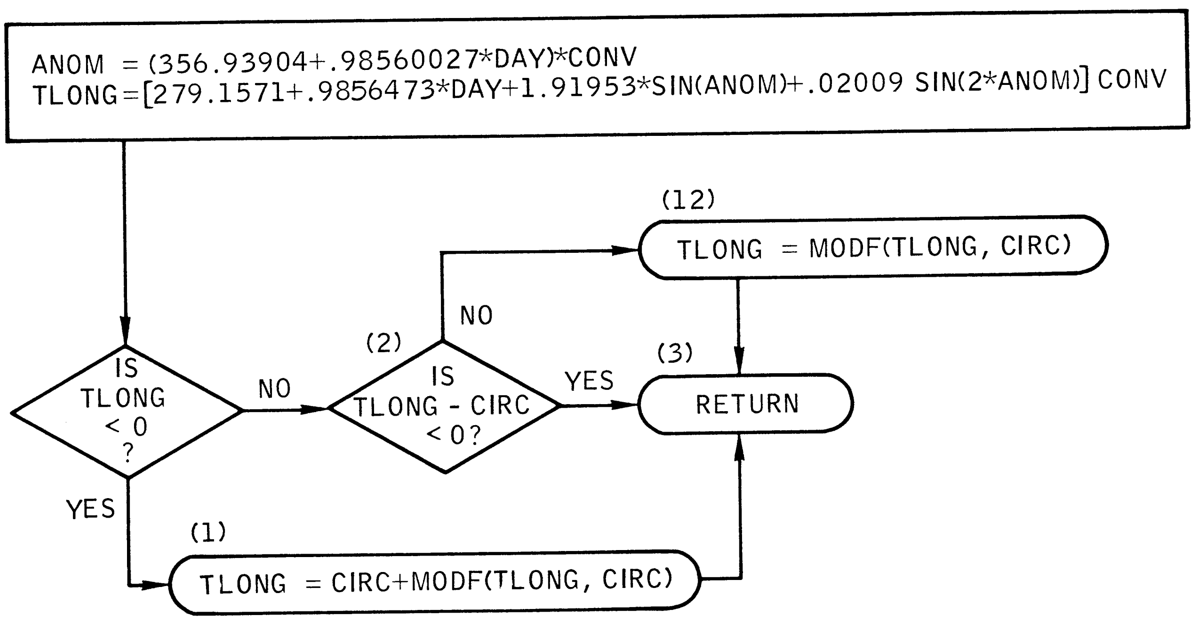
CHANGE (KK = 14)


1
ENTER
FL = L
IA = 21
Z(IA) = 0
JA = 111
JAA = K(JA)
IS JAA > 0 ?
YES
D(IA, JAA) = 0
(22)
IS JA = 118 ?
NO
JA = JA + 1
YES
NO
(23)
JA = 5
(20)
Z(IA) = Z(IA) + D(IA, JA)
IS JA = KE ?
YES
NO
JA = JA + 1
IS IA = 25 ?
NO
IA = IA + 1
YES
p = p + FL *Z(21)
e = e + FL *Z(22)
IS K(38) = 0 ?
YES
(102)
C(35) = p/(1 - E2)
C(36) = C(35)* (1 - e) - 1
IS C(36) - C(400) > 0 ?
NO
(100)
L = 1
K(10) = 1
K(20) = 1
K(21) = 1
K(38) = 1
K(39) = 1
RETURN
YES
NO
(101)
OMEGA = OMEGA + FL*Z(23)
ASCN = ASCN + FL*Z(24)
OINC = OINC + FL*Z(25)
PERIOD = TIME*a* √a
DDAY = FL*PERIOD
DAY = DAY + DDAY
IS e > 0 ?
NO
(162)
OMEGA = OMEGA + PI
e = |e|
YES
(163)
IS OMEGA < 0 ?
YES
(164)
OMEGA = CIRC + MODF (OMEGA, CIRC)
NO
(165)
OMEGA = MODF (OMEGA, CIRC)
IS OINC < 0 ?
YES
(166)
OINC = CIRC + MODF(OINC, CIRC)
NO
(167)
OINC = MODF (OINC, CIRC)
IS ASCN < 0 ?
YES
(168)
ASCN = CIRC + MODF (ASCN, CIRC)
NO
(169)
ASCN = MODF (ASCN, CIRC)
OST = TLONG (0)
ES = e2
RA = 1 - ES
a = p/RA
HTPER = a(1 - e) - 1
IS C(1) - .002 > 0 ?
NO
YRAM = XRAM
YDAM = XDAM
RETURN
YES
2

CHANGE (KK = 14) CONT.



TLONG


ANOM =(356.93904+.98560027*DAY)*CONV
TLONG=[279.1571+.9856473*DAY+1.91953*SIN(ANOM)+.02009 SIN(2*ANOM)] CONV
(12)
TLONG = MODF(TLONG, CIRC)
NO
(2)
IS TLONG - CIRC < 0?
YES
(3)
RETURN
IS TLONG < 0 ?
NO
YES
(1)
TLONG = CIRC+MODF(TLONG, CIRC)

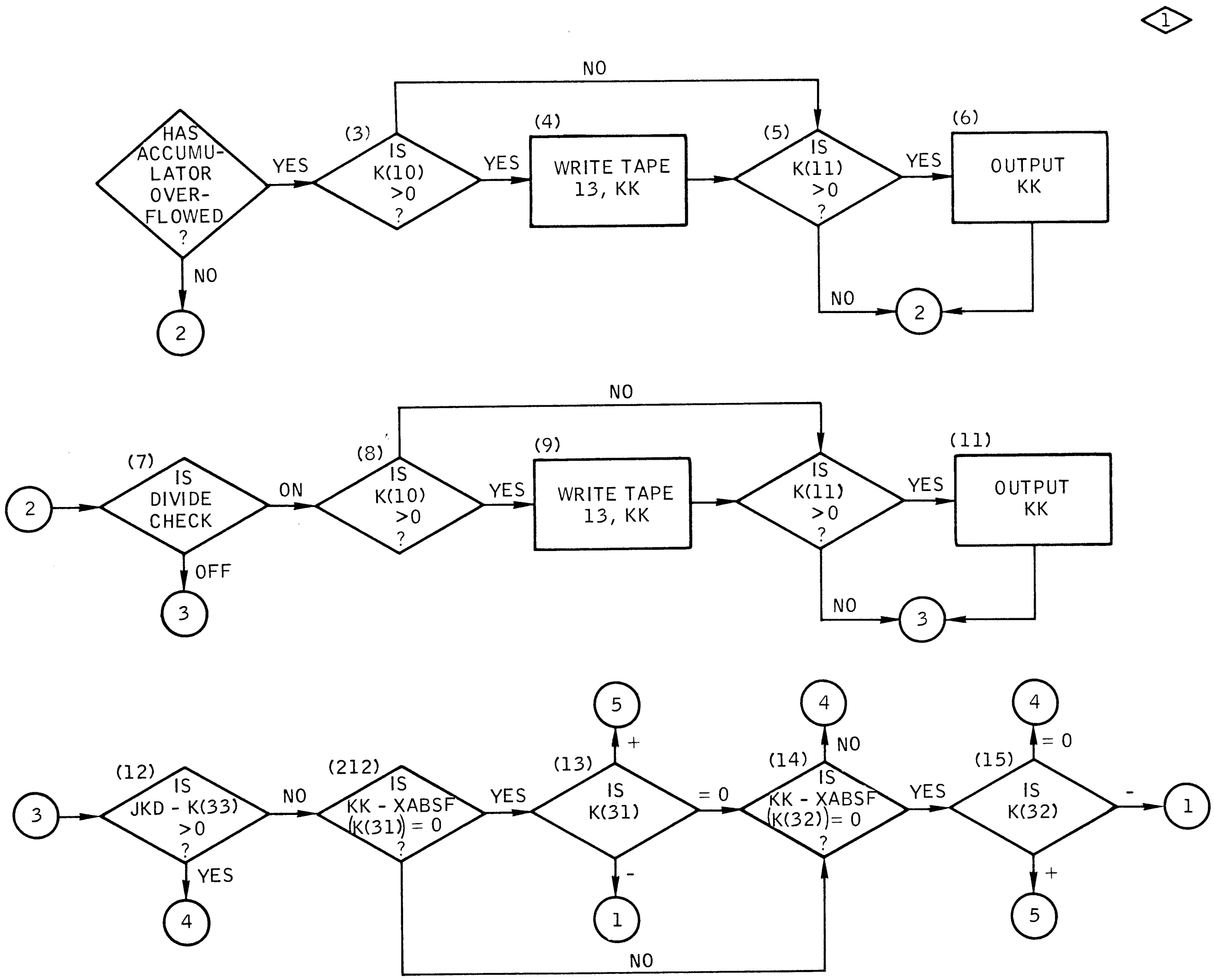
TDIST (Z)

1

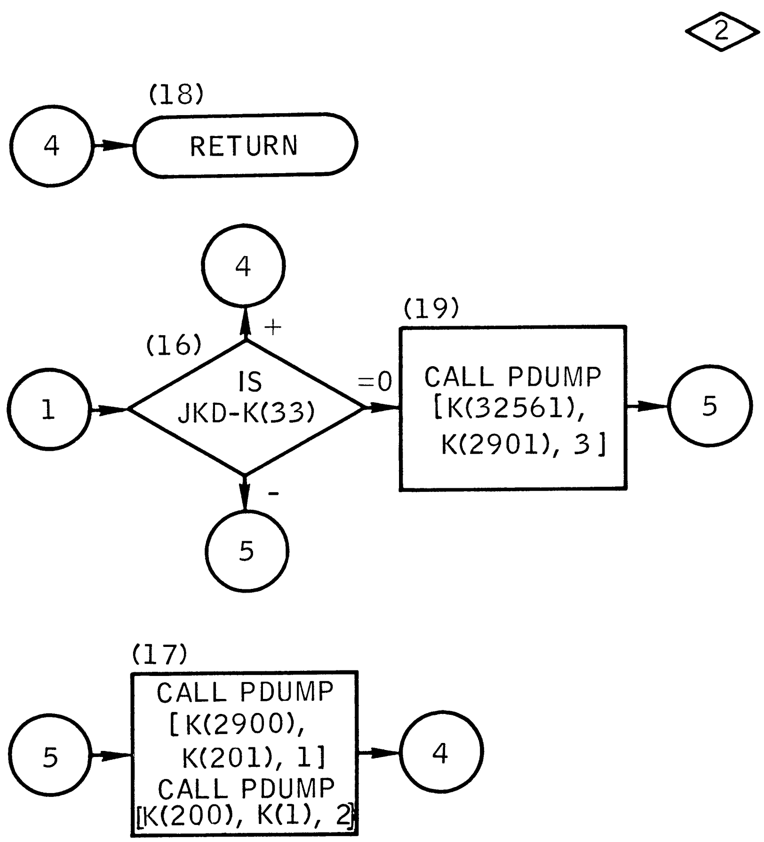
ANOM = (356.93904 + .98560027*DAY) CONV
X = .033478 COS (ANOM)
TDIST = .9994396 + X (1 + 1.25X)

RETURN

ODCHECK


1
NO
HAS ACCUMU- LATOR OVER- FLOWED ?
YES
NO
2
(3)
IS K(10) >0 ?
YES
(4)
WRITE TAPE 13, KK
(5)
IS K(11) >0 ?
YES
(6)
OUTPUT KK
NO
2
NO
2
(7)
IS DIVIDE CHECK
ON
OFF
3
(8)
IS K(10) >0 ?
YES
(9)
WRITE TAPE 13, KK
IS K(11) >0 ?
YES
(11)
OUTPUT KK
NO
3
3
(12)
IS JKD - K(33) >0 ?
NO
YES
4
(212)
IS KK - XABSF (K(31)) = 0 ?
YES
NO
(13)
IS K(31)
+
5
-
1
= 0
(14)
IS KK - XABSF (K(32)) = 0 ?
NO
4
YES
(15)
IS K(32)
= 0
4
-
1
+
5

ODCHEK CONT.


2
(18)
4
RETURN
4
+
(19)
(16)
IS
JKD-K(33)
=0
CALL PDUMP
[K(32561),
K(2901), 3]
1
5
-
5
(17)
CALL PDUMP
[K(2900),
K(201), 1]
CALL PDUMP
[K(200), K(1), 2]
5
4

Appendix A

EVALUATION OF INTEGRALS

In evaluating integrals of the type

$$I_k(n,m/\alpha,\beta) = \int_0^{2\pi} \frac{\cos^n v \sin^m v}{(\alpha+\beta \cos v)^k} dv \tag{A.1}$$

from a table of integrals

$$I_1(0,0) = 2\pi(a^2 - \beta^2)^{-1/2} \tag{A.2}$$

$$I_1(1,0) = 2\pi \beta^{-1}\left[1-\alpha(\alpha^2 - \beta^2)^{-1/2}\right] \tag{A.3}$$

then using the following form, with $K \geq n+1$

$$I_k(n,0) = \left. \frac{(-1)^{k-1} d^{k-1} I_1(0,0/\alpha,\beta)}{(k-1)! \, d\alpha^{k-n-1} d\beta^n} \right|_{\substack{\alpha = 1 \\ \beta = e}} \tag{A.4}$$

and

$$I_k(k,0) = \frac{(-1)^{k-1}}{(k-1)!} \left. \frac{d^{k-1} I_1(1,0/\alpha,\beta)}{d_\beta^{k-1}} \right|_{\substack{\alpha = 1 \\ \beta = e}} \tag{A.5}$$

the entire set of integrals required can be derived if $\sin^2 v = 1-\cos^2 v$ is used in evaluating integrals of the form $I_k(n,2k)$.

Since sin v is an odd function, a further simplification is possible, i.e.,

$$I_p(n,2k + 1) = 0 \qquad \text{all } n,\ k,\ p. \tag{A.6}$$

The list of the non-vanishing integrals of the above type with $p \geq (n + m)$ that are used are

$$I_1\ (0,0) = 2\ \pi\ (1 - e^2)^{-1/2}\ , \tag{A.7}$$

$$I_1\ (1,0) = 2\ \pi\ e^{-1} \left[1 - (1 - e^2)^{-1/2}\right]\ , \tag{A.8}$$

$$I_2\ (0,0) = 2\ \pi\ (1 - e^2)^{-3/2}\ , \tag{A.9}$$

$$I_2\ (0,2) = -2\ \pi\ e^{-2} \left[1 - (1 - 2^2)^{-1/2}\right]\ , \tag{A.10}$$

$$I_2\ (1,0) = -2\ \pi\ e\ (1 - e^2)^{-3/2}\ , \tag{A.11}$$

$$I_2\ (2,0) = 2\ \pi\ e^{-2} \left[1 - (1 - e^2)^{-3/2}\ (1 - 2\ e^2)\right] \tag{A.12}$$

$$I_3\ (0,0) = 3\ \pi\ (1 - e^2)^{-5/2}\ , \tag{A.13}$$

$$I_3\ (0,2) = 2\ \pi\ (1 - e^2)^{-3/2}\ , \tag{A.14}$$

$$I_3\ (1,0) = 3\ \pi\ e\ (1 - e^2)^{-5/2} \tag{A.15}$$

$$I_3\ (1,2) = -\ \pi\ e^{-3} \left[2 - (1 - e^2)^{-3/2} (2 - 3\ e^2)\right] \tag{A.16}$$

$$I_3\ (2,0) = \pi\ (1 - e^2)^{-5/2}\ (1 + 2\ e^2)\ , \tag{A.17}$$

$$I_3\ (3,0) = \pi\ e^{-3} \left[2 - (1 - e^2)^{-5/2}\ (2 - 5\ e^2 + 6\ e^4)\right]\ , \tag{A.18}$$

$$I_4\ (0,0) = \pi\ (1 - e^2)^{-7/2}\ (2 + 3\ e^2)\ , \tag{A.19}$$

$$I_4\ (0,2) = \pi\ (1 - e^2)^{-5/2}\ , \tag{A.20}$$

$$I_4\ (0,4) = \pi\ e^{-4} \left[2 - (1-e^2)^{-7/2}\ (2 - 7\ e^2 + 8\ e^4 - 3\ e^6)\right] \tag{A.21}$$

$$I_4\ (1,0) = -\pi\, e\, (1 - e^2)^{-7/2}\ (4 + e^2)\ , \tag{A.22}$$

$$I_4\ (1,2) = -\pi\, e\, (1 - e^2)^{-5/2}\ , \tag{A.23}$$

$$I_4\ (2,0) = \pi\, (1 - e^2)^{-7/2}\ (1 + 4\, e^2)\ , \tag{A.24}$$

$$I_4\ (2,2) = -\pi\, e^{-4} \left[2 - (1 - e^2)^{-7/2} (2 - 7e^2 + 9e^4 - 4e^6)\right], \tag{A.25}$$

$$I_4\ (3,0) = -\pi\, e\, (1 - e^2)^{-7/2}\ (3 + 2\, e^2)\ , \tag{A.26}$$

$$I_4\ (4,0) = \pi\, e^{-4} \left[2 - (1 - e^2)^{-7/2}\ (2 - 7e^2 + 8e^4 - 8e^6)\right]. \tag{A.27}$$

The other quantities that are evaluated in terms of the I's are

$$Q_1 = I_3(0,1) \tag{A.28}$$

$$Q_2 = I_3(1,0) \tag{A.29}$$

$$Q_3 = e\, I_3(0,1) + I_3(1,1) \tag{A.30}$$

$$Q_4 = I_3(0,0) + 2e\, I_3\ (1,0) + I_3(2,0) \tag{A.31}$$

$$Q_5 = 2I_3(0,0) + e\, I_3\ (1,0) - I_3(2,0) \tag{A.32}$$

$$Q_6 = I_3(1,1) \tag{A.33}$$

Appendix B

DERIVATION OF THE APPROXIMATE EXPRESSION FOR V_r

According to Eq. (108)

$$V_r = \left(\frac{\mu}{p}\right)^{1/2}\left[(1 + e^2 + 2e\cos v) - \frac{2p^{3/2}}{\sqrt{\mu}}\,\omega_E\cos i\right.$$

$$\left. + \frac{p^3\omega_E^2}{\mu(1 + e\cos v)^2}\left\{1 - \frac{1}{2}(1 - \cos^2 i)[1 - \cos(2\omega + 2v]\right\}\right]^{1/2}$$

$$= \left(\frac{\mu}{p}\right)^{1/2}(X + Y)^{1/2}$$

where

$$X = 1 + e^2 + 2e\cos v$$

$$Y = -\frac{2p^{3/2}}{\sqrt{\mu}}\,\omega_E\cos i$$

$$+ \frac{p^3\omega_E^2}{\mu(1 + e\cos v)^2}\left\{1 - \frac{1}{2}(1 - \cos^2 i)[1 - \cos(2\omega + 2v)]\right\}$$

For a typical close-in orbit of unspecified e and i we get

$$Y = -0.125\cos i + \frac{0.0038}{(1 + e\cos v)^2}\left\{1 - \frac{1}{2}(1 - \cos^2 i)[1 - \cos(2\omega + 2v)]\right\}$$

For small eccentricities (e of 0.1 or less) it is clear that in general the second terms of Y will be small compared to the first term for all

values of i except those near 90° or 270°. Compared to the magnitude of X the second term of Y will be small for all values of i, v, or ω. Proceeding under the assumption that the second term of Y is always small compared to X, it is neglected and now

$$Y \approx Y_1 = -2 \frac{p^{3/2}}{\sqrt{\mu}} \omega_E \cos i$$

The relative velocity is now

$$V_r = \left(\frac{\mu}{p}\right)^{1/2} \left(X + Y_1\right)^{1/2}$$

$$= \left(\frac{\mu}{p}\right)^{1/2} \left(X^{1/2} + \frac{Y_1}{2X^{1/2}} - \frac{1}{8} \frac{Y_1^2}{X^{3/2}} + \ldots\right)$$

For small e, as assumed above, the second- and higher-order terms in Y_1 can be neglected and the final form of V_r and the one programmed is

$$V_r \approx \sqrt{\frac{\mu}{p}} \left(\sqrt{X} + \frac{Y_1}{2\sqrt{X}}\right)$$

The exact and approximate expressions for V_r were evaluated for a wide range of values of e, i, ω, and v and the ratio

$$\Delta V_r = \frac{V_r(\text{approx}) - V_r(\text{exact})}{V_r(\text{exact})}$$

was computed. For i = 0 the ratio is independent of ω and is a maximum.

The following table shows the maximum values and the variation of the ratio ΔV_r for orbits with i = 0 and perigee distance of about 120 km, for various combinations of e and v.

It is interesting to note that the maximum value of the ratio occurs for $v = 180^{o}$, the position of the apogee point, and that this maximum value increases with e. The maximum value of ΔV_r is 0.2602 which corresponds to $v = 180^{o}$ and $e = 0.1$. This value of ΔV_r appears to be significant. However, the difference in the altitudes of perigee and apogee is large enough to cause the density of the atmosphere at apogee to be negligible compared to that at perigee. In fact, the product $\rho \Delta V_r^2$, which is directly proportional to drag acceleration, does not have an appreciable magnitude for any value of v.

Table 2

THE RATIO ΔV_r FOR VARIOUS COMBINATIONS OF v AND e

v	e, Eccentricity				
	0.0000	0.0001	0.0010	0.0100	0.100
0	$0.5313\text{x}10^{-6}$	$0.5313\text{x}10^{-6}$	$0.5522\text{x}10^{-6}$	$0.5391\text{x}10^{-6}$	$0.5051\text{x}10^{-6}$
45	$0.5313\text{x}10^{-6}$	$0.3253\text{x}10^{-4}$	$0.3295\text{x}10^{-3}$	$0.3254\text{x}10^{-2}$	$0.2831\text{x}10^{-1}$
90	$0.5313\text{x}10^{-6}$	$0.1123\text{x}10^{-3}$	$0.1128\text{x}10^{-2}$	$0.1124\text{x}10^{-1}$	0.1079
135	$0.5313\text{x}10^{-6}$	$0.1921\text{x}10^{-3}$	$0.1928\text{x}10^{-2}$	$0.1942\text{x}10^{-1}$	0.2093
180	$0.5313\text{x}10^{-6}$	$0.2252\text{x}10^{-3}$	$0.2260\text{x}10^{-2}$	$0.2287\text{x}10^{-1}$	0.2602
225	$0.5313\text{x}10^{-6}$	$0.1921\text{x}10^{-3}$	$0.1928\text{x}10^{-2}$	$0.1942\text{x}10^{-1}$	0.2093
270	$0.5313\text{x}10^{-6}$	$0.1123\text{x}10^{-3}$	$0.1128\text{x}10^{-2}$	$0.1124\text{x}10^{-1}$	0.1079
315	$0.5313\text{x}10^{-6}$	$0.3253\text{x}10^{-4}$	$0.3295\text{x}10^{-3}$	$0.3254\text{x}10^{-2}$	$0.2831\text{x}10^{-1}$

Appendix C

DISCUSSION OF MODEL ATMOSPHERES

The two atmosphere models currently used in the computer program are the U.S. Standard Atmosphere, 1962,[7] and an atmosphere model derived by Jacchia[6] and recently modified by Jacchia and Slowey.[8]

The U.S. Standard Atmosphere, 1962 is based on a combination of theory and measurements of satellites and rockets. The density as a function of altitude is entered into the 7090 computer program in tabular form. The altitude ranges from 0 to 700 km with altitude increments of 2 km.

The Jacchia and Slowey atmosphere model takes into account the bulge in the atmosphere caused by solar radiation (diurnal effect). It is based on observations by eight selected artifical satellites during the years 1958 through 1961. The satellites were in eccentric orbits with perigees between 350 and 750 km. These observations were used to derive an empirical formula for the atmospheric density which can be used above altitudes of 700 km (the maximum altitude for the U.S. Standard Atmosphere, 1963). The density is computed by the program by using the equation

$$\rho = \rho_o(z)\ F_{2o} \left[1 + \phi(z) \cos^6 \Psi/2)\right]$$

where

$$\log \rho_o(z) = -16.021 - 0.001985\ z + 6.363\ e^{-0.0026\ z}$$

$$\phi(z) = 4.6 + 4 \tan^{-1} \left[0.005\ (z - 600)\right]$$

and 4 is the angle between earth radii to the satellite and the center of the decimal bulge. The bulge lags the sun's longitude by about 30° due to the earth's rotation, and its effect on the atmospheric density

is almost zero at 200 km but increases with altitude. The quantity z is the height of the satellite above the oblate earth and is computed with Eq. (116).

The quantity F_{20} is the solar flux at 20 cm wave length. The measured flux values show erratic daily fluctuations in which the 27-day period of the solar rotation can often be recognized. The flux values are entered into the program in tabular form.

Appendix D

FORMULAE FOR THE DETERMINATION OF MOON POSITION

In order to evaluate the gravitational disturbances using Eqs. (45) through (57), the following quantities defining the moon's position must be determined (see Fig. 9).

1. i_M the inclination of the moon's orbit relative to the earth's equatorial plane.
2. Ω_M the angular position of the earth-moon line of nodes.
3. u_M the position of the moon relative to the nodal crossing.

From Ref. 9 the following mean orbital elements of the moon are obtained:

1. Geometric mean ecliptic longitude of the ascending node of the moon.

$$\Omega_{ME} = 3.7949497 - 9.2421914 \times 10^{-4} \text{ day} + 2.7185822 \times 10^{-14} \text{ day}^2 \qquad \text{(D-1)}$$

2. Mean longitude of perigee from the ecliptic of the moon.

$$\omega_{ME} = 5.5295486 + 2.8685814 \times 10^{-3} \text{ day} + 1.6226455 \times 10^{-13} \text{ day}^2 \qquad \text{(D-2)}$$

3. Mean anomaly of the moon.

$$M_M = 3.8 + 0.228 \text{ day} - \left[3.8 + 0.228 \text{ day}\right] + 0.0113663 + 2.7113928 \times 10^{-5} \text{ day} + 1.2075605 \times 10^{-3} \text{ day}^2 \qquad \text{(D-3)}$$

where day signifies the time after the Julian Date of 2436203.5, which is 31 December 1957, or GCT.

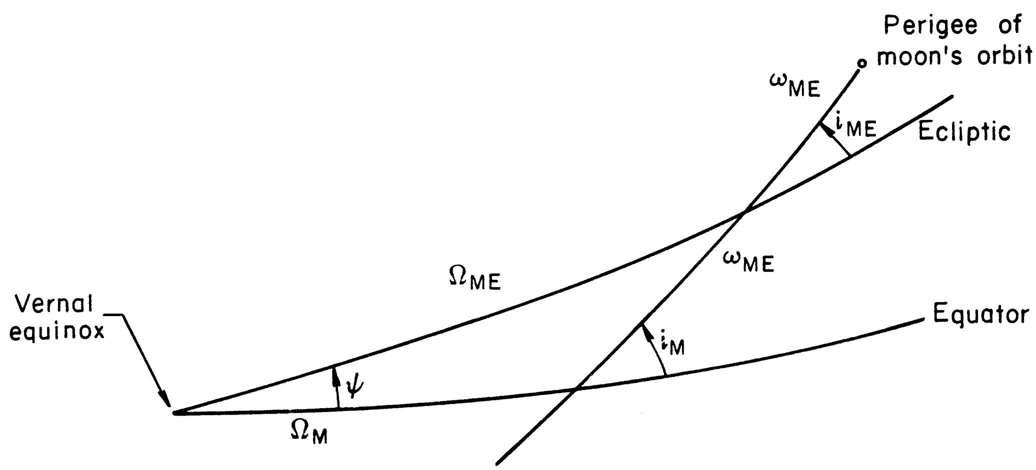


Fig. 9 — Typical orientation of the planes of the equator, ecliptic, and moon's orbit

Trigonometric functions of the angles used by the moon subroutine in the computer program are found by applying the spherical triangle sine and cosine relations. From Fig. 9

$$\sin \Omega_M = \frac{\sin i_{ME} \sin \Omega_{ME}}{\sin i_M} \tag{D-4}$$

$$\cos \Omega_M = \frac{\cos \Psi \sin i_{ME} \cos \Omega_{ME} + \sin \Psi \cos i_{ME}}{\sin i_M}$$

where i_{ME}, the orbital inclination of the moon to the ecliptic, and ψ, the inclination of the ecliptic plane to the equatorial plane, are constant and

$$\cos i_M = \cos \Psi \cos i_{ME} - \sin \Psi \sin i_{ME} \cos \Omega_{ME}$$

$$\sin i_M = \sqrt{1 - \cos^2 i_M} \tag{D-5}$$

Again from Fig. 9 and the spherical sine and cosine law

$$\sin \omega_M = \frac{\sin \Psi \left\{\cos i_{ME} \cos \Omega_{ME} \sin \omega_{ME} + \cos \omega_{ME} \sin \Omega_{ME}\right\}}{\sin i_M} + \frac{\cos \Psi \sin i_{ME} \cos \omega_{ME}}{\sin i_M} \tag{D-6}$$

$$\cos \omega_M = \frac{\sin \Psi \left\{\cos i_{ME} \cos \Omega_{ME} \cos \omega_{ME} - \sin \omega_{ME} \sin \Omega_{ME}\right\}}{\sin i_M} + \frac{\cos \Psi \sin i_{ME} \cos \omega_{ME}}{\sin i_M}$$

The true anomaly of the moon is obtained from the relations

$$\sin v_M = \frac{\sqrt{1 - e_M^2}\ \sin E_M}{1 - e_M \cos E_M}$$

$$\cos v_M = \frac{\cos E_M - e_M}{1 - e_M \cos E_M} \tag{D-7}$$

The eccentric anomaly E_M is found from a solution of Kepler's equation, which is

$$E_M - e_M \sin E_M = M_M \tag{D-8}$$

An expansion of E_M gives the first approximation

$$E_{oM} = M_M + 2\Sigma \frac{1}{n} J_n \text{ (ne) } \sin \text{ (nM)}$$

keeping powers of e_M^2

$$E_{oM} = M_M + e_M \sin M_M + e_M^2 \frac{\sin 2 M_M}{2} \tag{D-9}$$

Then by interpolation $E_{oM} + dE_M$ is the value of E_M which satisfies Eq. (D-8) accurately

$$E_{oM} + dE_M = e_M \sin (E_{Mo} + dE_M) = M_M$$

or

$$E_M = E_{oM} + \left\{ \frac{M_M - E_{oM} + e_M \sin (E_{oM})}{1 - e_M \cos (E_{oM})} \right\} \tag{D-10}$$

The distance from the earth to the moon may be obtained from

$$r_{EM} = a_M (1 - e_M \cos E_M) \tag{D-11}$$

By using Eqs. (D-9) and (D-10), r_{EM} can be expressed as a function of the quantities a_M, e_M and M_M, where a_M and e_M are given constants and M_M is computed from Eq. (D-3).

In a similar manner, the corresponding formulae for the sun's position can be obtained. These formulae, as are those for the moon's position, are given in Section III.

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