MEMORANDUM
RM-5323-PR
JULY 1967

THE CONSOLIDATION OF MAINTENANCE DURATIONS WITH APPLICATIONS TO AIRCRAFT SORTIES

N. Kaufman, S. H. Miller and H. J. Shukiar

PREPARED FOR:

UNITED STATES AIR FORCE PROJECT RAND

The RAND Corporation
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This research is supported by the United States Air Force under Project RAND—Contract No. F44620-67-C-0045—monitored by the Directorate of Operational Requirements and Development Plans, Deputy Chief of Staff, Research and Development, Hq USAF. Views or conclusions contained in this Memorandum should not be interpreted as representing the official opinion or policy of the United States Air Force.

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The RAND Corporation
1700 MAIN ST. • SANTA MONICA • CALIFORNIA • 90406

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Published by The RAND Corporation

PREFACE

This Memorandum is a RAND-initiated final report on a subtopic of maintenance information analysis. It describes in detail two computerized methods, CONCUR and CONVOL, that can be used to consolidate maintenance duration information. These methods are useful in preparing input for simulations, and enable considerable savings in computer time and space; in fact, certain simulations that were infeasible before may now be performed. Also, the output from CONCUR and CONVOL may be used as a simplifying input to analytic sortie-generation models.

The study is directed principally toward military operations research analysts who are interested in implementing maintenance information analysis. An understanding of elementary probability theory is necessary for thorough comprehension of the material herein.

SUMMARY

This Memorandum presents the details of two programmed techniques, CONCUR and CONVOL, that are being used to consolidate maintenance durations. Additionally, example applications of the techniques to aircraft systems are given.

CONCUR is used to determine statistical properties of the longest time required among a group of tasks that begin at the same time and proceed together (concurrently), i.e., the time to complete all tasks. CONVOL deals with the total time required to complete a set of tasks that occur one after the other (sequentially); again, the concern is with the time to complete all tasks. Used together, CONCUR and CONVOL allow probability distributions of times required for distinct maintenance actions to be compressed into a single distribution corresponding to the total down time for the system. In addition to revealing characteristics of the total down time (e.g., mean and variance), the consolidated distribution provides a compact input to sortie generation, simulation and analytic models.

This Memorandum is directed primarily toward those who may have pragmatic interests in the potential applications of the methods. Therefore, emphasis is on pertinent examples and the details of various calculations. The mathematical basis of this study rests upon two well established concepts from mathematical statistics: (1) the distribution of extreme values, and (2) the distribution of the sum of random variables. It is assumed throughout this Memorandum that the lengths of time required to perform distinct maintenance activities are statistically independent random variables.

ACKNOWLEDGMENTS

We gratefully acknowledge the assistance and suggestions provided by E. M. Scheuer and T. C. Smith.

CONTENTS

I. INTRODUCTION

The Office of the Secretary of Defense (OSD) and Headquarters USAF are interested in understanding the peacetime and wartime capabilities of weapon and support systems such as those for tactical fighter and cargo aircraft. To acquire a better understanding of unit capability, OSD recently directed several system studies. Among these was the "TAC Enhancement Study," completed by the Tactical Air Command with RAND participation.

The study examined three tactical fighter systems--the A-7, F-4 and F-111. It was conducted to find the mix of operational and maintenance policies and resources that would "enhance" future Tactical Air Command fighter units by giving them the capability to fly more combat sorties per aircraft per month. The cost, in terms of total obligational authority necessary to implement changes to the approved force in the USAF Force and Financial Plan, was estimated for each proposal so that the relative cost of enhancement could be weighed against the desired benefits. Study constraints were the total number of fighter unit equipment (UE), the number of fighter UE per squadron, and the number of squadrons per wing. SAMSOM II, a RAND-developed Monte Carlo simulation model, was used to simulate operations and maintenance interactions and to yield sortie capabilities and associated resource requirements and imbalances as different operational and maintenance policies were changed and analyzed.

This Memorandum describes some straightforward and relatively simple analytical concepts that were used as an adjunct to SAMSOM II in the TAC Enhancement Study. SAMSOM II will be used as the simulation model for Project Combat Sample,* and there may be other studies that will also use it. The concepts can be used by themselves to provide answers to questions dealing with the development of better maintenance

*Part of a TAC study "oriented toward demonstrating basic relationships between resources, missions, operational concepts and capability [of TAC airlift]." Hq TAC, "Condensed Data Collection Plan for Combat Sample," March 7, 1967.

and operational policies and/or mixes of maintenance and operational resources for existing (or future) weapon and support systems.

Section II describes the two analytical concepts, CONCUR and CONVOL. CONCUR is used for maintenance tasks that are started concurrently; CONVOL is used for maintenance tasks that are performed sequentially. Section III explains the basic probability considerations used to develop the concepts. SAMSOM II is discussed in Sec. IV. Section V deals with applications of CONCUR and CONVOL to maintenance planning factors; Sec. VI does the same with operational planning factors. Sections VII and VIII describe in detail the computer programs developed to implement these concepts. The programs are written in FORTRAN IV and produce results in a relatively short time.

This Memorandum is directed primarily toward those who may have pragmatic interests in the potential applications of the methods. Therefore, emphasis is on pertinent examples and the details of various calculations. The mathematical basis of this Memorandum rests upon two well-established concepts from mathematical statistics [2]: (1) the distribution of extreme values, and (2) the distribution of the sum of random variables. In both cases it is assumed that distinct random variables are mutually independent.

II. CONCUR AND CONVOL CONCEPTS

This section describes two well known analytical concepts that we call CONCUR and CONVOL. The authors discovered neither technique. Therefore each concept is named not to highlight new developments but rather to distinguish between the two and to make referencing easier. Both concepts, the techniques for applying them, and their computer programs are relatively easy to understand and use, and appear to have wide application.

Maintenance on aircraft (or other) systems usually consists of two kinds of tasks: (1) those that may be started at the same time and (2) those that, for safety or other reasons, must wait either for other tasks to be completed or for some time to elapse after other tasks have been started. Thus it is possible to have several groups of maintenance tasks of type two, where the first group must be completed before the second group starts, the second group completed before the third group starts, etc. For example, troubleshooting tasks precede unscheduled maintenance tasks which precede functional check maintenance tasks; each group is done in sequence and each except the first group waits its turn after an aircraft lands that requires attention.

CONCUR deals with the first kind of maintenance tasks (those that start at the same time), and is a technique for determining the distribution of time to complete all tasks. From this distribution it is possible to find the mean time (and other quantities, such as variance, etc.) to complete the longest task when all of them start together. CONVOL deals with the second kind of task, and is a technique for finding the distribution of time to complete tasks (or groups of tasks) that are performed sequentially. Again, the mean, etc. can be found from the distribution.

In addition to computing and printing the distribution of time to complete all tasks and the average or expected time for completing the tasks, the computer programs for CONCUR and CONVOL also provide for computing and printing the variance and standard deviation. For example, suppose after an aircraft lands that maintenance crews may

have to perform two troubleshooting tasks, three unscheduled maintenance tasks and one functional check task with characteristics as follows:

Troubleshooting

Task 1. Probability equals 1/2 that task takes 1 hour
Probability equals 1/2 that task takes 2 hours

Task 2. Probability equals 1/3 that task is not required
Probability equals 1/3 that task takes 1 hour
Probability equals 1/3 that task takes 2 hours

Unscheduled Maintenance

Task 1. Probability equals 1/2 that task takes 1 hour
Probability equals 1/4 that task takes 2 hours
Probability equals 1/4 that task takes 3 hours.

Task 2. Probability equals 1/2 that task takes 1 hour
Probability equals 1/2 that task takes 2 hours

Task 3. Probability equals 1/2 that task is not required
Probability equals 1/2 that task takes 1 hour

Functional Check

Task 1. Probability equals 1 that task takes 1 hour.

First,* to find the expected time to complete all tasks in each group, we would apply CONCUR to the tasks in the troubleshooting and unscheduled groups, respectively. CONCUR would not be required for functional checks since there is only one task in this group and it always takes an hour. For this elementary example, CONCUR produces the following results:

Troubleshooting

Expected value = 1.67 hours
Variance = 0.2 hours
Standard deviation = 0.447 hours
Cumulative distribution of time to complete troubleshooting tasks:
33.32 percent will be completed in 1 hour
100 percent will be completed in 2 hours or less

Unscheduled Maintenance

Expected value = 2 hours
Variance = 0.5 hours

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*In this example, as in most of the discussion, we assume no resource constraints, i.e., resources are available as required and no queues arise due to lack of maintenance resources.

Standard deviation = 0.71 hours
Cumulative distribution of time to complete unscheduled maintenance tasks:

25 percent will be completed in 1 hour
75 percent will be completed in 2 hours or less
100 percent will be completed in 3 hours or less

In the next section, we will explain the probabilistic concepts that produce these results. If we examine the troubleshooting and unscheduled maintenance tasks, however, the CONCUR results appear reasonable. For example, troubleshooting Task 1 always must be done after each aircraft lands, regardless of whether troubleshooting Task 2 is required or not. Since Task 1 requires a minimum of an hour, troubleshooting tasks will take at least an hour to complete. Since Tasks 1 and 2 never take longer than 2 hours in our hypothetical example, all aircraft will have their troubleshooting tasks completed within 2 hours after troubleshooting starts. A simple analysis shows that both the earliest and latest completion times of 1 and 2 hours, respectively, are consistent with the CONCUR troubleshooting results shown above. The mean time for completing all troubleshooting tasks is 1.7 hours, which lies between the shortest and longest troubleshooting task, as it should.

Similarly, for unscheduled maintenance we see that Tasks 1 and 2 each take a minimum of an hour. Therefore unscheduled maintenance cannot take less than an hour. Since unscheduled maintenance Task 1 may require 3 hours and no other task in this group could take longer, unscheduled maintenance cannot take longer than 3 hours since all of the tasks start at the same time, i.e., at the conclusion of troubleshooting. Again, analysis of the earliest and latest completion times for unscheduled maintenance is consistent with the times shown in CONCUR for unscheduled maintenance. The mean time for completing all unscheduled maintenance tasks is 2 hours--between the shortest time (1 hour) and longest time (3 hours), as it should be.

Second, in order to find the total elapsed time to complete all maintenance tasks (i.e., troubleshooting followed by unscheduled maintenance followed by functional check tasks), we would take the CONCUR results for troubleshooting and unscheduled maintenance and

use them with the functional check task data as inputs to CONVOL. CONVOL gives the following results for our elementary example:

Troubleshooting + Unscheduled Maintenance + Functional Check

Expected value = 4.67 hours

Variance = 0.72 hours

Standard deviation = 0.85 hours

8.33 percent of aircraft will have all maintenance required after flight completed in 3 hours.

41.66 percent of aircraft will have all maintenance required after flight completed in 4 hours or less.

83.33 percent of aircraft will have all maintenance required after flight completed in 5 hours or less.

100 percent of aircraft will have all maintenance required after flight completed in 6 hours or less.

If we consider (from the CONCUR results) that 1 hour each is the shortest time for completing each of troubleshooting and unscheduled maintenance, and functional check always takes an hour, then it can be seen that the sequence of troubleshooting, unscheduled maintenance and functional check cannot be completed in less than 3 hours. In the longest case, troubleshooting will take 2 hours, unscheduled maintenance 3 hours, and functional check 1 hour, for a total of 6 hours. The mean time (expected value) for all maintenance to be completed on an aircraft (4.67 hours) is between the shortest and longest maintenance times. Thus, in our example, CONVOL results are consistent with intuition.

III. PROBABILISTIC BASIS

In this section, we explain the basic probability considerations that were used to develop CONCUR and CONVOL. We shall assume that there are no resource constraints, i.e., the maintenance personnel, facilities, aerospace ground equipment (AGE), and parts required for each maintenance task are available when and where needed, and in the quantities required. This eliminates consideration of waiting times (i.e., queues) from the discussion that follows. We remind the reader that CONCUR and CONVOL may be considered partners to SAMSOM II, which is a powerful simulation tool to investigate the interaction of operations and maintenance (plus other support) actions and the effect of maintenance resources (or lack of maintenance resources) on waiting times. Therefore, except for a brief discussion of SAMSOM II in Sec. IV, relatively little attention will be given to the subject of queues and only in the case where resource availability is assured will SAMSOM II be discussed in any detail. Also, we shall consider first the single aircraft case. That is, we wish to determine the probability that a group or set of groups of maintenance tasks will be completed by or prior to a certain time after maintenance starts on an aircraft. When there are no resource constraints, most conclusions which apply to single aircraft can also be applied directly to n aircraft. This is generally not true when resource constraints exist.

CONCUR

We will now discuss how CONCUR functions when an aircraft requires maintenance after a sortie. When maintenance actions are required, start simultaneously and can be worked in parallel, consider k potential tasks. Let E_i denote that the ith task is required and $\bar{E}_i$ that it is not required. It is assumed that events $E_1, \ldots, E_k$ are mutually independent in a probability* sense. Associated with the

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*A more complete discussion of probability concepts may be found in Parzen [5], Feller [3], or other probability texts.

ith task is the probability, p_i, that E_i occurs and the probability, q_i, that $\bar{E}_i$ occurs, where $p_i + q_i = 1$. Let X_i be the time needed to complete task i; of course, if $\bar{E}_i$ occurs then $X_i = 0$. Let $G_i(x) =$ Prob $(X_i \leq x)$, the cumulative distribution function, and let $g_i(x)$ be the probability (density function) that it takes exactly x hours to complete task i. It is assumed here that X_i can take on only a finite set of values, $x_{i,1} < x_{i,2}, \ldots, < x_{i,m_i}$.

$F_i(x|E_i)$ is the probability that the ith task will be finished in x hours or sooner, given that event E_i occurs (i.e., given that task i is required). Since the time necessary to complete a task cannot be negative, $x \geq 0$. Similarly, $F(x|\bar{E}_i)$ is defined as the probability that task i will not take more than x hours, given $\bar{E}_i$ (i.e., given that task i is not required). Of course, in this case $F_i(x|\bar{E}_i) \equiv 1$ for all $x \geq 0$, since if task i is not required zero time is expended upon it. It is assumed that the maintenance times, $X_1, \ldots, X_k$, are mutually independent random variables. It is at least questionable whether the assumptions of mutual independence among the E_i's and mutual independence among the X_i's can be justified; however, these assumptions are part of the ground rules for this study. These assumptions may be realistic for many of the maintenance tasks; for some others, even imperfect data regarding statistical dependence are difficult to come by. Additionally, if dependence among certain tasks is specified, it may be possible to define certain joint events which, in conjunction with the remaining events, can be treated as mutually independent.

It is assumed that every task can be finished in finite time and no task in zero time; thus there is a time value, $X_i^* = \max x_{ij}$, such that $F_i(X_i^*|E_i) = 1$, and also that $F_i(0|E_i) = 0$. Furthermore $F_i(X_i^*|\bar{E}_i) = F_i(0|\bar{E}_i) = 1$, since $F_i(x|\bar{E}_i) \equiv 1$ for all $x \geq 0$.

Now task i either is or is not required when an aircraft lands. Therefore, events E_i and $\bar{E}_i$ are mutually exclusive and exhaustive, and therefore complementary events. The unconditional joint probability that task i will be required and that task i will be completed in x hours, or less, is given by the product,

$$p_i F_i(x|E_i).$$

The unconditional joint probability that task i will not be required and that task i will be completed in x hours, or less, is given by

$$q_i F_i(x|\bar{E}_i) = q_i \quad \text{for all } x \geq 0,$$

since

$$F_i(x|\bar{E}_i) \equiv 1 \quad \text{for all } x \geq 0.$$

$G_i(x)$, the unconditional probability that task i takes x hours or less to complete, may be obtained by summing the probabilities of the only two ways that this may occur, viz.,

$$G_i(x) = q_i F_i(x|\bar{E}_i) + p_i F_i(x|\bar{E}_i) = q_i + p_i F_i(x|E_i).$$

It follows that

$$G_i(X_i^*) = 1$$

and

$$G_i(0) = q_i.$$

Let T denote the time required to complete all maintenance tasks. Thus T equals the largest X_i over all tasks; i.e.,

$$T = \max\ (X_1, \ldots, X_k).$$

Due to independence the probability, $P(T \leq t)$, that all tasks (1 through k inclusive) will be completed by t hours or sooner is:

$$P(T \le t) = P[\max (X_1, \ldots, X_k) \le t]$$

$$= P(X_1 \le t, \ldots, X_k \le t)$$

$$= \prod_{i=1}^{k} G_i(t) = \prod_{i=1}^{k} [q_i + p_i F_i(t|E_i)];$$

(1) $$P(T \le t) = \prod_{i=1}^{k} [q_i + p_i F_i(t|E_i)] = G(t).$$

It follows that

$$G[\max (X_i^*)] = 1,$$

and that

$$G(0) = \prod_{i=1}^{k} q_i .$$

CONCUR's output includes the ordered values of $G(t)$ evaluated at all the distinct values that the X_i's ($i = 1, 2, \ldots, k$) may take on.

The probability that no maintenance at all will be required for any task is the product

(2) $$\prod_{i=1}^{k} q_i = Q.$$

Therefore the probability, P, that at least one of the maintenance tasks will be required is

(3) $$P = 1 - Q = 1 - \prod_{i=1}^{k} q_i .$$

We see, from formula (1) (and from its derivation) that the probability, $G(t)$, that all tasks will be completed in t hours or sooner is an unconditional probability. It is the sum of the probabilities of two mutually exclusive and exhaustive events: (1) completing all tasks in t hours or sooner, and maintenance is required for at least one of the k maintenance tasks, and (2) completing all tasks in t hours or less, because no maintenance is required for any task. But from formula (2), the probability of no maintenance is Q. Therefore, we can get the first probability by subtraction,

Prob (that at least one task is required <u>and</u> all maintenance is completed within t hours)

(4) $$= G(t) - Q = G(t) - \Pi\, q_i .$$

We know from probability theory, that for events A and B,

(5) $$P(A|B) = P(A \text{ and } B)/P(B).$$

That is, the conditional probability of event A, given that event B has occurred, equals the probability that A and B occur divided by the probability that B occurs. We now have all of the tools needed to answer a question of some interest: given that maintenance is required, what is the conditional probability, $C(t)$, that all tasks (i.e., longest job) will be finished prior to t hours after maintenance starts? Using formulas (1), (4), and (5) we get

(6) $$C(t) = \frac{G(t) - Q}{P} = \frac{\prod_{i=1}^{k} [q_i + p_i F_i(t|E_i)] - \prod_{i=1}^{k} q_i}{1 - \prod_{i=1}^{k} q_i} ,$$

where in formula (6)

$C(t)$, $[G(t) - Q]$, and P

correspond, respectively, to

$$P(A|B),\ P(A \text{ \underline{and} } B),\ \text{and } P(B).$$

Because of the discrete nature of the inputs, the cumulative distribution function is a step function and

$$F_i(t|E_i) = F_i(x_{i,j}|E_i) \quad \text{for } x_{i,j} \le t < x_{i,j+1},$$

and $j = 1, 2, \ldots, m_{i-1}$.

We now order all $x_{i,j}$ $i = 1, \ldots, k$, $j = 1, 2, \ldots, m_i$ from smallest to largest maintenance times (it is possible for two or more tasks to take the same length of time to completion) and relabel the distinct completion times, $x_{i,j}$, as y_ℓ, $\ell = 1, 2, \ldots, N$, where

$$0 \le y_1 < y_2 < \ldots < y_N = \max_{i,j} x_{i,j},$$

$$i = 1, 2, \ldots, k; \qquad j = 1, 2, \ldots, m_i.$$

We define the probability

$$g(y_\ell), \qquad \ell = 1, 2, \ldots, N$$

that all tasks in the group will be completed in exactly y_ℓ hours (this happens when the longest task will take exactly y_ℓ hours) as follows

$$g(y_\ell) = \begin{cases} G(y_\ell), & \ell = 1 \\ G(y_\ell) - G(y_{\ell-1}), & \ell = 2, \ldots, N. \end{cases}$$

Here also, we see that we have a probability density, $g(t)$, and a cumulative distribution function, $G(t)$, where the latter is also a step function and

$$G(t) = G(y_\ell) \quad \text{for } y_\ell \leq t < y_{\ell+1}, \qquad \ell = 1, 2, \ldots, N - 1.$$

We shall denote the random variable with density $g(y_\ell)$ by Y.

The expected value of the time (unconditional) to complete all the tasks, $E_U(t)$, is given by

$$E_U(t) = \int_0^\infty t \, dG(t),$$

which reduces to

$$E_U(t) = \sum_{\ell=1}^{N} y_\ell g(y_\ell) = E(Y)$$

because G(t) is a step function. The variance of time (unconditional) to complete all tasks is

$$E_U[t - E_U(t)]^2 = \sum_{\ell=1}^{N} y_\ell^2 g(y_\ell) - E_U^2(t) = \text{Var }(Y).$$

The standard deviation, SD(Y), is given by $SD(Y) = \sqrt{\text{Var }(Y)}$.

Now* $C(t) = aG(t) - b$, where

$$a = \frac{1}{1 - Q}$$

and

$$b = \frac{Q}{1 - Q}.$$

*See formula (6).

Therefore the expected value of the time (conditional) to complete all tasks, given that at least one task is required is

$$E_C(t) = \int_0^\infty t\ dC(t) = \int_0^\infty t\ d[aG(t) - b] = aE_U(t).$$

The variance of the time (conditional) to complete all tasks is

$$E_C[t - E_C(t)]^2 = E_C(t^2) - E_C^2(t) = \left[\int_0^\infty t^2\ dC(t)\right] - a^2E_U^2(t)$$

$$= \left\{\int_0^\infty t^2\ d[aG(t) - b]\right\} - a^2E_U^2(t),$$

Now, since $d[aG(t) - b] = adG(t)$, and

$$E_U[t - E_U(t)]^2 = \int_0^\infty [t - E_U(t)]^2\ dG(t) = \int_0^\infty t^2\ dG(t) - E_U^2(t),$$

it follows that

$$\left\{\int_0^\infty t^2\ d[aG(t) - b]\right\} - a^2E_U^2(t) = a\left\{E_U^2(t) + E_U[t - E_U(t)]^2\right\} - a^2E_U^2(t)$$

$$= aE_U^2(t)(1 - a) + aE_U[t - E_U(t)]^2.$$

In other words, the variance of the conditional time equals a, multiplied by the unconditional variance plus a(1 - a), multiplied by the square of the unconditional mean.

We will make a small digression here. Up to now, we have insisted that the times when a maintenance task may be completed comprise a finite set of time points, $(x_{i,1}, x_{i,2}, \ldots, x_{i,m_i})$, and that the probability $f_i(t|E_i)$ equals zero when $x_{i,j-1} < t < x_{i,j}$, where

$$f_i(t|E_i) = F_i(t|E_i) - F_i(x_{i,j-1}|E_i)$$

and

$$F_i(t|E_i) = F_i(x_{i,j-1}|E_i)$$

for

$$x_{i,j-1} \leq t < x_{i,j}$$

because of the step function nature of $F_i(t|E_i)$.

As previously indicated, if we select the discrete time points properly, we do not lose much accuracy nor does this cost us much in terms of computer time. Also, since the SAMSOM II "world" is also a "discrete world," CONCUR results based on discrete time points will be consistent with these SAMSOM II results. We know, however, that the real world approaches a continuous rather than a discrete situation. For example, a task that may be completed in 2 or 3 hours usually can be completed any time in between, say 2 hours and 46 minutes. Therefore, CONCUR is also programmed to allow results for tasks that will be completed in between the discrete time points that form the inputs for both CONCUR and/or SAMSOM II. This is accomplished in the computer by simple linear interpolation between the probabilities $F_i(x_{i,j}|E_i)$ and $F(x_{i,j-1}|E_i)$ to obtain $F_i(t|E_i)$ for

$$x_{i,j-1} < t < x_{i,j}.$$

Therefore, CONCUR has two sets of outputs: (1) based on linear interpolation, as explained above, called "Final Distribution" (interpolated probabilities); and (2) "Final Distribution" (uninterpolated probabilities) corresponding to the "discrete world" where jobs are finished only at specified times and never in between.

CONCUR Example

The following example is given to illustrate and further clarify some of CONCUR's probabilistic notions that have been discussed so far. We will use the set of three unscheduled maintenance tasks as described earlier.

It is assumed that $p_1 = p_2 = 1$, since Tasks 1 and 2 must be done on the aircraft after it lands, and $p_3 = \frac{1}{2}$. Therefore, $q_1 = q_2 = 1 - p_1 = 1 - p_2 = 0$ and $q_3 = 1 - p_3 = \frac{1}{2}$. The time, X_1, to complete task 1, is one of three possible values,

$$x_{1,1} = 1 \text{ hour},$$

$$x_{1,2} = 2 \text{ hours, or}$$

$$x_{1,3} = X_1^* = 3 \text{ hours.}$$

The time, X_2, to complete task 2 is

$$x_{2,1} = 1 \text{ hour, or}$$

$$x_{2,2} = X_2^* = 2 \text{ hours.}$$

The time, X_3, to complete task 3, given that maintenance is required, takes only one possible value, $x_{3,1} = 1$ hour.

The corresponding conditional cumulative distribution probabilities, $F_i(x_{i,j}|E_i)$, that a task will be completed in $x_{i,j}$ hours or sooner are as follows:

$$F_1(x_{1,1}|E_1) = F_1(1 \text{ hour}|E_1) = 0.5$$

$$F_1(x_{1,2}|E_1) = F_1(2 \text{ hours}|E_1) = 0.75$$

$$F_1(x_{1,3}|E_1) = F_1(3 \text{ hours}|E_1) = 1$$

$$F_2(x_{2,1}|E_2) = F_2(1 \text{ hour}|E_2) = 0.5$$

$$F_2(x_{2,2}|E_2) = F_2(2 \text{ hours}|E_2) = 1$$

$$F_3(x_{3,2}|E_3) = F_3(1 \text{ hour}|E_3) = 1$$

and

$$F_1(x_1|\bar{E}_1) = F_2(x_2|\bar{E}_2) = F_3(x_3|\bar{E}_3) = 1.$$

Also, the conditional probability densities, $f_i(x_{i,j}|E_i)$, that a task will be completed in exactly $x_{i,j}$ hours are

$$f_1(x_{1,1}|E_1) = f_1(1 \text{ hour}|E_1) = 0.5$$

$$f_1(x_{1,2}|E_1) = f_1(2 \text{ hours}|E_1) = F_1(x_{1,2}|E_1) - F_1(x_{1,1}|E_1) = 0.25$$

$$f_1(x_{1,3}|E_1) = f_1(3 \text{ hours}|E_1) = F_1(x_{1,3}|E_1) - F_1(x_{1,2}|E_1) = 0.25$$

$$f_2(x_{2,1}|E_2) = f_2(1 \text{ hour}|E_2) = 0.5$$

$$f_2(x_{2,2}|E_2) = f_2(2 \text{ hours}|E_2) = F_2(x_{2,2}|E_2) - F_2(x_{2,1}|E_2) = 0.5$$

$$F_3(x_{3,1}|E_3) = f_3(1 \text{ hour}|E_3) = 1.$$

The values of $x_{i,j}$, (i, j = 1, 2, 3) become the values of $y_\ell (\ell = 1, 2, 3)$, viz., all tasks will be completed by one of the following:

$$y_1 = 1 \text{ hour}$$

$$y_2 = 2 \text{ hours}$$

$$y_3 = 3 \text{ hours} = X_1^* = \max x_{i,j}.$$

The unconditional probabilities, $G(y_\ell)$, that all tasks will be completed by y_ℓ hours, or sooner, are:

$$G(y_1) = G(1\text{ hour}) = \prod_{i=1}^{3} [p_i F_i(1\text{ hour}|E_i) + (1 - q_i)]$$

$$= [p_1F_1(1\text{ hour}|E_1) + (1 - q_1)][p_2F_2(1\text{ hour}|E_2) + (1 - q_2)]$$

$$\bullet[p_3F_3(1\text{ hour}|E_3) + (1 - q_3)]$$

$$= [1 \bullet 0.5 + 0][1 \bullet 0.5 + 0][\tfrac{1}{2} \bullet 1 + \tfrac{1}{2}] = 0.25;$$

$$G(y_2) = G(2\text{ hours}) = \prod_{i=1}^{3} [p_i F_i(2\text{ hours}|E_i) + (1 - p_i)] = 0.75;$$

and

$$G(y_3) = G(3\text{ hours}) = 1.$$

The probability $G(0)$ that all tasks will be completed in zero hours is

$$G(\text{zero hours}) = Q = q_1q_2q_3 = 0 \bullet 0 \bullet 1 = 0\ ,$$

and

$$P = 1 - Q = 1.$$

The probabilities, $g(y_\ell)$, that the longest task in the group will take exactly y_ℓ hours are

$$g(y_1) = g(1\text{ hour}) = G(1\text{ hour}) = 0.25$$

$$g(y_2) = g(2\text{ hours}) = G(2\text{ hours}) - G(1\text{ hour}) = 0.75 - 0.25 = 0.5$$

$g(y_3) = g(3 \text{ hours}) = G(3 \text{ hours}) - G(2 \text{ hours}) = 1 - 0.75 = 0.25$

$g(0 \text{ hours}) = Q = 0.$

The expected value, $E(X_1|E_1)$, or mean time to complete Task 1, given that Task 1 is required, is

$$E(X_1|E_1) = \sum_{j=1}^{3} x_{1,j} f_1(x_{1,j}|E_1) = 1(0.5) + 2(0.25) + 3(0.25) = 1.75.$$

Similarly,

$$E(X_2|E_2) = \sum_{j=1}^{2} x_{2,j} f_2(x_{2,j}|E_2) = 1 \bullet 0.5 + 2 \bullet 0.5 = 1.5$$

and

$$E(X_3|E_3) = x_{3,1} f_3(x_{3,1}|E_3) = 1 \bullet 1 = 1.$$

The variances, Var $(X_i|E_i)$, are found as follows:

$$\text{Var } (X_1|E_1) = \sum_{j=1}^{3} x_{1,j}^2 f_1(x_{1,j}|E_1) - [E(X_1|E_1)]^2$$

$$= 1^2 \bullet 0.5 + 2^2 \bullet 0.25 + 3^2 \bullet 0.25 - (1.75)^2$$

$$= 1 \bullet 0.5 + 4 \bullet 0.25 + 9 \bullet 0.25 - 3.06 = 0.69$$

$$\text{Var } (X_2|E_2) = \sum_{j=1}^{2} x_{2,j}^2 f_2(x_{2,j}|E_2) - [E(X_2|E_2)]^2$$

$$= 1^2 \bullet 0.5 + 2^2 \bullet 0.5 - (1.5)^2 = 0.25;$$

and

$$\text{Var } (X_3|E_3) = x_{3,1}^2 f_3(x_{3,1}|E_3) - [E(X_3|E_3)]^2 = 1 - 1 = 0.$$

The standard deviations, $SD(x_i|E_i)$, are:

$$SD(X_1|E_1) = \sqrt{\text{Var } (X_1|E_1)} = \sqrt{0.69} = 0.83$$

$$SD(X_2|E_2) = \sqrt{\text{Var } (X_2|E_2)} = \sqrt{0.25} = 0.5$$

$$SD(X_3|E_3) = \sqrt{\text{Var } (X_3|E_3)} = 0.$$

The expected time, E(Y), to complete all three tasks, i.e., the longest, is obtained by

$$E(Y) = \sum_{\ell=1}^{3} y_\ell g(y_\ell) = 1 \bullet 0.25 + 2 \bullet 0.5 + 3 \bullet 0.25 = 2.$$

The variance, Var (Y), of the time to complete the longest task is

$$\text{Var } (Y) = \sum_{\ell=1}^{3} y_\ell^2 g(y_\ell) - [E(Y)]^2$$

$$= 1^2 \bullet 0.25 + 2^2 \bullet 0.5 + 3^2 \bullet 0.25 - 2^2 = 0.5,$$

and

$$SD(Y) = \sqrt{\text{Var } (Y)} = \sqrt{0.5} = 0.71.$$

In our example, because Q equals zero, it can be seen from formula (6), that the unconditional probabilities, G(t), of completing the longest task are equal to their corresponding conditional probabilities, C(t). However, this is usually not true.

CONVOL

The following CONVOL probability considerations apply to tasks that must be done sequentially.* Let $Z_i (i = 1, 2, \ldots, k)$ be the (nonnegative) time required to complete task i, where there are k sequential tasks and the ith task may be completed at n_i points in time (after initiation of maintenance). Our objective here is to obtain the probability distribution of the sum of the task times (ΣZ_i), i.e., $Pr[Z_1 + Z_2 + \ldots, Z_k \leq t]$. Now let $G_i(z) = Prob\ (Z_i \leq z)$, the cumulative distribution function, and let $g_i(z)$ be the probability (density function) that it takes exactly z to complete task i. Define $z_{i,j}$ as the jth time value that Z_i may take on, where $z_{i,1} < z_{i,2} < \ldots, < z_{i,n_i}$. It is assumed that the Z_i's are mutually independent. The probability of the joint event $(Z_i = z_{i,j}, Z_\ell \leq t) = g_i(z_{i,j})G_\ell(t)$ where $i \neq \ell$ and $\ell = 1, 2, \ldots, k$, $j = 1, 2, \ldots, n_i$ and $0 \leq t \leq z_{\ell,n_\ell}$.

$S_i = Z_i + Z_{i-1}$ is a new random variable, and the event $S_i \leq r$ occurs whenever $S_i = Z_i + Z_{i-1} \leq r$; that is, if any of the mutually exclusive events

$$(Z_i = z_{i,1}, Z_{i-1} \leq r - z_{i,1}), (Z_i = z_{i,2}, Z_{i-1} \leq r - z_{i,2}),$$

$$\ldots, (Z_i = z_{i,b}, Z_{i-1} \leq r - z_{i,b})$$

occur, where $i = 2, \ldots, k$, $b \leq n_i$ and $z_{i,b}$ is the largest of the timepoint values, $z_{i,1}, z_{i,2}, \ldots, z_{i,n_i}$, such that $z_{i,b} \leq r$. Therefore, the probability, $CV_i(r) = P(S_i \leq r)$, is obtained by

$$CV_i(r) = \sum_{j=1}^{b} g_i(z_{i,j})G_{i-1}(r - z_{i-1,j}). \tag{7}$$

Theoretically, r may be any nonnegative timepoint value. The reader will recall, however, that Z_i may assume any of n_i values and

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*See [3, p. 250].

that Z_{i-1} may be any of n_{i-1} timepoint values. Therefore, $S_i = Z_i + Z_{i-1}$ may assume $n_i n_{i-1}$ different values at most. Further, $G_i(t)$ is a step function where

$$G_i(t) = G_i(z_{i,j}) \quad \text{for } z_{i,j} \leq t < z_{i,j+1}.$$

Thus $CV_i(r)$ is also a step function.

Hence evaluating $CV_i(r)$ for all the dinstinct values that S_i may take on results in the cumulative distribution function that it takes r hours, or less, to complete the ith and (i - 1)st tasks, when they must be done in sequence. We denote the n_i distinct values of S_i by $r_{i,f}$ where

$$0 \leq r_{i,1} = z_{i,1} + z_{i-1,1} < r_{i,2}, \ldots, < r_{i,n_i} = z_{i,n_i} + z_{i-1,n_{i-1}}.$$

Let $cv_\ell(r_{\ell,f})$ be the probability density that it will take exactly $r_{\ell,f}$ hours to complete the (ℓ - 1)st and ℓth tasks in sequence. Then

$$\text{(8)} \qquad cv_\ell(r_{\ell,j}) = CV_\ell(r_{\ell,j}) - CV_\ell(r_{\ell,j-1}).$$

The cumulative distribution functions, $V_\ell(r)$, that the first ℓ tasks (ℓ = 1, ..., k) will be completed in r hours, or sooner, is given by

$$V_\ell(r) = P(Z_1 + Z_2 + \ldots + Z_\ell \leq r).$$

where Z_ℓ = time to complete task ℓ, ℓ = 1, 2, ..., k.

Now,

$$\text{(9)} \qquad V_\ell(r) = \begin{cases} G_1(r) \quad \text{[redacted]} & \text{for } \ell = 1 \\ \sum_{j=1}^{b} g_\ell(z_{\ell,j}) V_{\ell-1}(r - z_{\ell,j}) & \text{for } \ell = 2, \ldots, k, \end{cases}$$

whereas in the discussion preceding formula (7), $z_{\ell,b}$ is the largest timepoint value such that $z_{\ell,b} \leq r$. From formulas (8) and (9) we see that $V_\ell(r)$ is defined inductively and that each time a new task is added, the cumulative distribution obtained from the previous CONVOL process may be used as an input to the present CONVOL process.

Actually when working only with two tasks, ℓ and $\ell - 1$, it can be seen from formula (7) that, for computational purposes only--because of symmetry--either task may be considered as the ℓth or $(\ell - 1)$st task. Similarly in formula (8), again for computational purposes only, it does not matter in which order tasks are added so long as the CONVOL process is applied iteratively to all of the first ℓ applicable tasks.

The probability density, $v_\ell(r_{\ell,j})$, that it will take exactly $r_{\ell,j}$ hours to complete the first ℓ tasks $(\ell \leq k)$ in sequence, is obtained by

$$(10)\quad v_\ell(r_{\ell,j}) = \begin{cases} V_\ell(v_{\ell,1}) & \text{for } j = 1 \\ V_\ell(r_{\ell,j}) - V_\ell(r_{\ell,j-1}) & \text{for } j = 2, 3, \ldots, m_\ell, \end{cases}$$

where m_ℓ is the number of distinct time values at which the first ℓ tasks can be completed. If $R_\ell = (Z_1 + Z_2 + \ldots + Z_\ell)$, the expected values, $E(R_\ell)$, of the time, R_ℓ, it takes to complete the first ℓ tasks are obtained as follows:

$$(11)\qquad E(R_\ell) = \sum_{j=1}^{n_\ell} r_{\ell,j} v_\ell(r_{\ell,j}),$$

$$(12)\qquad \text{Var }(R_\ell) = \sum_{j=1}^{n_\ell} r_{\ell,j}^2 v_\ell(r_{\ell,j}) - [E(R_\ell)]^2,$$

$$(13)\qquad SD(R_\ell) = \sqrt{\text{Var }(R_\ell)}.$$

If r_{ℓ,m_ℓ} = max time possible to complete ℓ tasks in sequence then

$$r_{\ell,m_\ell} = z_{1,n_\ell} + z_{2,n_\ell} + \ldots + z_{\ell,n_\ell},$$

where $m_\ell \leq n_1 + n_2 + \ldots + n_\ell$, and z_{ℓ,n_ℓ} = max time possible to complete the ℓth task, $\ell = 1, 2, \ldots, k$.

Consider the case of tasks that may be grouped in k sets, where the tasks within each set can start at the same time, but each set (other than the initial one) must wait its turn in sequence. We now have a method, the CONCUR process, for "collapsing" or consolidating the tasks in each set. CONCUR will provide a cumulative distribution function for each collapsed set, and each may be treated as though it were an individual task in the CONVOL process which is then applied to determine planning factors for estimating the time it will take to complete all of the sequential sets of maintenance tasks.

CONVOL Example

We shall use the data from the same sequence of tasks (troubleshooting, unscheduled maintenance and functional checks) that we used earlier to illustrate the CONVOL process. CONCUR was used to collapse each set of tasks (as previously explained in detail for unscheduled maintenance) to provide the input data for the CONVOL process as shown below, where:

Task 1 is the set of troubleshooting tasks,
Task 2 is the set of unscheduled maintenance tasks, and
Task 3 is the set of functional check tasks.

The times, $z_{i,j}$, at which the ith task may be completed are:

$z_{1,1}$ = 1 hour and $z_{1,2}$ = 2 hours
$z_{2,1}$ = 1 hour, $z_{2,2}$ = 2 hours and $z_{2,3}$ = 3 hours
$z_{3,1}$ = 1 hour.

The cumulative distribution probabilities, $G_i(z_{i,j})$, that the ith task will be completed by $z_{i,j}$ hours, or sooner, are:

$$G_1(z_{1,1}) = G_1(1 \text{ hour}) = 0.33$$

$$G_1(z_{1,2}) = G_1(2 \text{ hours}) = 1.0$$

$$G_2(z_{2,1}) = G_2(1 \text{ hour}) = 0.25$$

$$G_2(z_{2,2}) = G_2(2 \text{ hours}) = 0.75$$

$$G_2(z_{2,3}) = G_2(3 \text{ hours}) = 1.0 .$$

The probability densities, $g_i(z_{i,j})$, that the ith task will take exactly $z_{i,j}$ hours are obtained using the cumulative distribution values:

$$g_1(z_{1,1}) = g_1(1 \text{ hour}) = G_1(1 \text{ hour}) = 0.33$$

$$g_1(z_{1,2}) = g_1(2 \text{ hours}) = G_1(2 \text{ hours}) - G_1(1 \text{ hour}) = 0.67$$

$$g_2(z_{2,1}) = g_2(1 \text{ hour}) = G_2(1 \text{ hour}) = 0.25$$

$$g_2(z_{2,2}) = g_2(2 \text{ hours}) = G_2(2 \text{ hours}) - G_2(1 \text{ hour}) = 0.5$$

$$g_2(z_{2,3}) = g_2(3 \text{ hours}) = G_2(3 \text{ hours}) - G_2(2 \text{ hours}) = 0.25$$

$$g_3(z_{3,1}) = g_3(1 \text{ hour}) = G_3(1 \text{ hour}) = 1.0 .$$

We wish to apply the CONVOL process first to Tasks 1 and 2 (troubleshooting and unscheduled maintenance), and then apply first results of CONVOL as inputs to a second CONVOL process by adding Task 3 (functional check). The timepoint values, $r_{2,j}$, for completing Tasks 1 and 2 in sequence are obtained by adding all the possible combinations of Z_1 (i.e., $Z_1 = z_{1,1} = 1$ and $Z_1 = z_{1,2} = 2$ hours) and Z_2 (i.e., $z_{2,1} = 1$, $z_{2,2} = 2$ and $z_{2,3} = 3$ hours), and eliminating duplicates. Thus r and $r_{2,j}$ may take on the following four unique values in formulas (7) through (13):

$$\min Z_1 + \min Z_2 = r_{2,1} = 2 \text{ hours}$$

$$r_{2,2} = 3 \text{ hours}$$

$$r_{2,3} = 4 \text{ hours}$$

$$\max Z_1 + \max Z_2 = r_{2,4} = 5 \text{ hours}.$$

The cumulative distribution probabilities, $CV_2(r)$, that Tasks 1 and 2 can be completed in r hours, or less, are:

$$CV_2(r = r_{2,1} = 2 \text{ hours}) = g_2(z_{2,1})G_1(2 - z_{2,1}) = g_2(1 \text{ hour})G_1(1 \text{ hour})$$

$$= 0.25 \bullet 0.33 = 0.08 = V_2(2 \text{ hours}),$$

$$CV_2(r = r_{2,2} = 3 \text{ hours}) = \sum_{j=1}^{2} g_2(z_{2,j})G_1(3 - z_{2,j})$$

$$= g_2(z_{2,1})G_1(3 - z_{2,1}) + g_2(z_{2,2})G_1(3 - z_{2,2})$$

$$= g_2(1 \text{ hour})G_1(2 \text{ hours}) + g_2(2 \text{ hours})G_1(1 \text{ hour})$$

$$= 0.25 \bullet 1.0 + 0.5 \bullet 0.33 = 0.42 = V_2(3 \text{ hours}),$$

$$CV_2(r = r_{2,3} = 4 \text{ hours}) = \sum_{j=1}^{3} g_2(z_{2,j})G_1(4 - z_{2,j})$$

$$= g_2(1 \text{ hour})G_1(3 \text{ hours}) + g_2(2 \text{ hours})G_1(2 \text{ hours})$$

$$+ g_2(3 \text{ hours})G_1(1 \text{ hour}) = 0.25 \bullet 1.0$$

$$+ 0.5 \bullet 1.0 + 0.25 \bullet 0.33 = 0.83 = V_2(4 \text{ hours}),$$

$$CV_2(r = r_{2,4} = 5 \text{ hours}) = \sum_{j=1}^{4} g_2(z_{2,j})G_1(4 - z_{2,j}) = 1.0 = V_2(5 \text{ hours}),$$

where

$$G_1(m \text{ hours}) = G_1(2 \text{ hours}) = 1.0 \quad \text{for } m \geq 2.$$

The probability density, $cv_2(r)$, that it will take exactly r hours to complete both Tasks 1 and 2 is obtained using formula (8) as follows:

$$cv_2(r_{2,1}) = cv_2(2 \text{ hours}) = CV_2(2 \text{ hours})$$

$$= 0.08 = v_2(2 \text{ hours}),$$

$$cv_2(r_{2,2}) = cv_2(3 \text{ hours}) = CV_2(3 \text{ hours}) - CV_2(2 \text{ hours})$$

$$= 0.42 - 0.08 = 0.34 = v_2(3 \text{ hours}),$$

$$cv_2(r_{2,3}) = cv_2(4 \text{ hours}) = CV_2(4 \text{ hours}) - CV_2(3 \text{ hours})$$

$$= 0.41 = v_2(4 \text{ hours}),$$

$$cv_2(r_{2,4}) = cv_2(5 \text{ hours}) = CV_2(5 \text{ hours}) - CV_2(4 \text{ hours})$$

$$= 0.17 = v_2(5 \text{ hours}).$$

Using formulas (11), (12), and (13), we find the expected value, the variance and standard deviation of the time, R_2, it will take to do Tasks 1 and 2:

$$E(R_2) = \sum_{j=1}^{4} r_{2,j} v(r_{2,j}) = 2(0.08) + 3(0.33)$$

$$+ 4(0.41) + 5(0.17) = 3.67,$$

$$\text{Var } (R_2) = \sum_{j=1}^{4} r_{2,j}^2 v(r_{2,j}) = 2^2 \bullet 0.08 + 3^2 \bullet 0.33$$

$$+ 4^2 \bullet 0.41 + 5^2 \bullet 0.17 = 0.72,$$

$$SD(R_2) = \sqrt{\text{Var } (R_2)} = 0.85.$$

We can apply CONVOL once again using the results just obtained and adding Task 3 (functional checks). We note, however, that all functional checks always take 1 hour and we can get the following results without calculation:

$r_{3,1}$ = 3 hours (minimum time to complete Tasks 1, 2 and 3)

$r_{3,2}$ = 4 hours

$r_{3,3}$ = 5 hours

$r_{3,4}$ = 6 hours (maximum time to complete Tasks 1, 2 and 3);

V_3(3 hours) = V_2(2 hours) = 0.08

V_3(4 hours) = V_2(3 hours) = 0.42

V_3(5 hours) = V_2(4 hours) = 0.83

V_3(6 hours) = V_2(5 hours) = 1.0.

Similarly,

$$v_3(r_{3,j}) = v_2(r_{2,j}) \qquad j = 1, 2, 3, 4.$$

In this special case, therefore, the expected time to complete all tasks is 1 hour greater than that to complete Tasks 1 and 2; i.e.,

$$E(R_3) = E(R_2) + 1 = 3.67 + 1 = 4.67 \text{ hours.}$$

The variance, Var (R_3), and standard deviation, $SD(r_3)$, do not change when the additional task takes a constant time to complete. Therefore, in our example,

$$\text{Var }(R_3) = \text{Var }(R_2) = 0.72$$

and

$$SD(R_3) = SD(R_2) = 0.85.$$

IV. SAMSOM II

SAMSOM II is a Monte Carlo simulation model programmed in SIMSCRIPT I. SAMSOM is an acronym standing for Support-Availability Multi-System Operations Model. Our purpose in introducing the reader to this model is threefold.

1. Those interested in aircraft maintenance planning factors may also be interested in an effective model for combining the factors to determine what effects their interactions have under varying operational and maintenance conditions.
2. SAMSOM II (or some other suitable simulation model) often offers the only practical way to obtain some aircraft system maintenance planning factors when working with operations and maintenance constraints on more than one aircraft type, with each type flying several kinds of missions, and at several bases, etc.
3. CONCUR and CONVOL may be used to develop aircraft system maintenance planning factors that can be used to check or supplement SAMSOM II output.

SAMSOM II is a very general simulation model that has been used effectively in studies of aircraft weapon systems (tactical fighters) and aircraft support systems (cargo aircraft). At present, SAMSOM II has three basic limitations. First it requires access to and use of an electronic computer in the 7044/7094 class; second it is limited by the core storage capacity of the 7044/7094 computer; and third it is programmed for examining aircraft direct maintenance support requirements only. The model is not readily adaptable to examining indirect maintenance requirements such as those that occur during shop repair.

Presently, SAMSOM II is available at the Research and Technology Division at Wright-Patterson Air Force Base and at The RAND Corporation. The Air Force has also installed it at the Pentagon for use in analysis of aircraft weapon and support systems. The RAND Memorandum describing SAMSOM II is in preparation. As more late model computers are installed, core storage, access to and use of computers should be less of a problem. Research is being conducted to extend SAMSOM II so that it can handle indirect maintenance as well. Therefore, in our opinion, the demand

for SAMSOM II (or other simulation models) for analysis and planning purposes is bound to increase.

SAMSOM II consists of three computer programs--(1) input, (2) simulation, and (3) output. The input program takes the inputs the user specifies and prepares them for the simulation. The inputs define the air bases, aircraft, missions, personnel, equipment, facilities, parts, etc., that are used in the scenario. Characteristics of each of these elements and their planned or programmed actions are also included in the inputs and indicate time between inspections, failure probabilities, time and resources needed to repair items (and their probability distributions), probability of weather levels (and their probability distributions), quantity and location of each resource, schedule of missions and sorties, mission routes, flying times between bases, and so forth. Finally, the inputs describe the operational policies such as mission priorities, number of aircraft on alert, minimum and maximum number of aircraft per sortie; the maintenance personnel working on aircraft at one time; the conflicting systems such as electrical and fire power, which may not be worked on simultaneously; and so forth.

The input program represents the combined effects resulting from Air Force decisions plus probabilistic phenomena in that:

1. It assigns aircraft to each base and assigns personnel and other resources to each base by work center.
2. It schedules sorties, personnel, and other resources by work center and shift throughout the entire period.
3. It sets the operational and maintenance policies by the time period in effect during the simulation.
4. It "ages" or assigns accumulated flying time to each aircraft to approximate a steady state condition.
5. It determines what type of weather will exist at each base throughout the period.
6. It decides when aircraft will be "scheduled" for possible standing failures. (A standing failure is related to calendar time rather than flying hours.)
7. It decides when aircraft will require maintenance after a sortie.

The simulation program, using SIMSCRIPT [4] and Monte Carlo routines, tries to accomplish the flying program "directed" by the input program. Each aircraft sortie has a well-defined maintenance "menu" spelling out the different types of maintenance (debriefing, troubleshooting, unscheduled maintenance, etc.) that it may require before and after each flight, and the hierarchy of maintenance tasks and resources that should be included in each type of maintenance. SAMSOM II accounts for the length of time that each maintenance resource is used, as determined by the Monte Carlo and SIMSCRIPT routines.

When available, resources are removed from their respective pools to work the specified time period as determined by their inputs and the Monte Carlo routines. So long as resources are available, work on an aircraft continues until it is all completed, at which time the aircraft is removed from a maintenance state and made available to the alert pool or for flying. The resources are then released to do additional work. When maintenance resources are required but are unavailable, SAMSOM II sets up a queue record and begins maintenance when the resources become available. The aircraft needing work are kept in the maintenance state during the queue time period and until the work is completed.

Within each maintenance type, tasks are started at the same time--when resources are available--although they can be scheduled sequentially. Thus debriefing must precede troubleshooting which precedes unscheduled maintenance which precedes functional check which precedes fuel servicing. Of course, if there are no tasks to be done for a particular maintenance type, SAMSOM II proceeds to the next maintenance type in sequence. The simulation program generates a transaction for each operations and maintenance action and records key information about the transaction, such as time of transaction, aircraft tail number, and so on.

The output program performs the function of evaluation and analysis, i.e., it processes the transaction data and generates operations and/or maintenance information. The user, by requesting the appropriate [redacted] reports, can get a detailed view and, for example, see how many teams in the hydraulics shops are working, idle, or requested but not available

at any point in simulated time. This can be done for any one resource, for every resource, for a selected set, and for any combination of timepoints the user desires for each resource. On the other hand, if the user wishes, he can get an overview and ask for summary data such as total sorties flown, total flying hours, etc. for all aircraft. Or the user can get both the detailed and summary data.

V. MAINTENANCE PLANNING FACTORS

In earlier sections we explained the basic concepts of CONCUR, CONVOL and SAMSOM II. Now we wish to show how CONCUR and CONVOL may be applied to obtain aircraft system maintenance planning factors.

One considerably important maintenance planning factor for new R&D aircraft that have been flown little or not at all is an estimate of the time-to-repair. Under the AFSC Systems Management Program, contractors of new aircraft systems are required to correlate maintenance functions and tasks (including frequency of occurrence, time for accomplishment, etc.) to personnel, maintenance ground equipment, and spares. For example for the A-7A, LTV Vought Aeronautics Division complies with the System Management Program by preparing detailed personnel planning data on each A-7A component. This is done via the MEARS (Maintenance Engineering Analysis and Recording System). For each task to be performed on the component, the information that AFSC directives require includes: nomenclature, designation by part number, MEARS control number, aircraft model, maintenance type, personnel specialty code and number of persons performing task, elapsed time (clock minutes) to complete task, and frequency of task.

The MEARS control number and other data enable the user to determine which work center or maintenance shop does each task. The frequency is usually specified by number of the task generated each flying hour. For example, task 1 on the pylon assembly, wing stations 1 and 8, is a remove and replace maintenance action that takes two Air Force Specialty Codes, 6511, an elapsed time of 75 minutes to do and occurs at a rate of 0.00020 per flying hour. If the A-7A flies 1.85 hours per sortie, the probability that this maintenance task will be required after a sortie is 0.000370 (based on Poisson distribution).

Table 1 gives the task number, probability per sortie that the task will be required, and required elapsed time it takes electrical shop (Work Center 3330) personnel to do tasks on the lighting system (System 44) of the A-7A. The team size is not shown since we do not need this information to find the time-to-repair cumulative distribution.

Table 1

UNSCHEDULED MAINTENANCE TASKS DONE BY THE ELECTRICAL SHOP (3330) ON THE LIGHTING SYSTEM (SYSTEM 44) OF THE A-7A

Task Number[a]	Probability per Sortie that Task is Required[b]	Elapsed Time to do Task (minutes)	Hours (t)
100	0.004	52	0.87
111	0.037	30	0.50
112A	0.002	102	1.70
112B	0.007	27	0.45
113	0.018	8	0.13
114	0.009	15	0.25
115	0.002	15	0.25
116	0.009	22	0.37
117	0.018	8	0.13
200	0.001	323	5.39
291A	0.004	29	0.48
291B	0.015	4	0.07
298	0.002	72	1.20

SOURCE: A-7A MEARS.

[a]Tasks with probability per sortie of less than 5 in 10,000 are not included here.

[b]Flying hours per sortie (one aircraft) times frequency per flying hour.

Table 1 contains the essential information to obtain the time-to-repair cumulative distributions for unscheduled maintenance tasks. The process we use with the Table 1 data is CONCUR, which produces the data shown in Table 2. CONCUR requires that the tasks be independent in the probability sense. If two tasks always occur together, they should be considered as one task with the elapsed time of the longer. When either one task or the other occurs, but not both, they should be combined and the weighted mean elapsed time used. More complicated task dependency combinations require more complex analysis involving conditional probabilities.

The cumulative probabilities in Table 2 are the unconditional probabilities, G(t), that the A-7A lighting system will require unscheduled maintenance by electrical shop maintenance teams and that it

Table 2

CUMULATIVE DISTRIBUTION OF UNCONDITIONAL TIME-TO-REPAIR THAT ELECTRICAL SHOP WORKS ON A-7A LIGHTING SYSTEM

Timepoint Value (Hours) (t)	Cumulative Probability G(t)
0.000	0.879
0.070	0.892
0.130	0.925
0.250	0.935
0.370	0.944
0.450	0.950
0.480	0.954
0.500	0.991
0.870	0.995
1.200	0.997
1.700	0.999
5.390	1.000

Expected value = 0.049
Variance = 0.052
Standard deviation = 0.228

Table 3

CUMULATIVE DISTRIBUTION OF CONDITIONAL TIME-TO-REPAIR THAT ELECTRICAL SHOP WORKS ON A-7A LIGHTING SYSTEM

Timepoint Value (Hours) (t)	Cumulative Probability C(t)
0.070	0.110
0.130	0.383
0.250	0.467
0.370	0.537
0.450	0.592
0.480	0.624
0.500	0.926
0.870	0.959
1.200	0.975
1.700	0.992
5.390	1.000

Expected value = 0.391
Variance = 0.289
Standard deviation = 0.538

will take t hours, or less, to complete all tasks (longest task). The probability (see formula 2--$Q = \Pi_{i=1}^{k} q_i$) that no tasks in the Table 1 group will be required is obtained from the first entry in Table 2, 0.879, which is the probability that all of these tasks take zero time (need not be done). The probability, P (break rate), that at least one task in the group will be required after an A-7A sortie is a maintenance planning factor of general interest. The break rate for the Table 1 tasks is 0.121.

The conditional cumulative time-to-repair probabilities, C(t), shown in Table 3 are obtained from Table 2 values (Ref. formula 6) by subtracting 0.879 from each cumulative probability and then dividing by 0.121. Each conditional cumulative probability in Table 3 is the probability that the electrical shop teams will complete all lighting system tasks in Table 1 in t hours, or less, given that maintenance is required (i.e., at least one Table 1 task is required). Other aircraft system planning factors of interest are the mean or expected value, variance, and standard deviation for the unconditional and conditional cases shown at the bottom of Tables 2 and 3, respectively.

The electrical shop performs unscheduled maintenance on the following A-7A aircraft systems.

Number	Name
12	Airframe
13	Landing Gear
14	Flight Control
42	Electrical
44	Lighting
46	Fuel
49	Misc. Utilities
63	UHF Communication
65	IFF
75	Weapon Delivery

For each of these shop-system combinations, it is possible to find the break rate, unconditional and conditional time-to-repair cumulative distributions, as well as applicable expected values, variances and standard deviations using CONCUR and appropriate formulas as demonstrated for the electrical shop, lighting system combination.

Table 4

CUMULATIVE DISTRIBUTION OF UNCONDITIONAL TIME-TO-REPAIR THAT ELECTRICAL SHOP WORKS ON ANY A-7A SYSTEM

Timepoint Value (Hours) (t)	Cumulative Probability G(t)
0.000	0.666
0.100	0.731
0.200	0.767
0.300	0.835
0.400	0.848
0.600	0.906
0.700	0.914
1.000	0.972
1.200	0.979
1.300	0.984
1.500	0.987
1.600	0.991
1.700	0.993
2.100	0.996
3.100	0.998
3.800	0.999
63.000	1.000

Expected value = 0.163
Variance = 0.190
Standard deviation = 0.437

Table 5

CUMULATIVE DISTRIBUTION OF CONDITIONAL TIME-TO-REPAIR THAT ELECTRICAL SHOP WORKS ON ANY A-7A SYSTEM

Timepoint Value (Hours) (t)	Cumulative Probability G(t)
0.100	0.195
0.200	0.303
0.300	0.505
0.400	0.545
0.600	0.720
0.700	0.743
1.000	0.918
1.200	0.937
1.300	0.951
1.500	0.962
1.600	0.972
1.700	0.980
2.100	0.988
3.100	0.996
3.800	0.998
63.000	1.000

Expected value = 0.676
Variance = 8.027
Standard deviation = 2.833

It is of interest to obtain for each maintenance shop the comparable aggregated planning factors that apply to the entire aircraft. These can be obtained directly via CONCUR using the break rate and conditional (or unconditional) time-to-repair cumulative distributions for each electrical shop-system combination. Table 4 shows the selected values that result using the unconditional cumulative distribution. We see from Table 4 that the probability of the electrical shop not having to do any A-7A unscheduled maintenance task after an aircraft sortie is 0.666 and that the probability that at least one electrical shop maintenance task will be required is 0.334.

Table 5, showing conditional time-to-repair cumulative probabilities for the electrical shop, is obtained from Table 4 in the same manner as Table 3 was obtained from Table 2.

VI. OPERATIONAL PLANNING FACTORS

We have shown how to use the CONCUR process with MEARS data for an R&D aircraft that has been flown little or not at all to estimate the following planning factors: (1) the probability that each maintenance shop will be required to do at least one unscheduled maintenance (or other maintenance type) task on an aircraft after one sortie; and (2) the unconditional and conditional unscheduled maintenance time-to-repair distributions for each maintenance shop. We shall subsequently show that it is possible to further aggregate by additional application of the CONCUR and/or CONVOL processes. But first, we shall discuss the comparable planning factors for aircraft that have operational and maintenance experience.

These aircraft pose a different sort of problem. The Air Force maintenance reporting systems, as directed by AFM 66-1, do not provide for clock-hour reporting. Therefore it is impossible to obtain time-to-repair distributions directly as might be the case if clock hours were reported. The Air Force, however, has used techniques developed by C. F. Bell, Jr., T. C. Smith [1], A. F. Sweetland [6], and others of The RAND Corporation, in several field tests to obtain augmented AFM 66-1 data including clock hours, team size, and other information for obtaining time-to-repair and break rates directly. For future worldwide application, the Air Force is currently testing modifications of maintenance data reporting systems that will include the augmented maintenance data.

One field test used to collect such data was the "Tackdown" C-130 field test, 5 January through 19 February 1965, at Pope Air Force Base, North Carolina. Table 6 shows maintenance data probabilities from this test for the electrical shop (work center 3330) on one C-130 aircraft after landing. The probability is 0.018 that the electrical shop will have to work on at least one task after a C-130 sortie. The unconditional time-to-repair distribution can be obtained from Table 6 by applying the CONCUR process and using 0.018 as the probability that the shop will be needed.

Table 6

CUMULATIVE DISTRIBUTION OF CONDITIONAL TIME-TO-REPAIR THAT ELECTRICAL SHOP PERFORMS TASK AFTER C-130 SORTIE
(One Aircraft)

Timepoint Values (Hours) (t)	Cumulative Probability G(t)
1.000	0.464
1.500	0.607
2.000	0.785
3.000	0.928
4.000	0.964
6.000	1.000

Expected value = 1.852
Variance = 1.594
Standard deviation = 1.263

Now, the next step in the process is to collapse these unscheduled maintenance planning factors by shop in order to determine the comparable factors for each C-130 sortie. To save space, we will use the distributions but will not show the detailed information as given in Table 6 for the Table 7 shops that also may need to do unscheduled maintenance on the C-130 after flight. Shops whose probability of being required is 0.005 or less are excluded.

Table 7

MAINTENANCE SHOPS THAT DO UNSCHEDULED MAINTENANCE ON C-130 AIRCRAFT

Maintenance Shop	Work Center Code
Crew Chief	21XX
Machine Shop	31XX
Engine	32XX
Prop	3220
Repair and Reclaim	3310
Fuel System	3320
Electrical	3330
Pneudraulic	3340
Instrument	3350
Mechanical Accessories	3360
Radio	4110
Radar	4120
Auto Pilot	4330

Let us assume that after each C-130 flight there is one mandatory task that occurs and always takes exactly two hours (this might be a minimum crew rest between flights, which starts with maintenance). All unscheduled work that the Table 7 shops do, as well as the mandatory maintenance tasks, all start at the same time. Therefore, we use CONCUR with the probability that each shop will have to do at least one unscheduled maintenance task with its conditional time-to-repair cumulative distribution. The result obtained is the unconditional cumulative distribution of the time-to-repair due to unscheduled maintenance of a C-130 aircraft after flight, as shown in Table 8.

Table 8

CUMULATIVE DISTRIBUTION OF UNCONDITIONAL TIME-TO-REPAIR AFTER C-130 SORTIE (ALL MAINTENANCE SHOPS)
(One Aircraft)

Timepoint Values (Hours) (t)	Cumulative Probability G(t)
2.000	0.904
2.500	0.927
2.900	0.939
3.000	0.942
3.100	0.944
3.500	0.953
3.700	0.959
3.800	0.961
4.000	0.967
4.500	0.971
4.800	0.974
5.000	0.975
6.000	0.984
6.500	0.986
7.500	0.989
8.600	0.992
9.700	0.994
10.800	0.995
13.300	0.997
19.500	0.999
20.600	1.000

Expected value = 2.273
Variance = 1.971
Standard deviation = 1.404

TURNAROUND TIME

For discussion purposes, aircraft turnaround time begins after touchdown when maintenance starts and ends when maintenance is completed and the aircraft is ready (in flying condition). For illustration, we consider a sequence of three "maintenance types"--[redacted] non-operationally ready supply (NORS), unscheduled maintenance, and launch service (upload and/or preflight).

In our example, the following ground rules apply: (1) if a NORS condition exists, no unscheduled maintenance starts until the parts arrive and the NORS condition is thereby eliminated; and (2) launch service starts immediately after unscheduled maintenance ends. With these conditions the CONVOL process, used with the unconditional time required distributions of each "maintenance type" applying to a [redacted] specific Air Force base, will give the turnaround maintenance planning factors for the C-130 aircraft at that base. (In cases where two or more maintenance types are concurrent rather than sequential, we would use CONCUR and then apply CONVOL to the aggregated maintenance types that must be done sequentially.) Table 9 shows the unconditional time distributions for NORS and launch service that are used in CONVOL along with the Table 8 data to obtain the C-130 turnaround planning factors for a selected air base. These factors are shown in Table 10.

MULTIPLE SORTIES

One Aircraft

Thus far we have been concerned with the turnaround time following a single sortie. Having once derived the turnaround distribution, however, CONVOL may be applied to obtain the probability that a single aircraft can fly m sorties during a specified time interval (T). Because the second flight cannot begin until maintenance (if necessary) after the first flight is completed, the third flight cannot start until the maintenance on the second is completed, etc., we have an ideal setup for using CONVOL and this will be our line of attack.

Table 9

CUMULATIVE DISTRIBUTION OF UNCONDITIONAL TIME-TO-REPAIR AFTER C-130 SORTIE (ALL MAINTENANCE SHOPS)
(One Aircraft)

NORS[a]		Launch Service[b]	
Timepoint Value (Hours) (t)	Cumulative Probability G(t)	Timepoint Value (Hours) (t)	Cumulative Probability G(t)
0.000	0.972	0.000	0.963
14.900	1.000	0.500	0.969
		1.500	0.973
		2.000	0.978
		2.500	0.980
		3.000	0.984
		3.500	0.988
		4.000	0.992
		4.500	0.995
		5.000	0.997
		6.000	1.000

[a]Expected value = 0.417; variance = 6.042; standard deviation = 2.458.

[b]Expected value = 0.108; variance = 0.399; standard deviation = 0.632.

Table 10

UNCONDITIONAL TURNAROUND CUMULATIVE DISTRIBUTIONS (ALL MAINTENANCE SHOPS) AT HICKAM AIR FORCE BASE AFTER C-130 SORTIE
(One Aircraft)

Timepoint Value (Hours) (t)	Cumulative Probability G(t)	Timepoint Value (Hours) (t)	Cumulative Probability G(t)
2.000	0.8457	8.000	0.9604
2.500	0.8728	10.500	0.9637
3.000	0.8873	11.200	0.9670
3.100	0.8893	17.300	0.9687
3.500	0.9012	17.400	0.9931
3.600	0.9013	18.700	0.9947
3.700	0.9065	19.400	0.9952
4.000	0.9184	19.500	0.9973
4.500	0.9248	20.600	0.9988
5.000	0.9323	21.500	0.9992
5.500	0.9360	22.400	0.9994
6.000	0.9480	23.500	0.9997
6.500	0.9527	25.000	0.9998
7.000	0.9547	34.000	0.9999
7.500	0.9576	41.000	1.0000

Expected value = 2.832; variance = 9.243; standard deviation = 3.040.

Let X_i equal the length of the ith repair and U_i equal the length of the ith flight. Then the probability, $COMP_T(m)$, of completing at least m sorties in a time interval of length T, with one initially ready aircraft is

$$(14)\quad COMP_T(m) = \begin{cases} 1, & \text{for } T \geq U_1 \text{ and } m = 1 \\ 0, & \text{for } T < U_1 \text{ and } m = 1, 2, 3, \ldots \\ Pr(X_1 + X_2 + \ldots + X_{m-1} \leq T - U_1 - U_2 - \ldots - U_m) & \\ \quad \text{for } T \geq U_1 \text{ and } m = 2, 3, \ldots & \end{cases}$$

CONVOL provides the method to evaluate formula (14) directly. To solve for m = 2, we apply CONVOL once; for m = 3 we apply CONVOL twice, etc.

Table 11 shows some selected values from the computer run for the twofold through fourfold case for the C-130 Hickam example. Figure 1 contains the graphs of the onefold through tenfold cases. For example, for a constant flying time of one hour per flight if we wish to find the probability, $COMP_{19}(2)$, of completing at least two sorties with one ready Hickam C-130 aircraft in a 19-hour flying day, we use the onefold CONVOL result (Table 10) with T = FH = 19 hours and $U_1 = U_2 = 1$. We enter at the Table 10 timepoint value of

$$t = T - U_1 - U_2 = 17 \text{ hours}$$

to find

$$COMP_{19}(2) = 0.967.$$

To find the probability, $COMP_{19}(5)$, of completing at least five sorties with one ready C-130 Hickam aircraft in a 19-hour flying day, we use the fourfold CONVOL result (Table 11) with

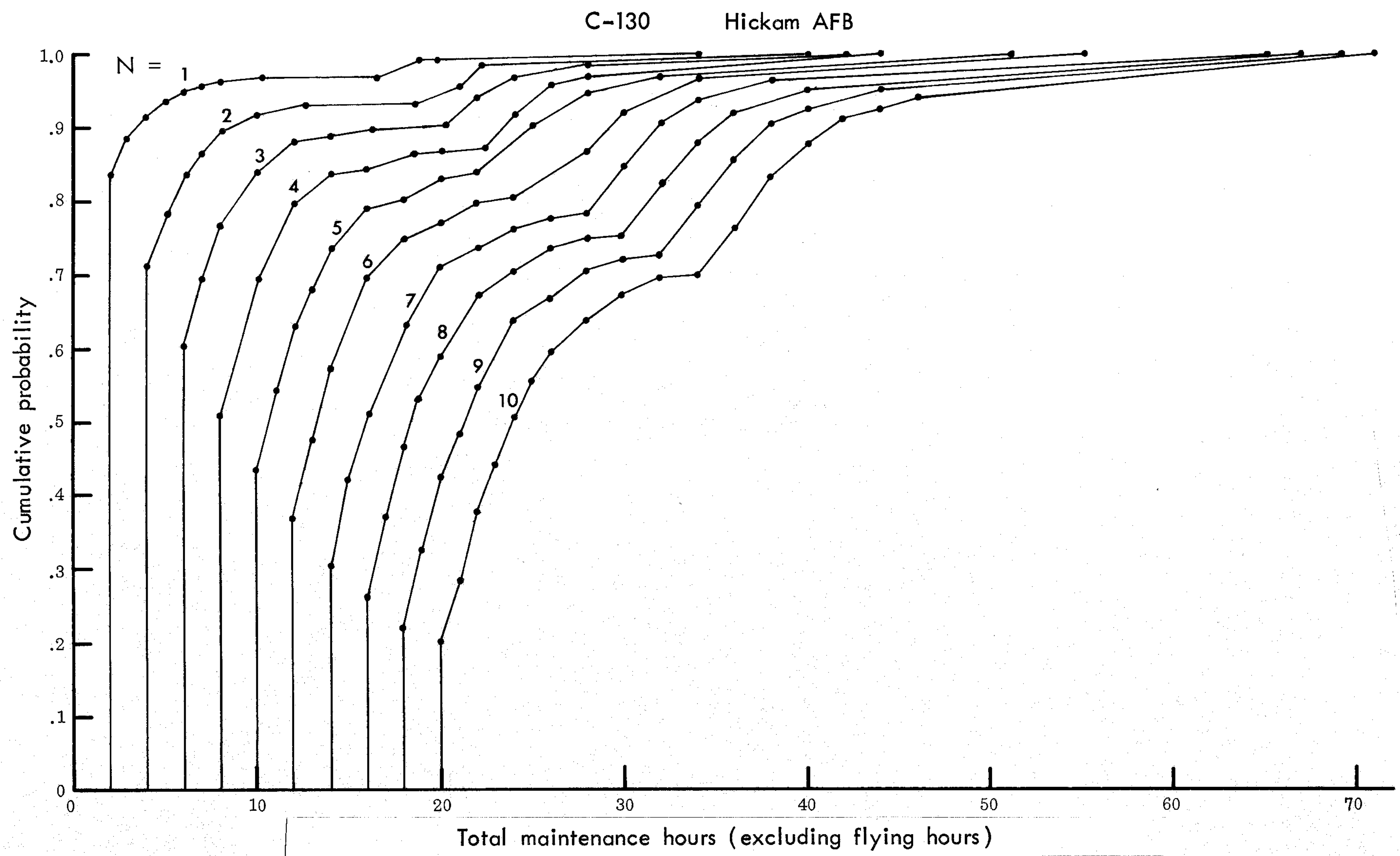

Fig. 1 -- Probability of at least N recoveries of one initially ready aircraft

Table 11

UNCONDITIONAL CUMULATIVE DISTRIBUTION FOR TOTAL MAINTENANCE TIME (ALL SHOPS) REQUIRED TO TURNAROUND A C-130 AIRCRAFT AT LEAST "N" TIMES AT HICKAM AIR FORCE BASE[a]

N=2 [b]		N=3 [c]		N=4 [d]	
Timepoint Value (Hours) (t)	Cumulative Probability G(t)	Timepoint Value (Hours) (t)	Cumulative Probability G(t)	Timepoint Value (Hours) (t)	Cumulative Probability G(t)
4.000	0.715	6.000	0.605	8.000	0.512
5.000	0.786	7.000	0.696	9.000	0.615
5.700	0.820	7.700	0.739	10.000	0.698
6.500	0.853	9.000	0.803	11.000	0.743
7.500	0.874	11.000	0.862	12.500	0.806
8.500	0.904	12.500	0.880	14.000	0.837
9.500	0.914	14.500	0.892	15.500	0.844
11.000	0.921	20.500	0.901	17.500	0.864
13.500	0.931	21.500	0.924	22.500	0.871
19.800	0.958	22.500	0.946	24.000	0.925
20.700	0.980	23.200	0.968	27.000	0.973
21.400	0.984	24.000	0.978	30.000	0.986
22.600	0.993	27.000	0.991	34.000	0.991
40.000	1.000	42.000	1.000	44.000	1.000

[a] Selected values from computer run; see Table 10 and Fig. 2 for N=1 case.

[b] Expected value = 5.753; variance = 21.260; standard variation = 4.611.

[c] Expected value = 8.602; variance = 30.913; standard variation = 5.560.

[d] Expected value = 11.464; variance = 40.688; standard variation = 6.379.

$$t = T - U_1 - U_2 - U_3 - U_4 - U_5 = 19 - 5 = 14 \text{ hours}$$

to obtain

$$COMP_{19}(5) = 0.837.$$

Table 12 shows the values obtained from the CONVOL results for m = 1, 2, ..., 8. Since $COMP_T(m)$ is the probability of completing at least m sorties during T, the probability of completing exactly m sorties during T, $comp_T(m)$, equals $COMP_T(m) - COMP_T(m+1)$, m = 0, 1, ...

Table 12

PROBABILITY OF COMPLETING AT LEAST "m" C-130 1-HOUR SORTIES IN A 19-HOUR FLYING DAY AT HICKAM AIR FORCE BASE WITH ONE READY C-130 AIRCRAFT

Number of Sorties (m)	Probability $COMP_{19}(m)$	Hours "Available" for Turnaround (19-m)
1	1.000	18
2	0.967	17
3	0.931	16
4	0.892	15
5	0.837	14
6	0.685	13
7	0.366	12
8 or more	0.0	11

K Aircraft

Define $COMP_{T,K}(m)$ as the probability that K initially ready aircraft can complete at least m sorties during time T.* Then the probability of completing exactly m sorties, $comp_{T,K}(m)$, during T with K aircraft equals

$$comp_{T,K}(m) = COMP_{T,K}(m) - COMP_{T,K}(m + 1), \qquad m = 0, 1, \ldots .$$

and

$$COMP_{T,K}(m) = \sum_{j=0}^{m} COMP_{T,1}(j)\, comp_{T,K-1}(m - j).$$

CONVOL may be used iteratively to make the above calculations. Figure 2 shows the results for our C-130 example for values of K from one through ten (the actual computer outputs are $1 - COMP_{19}(m)$).**

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*$COMP_T(m) \equiv COMP_{T,1}(m)$.

**When using the computer program to find $COMP_{T,M}(m)$ for $M > 1$, the input distribution should be $comp_{T,1}(m)$, not $COMP_{T,1}(m)$.

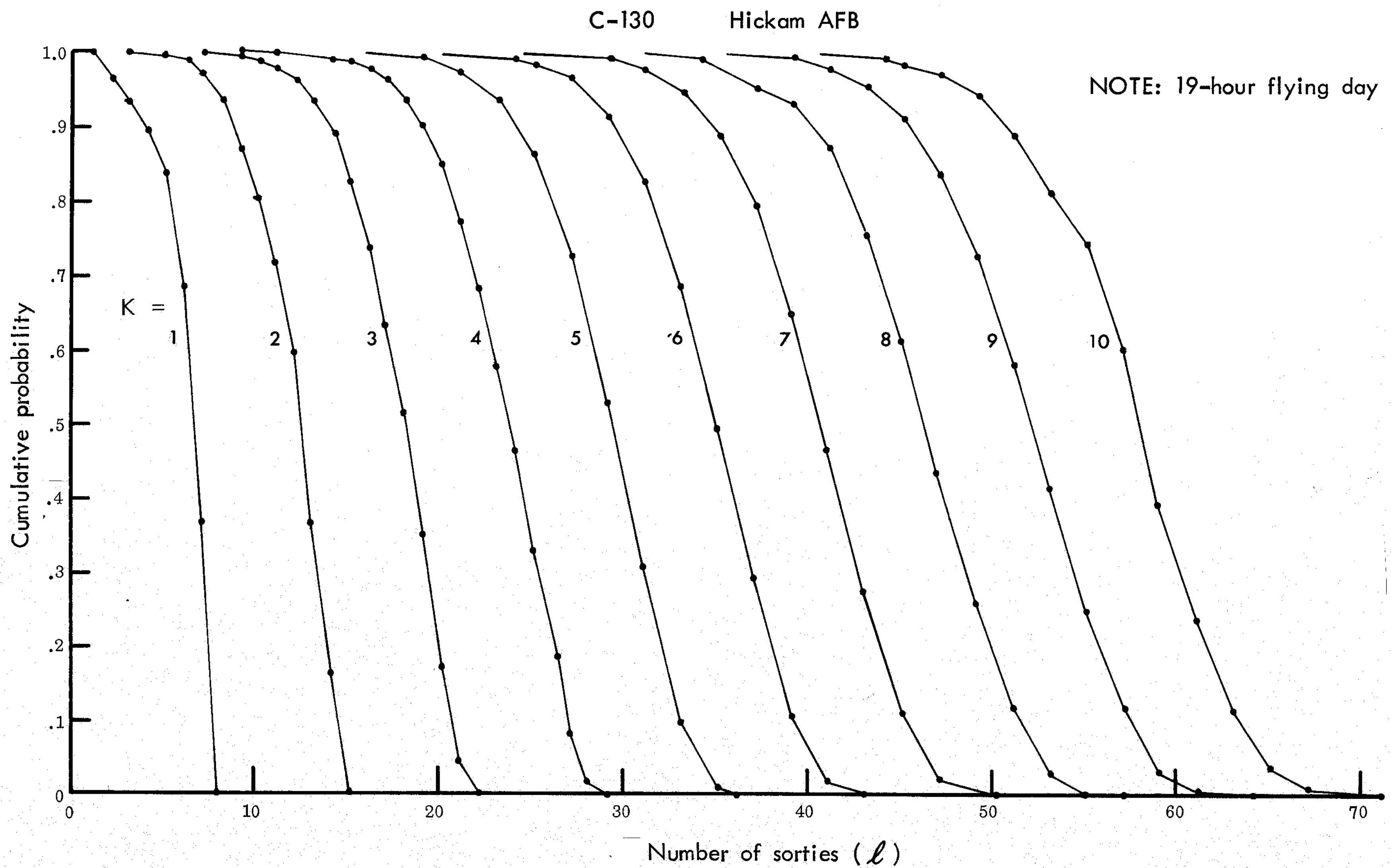

Fig. 2 -- Probability that K aircraft fly at least ℓ sorties

Because of the 2-hour minimum turnaround time and 1-hour constant flying time, we have seen that the probability is zero that eight or more sorties will be flown by one ready C-130 aircraft in a 19-hour flying day. For two ready C-130 aircraft, the probability is one that they will fly at least two 1-hour sorties and zero that they will fly 15 or more sorties, since the maximum number of sorties per C-130 aircraft per day in our example is seven. There is a probability of 0.5806 that they will fly a total of 12 sorties or more. This is the sum of probabilities that the first aircraft will fly exactly zero sorties and the second 12 or more; the first exactly one and the [redacted] second 11 or more, etc.

The expected number of completed sorties by M (initially ready)* aircraft in time T, $E_T(M)$, equals**

$$\sum_{m=0}^{\infty} m[\mathrm{COMP}_{T,K}(m) - \mathrm{COMP}_{T,K}(m+1)] = \sum_{m=0}^{\infty} \mathrm{COMP}_{T,K}(m) = E_T(K).$$

For the case with unlimited resources, $E_T(K) = KE_T(1)$. In our example at Hickam, the expected number of sorties one aircraft completes equals,

$$\sum \mathrm{COMP}_{19,1}(m) = 1.000 + 0.967 + 0.931 + 0.892 + 0.837$$

$$+ 0.685 + 0.366 + 0.0 = 5.678 = E_{19,1}(1).$$

where values of $\mathrm{COMP}_{19,1}(m) = \mathrm{COMP}_{19}(m)$ are from Table 12,

---

*For the general case covering aircraft initially ready or not see Ref. 7.

**Assumes in addition to unlimited resources that the NORS distribution for each plane is not dependent upon the number of aircraft (K).

TOTAL MISSION TIME (AIRCRAFT RECOVERY PLUS FLYING)

We will now return to the case of single aircraft with no resource constraints (i.e., no queues). We have shown (1) how to compute the time-to-repair using CONCUR for tasks that start at the same time, and (2) how to aggregate by maintenance type (e.g., unscheduled maintenance) to obtain the time-to-repair distributions. Now, for operational aircraft, this can be done for each Air Force base where the required data are available, if there is reason to believe that the time-to-repair distribution differs significantly for one or more bases.

We will consider that this has been done. We wish to determine the total turnaround time for a C-130 aircraft at Travis, starting with launch service before it lifts off on a Pacific route and ending with the completion of maintenance at the home base, Travis. The Pacific route includes stops at Hickam, Wake, Kadena, Saigon, Kadena, Midway, and ends at Travis. Table 13 shows the planned flying time between stops. At some bases there may be delays for parts (NORS). There may also be delays due to weather, but this can be allowed for in our unscheduled maintenance time-to-repair distribution if we desire, so we will not discuss weather delays further.

The "maintenance type" time-to-repair distributions require a lot of space. Thus, in Table 13, we show only the expected values for each maintenance type, by air base; these will be used in our discussion of the methods required to obtain the maintenance (and operations) planning factor, total mission (aircraft recovery plus flying) time. In Table 13, notice that when the C-130 returns to Travis, it goes into periodic maintenance if 500 flying hours have accrued since the last periodic; if not, it will go into postflight maintenance if 125 flying hours have accrued since its last postflight or periodic. If neither postflight nor periodic maintenance is required, then unscheduled maintenance is done if required.

Our first job is to collapse the last three maintenance type distributions into one consolidated distribution, then all "maintenance types" will be sequential and we will be able to use CONVOL.

Table 13

C-130 PACIFIC CARGO MISSION ROUTE

Airbase	Maintenance Type	Expected Time-to-Repair (Hours)	Flying Time to Next Base (Hours)
Travis	Launch service	1.374	7.93
Hickam	NORS	0.417	
Hickam	Unscheduled maintenance	2.273	
Hickam	Launch service	0.108	7.43
Wake	NORS	0.848	
Wake	Unscheduled maintenance	2.273	
Wake	Launch service	0.108	8.17
Kadena	NORS	1.428	
Kadena	Unscheduled maintenance	2.273	5.87
Kadena	Launch service	0.108	
Saigon[a]	Unscheduled maintenance	2.273	
Saigon	Launch service	0.108	5.87
Kadena	NORS	1.428	
Kadena	Unscheduled maintenance	2.273	
Kadena	Launch service	0.108	10.77
Midway	NORS	0.848	
Midway	Unscheduled maintenance	2.273	
Midway	Launch service	0.108	10.47
Travis	NORS	0.461	
Travis	Unscheduled maintenance	1.819	
Travis	Periodic (500 hours)	9.085	
Travis	Postflight (125 hours)	6.591	
Total		38.581	56.51

[a]NORS less than 0.001 probability at Saigon.

The total route or mission flying time (as shown in Table 13) is 56.51 hours. Therefore, in nine complete trips on this route a C-130 flies a total of 508.59 hours and completes one periodic, three postflights and five unscheduled maintenance tasks. Therefore, when a C-130 lands at Travis at the end of the mission, on the average there is a one-ninth probability it will require periodic, three-ninths probability it will require postflight, and five-ninths probability that it will require unscheduled maintenance.

By using these probabilities in CONCUR, with the associated unconditional time-to-repair distribution for each of the three

maintenance types, we can obtain the consolidated, unconditional time-to-repair distribution that will be an input to CONVOL. CONVOL, with the consolidated and other distributions as inputs, provides the unconditional distribution of the total mission turnaround (aircraft recovery) time which is graphically shown by the thin line in Fig. 3. The total mission (aircraft recovery plus flying) time is also shown in Fig. 3. It is obtained by adding the route flying time of 56.51 hours to each possible total turnaround time at all bases. The expected total mission time is approximately 96 hours in our example.

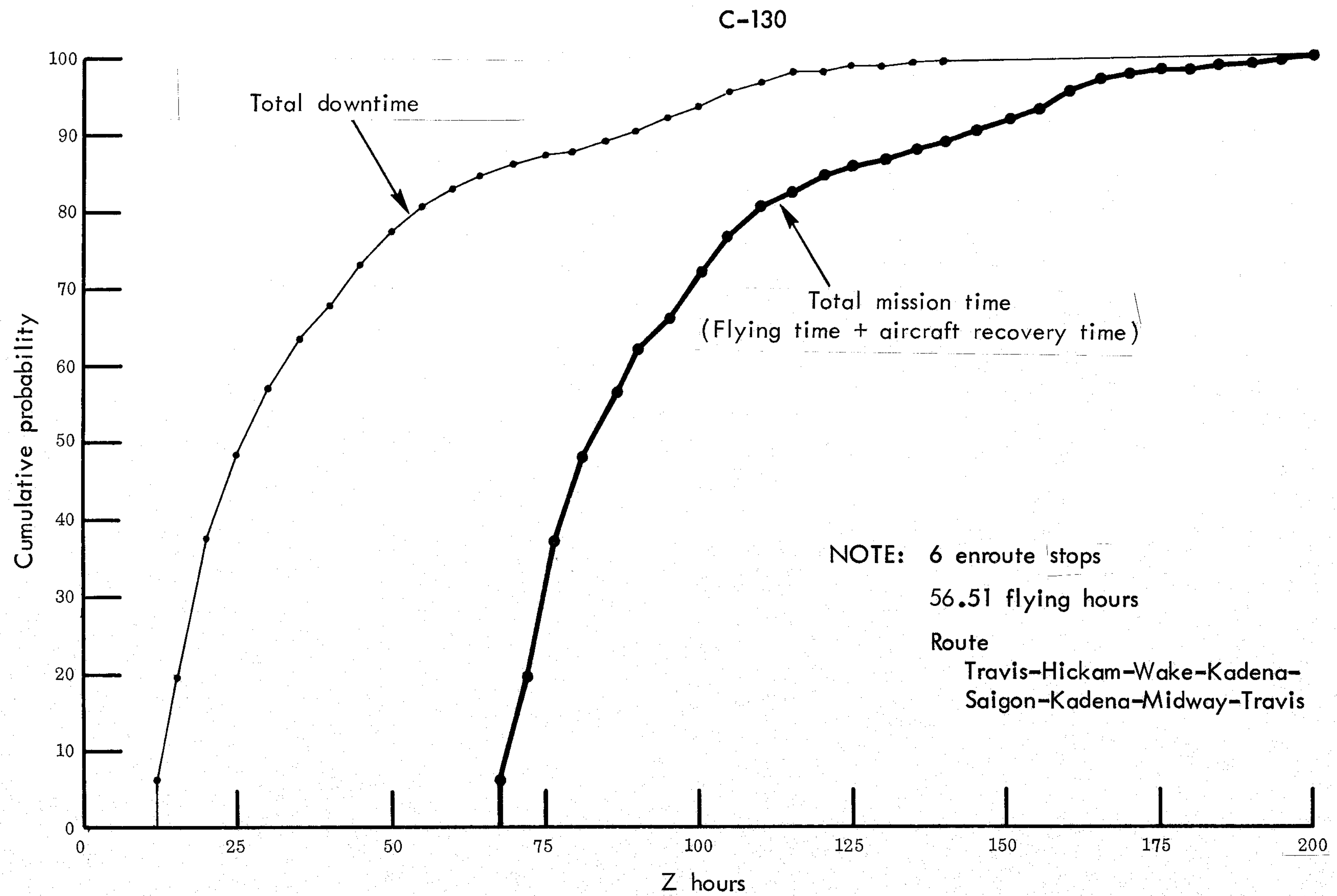

Fig. 3 -- Probability of completing Pacific Route in not more than Z hours

VII. CONCUR OPERATING INSTRUCTIONS

This section explains how to use the CONCUR computer program. It details how to prepare inputs to and interpret outputs from the program.

DESCRIPTION OF CONCUR INPUT DECK

The CONCUR input deck contains a master distribution deck, several system decks, a header card, and several input terminators. Figure 4 shows the physical location of these items in the CONCUR input deck.

The System Deck

Each system deck represents a set of tasks that are to be performed simultaneously; each card of the deck except the first represents one task. Figure 5, which is identical to input form six of SAMSOM II, contains the task card format. The headings of all non-relevant fields of the input form have been shaded.

Column 3 of all system deck cards must contain the number 6. The first card of each system deck must otherwise be blank. The probability that a task is required must be entered into Cols. 27-31 of each task card, the Failure Probability field. The duration of the task must be entered into Cols. 37-43, the Duration field. If a distribution of durations is to be used, then the negative of the distribution's number must be entered into the Duration field. CONCUR saves Cols. 24-26, the System List field, so that, in case a requested distribution cannot be found in the master distribution deck or in case the distribution is not acceptable to CONCUR, the program can inform the user of the task card that requested the distribution. Therefore the user should enter into the System List field of each task card a unique card identifier. Columns 5-20 of the last task card of the system deck are saved and printed at the head of each final distribution. All other fields of the system deck are ignored and may therefore be used for comments. All columns of each task card are printed when CONCUR lists the input deck.

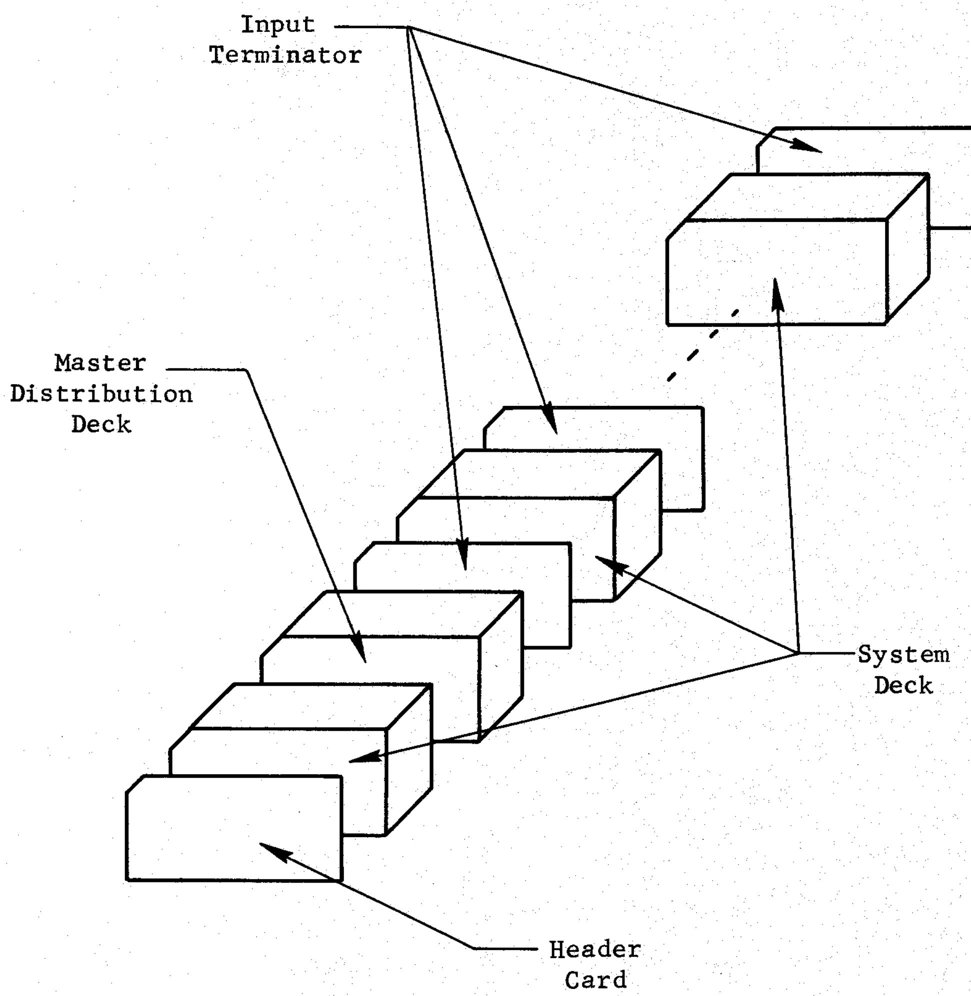

Fig. 4 -- Physical structure of the input deck

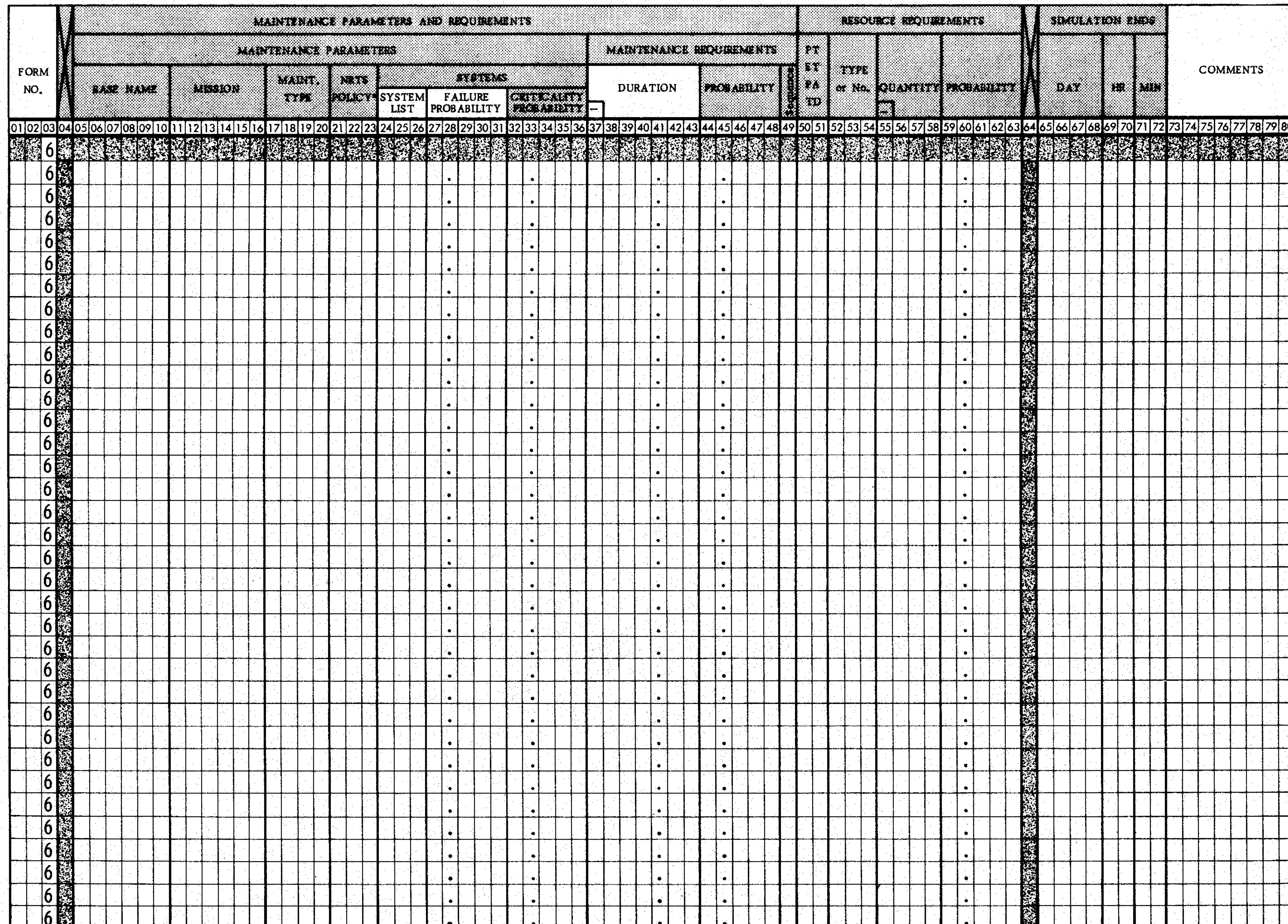

Fig. 5 -- System deck input form

A system deck can contain no more than 50 task cards; and if the deck contains more than 50, only the first 50 are processed.

The Master Distribution Deck

The master distribution deck must contain all the distributions that any system deck will request. CONCUR will write the master distribution deck onto an intermediate storage device and retrieve from the device only the distributions requested by the then current system deck. Figure 6, which is identical to input form 7 of SAMSOM II, contains the format of the master distribution deck.

Column 3 of all master distribution deck cards must contain the number 7. The first card of the deck must otherwise be blank. The distribution number must appear on the first card of each distribution and may appear on all cards of the distribution. Both cumulative and density probability distributions may be used. If a density distribution is used, then its probabilities must add to unity. CONCUR will convert all distributions to cumulative ones with increasing time values. No distribution may contain more than 104 entries, and if one does the program will ignore all but the first 104 entries. If the over-sized distribution is cumulative, however, then the program will set the 104th probability to unity.

The Header Card

The first card of the input deck must be the header card that normally will be blank but that permits the specification of several options if desired. These options and their methods of specification will be explained under the major heading "User Options" in this section.

The Input Terminator

To inform CONCUR that the end of a system deck has been reached, and that computations should begin, the input terminator must be used. This is nothing more than a blank card. The input terminator must follow the last card of all but the first system deck. In this case it must follow the master distribution deck.

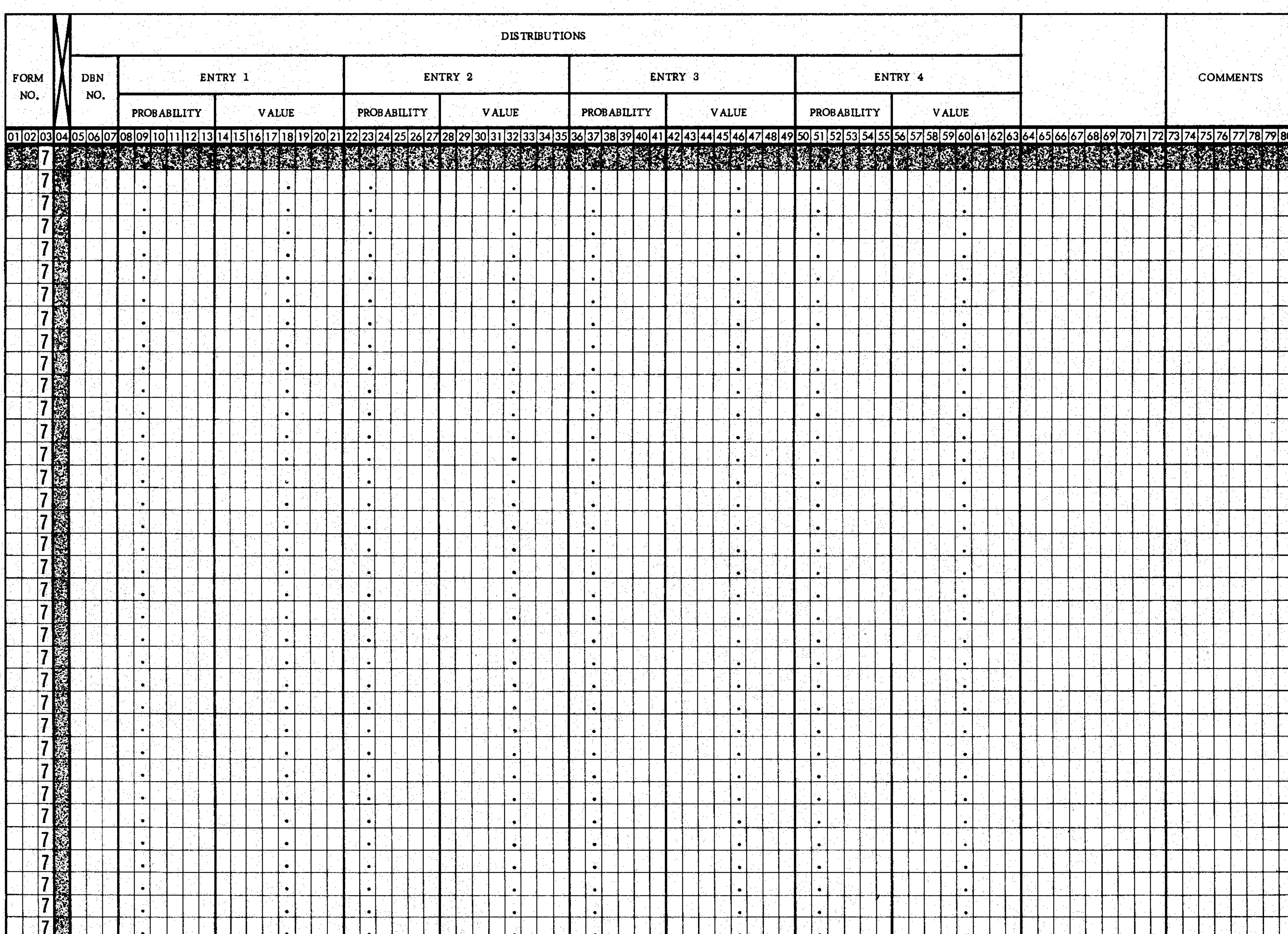

Fig. 6 -- Master distribution deck input form

SAMPLE PROBLEMS--INPUTS

At this point two sample CONCUR problems will be defined, and the required inputs will be presented.

Sample Problem 1

Section II described three groups of tasks--troubleshooting, unscheduled maintenance and functional check. Sample problem 1 deals with those groups.

Troubleshooting. Troubleshooting is divided into two tasks. The probability that the first task is required is unity. The distribution of durations given that the task is required is

0.5	1 hr.
1.0	2 hr.

The probability that the second task is required is two-thirds, which must be rounded to 0.667. The distribution of durations given that the second task is required is

0.5	1 hr.
1.0	2 hr.

Unscheduled Maintenance. Unscheduled maintenance is divided into three tasks. The probability that the first task is required is unity. The distribution of durations given that the task is required is

0.5	1 hr.
0.75	2 hr.
1.0	3 hr.

The probability that the second task is required is also equal to unity. The distribution of durations given that the task is required is

0.5	1 hr.
1.0	2 hr.

The probability that the third task is required is 0.5. Whenever it is required, the third task lasts one hour.

Functional Check. Functional check contains only one task; therefore CONCUR need not be applied.

We are now ready to prepare the input forms for CONCUR. First we will prepare the master distribution deck, which requires two distributions. The first, assigned distribution number 1, is

0.5	1 hr.
1.0	2 hr.

The second, designated distribution 2, is

0.5	1 hr.
0.75	2 hr.
1.0	3 hr.

Figure 7 contains the master distribution deck.

Two system decks are required, one for troubleshooting and one for unscheduled maintenance. Figure 8 contains the two system decks. Five items of interest should be noted. First, note that the first card of each system deck is blank except for the number 6 entered in Col. 3. Second, note the unique identifiers appearing in each card's System List field. Third, note that the Duration fields of all but one task card contain negative integers, thus indicating that duration distributions are to be used. For example, TS1 uses distribution one because "-1." appears in TS1's Duration field. Fourth, note that several task cards use the same distribution. Finally, note that comments have been entered on the task cards.

The outputs of this and the next sample problem will be explained under the major heading entitled "Description of Outputs."

Sample Problem 2

In sample problem 2 the CONCUR process will be approached with a different perspective. Suppose that four systems of an aircraft, SY1, SY2, SY3 and SY4, can be repaired simultaneously. Suppose too that the failure probabilities, P, of the systems are

$$P(SY1) = 0.007$$
$$P(SY2) = 0.201$$
$$P(SY3) = 0.089$$
$$P(SY4) = 0.013$$

FORM NO.	DBN NO.	DISTRIBUTIONS: ENTRY 1 PROBABILITY	ENTRY 1 VALUE	ENTRY 2 PROBABILITY	ENTRY 2 VALUE	ENTRY 3 PROBABILITY	ENTRY 3 VALUE	ENTRY 4 PROBABILITY	ENTRY 4 VALUE		COMMENTS
01 02 03	04 05 06 07	08 09 10 11 12 13	14 15 16 17 18 19 20 21	22 23 24 25 26 27	28 29 30 31 32 33 34 35	36 37 38 39 40 41	42 43 44 45 46 47 48 49	50 51 52 53 54 55	56 57 58 59 60 61 62 63	64 65 66 67 68 69 70 71 72	73 74 75 76 77 78 79 80
7											
7	1	.5	1.	1.	2.	.	.	.	.		
7	2	.5	1.	.75	2.	1.	3.	.	.		
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Fig. 7 -- Master distribution deck for sample problem 1

MAINTENANCE PARAMETERS AND REQUIREMENTS — RESOURCE REQUIREMENTS — SIMULATION ENDS — COMMENTS

	FORM NO. (01–03)	BASE NAME / MISSION / MAINT. TYPE (05–20)	NRTS POLICY* (21–23)	SYSTEM LIST (24–26)	FAILURE PROBABILITY (27–31)	CRITICALITY PROBABILITY (32–36)	DURATION (37–43)	PROBABILITY / Sequence / PT ET PA TD / TYPE or No. / QUANTITY / PROBABILITY (44–63)	DAY (65–68)	HR (69–70)	MIN (71–72)	COMMENTS (73–80)
System Deck 1	6											
	6			TS11	.	.	-1.	SYSTEM DECK ONE IS				
	6	TROUBLESHOOTING		TS2	.667	.	-1.	FOR TROUBLESHOOTS				
	6				.	.	.	. .				
	6				.	.	.	. .				
System Deck 2	6				.	.	.	. .				
	6			UM11	.	.	-2.	SYSTEM DECK TWO IS				
	6			UM21	.	.	-1.	FOR UNSCHEDULED .				
	6	UNSCHED. MAINT.		UM3	.5	.	1.	MAINTENANCE .				
	6				.	.	.	. .				
	6				.	.	.	. .				
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Fig. 8 -- System decks for sample problem 1

Finally, suppose that the repair time distributions for the systems are, respectively,

SY1		SY2		SY3		SY4	
0.5 hr.	0.073	1.0 hr.	0.38	2 hr.	0.074	4 hr.	0.25
3.5 hr.	0.309	2.5 hr.	0.74	3 hr.	0.305	5 hr.	0.37
4.0 hr.	0.502	4.6 hr.	0.82	7 hr.	0.82	10 hr.	0.83
5.0 hr.	0.684	5.2 hr.	0.87	8 hr.	0.94	11 hr.	0.95
7.0 hr.	0.837	6.0 hr.	0.95	12 hr.	1.0	15 hr.	1.0
10.0 hr.	1.0	7.0 hr.	0.99				
		7.3 hr.	1.0				

Given the above data we would like to determine the distribution of repair times for all the systems. Figures 9 and 10 contain the requisite system and master distribution decks for this problem.

Conditional and Unconditional Probability Distributions

The reader may have noticed that in the two sample problems only conditional probability distributions have been used. That is, the distributions represent durations of actual repair times given that a failure occurs. Frequently, however, data is of the unconditional type, and it would be nice if the user could use the data directly without having to extract a failure probability and a conditional distribution. Fortunately, it is possible to do so. The user need only enter into the failure probability field of the appropriate task card a value of 1.0. The unconditional distribution can then be used directly.

DESCRIPTION OF OUTPUTS

At this point CONCUR outputs will be examined. The outputs of the two sample problems will also be discussed. The first output CONCUR produces is a listing of the options specified on the header card. Since the header card and its options are explained under the major heading "User Options," discussion of this output will be deferred. The next output produced is a listing of the system deck. Each card of this deck is printed, along with all error and warning messages deemed necessary.

FORM NO. (01–03)	BASE NAME (05–10)	MISSION (11–16)	MAINT. TYPE (17–20)	NRTS POLICY* (21–23)	SYSTEM LIST (24–26)	FAILURE PROBABILITY (27–31)	CRITICALITY PROBABILITY (32–36)	DURATION (37–43)	PROBABILITY (44–48)	Sequence (49)	PT ET PA TD (50–51)	TYPE or No. (52–54)	QUANTITY (55–58)	PROBABILITY (59–63)	DAY (65–68)	HR (69–70)	MIN (71–72)	COMMENTS (73–80)
6																		
6					SY1	.007	.	-1.	.					.				
6					SY2	.201	.	-2.	.					.				
6					SY3	.089	.	-3.	.					.				
6	FØUR SIMUL. SYS.				SY4	.013	.	-4.	.					.				
6						.	.	.	.					.				
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Fig. 9 -- System deck for sample problem 2

FORM 20
8 FEB 66

FORM NO.	DBN NO.	ENTRY 1 PROBABILITY	ENTRY 1 VALUE	ENTRY 2 PROBABILITY	ENTRY 2 VALUE	ENTRY 3 PROBABILITY	ENTRY 3 VALUE	ENTRY 4 PROBABILITY	ENTRY 4 VALUE		COMMENTS
01 02 03	05 06 07	08 09 10 11 12 13	14 15 16 17 18 19 20 21	22 23 24 25 26 27	28 29 30 31 32 33 34 35	36 37 38 39 40 41	42 43 44 45 46 47 48 49	50 51 52 53 54 55	56 57 58 59 60 61 62 63	64 65 66 67 68 69 70 71 72	73 74 75 76 77 78 79 80
7											
7	1	.073	.5	.309	3.5	.502	4.	.684	5.		
7	1	.837	7.	1.	10.	.	.	.	.		
7	2	.38	1.	.74	2.5	.82	4.6	.87	5.2		
7		.95	6.	.99	7.	1.	7.3	.	.		
7	3	.074	2.	.305	3.	.82	7.	.94	8.		
7		1.	12.	.	.	.	.	.	.		
7	4	.25	4.	.37	5.	.83	10.	.95	11.		
7		1.	15.	.	.	.	.	.	.		
7		.	.	.	.	.	.	.	.		
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Fig. 10 -- Master distribution deck for sample problem 2

After the system deck is printed, CONCUR attempts to retrieve all requested distributions. The retrieved distributions, their expected values, variances, and standard deviations are then printed. If a distribution is requested, but for some reason cannot be retrieved, then an appropriate error message is printed indicating the cause of the difficulty.

When all requested distributions have been retrieved, CONCUR computes the two final distributions--one with interpolated and the other with uninterpolated probabilities. The distinctions between the two will be examined when the outputs for sample problem 2 are discussed. Each distribution, its expected value, variance, and stardard deviation are printed. In addition, a centile distribution is printed for each final distribution. The centile distribution is one whose probability values are multiples of 0.1 and whose time values are interpolated from the final distribution. The centile distribution will be discussed more completely when sample problem 1 is explained.

The computer program will continue in this manner until all system decks have been processed.

Sample Problem 1

Figures 11, 12 and 13 contain CONCUR output for the troubleshoot group of sample problem 1. Figure 11 lists the inputs. Figure 12 contains the interpolated probability final distribution and the corresponding centile distribution for troubleshoot. The centile distribution is computed by linear interpolation of the final distribution. For instance, in Fig. 12 there is a centile distribution entry of

0.6 1.40007.

This is computed by linear interpolation between the two points

0.33325	1.0 hr.
1.0	2.0 hr.

```
6
6                                TS1   1.000           -1.00 SYSTEM DECK ONE IS
6   TROUBLESHOOTING              TS2   0.667           -1.00 FOR TROUBLESHOOTS
7                                      -.000           -0.00
7   1   0.5000   1.000   1.0000  2.000   -.0000  -0.000   -.0000  -0.000

EXPECTED VALUE =    1.50000
VARIANCE =    0.25000
STANDARD DEVIATION =    0.50000
```

Fig. 11 -- System deck and requested distribution for the troubleshoot task group, sample problem 1

FINAL DISTRIBUTION (INTERPOLATED PROBABILITIES)

TROUBLESHOOTING

CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE
0.00000	0.00000	0.33325	1.00000	1.00000	2.00000						

EXPECTED VALUE = 1.66675
VARIANCE = 0.22219
STANDARD DEVIATION = 0.47138

Centile distribution

0.10000	0.30008
0.20000	0.60015
0.30000	0.90023
0.40000	1.10011
0.50000	1.25009
0.60000	1.40007
0.70000	1.55006
0.80000	1.70004
0.90000	1.85002
1.00000	2.00000

Fig. 12 -- Interpolated final distribution for the troubleshoot task group, sample problem 1

FINAL DISTRIBUTION (UNINTERPOLATED PROBABILITIES)

TROUBLESHOOTING

CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE
0.00000	0.00000	0.33325	1.00000	1.00000	2.00000						

EXPECTED VALUE = 1.66675
VARIANCE = 0.22219
STANDARD DEVIATION = 0.47138

0.10000	0.30008
0.20000	0.60015
0.30000	0.90023
0.40000	1.10011
0.50000	1.25009
0.60000	1.40007
0.70000	1.55006
0.80000	1.70004
0.90000	1.85002
1.00000	2.00000

Fig. 13 -- Uninterpolated final distribution for the troubleshoot task group, sample problem 1

of the final distribution, that is, by solving for T in the proportion

$$\frac{2 - 1}{1 - 0.33325} = \frac{2 - T}{1 - 0.6}$$

thus yielding

$$T = 2 - 0.4/0.66675 = 1.40007.$$

Figure 13 contains the uninterpolated probability final distribution and its corresponding centile distribution.

Figures 14, 15 and 16 contain the CONCUR output for the unscheduled maintenance group of sample problem 1. Figure 14 lists the inputs. Note that a warning message has been printed for UM3, informing the user that a fix time has been entered into the task card's duration field. Figures 15 and 16 contain the interpolated probability and uninterpolated probability final distributions and their corresponding centile distributions.

Sample Problem 2

Figures 17, 18 and 19 contain the outputs for sample problem 2. A comparison of the expected values in Figs. 18 and 19 indicates that there is indeed a difference between the interpolated and uninterpolated final distributions. To understand the difference between the two distributions, one must understand how CONCUR selects the time values of final distributions and how it computes the probabilities associated with each selected value. The selection process is rather simple. All time values of the requested distributions become time values of the final distribution. Figure 20 contains the requested distributions and their merged time values. To compute the probability associated with each time value, CONCUR uses the formula

$$G(t) = \Pi(p_i P_i(t) + 1 - p_i),$$

```
 6
 6                               UM1    1.000           -2.00 SYSTEM DECK TWO IS
 6                               UM2    1.000           -1.00 FOR UNSCHEDULED
 6   UNSCHED. MAINT.             UM3    0.500            1.00 MAINTENANCE
THE DISTRIBUTION SPECIFIER       1.00 IS NOT NEGATIVE.  THEREFORE IT WILL BE CONSIDERED A FIX TIME.
                                        -.000           -0.00
 7   1    0.5000   1.000   1.0000   2.000   -.0000  -0.000   -.0000  -0.000

 EXPECTED VALUE =    1.50000
 VARIANCE =   0.25000
 STANDARD DEVIATION =   0.50000


 7   2    0.5000   1.000   0.7500   2.000   1.0000   3.000   -.0000  -0.000

 EXPECTED VALUE =    1.75000
 VARIANCE =   0.68750
 STANDARD DEVIATION =   0.82916
```

Fig. 14 -- Input listing for the unscheduled maintenance task group, sample problem 1

FINAL DISTRIBUTION (INTERPOLATED PROBABILITIES)

UNSCHED. MAINT.

CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE
0.00000	0.00000	0.25000	1.00000	0.75000	2.00000	1.00000	3.00000				

EXPECTED VALUE = 2.00000
VARIANCE = 0.50000
STANDARD DEVIATION = 0.70711

0.10000	0.40000
0.20000	0.80000
0.30000	1.10000
0.40000	1.30000
0.50000	1.50000
0.60000	1.70000
0.70000	1.90000
0.80000	2.20000
0.90000	2.60000
1.00000	3.00000

Fig. 15 -- Interpolated final distribution for the unscheduled maintenance task group, sample problem 1

FINAL DISTRIBUTION (UNINTERPOLATED PROBABILITIES)

UNSCHED. MAINT.

CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE
0.00000	0.00000	0.25000	1.00000	0.75000	2.00000	1.00000	3.00000				

EXPECTED VALUE = 2.00000
VARIANCE = 0.50000
STANDARD DEVIATION = 0.70711

0.10000	0.40000
0.20000	0.80000
0.30000	1.10000
0.40000	1.30000
0.50000	1.50000
0.60000	1.70000
C.70000	1.90000
0.80000	2.20000
0.90000	2.60000
1.00000	3.00000

Fig. 16 -- Uninterpolated final distribution for the unscheduled maintenance task group, sample problem 1

```
 6
 6                                SY1   0.007          -1.00
 6                                SY2   0.201          -2.00
 6                                SY3   0.089          -3.00
 6   FOUR SIMUL. SYS.             SY4   0.013          -4.00
 7                                      -.000          -0.00
 7   1   0.0730   0.500   0.3090   3.500   0.5020   4.000   0.6840   5.000
 7   1   0.8370   7.000   1.0000  10.000   -.0000  -0.000   -.0000  -0.000

EXPECTED VALUE =   5.24550
VARIANCE =   6.82898
STANDARD DEVIATION =   2.61323


 7   2   0.3800   1.000   0.7400   2.500   0.8200   4.600   0.8700   5.200
 7  -0   0.9500   6.000   0.9900   7.000   1.0000   7.300   -.0000  -0.000

EXPECTED VALUE =   2.74100
VARIANCE =   3.53462
STANDARD DEVIATION =   1.88006


 7   3   0.0740   2.000   0.3050   3.000   0.8200   7.000   0.9400   8.000
 7  -0   1.0000  12.000   -.0000  -0.000   -.0000  -0.000   -.0000  -0.000

EXPECTED VALUE =   6.12600
VARIANCE =   6.40213
STANDARD DEVIATION =   2.53024


 7   4   0.2500   4.000   0.3700   5.000   0.8300  10.000   0.9500  11.000
 7  -0   1.0000  15.000   -.0000  -0.000   -.0000  -0.000   -.0000  -0.000

EXPECTED VALUE =   8.27000
VARIANCE =  10.37710
STANDARD DEVIATION =   3.22135
```

Fig. 17 -- Inputs for sample problem 2

FINAL DISTRIBUTION (INTERPOLATED PROBABILITIES)

FOUR SIMUL. SYS.

CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE
0.71340	0.00000	0.74954	0.50000	0.78569	1.00000	0.83312	2.00000	0.86504	2.50000	0.87876	3.00000
0.88830	3.50000	0.89886	4.00000	0.91126	4.60000	0.92304	5.00000	0.92876	5.20000	0.95441	6.00000
0.97519	7.00000	0.98081	7.3C000	0.98933	8.00000	0.99513	10.00000	0.99802	11.00000	1.00000	12.00000

EXPECTED VALUE = 1.01346
VARIANCE = 4.52954
STANDARD DEVIATION = 2.12827

0.80000	1.30178
0.9C000	4.05521
1.CC000	12.C0C00

Fig. 18 -- Interpolated final distribution for sample problem 2

FINAL DISTRIBUTION (UNINTERPOLATED PROBABILITIES)

FOUR SIMUL. SYS.

CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE
0.71340	0.00000	0.78200	1.00000	0.78765	2.00000	0.85276	2.50000	0.87186	3.00000	0.87331	3.50000
0.87738	4.00000	0.89227	4.60000	0.89481	5.00000	0.90414	5.20000	0.91907	6.00000	0.97285	7.00000
0.97481	7.30000	0.98539	8.00000	0.99246	10.00000	0.99401	11.00000	1.00000	12.00000		

EXPECTED VALUE = 1.17566
VARIANCE = 5.71290
STANDARD DEVIATION = 2.39017

0.80000	2.09484
0.90000	5.11117
1.00000	12.00000

Fig. 19 -- Uninterpolated final distribution for sample problem 2

DB1		DB2		DB3		DB4		Merged Time Values
.073	.5							.5
		.38	1.0					1.0
				.074	2.0			2.0
		.74	2.5					2.5
				.305	3.0			3.0
.309	3.5							3.5
.502	4.0					.25	4.0	4.0
		.82	4.6					4.6
.684	5.0					.37	5.0	5.0
		.87	5.2					5.2
		.95	6.0					6.0
.837	7.0	.99	7.0	.82	7.0			7.0
		1.0	7.3					7.3
				.94	8.0			8.0
1.0	10.0					.83	10.0	10.0
						.95	11.0	11.0
				1.0	12.0			12.0
						1.0	15.0	15.0

Fig. 20 -- Requested distributions and final distribution time values for sample problem 2

where $G(t)$ is the final distribution probability associated with the time value t; p_i is the probability that task i is required; and $P_i(t)$ is a probability derived from the distribution requested by task i.

It is the way in which P_i is derived that yields the two final distributions. In the interpolated case $P_i(t)$ is defined by the linear interpolation proportion:

$$\frac{P_i(t) - F_i(t_j)}{t - t_j} = \frac{F_i(t_{j+1}) - F_i(t_j)}{t_{j+1} - t_j} \qquad t_j < t \le t_{j+1},$$

where t_j and t_{j+1} are consecutive time values of the distribution requested by task i; and $F_i(t_j)$ and $F_i(t_{j+1})$ are the probability entries of the requested distribution that correspond to t_j and t_{j+1}, respectively. When t is greater than the requested distribution's largest time, then $P_i(t)$ is set equal to 1.0. When t is less than the requested distribution's smallest time, then interpolation occurs between the points (0, 0) and $[F_i(t_1), t_1]$.

In the uninterpolated case, $P_i(t)$ is defined by

$$P_i(t) = \begin{cases} 0 & \text{for } t < \min(t_j) \\ F_i(t_j) & t_j \le t < t_{j+1} \\ 1 & t \ge \max(t_j) \end{cases},$$

where $\min(t_j)$ and $\max(t_j)$ are, respectively, the smallest and largest time values in the requested distribution.

In the event that $t = t_j$, the two values of $P_i(t)$ will be [redacted] identical. Therefore, if all requested distributions have a common time value t, then the two values of $G(t)$ will be identical. Sample problem 1 is a good example of this in that the two final distributions of each task group are identical.

USER OPTIONS

We have mentioned that the header card may be used to select various options during a CONCUR run. At this point the options and the methods by which they are selected will be described. Figure 21 contains the format of the header card.

Converted Distribution Output

Any type of discrete probability distribution may be entered into the master distribution deck, be it a density function or a cumulative one. CONCUR automatically converts all requested distributions into the cumulative form. If the user wishes that the converted distribution be printed, he may so indicate by placing the number 2 into column one of the header card. Each retrieved distribution will then be printed twice more, the first in the unconverted form and the second in the converted form.

If the user places the number 1 into Col. 1, then a special purpose debugging dump will be printed. The dump generates a good deal of output and is of scant use to the general user. Therefore, it will not be explained. It is mentioned only as a warning to the user.

Punching Final Distributions

If the user desires, he may request that the final distributions be punched. To select this option, a nonzero integer must be placed into card columns two through four of the header card. If n is the number entered into the field, then the distribution number of the first final distribution will be $|n|$, the second's will be $|n|+1$, etc. The cards will be in a format suitable for input to either CONCUR, CONVOL or SAMSOM II.

Minimum Difference Between Probabilities

A preliminary version of CONCUR indicated that very often, when actual data were used and when many distributions were being cumulated, the difference between consecutive final distribution probabilities was

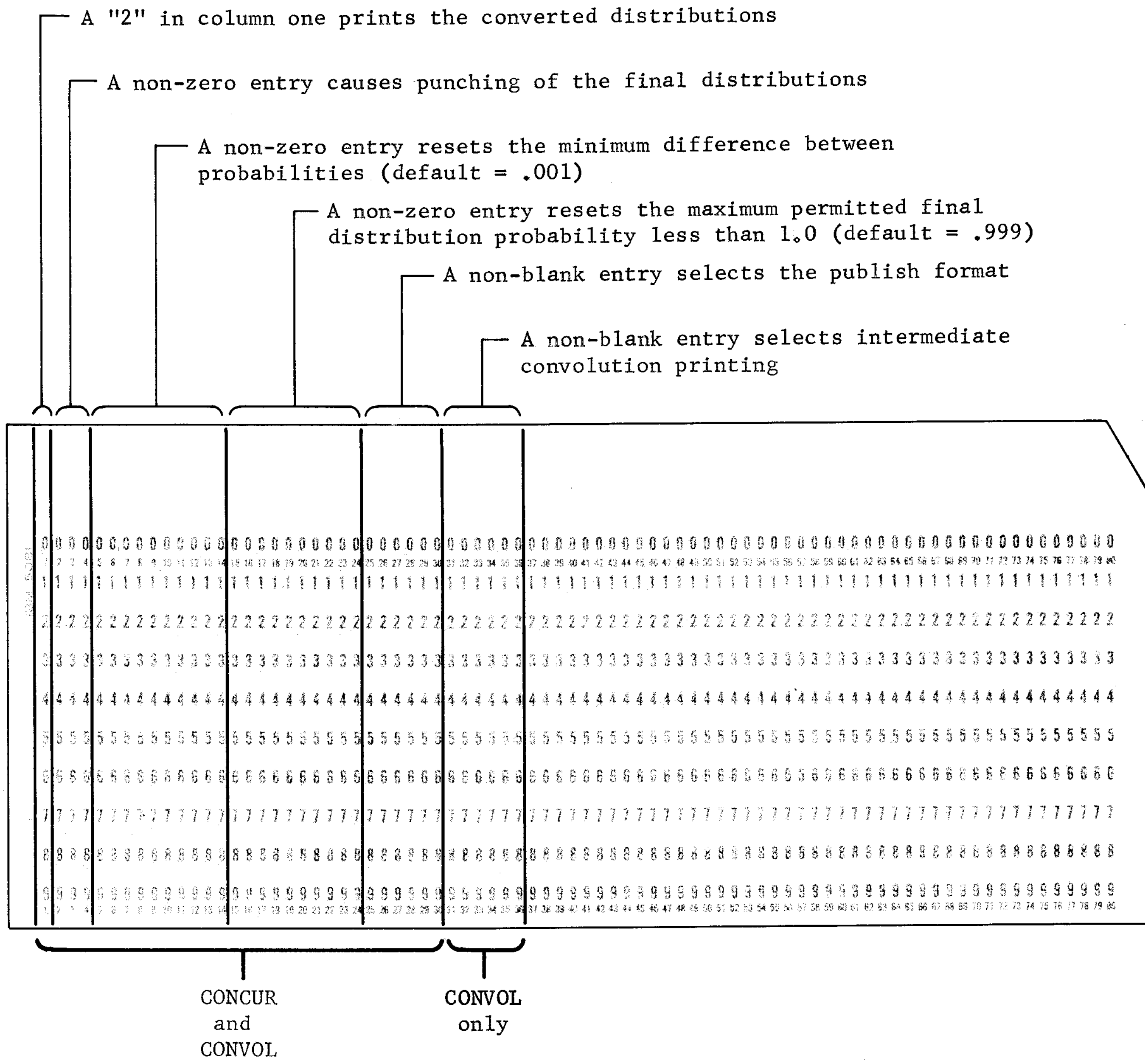

Fig. 21 -- Header card format

very small. Therefore, a point elimination algorithm was implemented to discard the small probability points. Thus if two consecutive points have probabilities that differ by no greater than 0.001, then the larger of the two points is discarded from the final distribution.

The user may alter the minimum difference value or eliminate the algorithm completely. If an alternate value is desired, then the value must be entered on the header card in Cols. 5-14. If the algorithm is to be eliminated completely, then a negative number must be entered into the field.

Maximum Permitted Nonunity Probability

In Figs. 18 and 19, the final distributions for sample problem 2 are presented. Both figures indicate that the probability is unity that all tasks are completed in twelve hours or less. If we examine Fig. 20, however, we see that the fourth distribution listed contains a time value of 15 hours. There seems to be an inconsistency between the final and input distributions. Actually this example displays another CONCUR option.

CONCUR provides the user with the option to eliminate from the final distributions all nonunity probabilities that are greater than a given value. CONCUR changes the first probability greater than the given value to unity. If the user does not specify to the contrary on the header card, then a default value of 0.999 is assumed. The default value was used in sample problem 2, and thus the 15-hour point was eliminated.

The user may specify that another value is to be used, or that no point elimination is to take place. If a different value is to be used, then the user must enter the value into Cols. 15-24 of the header card. If no point elimination is desired, then a negative number must be entered into the field.

The Publish Format

Frequently, computer output becomes a part of a research document, and since computer output is usually on wide paper, the output must be reduced or wide paper must be incorporated into the document.

To avoid the necessity and cost of such techniques, CONCUR provides the user with a publishing format for the final distributions. To activate the format, a nonblank character must appear in Cols. 25-30 of the header card.

User Option Examples

Figures 22 and 23 contain the final distributions for sample problem 2 with both point elimination techniques ignored and with the publishing format selected. Note the presence of the 15-hour entry. Figure 24 contains distribution conversion output for sample problem 2.

Note that if the maximum permitted probability is reset to a value larger than 0.999 without altering the minimum difference value, then the maximum probability is still 0.999, because if a greater probability were encountered, the difference between it and 1.0 would be less than 0.001. Hence the point's probability would be set to unity, and all subsequent points would be ignored.

MAXIMUM FINAL DISTRIBUTION SIZE

The final distribution may contain a maximum of four thousand points. If more than four thousand distinct points exist during a run, then all but the first four thousand are ignored. A warning message is printed to inform the user of the condition.

FINAL DISTRIBUTION (INTERPOLATED PROBABILITIES)

FOUR SIMUL. SYS.

CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE
0.71340	0.00000	0.74954	0.50000	0.78569	1.00000
0.83312	2.00000	0.86504	2.50000	0.87876	3.00000
0.88830	3.50000	0.89886	4.00000	0.91126	4.60000
0.92304	5.00000	0.92876	5.20000	0.95441	6.00000
0.97519	7.00000	0.98081	7.30000	0.98933	8.00000
0.99513	10.00000	0.99802	11.00000	0.99951	12.00000
1.00000	15.00000				

EXPECTED VALUE = 1.01492
VARIANCE = 4.56605
STANDARD DEVIATION = 2.13683

0.80000	1.30178
0.90000	4.05521
1.00000	15.00000

Fig. 22 -- Publish format and no point elimination for the interpolated final distribution, sample problem 2

FINAL DISTRIBUTION (UNINTERPOLATED PROBABILITIES)

FOUR SIMUL. SYS.

CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE
0.71340	0.00000	0.71376	0.50000	0.78200	1.00000
0.78765	2.00000	0.85276	2.50000	0.87186	3.00000
0.87331	3.50000	0.87738	4.00000	0.89227	4.60000
0.89481	5.00000	0.90414	5.20000	0.91907	6.00000
0.97285	7.00000	0.97481	7.30000	0.98539	8.00000
0.99246	10.00000	0.99401	11.00000	0.99935	12.00000
1.00000	15.00000				

EXPECTED VALUE = 1.17742
VARIANCE = 5.76111
STANDARD DEVIATION = 2.40023

0.80000	2.09484
0.90000	5.11117
1.00000	15.00000

Fig. 23 -- Publish format and no point elimination for the uninterpolated final distribution, sample problem 2

```
6
6                          SY1   0.007          -1.00
6                          SY2   0.201          -2.00
6                          SY3   0.089          -3.00
6   FOUR SIMUL. SYS.       SY4   0.013          -4.00
7                                -.000          -0.00
7   1   0.0730   0.500   0.3090   3.500   0.5020   4.000   0.6840   5.000
7   1   0.8370   7.000   1.0000  10.000   -.0000  -0.000   -.0000  -0.000

THIS DISTRIBUTION CONTAINS  6 ENTRIES
  0.07300   0.50000   0.30900   3.50000    0.50200   4.00000    0.68400   5.00000    0.83700   7.00000    1.00000  10.00000

THIS DISTRIBUTION CONTAINS  6 ENTRIES
  0.07300   0.50000   0.30900   3.50000    0.50200   4.00000    0.68400   5.00000    0.83700   7.00000    1.00000  10.00000

 EXPECTED VALUE =   5.24550
 VARIANCE =   6.82898
 STANDARD DEVIATION =   2.61323


7   2   0.3800   1.000   0.7400   2.500   0.8200   4.600   0.8700   5.200
7  -0   0.9500   6.000   0.9900   7.000   1.0000   7.300   -.0000  -0.000

THIS DISTRIBUTION CONTAINS  7 ENTRIES
  0.38000   1.00000   0.74000   2.50000    0.82000   4.60000    0.87000   5.20000    0.95000   6.00000    0.99000   7.00000
  1.00000   7.30000

THIS DISTRIBUTION CONTAINS  7 ENTRIES
  0.38000   1.00000   0.74000   2.50000    0.82000   4.60000    0.87000   5.20000    0.95000   6.00000    0.99000   7.00000
  1.00000   7.30000

 EXPECTED VALUE =   2.74100
 VARIANCE =   3.53462
 STANDARD DEVIATION =   1.88006


7   3   0.0740   2.000   0.3050   3.000   0.8200   7.000   0.9400   8.000
7  -0   1.0000  12.000   -.0000  -0.000   -.0000  -0.000   -.0000  -0.000

THIS DISTRIBUTION CONTAINS  5 ENTRIES
  0.07400   2.00000   0.30500   3.00000    0.82000   7.00000    0.94000   8.00000    1.00000  12.00000

THIS DISTRIBUTION CONTAINS  5 ENTRIES
  0.07400   2.00000   0.30500   3.00000    0.82000   7.00000    0.94000   8.00000    1.00000  12.00000

 EXPECTED VALUE =   6.12600
 VARIANCE =   6.40213
 STANDARD DEVIATION =   2.53024


7   4   0.2500   4.000   0.3700   5.000   0.8300  10.000   0.9500  11.000
7  -0   1.0000  15.000   -.0000  -0.000   -.0000  -0.000   -.0000  -0.000

THIS DISTRIBUTION CONTAINS  5 ENTRIES
  0.25000   4.00000   0.37000   5.00000    0.83000  10.00000    0.95000  11.00000    1.00000  15.00000

THIS DISTRIBUTION CONTAINS  5 ENTRIES
  0.25000   4.00000   0.37000   5.00000    0.83000  10.00000    0.95000  11.00000    1.00000  15.00000

 EXPECTED VALUE =   8.27000
 VARIANCE =  10.37710
 STANDARD DEVIATION =   3.22135
```

Fig. 24 -- Input listing with conversion of requested distributions, sample problem 2

VIII. CONVOL OPERATING INSTRUCTIONS

CONVOL operating procedures are almost identical to those of CONCUR. Therefore only the distinctions will be discussed in detail.

DESCRIPTION OF CONVOL INPUT DECK

The CONVOL input deck contains the same items as the CONCUR input deck. Figure 4 contains the input deck's general physical structure.

The Header Card

All user options available in CONCUR are available in CONVOL, and their selection rules are identical. CONVOL does permit one additional user option--the printing of intermediate convolutions. This option is made possible because CONVOL operates on only two distributions at a time. To select the intermediate convolution output, the user must enter into Cols. 31-36 of the header card any nonblank character string except the character string "DUMP", which triggers a special purpose debugging dump. The dump produces a good deal of output and therefore should be avoided. Because it is of scant use to the general user, it will not be described.

The System Deck

The CONVOL system deck is almost identical to that of CONCUR, the only difference being that CONVOL does not require an entry into the failure probability field of the task card. CONVOL ignores the field completely. Each task card of the CONVOL system deck represents a task (or group of tasks) that must be performed after the tasks whose cards precede it and before the tasks whose cards follow it. Hence the order of the task cards in the system deck determines the order in which the tasks are performed.

The Master Distribution Deck

The CONVOL master distribution deck is identical to its counterpart in CONCUR. Since CONVOL does not permit the entry of failure probabilities on task cards, however, the distributions in the master deck are unconditional. Therefore if the user wishes to employ an element of uncertainty for a task group (or task), he must incorporate the uncertainty into the distribution itself.

The Input Terminator

The CONVOL input terminator is identical to its CONCUR counterpart, being simply a blank card.

SAMPLE PROBLEM

Section II presented an example where three groups of tasks--troubleshoot, unscheduled maintenance and functional check--were defined and the corresponding cumulative and convoluted distributions were presented. In the CONCUR section, the first two groups of tasks were processed yielding unconditional probability distributions of fix times for each task group. In this section the example will be completed by applying CONVOL to the three task groups.

The output for this problem is given in Figs. 25 and 26; the intermediate convolutions have been printed. Two items should be noted in Fig. 25. First, the first intermediate convolution distribution is no more than the first requested distribution, distribution 1. (In general the two distributions will not be identical, since point elimination and time value rounding are performed on the intermediate convolution. See the following subsection.) Second, the last intermediate convolution distribution is identical to the immediately preceding one except that the time values have been increased by one hour, thus reflecting the fact that the functional check always takes one hour. In fact, whenever an absolute time is specified, CONVOL increases the time values of the convoluted distribution by the absolute fix time instead of going through the convolution process.

```
 6
 6                           TS                      -1.0C      TROUBLESHOOTING
 6                           UM                      -2.0C      UNSCHEDULED MAINTENANCE
 6   TS + UM + FS            FS                       1.0C      FUNCTIONAL CHECKS
THE DISTRIBUTION SPECIFIER   1.00 IS NOT NEGATIVE.  THEREFORE IT WILL BE CONSIDERED A FIX TIME.
 7                                                   -0.00
 7   1   0.3332   1.000   1.0000   2.000   -.0000  -0.000   -.000C  -0.000

EXPECTED VALUE =   1.66680
VARIANCE =   0.22218
STANDARD DEVIATION =   0.47136


 7   2   0.2500   1.000   0.7500   2.000   1.0000   3.000   -.0000  -0.000

EXPECTED VALUE =   2.00000
VARIANCE =   0.50000
STANDARD DEVIATION =   0.70711


  TS + UM + FS

  CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE

  0.33320   1.00000   1.00000   2.00000

EXPECTED VALUE =   1.66680
VARIANCE =   0.22218
STANDARD DEVIATION =   0.47136


  TS + UM + FS

  CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE

  0.08330   2.00000   0.41660   3.00000   C.83330   4.00000   1.00000   5.00000

EXPECTED VALUE =   3.66680
VARIANCE =   0.72218
STANDARD DEVIATION =   0.84981


  TS + UM + FS

  CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE     CUMPROB   VALUE

  0.08330   3.00000   C.41660   4.00000   0.83330   5.00000   1.00000   6.00000

EXPECTED VALUE =   4.66680
VARIANCE =   C.72218
STANDARD DEVIATION =   0.84981
```

Fig. 25 -- Input listing and intermediate convolutions for CONVOL sample problem

CONVOLUTED DISTRIBUTION

TS + UM + FS

CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE	CUMPROB	VALUE
0.08330	3.00000	0.41660	4.00000	0.83330	5.00000	1.00000	6.00000				

EXPECTED VALUE = 4.66680
VARIANCE = 0.72218
STANDARD DEVIATION = 0.84981

Fig. 26 -- Final distribution for CONVOL sample problem

FURTHER CONVOL CONSIDERATIONS

The maximum number of points permitted in the final distribution is one thousand. At first glance this seems to be quite unreasonable. For suppose that three distributions are to be convoluted, each containing twenty points. The maximum number of possible distribution entries is eight thousand in this case (20^3), and it is quite possible that more than one thousand distinct entries exist.

To overcome this problem, CONVOL has three built-in rules. First, after each intermediate convolution, the point elimination techniques controlled by the header card are applied. Thus consecutive probabilities are kept at least a minimum distance apart, and the maximum nonunity probability rule is applied (the user can of course indicate via the header card options that the rules are not to be applied).

Second, each distribution time value is rounded to the nearest 1/20 if the time value is less than 24, and to the nearest unit if the value is greater than 24. Thus, if the time values represent hours, then values are rounded to the nearest multiple of three minutes for all time values less than 24 hours, and to the nearest hour for all time values greater than 24 hours. The rounding technique greatly increases the probability that "duplicates" will occur when the time values are being determined.

The two techniques do in fact eliminate a large number of unnecessary and costly (in terms of computer time) points from the intermediate distributions--in actual cases more than 50 percent of the points have been eliminated, yielding insigificant changes in the expected value and variance of the final distributions. These techniques, however, do not preclude the possibility that the maximum distribution size is exceeded. Therefore a third rule has been built into CONVOL that selects points to be included into the intermediate distributions if the maximum size is exceeded. If the maximum distribution size has been reached, and if more points remain, then:

1. If the time value currently being considered is the largest thus far, it will replace the previous largest time value, and the latter will be discarded.

2. If the time value is not the largest thus far but is greater than 24, it is discarded.
3. If the time value is not the largest thus far and is less than 24, it is retained and the second largest value is discarded.

Fortunately in most cases, the distribution size does not exceed the maximum.

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