NOTES ON INCENTIVE CONTRACTING

K. L. Deavers and J. J. McCall

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PREFACE

This Memorandum is part of a continuing RAND study of Air Force procurement policy. In this particular effort, some of the difficulties associated with incentive contracting are discussed and the results of 252 completed Air Force incentive contracts are analyzed. Four or five years ago, a study of this nature would have been of little interest, for then the use of fixed-price incentive contracts was quite small. However, data indicate that this type of contract has grown from 22.8 percent of net contract obligations in fiscal 1962, to 41.5 percent in fiscal 1964. The growing importance of incentive contracts clearly calls for some efforts at interpreting the available data and the results of their use. It is in this spirit, and as an aid and caution to other researchers, that this Memorandum is presented. The study should interest Air Force personnel holding procurement responsibilities.
This Memorandum discusses two related aspects of incentive contracting. First it describes the researcher's problems in interpreting the results of such contracting. These problems center around the effect of risk on contractors in weapon procurements; and on the unknown, and presently undeterminable, effect of different sharing rates on the initial targets that firms submit on incentive contracts. It is impossible to distinguish between these two effects with data currently available.

Second, an empirical analysis of 252 Air Force incentive contracts is presented. The purpose is to explore the relationship between sharing rates and different aspects of contract cost, with a view to testing some of the economic hypotheses developed in the first part. The results of these tests are mixed, with some of the results tending to support our hypotheses, and others rejecting them. Thus, for the 1962-63 data, we could find no statistical evidence to suggest that the contracts with higher sharing rates tended to be either more efficient or more risky, in that the relative differences between actual and target costs were not significantly different for the higher and lower sharing rates. On the other hand, we did find that the difference between actual and target costs for the contracts during 1959-1962 become increasingly negative with increasing sharing rate which is consistent with the risk-efficiency hypothesis. Other differences in outcome were also noted between the two time periods, possibly suggesting some difference in governmental and contractor behavior with experience in incentive-type contracts.
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I. INTRODUCTION

This Memorandum discusses two related aspects of incentive contracting. Section II describes the researcher's problems in interpreting the results of incentive contracting. These problems center around the effect of risk on contractors in weapons procurements; and on the unknown, and presently undeterminable, effect of different sharing rates on the initial targets that firms submit on incentive contracts.

Recognizing these difficulties, Sec. III uses the results of 252 completed Air Force fixed-price incentive contracts to examine the impact on contractor behavior of different sharing rates. This analysis is interesting because of the apparently persistent and consistent effects it illustrates, and because of the questions these effects raise about the use of incentive contracting as a tool for "risk sharing." The effect of risk on target costs or bids is presented in the Appendix for firms with three different utility functions.

A study of this nature would have been of little interest 4 or 5 years ago, for then the use of fixed-price incentive contracts was quite small. However, data presented in another RAND study indicate that this type of contract has grown from 22.8 percent of new contract obligations in fiscal 1962, to 41.5 percent in fiscal 1964. The growing importance of incentive contracts clearly calls for some efforts

at interpreting the available data and the results of their use. It is in this spirit, and as an aid and caution to other researchers, that this Memorandum is presented.
II. RISK AND INCENTIVE CONTRACTING

Two assumptions underlie almost all studies of defense procurement policy. The first is that defense contracts are more uncertain than production contracts in the private sector of the economy; and the second is that business firms are risk averse, that is, firms demand risk premiums that are positively correlated with contract risk.*

The presence of large and formidable uncertainty has attracted considerable attention in the literature of defense procurement policy. The discovery and implementation of novel technology is frequently mentioned as the main determinant of this uncertainty. Of course, the impact of technological change on a purely research contract appears direct and obvious; either a technological advance occurs or it does not. However, uncertainty consists not only of the inability to predict whether or not an event will occur but also to define the event before it occurs. In production contracts, uncertainty is usually allocated among three variables: the product's cost, the delivery date, and the performance. Nevertheless problems in product definition still constitute a major source of uncertainty. The initial specification of the product assumes an adequate knowledge of the available technology and also information regarding product use or mission. Specification changes occurring during the production period are caused by an insufficient knowledge of initial technology and product use, and changes in technology and mission during the production period. These changes

*The effect of risk on target costs or bids is presented in the Appendix for vendors having three different utility functions.
in turn affect the product's cost, delivery date, and performance, and account for a significant amount of contractual uncertainty. The remainder of this Section considers several problems of military production contracts.

Typically, procurement literature asserts that weapon production is inherently more risky than private production. The documentation used to back up this assertion often consists of a sample of cost outcomes on a very small, highly selected series of weapons procurements. Neglecting for the moment problems associated with definitional changes in the weapon and their appropriate treatment, it is not clear that a more inclusive or "average" sample of weapon procurements would have cost outcomes more uncertain than those for private sector manufacturing or construction. Indeed, the opportunity to adjust the initial bid during production is prima facie evidence that defense contracting is less risky than the underlying uncertainty suggests. Also, the relevant alternative to weapons manufacture is not that of engaging in projects with no underlying uncertainty.

Much of the risk present in weapon production contracts can be attributed to changes in specifications during the production process. There are several reasons for these changes. Sometimes the product is poorly defined; when this is recognized, alterations occur. Alternatively, the initial product specification may include all information (technology) available, but further technological changes may occur during production. It is important to distinguish between risk imposed by the buyer via specification changes and risk that is caused by other forces. When this distinction is not made, each specification change provides an opportunity for charging the Government for not only
the cost of the change but also the cost of other adverse occurrences.

In the private sector, risk aversity on the part of a seller necessitates paying a risk premium as a compensation for risk bearing. In the defense sector, risk aversity has led to the development of "incentive contracts." In these contracts the seller submits a target cost estimate. If actual costs are less (greater) than target costs, the seller captures (bears) a fixed percentage $\beta$ of the "profits" ("losses") -- difference between actual and target costs. The remaining "profits" or "losses," $(1-\beta)$ of the total, go to the Government.

Incentive contracts can be justified on the following grounds. Sellers are so risk averse that extraordinary risk premiums would be demanded if they were required to bear all the risk, that is, if the sharing proportion were set equal to unity. When the sharing proportion is less than unity, the contractor's risk is reduced and a smaller risk premium is required. For example, if the underlying distribution of profits and losses is normal with mean $\mu$ and variance or risk $\sigma^2$, the variance can be reduced to $\beta^2 \sigma^2$ by establishing a sharing proportion less than unity. The risk premium $\rho(\beta; \sigma^2)$ for risk averse firms is a monotone increasing function of $\beta$, the sharing proportion. The risk premium is maximum when $\beta=1$ and minimum when $\beta=0$. Assuming that the Government is insensitive to risk and wishes to minimize the expected cost of a particular product, the sharing proportion at which the risk premium is minimum is of sole interest only if other costs are insensitive to the sharing proportion.

There are two reasons to expect other costs to be negatively related to the sharing proportion. First, as the sharing proportion decreases it
becomes increasingly difficult for the Government to discriminate between low and high cost firms on the basis of their target costs. Consequently, expected costs increase as the sharing proportion declines. Secondly, even if the Government chooses the most efficient firm, the firm's motivation to reduce costs is blunted as the sharing proportion declines from unity to zero. This is especially evident for firms that produce in both the private and defense sectors. Any cost reduction in the private sector is reflected in increased profits in the private sector $\beta = 1$. On the other hand, cost reductions obtained in the public sector must be shared with the Government; the sharing proportion is less than unity. It would not be surprising if such a company concentrated its cost reduction activities in the private sector.

A final, unique risk that a firm runs in dealing with the Government, especially on incentive-type contracts, is renegotiation. If the firm does particularly well in cutting contract costs, it not only must share a portion of its profits with the Government, but a separate Government agency may sue to recover additional profits, thus effectively changing the sharing rate.

The upshot of this discussion is that determining an optimal sharing proportion -- one that minimizes expected costs to the Government -- is, in practice, an exceedingly difficult matter.** Both the relation

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**Formally the problem is straightforward. Let $c(\beta)$ denote the expected costs of a contract exclusive of the risk premium as a function of the sharing proportion. For the reasons mentioned, $c(\beta)$ is a monotone decreasing function of $\beta$, $c'(\beta) < 0$. On the other hand, $\rho(\beta; \sigma^2)$,
between $\rho$ and the firm's efficiency and the relation between the risk premium and $\rho$ must be estimated. Procurement data currently available are not appropriate for either problem.

In addition to "incentives" for lower costs, military incentive contracts have been modified to include "incentives" for high performance and prompt delivery. These multiple incentive contracts specify three sharing proportions, $\beta$, $\gamma$, and $\delta$, for cost, performance and delivery, respectively. Frequently, $\gamma$ is further decomposed into several proportions, $\gamma_1$, $\gamma_2$, $\cdots$, $\gamma_n$, and applied to significant performance characteristics like reliability, maintainability, etc. The determination of each "optimal" sharing proportion faces the same difficulties as before. In addition, the optimal set of proportions can be established only after the relative values of lower cost, earlier delivery and higher performance have been assessed. Needless to say, this assessment is no trivial matter. The data currently available cannot be used to examine the effect of performance and schedule incentives.

In summary, the problem in interpreting the results of current incentive contracts is the effect of risk on the target costs submitted by firms under different sharing rates.

(a) First, as $\rho$ increases from 0 to 1, the share of risk borne by the firm increases. If the firm is risk averse, it will

the risk premium is a monotone increasing function of $\beta, \rho'(\beta; \sigma^2) > 0$. The total expected contract cost is denoted by $K(\beta; \sigma^2) > 0$.

$$K(\beta; \sigma^2) = c(\beta) + \rho(\beta; \sigma^2).$$

Total costs are minimized when $- c'(\beta) = \rho'(\beta; \sigma^2)$ provided that $K'' > 0$. If $K'' < 0$, minimum expected costs occur at $\beta = 0$ or $\beta = 1$. 
adjust its bid upward to compensate. Thus, for contracts with equal risk, the proportion of overruns should decline as the sharing rate increases.

(b) Also, as $\beta$ increases from 0 to 1, it will be worthwhile for the firm to spend more to decrease the uncertainty about the weapon's production costs. Thus, two contracts with different sharing rules, written for weapons that initially have the same uncertainty, will lead the firms to spend differently on learning about production costs. This will also cause the proportion of overruns to decline as the sharing rate increases.

(c) Finally, because of policy in the use of incentive contracts, one would expect fewer risky contracts as $\beta$ approaches 1. This would have the same result as (a) and (b) on the proportion of overruns observed.

Since all these factors operate in the same way, interpretation of the observed decreasing proportion of overruns as $\beta$ moves from 0 to 1 should be most cautious. This is especially true when one realizes that different policy implications result, depending on which factor dominates in the observed outcome.

There are also additional theoretical complications. Calculation of a risk premium is appropriate when the firm's utility function is unaffected by third and higher order moments of the probability distribution of profits. For example, the firm may be affected by the skewness of the distribution. More complicated utility functions would, in general, require calculating a distribution premium that depended on the entire distribution of profits and not only the variance.
The effect of different assumptions about the shape of the firms' utility function on optimal incentive rates is being explored. Preliminary results indicate the optimal rates are affected not only by the firm's (and Government's) utility function, as one would expect, but also by the precise form of the distribution of production costs. This being so, one is certainly skeptical of any optimizing scheme that ignores these factors. One must also be skeptical of using multiple, nonsymmetric, variable, etc., sharing proposals in the midst of the ignorance that exists concerning the real incentive effects of the most elementary sharing schemes. An attempt to discuss these effects is made in Sec. III.
III. EMPIRICAL ANALYSIS OF INCENTIVE CONTRACTING

In this Section, data are presented for two groups of incentive contracts. The first contains the final settlement reports for 126 contracts completed between 1959 and 1962; the second contains final settlement reports for 126 contracts completed in the 1962-1963 period. Information in these final settlement reports includes: the initial and adjusted target costs, actual costs, sharing proportions, and target profits. This information is not quite complete in the first group since, in some cases, the initial or adjusted costs were not recorded. Most contracts are for basic weapon systems or major subsystems. The production costs all exceed one million dollars.

The data were not pooled into a single sample first because they were collected separately and each contained about the same number of observations. More importantly, separate groups were retained to observe the behavior of contracts over time. Incentive contracts were almost never used before 1958, hence the first group of contracts reflects initial responses for both the government and the firms. The second group reflects a more informed response, and it is important to investigate whether this learning process caused either the Government or the firms to alter their contracting behavior. Several contracting characteristics indicate that some learning did occur.

The contracts in each group are divided into three subgroups. The first contains all contracts that have sharing proportions between

*In some of the analyses pertinent information is available for only 124 of these contracts.
0.025 and 0.15; the second have a proportion of 0.20; and the third have proportions between 0.25 and 0.50.

The initial target cost is established before any production is begun. During the course of production various modifications are performed. These give rise to supplementary agreements that alter the initial target cost. As the contract progresses, an adjusted target cost is calculated that incorporates the effects of these supplementary agreements. Unfortunately, no data are readily available on the number and sizes of the individual supplementary agreements. In some cases the firm suggests modifications; other times the Government initiates the changes. For each contract it would clearly be desirable to know what proportion of changes the firm proposed and also how frequently it proposed them. The firm could use these modification proposals to offset costs that were unanticipated when the initial target cost was specified. Since the incentive to offset unanticipated costs is positively related to the sharing proportion, the frequency of modification proposals should also bear a positive relation to the sharing proportion.* However, data are not available for testing this.

*More precisely, let \( X_0 \) and \( Y_0 \) denote, respectively, the target cost and the actual cost of producing the initially specified product; let \( X_i \) and \( Y_i \), \( i = 1, 2, \ldots, n \), be the respective modification costs. If \( Y_0 > X_0 \), the firm underestimates actual costs, then it behooves the firm to propose modifications and to overestimate their cost. Thus, when \( Y_0 > X_0 \), the firm makes \( n \) modifications and \( Y_1 < X_1, Y_2 < X_2, \ldots, Y_n < X_n \), so that

\[
\sum_{i=0}^{n} Y_i < \sum_{i=0}^{n} X_i.
\]

The number of modification proposals, \( n \), and the size of the overestimate on each \( Y_i - X_i \), \( i = 1, \ldots, n \), should be positively related to the sharing proportion.
conjecture. The only data available on modifications are the differences between the adjusted target cost and the initial target cost.

This difference gives only a gross measure of total modification costs. It would be most desirable to know the firm's expected costs when the initial target is submitted. The ratio of the difference between expected actual costs and initial target cost to expected active cost could then be calculated. This is a much more meaningful statistic than the ratio that is calculated.* Unfortunately, information on expected costs will probably never be available.

No data were available on either product quality or deviations from the target delivery date. If these data were available, the findings of this study could be significantly altered.

The data limitations enumerated indicate the constraints of this study and the tentative nature of the conclusions. The reader is urged to evaluate the following empirical analyses with these limitations in mind.

ANALYSIS OF INITIAL COST ESTIMATES

The relative size of each difference between actual cost and initial estimate is calculated by dividing the difference by the actual cost. This will be referred to as ratio 1. Positive values of ratio 1 indicate that actual cost is greater than initial cost; negative values indicate that initial cost is greater than actual cost. The sample mean ($\bar{x}$) and variance(s) of this statistic are

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*The calculated ratio is actual cost minus initial cost divided by actual cost.
calculated with different sharing proportions for each of the three groups. These calculations are summarized in Table 1 for the 124 observations comprising the 1959-1962 data and for the 126 observations comprising the 1962-1963 data. The sample size of each group and the sample variance of the mean are also presented. For the 1959-1962 data, values of ratio 1 are a decreasing function of the sharing proportion; for the 1962-1963 data, values of ratio 1 are an increasing function of the sharing proportion. Notice also that in letting the variance of the mean of ratio 1 (column 5, Table 1) measure uncertainty, the more uncertain contracts tend to have high sharing proportions. This result is not consistent with the hypothesis that low sharing proportions are assigned to the most risky contracts.

Table 1

<table>
<thead>
<tr>
<th>Sharing Proportion</th>
<th>Mean x</th>
<th>Variance $s^2$</th>
<th>Sample Size N</th>
<th>Variance of $\bar{x}$ $s^2_{\bar{x}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959-1962</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>\leq .15</td>
<td>5.06</td>
<td>338.42</td>
<td>38</td>
<td>8.91</td>
</tr>
<tr>
<td>= .20</td>
<td>-1.33</td>
<td>1123.80</td>
<td>71</td>
<td>16.05</td>
</tr>
<tr>
<td>\geq .25</td>
<td>-2.78</td>
<td>576.22</td>
<td>15</td>
<td>38.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>1962-1963</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>\leq .15</td>
<td>0.35</td>
<td>258.18</td>
<td>54</td>
<td>4.78</td>
</tr>
<tr>
<td>= .20</td>
<td>0.52</td>
<td>192.03</td>
<td>53</td>
<td>3.62</td>
</tr>
<tr>
<td>\geq .25</td>
<td>3.37</td>
<td>335.08</td>
<td>19</td>
<td>17.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>126</td>
<td></td>
</tr>
</tbody>
</table>
A one-way analysis of variance is employed to test whether the average values of ratio \( I \) are significantly different. The resulting F-ratio is not significant at 10 percent for either set of data. In fact, in both cases the within-group variability was greater than the between-group variability. This analysis is summarized in Table 2.

Table 2

<table>
<thead>
<tr>
<th>Variation</th>
<th>SSD</th>
<th>df</th>
<th>( s^2 )</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1959-1962</td>
</tr>
<tr>
<td>Between Groups</td>
<td>1,189.32</td>
<td>2</td>
<td>594.66</td>
<td>( F(2122) = 0.726 )</td>
</tr>
<tr>
<td>Within Groups</td>
<td>99,829.54</td>
<td>122</td>
<td>818.27</td>
<td>Not significant at 10 percent</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1962-1963</td>
</tr>
<tr>
<td>Between Groups</td>
<td>195.15</td>
<td>2</td>
<td>97.58</td>
<td>( F(2124) = 0.407 )</td>
</tr>
<tr>
<td>Within Groups</td>
<td>29,700.46</td>
<td>124</td>
<td>239.52</td>
<td>Not significant at 10 percent</td>
</tr>
</tbody>
</table>

Table 3 shows the proportion of negative values of ratio \( I \) in each group for the 1959-1962 and the 1962-1963 data. Both sets show that the proportion of negative values of ratio \( I \) is highest for the group with a 0.2 sharing proportion; otherwise the findings are incompatible. However, none of the proportions for the 1962-1963 data are significantly different from 0.5. The proportion of negative values of ratio \( I \) for the group with the lowest sharing proportion for the 1959-1962 data is significantly (at .05 percent) less than 0.5.

ANALYSIS OF ADJUSTED COST ESTIMATES

Essentially the same analysis is applied to the adjusted target costs. It will be recalled that these targets include the cost of
Table 3
PROPORTION OF NEGATIVE VALUES OF RATIO 1
FOR EACH CONTRACT GROUP

<table>
<thead>
<tr>
<th>Sharing Proportion</th>
<th>Proportion of Negative Values</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \leq .15 )</td>
<td>.32</td>
<td>38</td>
</tr>
<tr>
<td>( = .20 )</td>
<td>.48</td>
<td>71</td>
</tr>
<tr>
<td>( \geq .25 )</td>
<td>.47</td>
<td>15</td>
</tr>
<tr>
<td>( \leq .15 )</td>
<td>.52</td>
<td>54</td>
</tr>
<tr>
<td>( = .20 )</td>
<td>.60</td>
<td>53</td>
</tr>
<tr>
<td>( \geq .25 )</td>
<td>.47</td>
<td>19</td>
</tr>
</tbody>
</table>

various supplemental agreements that occur between initiation and completion of the contract. Note that the tendency for positive values of ratio 2 (actual cost minus adjusted cost divided by actual cost) to be less than positive values of ratio 1 is a logical consequence of the supplemental agreements. Recall that ratio 1 is defined as actual cost minus initial cost divided by actual cost. The actual product cost may be divided into two parts, say \( x \) and \( y \), where \( x \) and \( y \) respectively denote the actual cost of producing the initially specified product and the actual cost of the supplemental agreements. Assume that both the firm's initial and adjusted targets, \( \hat{x} \) and \( \hat{x} + \hat{y} \), are estimates of \( x \) and \( x + y \), respectively. That is, the initial estimate makes no allowance for any future supplemental agreements and, correspondingly, the estimated cost of the supplemental agreements is independent of the accuracy of the initial estimates. Then \( r_1 \), ratio 1, is given by

\[
r_1 = \frac{x+y - \hat{x}}{x+y},
\]
whereas $r_2$, ratio 2 is

$$r_2 = \frac{x+y - (\hat{x}+\hat{y})}{x+y}.$$ 

The empirical analysis will show that when ratio 1 is positive, ratio 2 tends to be smaller, that is,

$$r_1 > r_2, \text{ when } r_1 > 0.$$ 

But all that this inequality implies is

$$\frac{x+y - \hat{x}}{x+y} > \frac{x+y - (\hat{x}+\hat{y})}{x+y},$$

or

$$\hat{y} > 0.$$ 

All that is necessary for ratio 1 to exceed ratio 2 is that the estimated cost of supplemental agreements be positive.

The behavior of contracts for negative values of $r_1$ can be interpreted in the same way. A negative value of $r_1$ means simply that $\hat{x}$, the initial estimate, is larger than $x+y$, the actual cost. The empirical analyses show that contracts with negative values of ratio 1 tended to preserve or increase the negative value via supplemental agreements, that is,

$$r_1 \geq r_2, \text{ when } r_1 < 0,$$

which can be rewritten

$$\frac{x+y - \hat{x}}{x+y} \geq \frac{x+y - (\hat{x}+\hat{y})}{x+y},$$
or

\[ y \geq 0. \]

A priori, it might be expected that when ratio 1 is negative, modifications would be implemented so that the adjusted target, \( \hat{x} + \hat{y} \), would decline, i.e., \( r_2 \geq r_1 \) and \( \hat{y} \leq 0 \). As we will see, modifications like this rarely occur.

These algebraic relations between ratio 1 and ratio 2 say nothing about the effect of a changing sharing proportion. Table 4 shows this effect for the two groups of data. The effect is the same for both--the tendency for negative values of ratio 2 is an increasing function of the sharing proportion.

Table 4

<table>
<thead>
<tr>
<th>Sharing Proportion</th>
<th>Mean ( \hat{x} )</th>
<th>Variance ( S^2 )</th>
<th>Sample Size ( N_i )</th>
<th>Variance of ( \hat{x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959-1962</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \leq .15 )</td>
<td>0.73</td>
<td>186.34</td>
<td>38</td>
<td>4.90</td>
</tr>
<tr>
<td>( = .20 )</td>
<td>-1.82</td>
<td>58.95</td>
<td>74</td>
<td>0.80</td>
</tr>
<tr>
<td>( \geq .25 )</td>
<td>-12.50</td>
<td>336.02</td>
<td>14</td>
<td>24.00</td>
</tr>
<tr>
<td>1962-1963</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \leq .15 )</td>
<td>-2.01</td>
<td>161.33</td>
<td>54</td>
<td>2.99</td>
</tr>
<tr>
<td>( = .20 )</td>
<td>-3.77</td>
<td>38.67</td>
<td>53</td>
<td>0.73</td>
</tr>
<tr>
<td>( \geq .25 )</td>
<td>-5.94</td>
<td>67.19</td>
<td>19</td>
<td>3.54</td>
</tr>
</tbody>
</table>

A one-way analysis of variance was performed on both sets of data summarized in Table 4 and the results are presented in Table 5. Results for the 1959-1962 data indicate that the average of negative values of ratio 2 are significantly different (at 1 percent). For the 1962-1963
Table 5

ANALYSIS OF VARIANCE OF EACH CONTRACT GROUP
(Positive and Negative Values of Ratio 2)

<table>
<thead>
<tr>
<th>Variation</th>
<th>SSD</th>
<th>df</th>
<th>Mean Square</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1959-1962</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Between Groups</td>
<td>1,822.67</td>
<td>2</td>
<td>911.34</td>
<td>F(2124) = 7.259</td>
</tr>
<tr>
<td>Within Groups</td>
<td>15,566.24</td>
<td>124</td>
<td>125.53</td>
<td>Significant at 1%</td>
</tr>
<tr>
<td></td>
<td>1962-1963</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Between Groups</td>
<td>255.19</td>
<td>2</td>
<td>112.59</td>
<td>F(2124) = 1.186</td>
</tr>
<tr>
<td>Within Groups</td>
<td>11,771.02</td>
<td>124</td>
<td>94.93</td>
<td>Significant at 25%</td>
</tr>
</tbody>
</table>

data, the average of negative values of ratio 2 was not different at any reasonable level of significance.

Table 6 shows the proportion of negative values of ratio 2 as a function of the sharing proportion. Again the two sets of data yield similar results. The proportion of negative values is an increasing function of the sharing proportion. And in both, the intermediate and
highest sharing proportions have proportions of negative values that are significantly (at 5 percent) greater than 0.5.

THE INFLUENCE OF CONTRACT SIZE

The analysis of both initial and adjusted target costs has not measured the influence of contract size on the accuracy of the estimates. Using final cost as a measure of contract size, Table 7 presents the average contract size for each sharing proportion group for both sets of data. For the 1959-1962 group this tabulation suggests that the tendency for positive values of ratios 1 and 2 is a decreasing

<table>
<thead>
<tr>
<th>Sharing Proportion</th>
<th>Positive Values of Ratio 1</th>
<th>Positive Values of Ratio 2</th>
<th>Average Size ($ millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959-1962</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>≤ .15</td>
<td>5.06</td>
<td>0.73</td>
<td>36.31</td>
</tr>
<tr>
<td>= .20</td>
<td>1.33</td>
<td>-1.82</td>
<td>55.43</td>
</tr>
<tr>
<td>≥ .25</td>
<td>2.78</td>
<td>12.50</td>
<td>82.03</td>
</tr>
<tr>
<td>1962-1963</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>≤ .15</td>
<td>0.35</td>
<td>-2.01</td>
<td>14.59</td>
</tr>
<tr>
<td>= .20</td>
<td>0.52</td>
<td>-3.77</td>
<td>32.50</td>
</tr>
<tr>
<td>≥ .25</td>
<td>3.37</td>
<td>-5.94</td>
<td>26.21</td>
</tr>
</tbody>
</table>

function of contract size, the larger average sizes being associated with large negative values. Closer examination, however, reveals that this association is illusory. First, a nonparametric test is unable to reject (at 10 percent) the null hypothesis that the sample of size observations in the 0.25 to 0.50 class is drawn from the same population as the sample of observations in the 0.025 to 0.15 class. More importantly, when the sharing proportion is held constant, large contracts tend to exhibit large positive values of ratios 1 and 2. This
relation is obtained by calculating the Spearman rank correlation coefficient between contract size and positive values of the ratios for each sharing proportion group. The ensuing correlation coefficients are enumerated in Table 8 for both initial and adjusted estimates. The coefficients indicate a generally weak but persistent positive relationship between contract size and positive values of the ratios.

Table 8
CORRELATIONS BETWEEN RATIO 1 AND RATIO 2
AND CONTRACT SIZE

<table>
<thead>
<tr>
<th>Sharing Proportion</th>
<th>Initial Correlation</th>
<th>Adjusted Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959-1962</td>
<td></td>
<td></td>
</tr>
<tr>
<td>≤ .15</td>
<td>.14^a</td>
<td>.05^a</td>
</tr>
<tr>
<td>= .20</td>
<td>.18^b</td>
<td>.13^a</td>
</tr>
<tr>
<td>≥ .25</td>
<td>.34^a</td>
<td>.17^a</td>
</tr>
<tr>
<td>1962-1963</td>
<td></td>
<td></td>
</tr>
<tr>
<td>≤ .15</td>
<td>.02^a</td>
<td>-.02^a</td>
</tr>
<tr>
<td>= .20</td>
<td>.02^a</td>
<td>-.09^a</td>
</tr>
<tr>
<td>≥ .25</td>
<td>.26^a</td>
<td>-.36^b</td>
</tr>
</tbody>
</table>

^a Not significantly different from zero.
^b Significantly different at 10 percent.

For the 1962-1963 data, Table 7 tabulations suggest that there is not even an apparent relationship between contract size and positive values of the ratios. Moreover, the Spearman rank correlation coefficients between contract size and positive ratios exhibit no consistent tendency for positive values. In Table 8, for two cases there is a negative correlation between contract size and ratio 2. However, only one of the observed correlations is significant at the 10-percent level.
RELATION BETWEEN INITIAL AND ADJUSTED COST ESTIMATES

When it appears likely that actual costs will exceed initial cost estimates (the firm is incurring losses relative to the initial cost estimate), firms will probably make this fact known to the Government when estimating adjusted costs. Efforts will be made to increase the adjusted estimate above the initial estimate. Indeed, extraordinary efforts should be observed when the sum of the sharing and target proportions is large. For as this sum increases, the rewards associated with the alternative activities of the firm's management become relatively inferior to those of campaigning for high adjusted costs.

Table 9 attempts to measure this effect. Column 2 presents a measure of the relative amount by which adjusted costs exceed initial costs when actual costs exceed initial costs. Column 3 shows the relative

Table 9

<table>
<thead>
<tr>
<th>Sharing Proportion</th>
<th>D1 + D2</th>
<th>Actual Costs Greater Than Initial Estimate</th>
<th>Actual Costs Less Than Initial Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959-1962</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>≤ .15</td>
<td>0.55</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>= .20</td>
<td>0.99</td>
<td>0.57</td>
<td></td>
</tr>
<tr>
<td>≥ .25</td>
<td>1.43</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>1962-1963</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>≤ .15</td>
<td>0.44</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>= .20</td>
<td>1.08</td>
<td>-0.10</td>
<td></td>
</tr>
<tr>
<td>≥ .25</td>
<td>1.58</td>
<td>0.22</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: D1 (adjusted costs - initial costs) + D2 (actual costs - initial costs).
amount by which initial costs exceed adjusted cost when initial costs exceed actual costs. Both calculations are made for each type of incentive contract. For each class of contracts, the percentage movement toward actual costs is substantially greater when actual costs exceed initial costs than when initial costs exceed actual costs. For the 1962-1963 data, it appears from Column 2 that the pressure the firm exerts is an increasing function of the sharing proportion when actual costs exceed initial estimates. Column 3 reveals that the Government does not respond to the sharing proportion in any systematic fashion when actual costs are less than initial estimates.

**CONTRACT GROWTH**

The influence sharing proportion has on contract growth will now be measured. The adjusted cost estimate includes the costs of supplemental agreements; hence a relative measure of contract growth is adjusted cost minus initial cost divided by adjusted cost. Assume that the firm's behavior is guided by purely economic considerations. Then contracts with high sharing proportions are likely to grow more than contracts with low sharing proportions.

Table 10 shows the relationship between contract growth and sharing proportion for both sets of data. For 1959-1962, the most growth occurs for the contracts with the highest sharing proportion. This confirms the purely economic hypothesis. However, contracts with the lowest sharing proportion grow more than those with a 20-percent proportion. The 1962-1963 data give unambiguous support to the purely economic hypothesis; contract growth is positively related to sharing.
proportion. It may be tentatively concluded that between the two time periods, differences occurred among firms having contracts with a 20-percent sharing proportion.

Table 10

RELATIONSHIP BETWEEN CONTRACT GROWTH AND SHARING PROPORTION FOR BOTH SETS OF DATA

<table>
<thead>
<tr>
<th>Sharing Proportion</th>
<th>Relative Contract Growth\textsuperscript{a}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1959-1962</td>
</tr>
<tr>
<td>.05</td>
<td>4.36</td>
</tr>
<tr>
<td>.20</td>
<td>0.48</td>
</tr>
<tr>
<td>.25</td>
<td>8.64</td>
</tr>
</tbody>
</table>

\textsuperscript{a}The ratio of adjusted minus initial to adjusted expressed as a percentage in the table can be calculated directly from \( r_1 \) and \( r_2 \). The percentage contract growth, \( G \), is given by:

\[
G = \frac{\text{adjusted} - \text{initial}}{\text{adjusted}} \times 100
\]

\[
= (r_1 - r_2)(1 - r_2)^{-1}(100).
\]
IV. CONCLUSIONS

This Memorandum discussed two different aspects of incentive contracting. The first was a presentation of several interpretative problems that arise in implementing incentive contracts. One of the main difficulties is predicting both the efficiency and risk effects of different sharing arrangements. These effects are present and intertwined in all incentive contracts. Data that are currently collected are not sufficient for distinguishing between risk and incentive effects. Multiple incentive contracts, where incentives are awarded not only to low costs but also to prompt delivery and high performance, are plagued by similar difficulties.

The second part of the Memorandum presented an empirical analysis of 252 incentive contracts, 126 contracts having been completed during 1959-1962, and 126 completed in 1962-1963. The purpose was to explore the relationship between sharing rates and different aspects of contract cost, with a view to testing some of the economic hypotheses developed in the first part. The results of these tests are mixed, with some of the results tending to support our hypotheses, and others rejecting them. Thus, for the 1962-1963 data, we could find no statistical evidence to suggest that the contracts with higher sharing rates tended to be either more efficient or more risky, in that the relative difference between actual and target costs were not significantly different for the higher and lower sharing rates. On the other hand, we did find that the difference between actual and target costs for the contracts during 1959-1962 become increasingly negative with increasing sharing rate which is consistent with the risk-efficiency
hypothesis. Other differences in outcome were also noted between the two time periods, possibly suggesting some difference in governmental and contractor behavior with experience in incentive-type contracts.

The implication of this study is that the factors influencing contract outcomes are very complex, and that a study utilizing more complex economic models and richer sources of contract data may be required to secure better agreement between theory and experience. Considering the large administrative effort now expended on contract negotiation and administration, as well as the significance of the product and its cost to our national security, it behooves us to undertake such further investigations.
APPENDIX

This Appendix calculates a lower bound on the bid a vendor submits for an incentive type contract. If the contracts are let under competitive bidding the bids will tend to be equal to this lower bound. The vendor is assumed to have a constant attitude toward risk; that is, the risk premium required for a particular risk is independent of the firm's wealth. The firm may have constant risk aversion, constant risk indifference, or constant risk preference. Bids that firms submit are conditioned by the alternative opportunities available to them. In particular, it is assumed that the bid is calculated so that the expected utility of the incentive contract is no less than the expected utility of the best alternative opportunity. A lower bound on the bid is obtained by equating these expected utilities. The cost, $C$, of producing for the incentive contract is a random variable with an arbitrary probability distribution. Similarly, the profits, $\Pi$, obtainable on the best alternative opportunity are also a random variable with an arbitrary probability distribution.

CONSTANT RISK AVERSION

When the firm has constant risk aversion its utility function with respect to profits, $\Pi$, may be denoted by

$$u_1(\Pi) = a_1 - b_1 e^{-\rho \Pi}, \quad \rho > 0,$$
where \( \rho \) measures risk aversion, i.e., the risk aversion function is simply

\[
\rho(\Pi) = -\frac{u''_1}{u'_1} = \rho.
\]

Profits, \( \Pi_2 \), from the incentive contract are

\[
\Pi_2 = \alpha p + \beta(p-C), \quad 0 \leq \alpha < 1, \quad 0 < \beta < 1,
\]

where \( \alpha \) is the target proportion, \( p \) is the bid or target cost, \( \beta \) is the sharing proportion, and \( C \) denotes the actual costs. If \( \Pi_1 \) is opportunity profits, then both are random variables with arbitrary cumulative distribution functions (c.d.f.), say \( F_1 \) and \( F_2 \), respectively.

A lower bound on the bid, \( p \), submitted by the firm is obtained by equating the expected value of \( u_1(\Pi_1) \) with the expected value of \( u_1(\Pi_2) \) and solving for \( p \), where the first expectation is with respect to \( \Pi_1 \) and the second expectation is with respect to \( C \). Symbolically,

\[
E_{\Pi_1} u_1(\Pi_1) = a_1 - b_1 E_{\Pi_1} e^{-\rho\Pi_1},
\]

\[
E_C u_1(\Pi_2) = a_1 - b_1 E_C e^{-\rho(\alpha p + \beta(p-C))}.
\]

Equating \( E_{\Pi_1} u_1(\Pi_1) \) and \( E_C u_1(\Pi_2) \) and solving for \( p \) yields

\[
p = \frac{\ln \psi(S_2) - \ln \psi(S_1)}{\rho(\alpha + \beta)},
\]

where \( \psi(S_1) \) denotes the moment generating function of \( F_1 \) (the c.d.f. of \( \Pi_1 \)), \( \psi(S_2) \) denotes the moment generating function of \( F_2 \) (the c.d.f. of \( C \)), \( S_1 = -\rho \), and \( S_2 = \rho \beta \). For example, if \( F_1 \) and \( F_2 \) are normal with mean and variance, \( m_1, \sigma_1^2 \), and \( m_2, \sigma_2^2 \), respectively, then

\[
\psi(S_1) = e^{S_1 m_1 + \frac{1}{2} S_1^2 \sigma_1^2} = e^{-\rho m_1 + \frac{1}{2} \rho^2 \sigma_1^2}
\]

and

\[
\psi(S_2) = e^{S_2 m_2 + \frac{1}{2} S_2^2 \sigma_2^2} = e^{\rho \beta m_2 + \frac{1}{2} \rho^2 \beta \sigma_2^2}.
\]

It follows that *

\[
p = \begin{cases} 
\frac{m_1 + \beta m_2 + \frac{1}{2} \beta [\beta \sigma_2^2 - \sigma_1^2]}{\alpha + \beta}, & m_1 > 0 \\
\frac{\beta m_2 + \frac{1}{2} \beta [\beta \sigma_2^2 - \sigma_1^2]}{\alpha + \beta}, & m_1 < 0,
\end{cases}
\]

where \( \frac{1}{2} \beta (\beta \sigma_2^2 - \sigma_1^2) \) can be interpreted as the risk premium. Notice that when \( \beta = \sigma_1/\sigma_2 \), the risk on the incentive contract is identical to that of its alternative opportunity and the risk premium is zero. The risk premium is positive when \( \beta > \sigma_1/\sigma_2 \) and negative when \( \beta < \sigma_1/\sigma_2 \).

*See McCall, op. cit. It is assumed that if expected alternative profits are negative, the firm will not produce and will reduce its expected profits to zero.*
Also, assuming $m_1 > 0$, the bid for a fixed-price contract ($\beta = 1$) with $\alpha > 0$ is given by
\[
p = m_1 + m_2 + \frac{1}{\alpha}(\sigma_2^2 - \sigma_1^2).
\]

Similarly, the bid for a cost-plus-fixed-fee contract ($\beta = 0$) with $\alpha > 0$ is given by
\[
p = \frac{m_1 - \frac{1}{\alpha}\sigma_1^2}{\sigma},
\]
which, of course, may be negative.

**CONSTANT RISK INDIFFERENCE**

For constant risk indifference,
\[
\mathbb{E}u_2(\Pi) = a_2 + b_2 \mathbb{E} \Pi
\]
and
\[
\mathbb{E}u_2(\Pi) = a_2 + b_2 \mathbb{E} \Pi.
\]

It follows that a lower bound on the bid the firm will submit is
\[
p = \begin{cases} 
\frac{\mathbb{E}\Pi - \beta E}{\alpha + \beta}, & \mathbb{E}\Pi > 0 \\
\frac{\beta E}{\alpha + \beta}, & \mathbb{E}\Pi < 0.
\end{cases}
\]
CONSTANT RISK PREFERENCE

Finally, if the firm's utility function exhibits constant risk preference,

\[ u_3(\Pi) = a_3 + b_3 e^{\rho \Pi}, \rho > 0 \]

\[ E_{\Pi_1} u_3(\Pi_1) = a_3 + b_3 E_{\Pi_1} e^{\rho \Pi_1} \]

\[ E_{\Psi} u_3(\Pi_2) = a_3 + b_3 E_{\Psi} e^{\rho(\alpha \Psi + \beta (\Psi - C))} \]

and

\[ p = \frac{\ln \Psi(S_1) - \ln \Psi(S_2)}{\rho(\alpha + \beta)} \]

where \( S_1 = \rho \), and \( S_2 = \rho \beta \). For example, if \( F_1 \) and \( F_2 \) are normal with mean and variance, \( m_1, \sigma_1^2 \), and \( m_2, \sigma_2^2 \), respectively, then

\[ \Psi(S_1) = e^{\rho m_1 + \frac{1}{2} \rho \sigma_1^2} \]

and

\[ \Psi(S_2) = e^{-\rho \beta m_2 + \frac{1}{2} \rho \beta \sigma_2^2} \]

It follows that a lower bound on the bid this risk preferent firm submitted is given by

\[ p = \begin{cases} 
\frac{m_1 + \beta m_2 + \frac{1}{2} \rho \sigma_1^2 \left[ \frac{1}{2} \beta \sigma_2^2 \right]}{\alpha + \beta}, & m_1 > 0 \\
\frac{\beta m_2 + \frac{1}{2} \rho \sigma_1^2 \left[ \frac{1}{2} \beta \sigma_2^2 \right]}{\alpha + \beta}, & m_1 < 0.
\end{cases} \]