MEMORANDUM
RM-5627-PR
MAY 1968

USER'S INSTRUCTIONS FOR 0-1 INTEGER LINEAR PROGRAMMING CODE RIP30C

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SANTA MONICA • CALIFORNIA

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PREFACE

The authors have received numerous requests for copies of the experimental code used to obtain the computational experience reported in Reference 2. Although not a production code, it has proven capable of solving 0-1 integer linear programs of practical size quickly enough to make it among the fastest, if not the fastest, code of its kind in existence at present. It has therefore been deemed advisable to make the code available to potential users. This Memorandum furnishes the necessary instructions. Familiarity is assumed with References 1 and 2.

A. M. Geoffrion is a consultant to The RAND Corporation.

SUMMARY

This Memorandum furnishes the information required in order to use 0-1 integer linear programming code RIP30C: a discussion of input and output, an example, and a listing of the code.

CONTENTS

I. INTRODUCTION

The experimental code used to obtain the computational experience reported in [2] is not a production code. It was developed to test the usefulness of certain innovations applied to a simple Balasian algorithm. The central concern was the rate of increase of solution time as a function of the number of variables, rather than how to achieve the smallest possible execution time for particular problems. For this reason, the simplest possible Balasian algorithm was used as the starting point, and concessions were freely made to programming expediency (e.g., no machine language). It would not be difficult to improve the code substantially by reprogramming or introducing some of the more sophisticated tests already available in the literature.

The program solves integer linear programs of the form

$$\text{(P)} \qquad \text{Minimize } cx \text{ subject to } b + Ax \geq 0$$

$$x_j = 0 \text{ or } 1$$

where c and x are n-vectors, b is an m-vector, and A is m by n. Any bounded integer linear program can be written in this form, using elementary manipulations if necessary. The general features of the algorithm employed are described in [2], which in turn depends heavily on [1]. Familiarity with these references is presumed here.

Input and various options of the program are discussed in Sec. I, and output is discussed in Sec. II. We give a numerical example in Appendix A, and a liberally commented program listing in Appendix B.

II. INPUT AND OPTIONS

The following parameter and data cards appear for each problem to be run:

(a) Parameter card

(b) S-card(s)

(c) C-card(s)

(d) B-card(s)

(e) A-card(s)

(f) Blank card.

Problems can be stacked by repeating cards a through f.

PARAMETER CARD

The input parameters are:

M	The number of constraints (see Remark 1).
N	The number of variables (see Remark 1).
L	The number of variables in the initial partial solution (L must correspond to the number of entries on the S-card). If L = 0, the initial partial solution is empty. If L < 0, an "LP start" is used (see Remark 2).
SC	Punch 0 if no imbedded linear program is desired (the algorithm then reduces to a simple Balasian algorithm), and 1 if the imbedded linear program is to be used.
KENUM	When intermediate output is used (NØP = 0), the fraction of all 2^n possible solutions that have been implicitly enumerated is printed out every KENUM times that backtracking occurs. KENUM = 20 is reasonable.
ZBAR	If an upper bound $\bar{z}$ on the optimal value of the objective function of (P) is known, put ZBAR = $\bar{z}$ - gcd + 0.0001, where gcd is the greatest common divisor of the cost coefficients c_j (we assume that $\bar{z}$ is a multiple of gcd). Hence, if all c_j (and $\bar{z}$) are integer, put ZBAR = $\bar{z}$ - .9999. The effect will be that the program looks only for feasible solutions with value < ZBAR. If no upper bound is known, put ZBAR = 0. See Remark 3 below.
ISCMAX	The maximum number of surrogate constraints that will be carried. ISCMAX = 4 is reasonable.

ISCFR — The frequency with which the imbedded linear program is used. ISCFR = 0 means that it will never be used; ISCFR = j, j a positive integer, means that it will be used every j^{th} time. ISCFR = 1 has proven effective, but frequently a value of 8 or so is even better.

MAXC — If equal to 0, nothing will happen. If equal to 1, all signs on the C- and A-Cards will be reversed automatically when these cards are read in. This is purely a convenience for manuscripting and keypunching for problems with a preponderance of minus signs in C and A.

MAXT — Terminates the calculations after MAXT seconds.

NØP — If equal 1, intermediate output will be suppressed; if equal 0, intermediate output will appear. Normally NØP will be set at 1.

ZKBAR — Put equal to gcd (see ZBAR) minus 1. Thus, if all c_j are integer, put ZKBAR = 0. The effect is that the program looks only for feasible solutions with value at least (ZKBAR + .99999) less than the best feasible solution currently known; this doesn't exclude any optimal solutions. (A solution within Δ of the optimum can be found if desired by increasing the above value of ZKBAR by Δ.)

ITB — Determines the "augmentation" rule. ITB = 0 specifies the simplified version of Balas' rule described in [2]. ITB = 1 specifies the same rule except that only those free variables corresponding to fractional dual variables of (LP_s) are considered as candidates.

H1,H2 — Arbitrary problem identifiers.

Remark 1: The program is currently dimensioned to use 32,000 words of core in such a way that the following limits must be observed:

$$M + ISCMAX \leq 50$$

$$N \leq 90.$$

If the program is to be redimensioned for any reason, such as the availability of additional core, it will be useful to know that the number of words required is approximately $N^2 + N(3M + 18) + 9{,}000$.

Remark 2: The "LP start" is achieved at no additional computational expense over the L = 0 case. It amounts to first solving the continuous approximation to (P) ($0 \leq x_j \leq 1$, but x_j not necessarily integer) by

linear programming, and then taking the initial partial solution as the one determined by the variables that have the value 0 or 1. The net effect is to examine all roundings of the continuous solution first.

Remark 3: If any c_j are negative (after MAXC has changed the input signs, if it has value 1), the program internally makes a trivial change of variables to make such c_j nonnegative: replace x_j by $y_j = (1 - x_j)$ if $c_j < 0$. The problem is solved in terms of the new variables, and the reverse transformation is made at final output in order to recover the solution to the original problem. ZBAR must be set at a value corresponding to the transformed problem when it is desired to use a known upper bound; hence, when $c_j < 0$ for $j \epsilon J$, put ZBAR = $\bar{z}$ - gcd + .0001 + $\sum_{j \epsilon J} |c_j|$.

The fields and formats of the parameter card are as follows:

Parameter	Column	Format
M	1-3	Integer
N	4-6	"
L	7-9	"
SC	10-12	"
KENUM	13-17	"
ZBAR	18-23	E
ISCMAX	24-26	Integer
ISCFR	27-29	"
MAXC	30-32	"
MAXT	33-37	"
NØP	38-40	"
ZKBAR	41-46	E
ITB	47-49	Integer
H1	50-55	Hollerith
H2	56-61	"

S-CARD(S)

The algorithm can start with any initial partial solution (see [1]). When the initial partial solution is desired to be nonempty (L > 0), if x_j is to be fixed at the value one (zero) then "j" ("-j") is entered on the S-card, followed by "B" when an underline is desired. The S-card is divided into 12 fields of 5 columns each: 1-5, 6-10, ..., 66-70. Only the first four columns of each field are to be used except

when underlines are desired, in which case "B" must appear in the fifth column of the field.

The special instruction given above in Remark 3 for ZBAR, when a change of variables is made, also applies here. That is, the sign of ± j or ± jB must be changed when $c_j < 0$.

C-CARD(S)

The values of the c_j must be entered in order (negative values are permissible, as noted above). Each card has six fields of eleven columns read in E-format. The fields are separated by an unread column so that the values of the c_j are in columns 1-11, 13-23, ..., 61-71.

B-CARD(S)

The values of the b_i must be entered in order. The format is exactly the same as for the C-cards.

A-CARD(S)

Only nonzero a_{ij} need be entered, and they may be entered in any order. Each value is identified by its row and column. There are four or fewer entries on each of the "A" cards. Each entry has a seventeen-column field.

	Columns	Format
Row	1-3	Integer
Column	4-6	Integer
Value	7-17	E

The fields are separated by an unread column so that the matrix subscripts and values of the a_{ij} are in columns 1-17, 19-35, 37-53, 55-71.

III. OUTPUT

The preliminary, intermediate, and final outputs are as follows:

The "Parameter," "S," "C," and "B" cards are printed in that order (six values to a line for the "C" and "B" cards). Then the complete A matrix is printed (with zeros), row by row. If MAXC = 1, the sign reversals in "C" and "A" will have occurred.

If NØP = 0, intermediate output is produced to reveal the course of the calculations--each feasible solution found, each new surrogate constraint, data concerning each imbedded linear program, and a summary of progress to date after each KENUM "backtrackings." Since this information is likely to be of little incremental value to the user over the final output information, no detailed explanation is given here.

The final output gives the following: the problem designation; the message "implicit enumeration complete" or "time exceeded" according to whether termination did or did not occur within MAXT seconds; the total execution time in seconds; the solution (obj fc value and a list of which variables equal 1); and some statistical information on the course of the algorithm, such as the number of feasible solutions found, the number of times the imbedded linear program was called, the number of times its dual variables were all integers, the number of iterations, and the time at which the last feasible solution was found. If no feasible solutions were found, this is indicated by the statistic "no. feasible solutions 0." If the time limit was exceeded, the final output is preceded by a brief report giving the proportion of all 2^n possible solutions that have been accounted for and the final "state" vector [1], with "B" signifying an underline. In this case the solution printed out is the best feasible solution so far found (if any), and all the information needed to restart the calculations is available: make the S-card correspond to the final state vector (set L accordingly), and put ZBAR equal to the formula in Remark 3 with LEAST Z BEFORE VARIABLE CHANGE in the role of $\bar{z}$.

Appendix A

EXAMPLE

We shall illustrate the above by solving Petersen's fourth example [3].

For this problem, M = 10 and N = 20. We shall take L = 1, SC = 1, KENUM = 10, ZBAR = 0 (since we will not bother to determine a bound on the objective function), ISCMAX = 4, ISCFR = 1, MAXC = 1 (since we wish to avoid keypunching all the minus signs for c and A), MAXT = 60, NØP = 1, ZKBAR = 4 (since the greatest common divisor of the c_j is 5), ITB = 0, and H1 = PETE 5.

The S-card will contain the number -20 (we have elected an initial partial solution that is empty except for x_{20} at the value 1--remember the sign reversal required here because C_{20} is negative).

The input and output are reproduced below.

```
10 20  1  1   10 0.0    4  1  1  600  1  4.0   0 PETE 4
-20
1.00   E  2/ 2.20   E  2/ 9.0    E  1/ 4.00   E  2/ 3.00   E  2/ 4.00   E  2/
2.05   E  2/ 1.20   E  2/ 1.60   E  2/ 5.80   E  2/ 4.00   E  2/ 1.40   E  2/
1.00   E  2/ 1.300  E  3/ 6.50   E  2/ 3.20   E  2/ 4.80   E  2/ 8.0    E  1/
6.0    E  1/ 2.550  E  3/
5.50   E  2/ 7.00   E  2/ 1.30   E  2/ 2.40   E  2/ 2.80   E  2/ 3.10   E  2/
1.10   E  2/ 2.05   E  2/ 2.60   E  2/ 2.75   E  2/
 1  1 8.     E  0/  2  1 8.     E  0/  3  1 3.     E  0/  4  1 5.     E  0/
 1  2 2.4    E  1/  2  2 4.4    E  1/  3  2 6.     E  0/  4  2 9.     E  0/
 1  3 1.3    E  1/  2  3 1.3    E  1/  3  3 4.     E  0/  4  3 6.     E  0/
 1  4 8.0    E  1/  2  4 1.00   E  2/  3  4 2.0    E  1/  4  4 4.0    E  1/
 1  5 7.0    E  1/  2  5 1.00   E  2/  3  5 2.0    E  1/  4  5 3.0    E  1/
 1  6 8.0    E  1/  2  6 9.0    E  1/  3  6 3.0    E  1/  4  6 4.0    E  1/
 1  7 4.5    E  1/  2  7 7.5    E  1/  3  7 8.     E  0/  4  7 1.6    E  1/
 1  8 1.5    E  1/  2  8 2.5    E  1/  3  8 3.     E  0/  4  8 5.     E  0/
 1  9 2.8    E  1/  2  9 2.8    E  1/  3  9 1.2    E  1/  4  9 1.8    E  1/
 1 10 9.0    E  1/  2 10 1.20   E  2/  3 10 1.4    E  1/  4 10 2.4    E  1/
 1 11 1.30   E  2/  2 11 1.30   E  2/  3 11 4.0    E  1/  4 11 6.0    E  1/
 1 12 3.2    E  1/  2 12 3.2    E  1/  3 12 6.     E  0/  4 12 1.6    E  1/
 1 13 2.0    E  1/  2 13 4.0    E  1/  3 13 3.     E  0/  4 13 1.1    E  1/
 1 14 1.20   E  2/  2 14 1.60   E  2/  3 14 2.0    E  1/  4 14 3.0    E  1/
 1 15 4.0    E  1/  2 15 4.0    E  1/  3 15 5.     E  0/  4 15 2.5    E  1/
 1 16 3.0    E  1/  2 16 6.0    E  1/  3 16 0.     E  0/  4 16 1.0    E  1/
 1 17 2.0    E  1/  2 17 5.5    E  1/  3 17 5.     E  0/  4 17 1.3    E  1/
 1 18 6.     E  0/  2 18 1.0    E  1/  3 18 3.     E  0/  4 18 5.     E  0/
 1 19 3.     E  0/  2 19 6.     E  0/  3 19 0.     E  0/  4 19 1.     E  0/
 1 20 1.80   E  2/  2 20 2.40   E  2/  3 20 2.0    E  1/  4 20 8.0    E  1/
 5  1 5.     E  0/  6  1 5.     E  0/  7  1 0.     E  0/  8  1 3.     E  0/
 5  2 1.1    E  1/  6  2 1.1    E  1/  7  2 0.     E  0/  8  2 4.     E  0/
 5  3 7.     E  0/  6  3 7.     E  0/  7  3 1.     E  0/  8  3 5.     E  0/
 5  4 5.0    E  1/  6  4 5.5    E  1/  7  4 1.0    E  1/  8  4 2.0    E  1/
 5  5 4.0    E  1/  6  5 4.0    E  1/  7  5 4.     E  0/  8  5 1.4    E  1/
 5  6 4.0    E  1/  6  6 4.0    E  1/  7  6 1.0    E  1/  8  6 2.0    E  1/
 5  7 1.9    E  1/  6  7 2.1    E  1/  7  7 0.     E  0/  8  7 6.     E  0/
 5  8 7.     E  0/  6  8 9.     E  0/  7  8 6.     E  0/  8  8 1.2    E  1/
 5  9 1.8    E  1/  6  9 1.8    E  1/  7  9 0.     E  0/  8  9 1.0    E  1/
 5 10 2.9    E  1/  6 10 2.9    E  1/  7 10 6.     E  0/  8 10 1.8    E  1/
 5 11 7.0    E  1/  6 11 7.0    E  1/  7 11 3.2    E  1/  8 11 4.2    E  1/
 5 12 2.1    E  1/  6 12 2.1    E  1/  7 12 3.     E  0/  8 12 9.     E  0/
 5 13 1.7    E  1/  6 13 1.7    E  1/  7 13 0.     E  0/  8 13 1.2    E  1/
 5 14 3.0    E  1/  6 14 3.5    E  1/  7 14 7.0    E  1/  8 14 1.00   E  2/
 5 15 2.5    E  1/  6 15 2.5    E  1/  7 15 1.0    E  1/  8 15 2.0    E  1/
 5 16 1.5    E  1/  6 16 2.0    E  1/  7 16 0.     E  0/  8 16 5.     E  0/
 5 17 2.5    E  1/  6 17 2.5    E  1/  7 17 0.     E  0/  8 17 6.     E  0/
 5 18 5.     E  0/  6 18 5.     E  0/  7 18 0.     E  0/  8 18 4.     E  0/
 5 19 1.     E  0/  6 19 2.     E  0/  7 19 0.     E  0/  8 19 1.     E  0/
 5 20 1.00   E  2/  6 20 1.10   E  2/  7 20 0.     E  0/  8 20 2.0    E  1/
 9  1 3.     E  0/ 10  1 3.     E  0/
 9  2 6.     E  0/ 10  2 8.     E  0/
 9  3 9.     E  0/ 10  3 9.     E  0/
 9  4 3.0    E  1/ 10  4 3.5    E  1/
 9  5 2.9    E  1/ 10  5 2.9    E  1/
 9  6 2.0    E  1/ 10  6 2.0    E  1/
 9  7 1.2    E  1/ 10  7 1.6    E  1/
 9  8 1.2    E  1/ 10  8 1.5    E  1/
 9  9 1.0    E  1/ 10  9 1.0    E  1/
 9 10 3.0    E  1/ 10 10 3.0    E  1/
 9 11 4.2    E  1/ 10 11 4.2    E  1/
 9 12 1.8    E  1/ 10 12 2.0    E  1/
 9 13 1.8    E  1/ 10 13 1.8    E  1/
 9 14 1.10   E  2/ 10 14 1.20   E  2/
 9 15 2.0    E  1/ 10 15 2.0    E  1/
 9 16 1.5    E  1/ 10 16 2.0    E  1/
 9 17 1.8    E  1/ 10 17 2.2    E  1/
 9 18 7.     E  0/ 10 18 7.     E  0/
 9 19 2.     E  0/ 10 19 3.     E  0/
 9 20 4.0    E  1/ 10 20 5.0    E  1/
```

```
10 20  1  1   10  0.          4  1  1  600  1 0.4000E 01   0  PETE 4
M= 10  N= 20


  -20

-1.0000000 E 02  -2.2000000 E 02  -9.0000000 E 01  -4.0000000 E 02  -3.0000000 E 02  -4.0000000 E 02
-2.0500000 E 02  -1.2000000 E 02  -1.6000000 E 02  -5.8000000 E 02  -4.0000000 E 02  -1.4000000 E 02
-1.0000000 E 02  -1.3000000 E 03  -6.5000000 E 02  -3.2000000 E 02  -4.8000000 E 02  -8.0000000 E 01
-6.0000000 E 01  -2.5500000 E 03

 5.5000000 E 02   7.0000000 E 02   1.3000000 E 02   2.4000000 E 02   2.8000000 E 02   3.1000000 E 02
 1.1000000 E 02   2.0500000 E 02   2.6000000 E 02   2.7500000 E 02

-8.0000000 E 00  -2.4000000 E 01  -1.3000000 E 01  -8.0000000 E 01  -7.0000000 E 01  -8.0000000 E 01
-4.5000000 E 01  -1.5000000 E 01  -2.8000000 E 01  -5.0000000 E 01  -1.3000000 E 02  -3.2000000 E 01
-2.0000000 E 01  -1.2000000 E 02  -4.0000000 E 01  -3.0000000 E 01  -2.0000000 E 01  -6.0000000 E 00
-3.0000000 E 00  -1.8000000 E 02

-8.0000000 E 00  -4.4000000 E 01  -1.3000000 E 01  -1.0000000 E 02  -1.0000000 E 02  -9.0000000 E 01
-7.5000000 E 01  -2.5000000 E 01  -2.8000000 E 01  -1.2000000 E 02  -1.3000000 E 02  -3.2000000 E 01
-4.0000000 E 01  -1.6000000 E 02  -4.0000000 E 01  -6.0000000 E 01  -5.5000000 E 01  -1.0000000 E 01
-6.0000000 E 00  -2.4000000 E 02

-3.0000000 E 00  -6.0000000 E 00  -4.0000000 E 00  -2.0000000 E 01  -2.0000000 E 01  -3.0000000 E 01
-8.0000000 E 00  -3.0000000 E 00  -1.2000000 E 01  -1.4000000 E 01  -4.0000000 E 01  -6.0000000 E 00
-3.0000000 E 00  -2.0000000 E 01  -5.0000000 E 00  -0.             -5.0000000 E 00  -3.0000000 E 00
-0.              -2.0000000 E 01

-5.0000000 E 00  -9.0000000 E 00  -6.0000000 E 00  -4.0000000 E 01  -3.0000000 E 01  -4.0000000 E 01
-1.6000000 E 01  -5.0000000 E 00  -1.8000000 E 01  -2.4000000 E 01  -6.0000000 E 01  -1.6000000 E 01
-1.1000000 E 01  -3.0000000 E 01  -2.5000000 E 01  -1.0000000 E 01  -1.3000000 E 01  -5.0000000 E 00
-1.0000000 E 00  -8.0000000 E 01

-5.0000000 E 00  -1.1000000 E 01  -7.0000000 E 00  -5.0000000 E 01  -4.0000000 E 01  -4.0000000 E 01
-1.9000000 E 01  -7.0000000 E 00  -1.8000000 E 01  -2.9000000 E 01  -7.0000000 E 01  -2.1000000 E 01
-1.7000000 E 01  -3.0000000 E 01  -2.5000000 E 01  -1.5000000 E 01  -2.5000000 E 01  -5.0000000 E 00
-1.0000000 E 00  -1.0000000 E 02

-5.0000000 E 00  -1.1000000 E 01  -7.0000000 E 00  -5.5000000 E 01  -4.0000000 E 01  -4.0000000 E 01
-2.1000000 E 01  -9.0000000 E 00  -1.8000000 E 01  -2.9000000 E 01  -7.0000000 E 01  -2.1000000 E 01
-1.7000000 E 01  -3.5000000 E 01  -2.5000000 E 01  -2.0000000 E 01  -2.5000000 E 01  -5.0000000 E 00
-2.0000000 E 00  -1.1000000 E 02

-0.              -0.              -1.0000000 E 00  -1.0000000 E 01  -4.0000000 E 00  -1.0000000 E 01
-0.              -6.0000000 E 00  -0.              -6.0000000 E 00  -3.2000000 E 01  -3.0000000 E 00
-0.              -7.0000000 E 01  -1.0000000 E 01  -0.              -0.              -0.
-0.              -0.

-3.0000000 E 00  -4.0000000 E 00  -5.0000000 E 00  -2.0000000 E 01  -1.4000000 E 01  -2.0000000 E 01
-6.0000000 E 00  -1.2000000 E 01  -1.0000000 E 01  -1.8000000 E 01  -4.2000000 E 01  -9.0000000 E 00
-1.2000000 E 01  -1.0000000 E 02  -2.0000000 E 01  -5.0000000 E 00  -6.0000000 E 00  -4.0000000 E 00
-1.0000000 E 00  -2.0000000 E 01

-3.0000000 E 00  -6.0000000 E 00  -9.0000000 E 00  -3.0000000 E 01  -2.9000000 E 01  -2.0000000 E 01
-1.2000000 E 01  -1.2000000 E 01  -1.0000000 E 01  -3.0000000 E 01  -4.2000000 E 01  -1.8000000 E 01
-1.8000000 E 01  -1.1000000 E 02  -2.0000000 E 01  -1.5000000 E 01  -1.8000000 E 01  -7.0000000 E 00
-2.0000000 E 00  -4.0000000 E 01

-3.0000000 E 00  -8.0000000 E 00  -9.0000000 E 00  -3.5000000 E 01  -2.9000000 E 01  -2.0000000 E 01
-1.6000000 E 01  -1.5000000 E 01  -1.0000000 E 01  -3.0000000 E 01  -4.2000000 E 01  -2.0000000 E 01
-1.8000000 E 01  -1.2000000 E 02  -2.0000000 E 01  -2.0000000 E 01  -2.2000000 E 01  -7.0000000 E 00
-3.0000000 E 00  -5.0000000 E 01
```

```
      PETE 4

IMPLICIT ENUMERATION COMPLETE      TOTAL TIME=   2.298

LEAST Z BEFORE VARIABLE CHANGE =-6.1200000 E 03
     1       0       0       0       0       0       0       0       0      10       0       0       0      14      15
    16      17      18      19      20

NO. FEASIBLE SOLUTIONS     5
ZS GE ZBAR     4 TIMES
CONSTRAINT INFEASIBLE     9 TIMES
AUGMENTATION IMPOSSIBLE     0 TIMES
AUGMENTATION POSSIBLE    13 TIMES
INTEGER DUALS     C TIMES
NO. OF ROUNDED INT. DUALS     4
LP FATHOMED     1 TIMES
LP CALLED    14 TIMES
NO. ITERATIONS    29
LAST FEASIBLE SOLUTION AT    2.239  SECONDS
```

Appendix B

LISTING OF RIP30C

```
$IBFTC RIP30C                                                                   00010
      DIMENSION A(50,90),JF(50,90)                                              00020
      DIMENSION B(100),C(100),BS(100),S(100),SB(100),NS(100),NF(100)            00030
      DIMENSION ITEMP(4),JTEMP(4),ATEMP(4),SMAX(100),SMAXB(100),T(100)          00040
      DIMENSION CS(100)                                                         00050
      DIMENSION XL(90),D(90),E(90,90)                                           00060
      DIMENSION JH(100),XX(100),Y(100),PE(100),KO(6)                            00070
      INTEGER S,SMAX,SC,T                                                       00080
      COMMON /BLS/MS(90),ZBAR                                                   00090
      DATA BCIB/6HB      /                                                      00100
      DATA BLANK/6H      /                                                      00110
  100 DO 110 I=1,90                                                             00120
      B(I)=0.0                                                                  00130
      C(I)=0.0                                                                  00140
      BS(I)=0.0                                                                 00150
      S(I)=0                                                                    00160
      SB(I)=BLANK                                                               00170
      NS(I)=0                                                                   00180
      NF(I)=0                                                                   00190
      SMAX(I)=0                                                                 00200
      SMAXB(I)=BLANK                                                            00210
      T(I)=0                                                                    00220
      DO 110 J=1,50                                                             00230
      A(J,I)=0.0                                                                00240
      JF(J,I)=0.0                                                               00250
  110 CONTINUE                                                                  00260
      II=0                                                                      00270
      NCON=0                                                                    00280
      NRED=0                                                                    00290
      NAUG=0                                                                    00300
      NOPT=0                                                                    00310
      NID=0                                                                     00320
      NAP=0                                                                     00330
      NLPF=0                                                                    00340
      NSIMP=0                                                                   00350
      NFATH=0                                                                   00360
      NENUM=0                                                                   00370
      NTCE=0                                                                    00380
      NCID2=0                                                                   00390
      IPOST=1                                                                   00400
      IFRST=1                                                                   00410
      IINS=5                                                                    00420
C  READ A NEW SET OF DATA                                                       00430
C  PARAMETER CARD FIRST                                                         00440
C  'S' CARD SECOND                                                              00450
C  'C','B','A' MATRICES FOLLOW 'S'                                              00460
C  MINIMIZE SUM C(J)*X(J)                                                       00470
C  CONSTRAINTS ARE B(I)+SUM A(I,J)*X(J) GE ZERO                                 00480
      READ 9000,M,N,L,SC,KENUM,ZBAR,ISCMAX,ISCFR,MAXC,MAXT,                    00490
     *          NOP,ZKBAR,ITB,H1,H2                                             00500
 9000 FORMAT (4I3,I5,E6.0,3I3,I5,I3,E6.0,I3,2A6)                                00510
      PRINT 9993                                                                00520
      PRINT 9001,M,N,L,SC,KENUM,ZBAR,ISCMAX,ISCFR,MAXC,MAXT,                    00530
     *          NOP,ZKBAR,ITB,H1,H2                                             00540
 9001 FORMAT (4I3,I5,1X,E11.4,3I3,I5,I3,E11.4,1X,I3,1X,2A6)                     00550
      IF (MAXT.EQ.0) MAXT=999999                                                00560
      MAXT=1000*MAXT                                                            00570
      MO=M                                                                      00580
      M1=MO+1                                                                   00590
```

```
      JSCFR=ISCFR                                                       00600
      ZKBAR=ZKBAR+.99999                                                00610
      PRINT 9010,M,N                                                    00620
 9010 FORMAT (3HOM=,I3,2X,2HN=,I3)                                      00630
      PRINT 9992                                                        00640
 9991 FORMAT (1H )                                                      00650
 9992 FORMAT (1HO)                                                      00660
 9993 FORMAT (1H1)                                                      00670
      L1=L                                                              00680
      IF (L.LE.0) L1=0                                                  00690
      READ 9100,((S(K),SB(K)),K=1,L1)                                   00700
 9100 FORMAT (14(I4,A1))                                                00710
      IF (L.GE.0) GO TO 130                                             00720
      L=0                                                               00730
      IFRST=0                                                           00740
  130 CONTINUE                                                          00750
      READ 9200,(C(J),J=1,N)                                            00760
 9200 FORMAT (6(E11.0,1X))                                              00770
C*********************************                                    00780
      IF (MAXC.EQ.0) GO TO 141                                          00790
      DO 140 J=1,N                                                      00800
  140 C(J)=-C(J)                                                        00810
  141 CONTINUE                                                          00820
      READ 9200,(B(I),I=1,M)                                            00830
  200 READ 9400,((ITEMP(K),JTEMP(K),ATEMP(K)),K=1,4)                    00840
 9400 FORMAT (4(2I3,E11.0,1X))                                          00850
      END=0.0                                                           00860
      DO 250 K=1,4                                                      00870
      KI=ITEMP(K)                                                       00880
      KJ=JTEMP(K)                                                       00890
      IF (KI.EQ.0) GO TO 250                                            00900
      IF (KJ.EQ.0) GO TO 250                                            00910
      KJF=NF(KI)+1                                                      00920
      NF(KI)=KJF                                                        00930
      JF(KI,KJF)=KJ                                                     00940
C*********************************                                    00950
      IF (MAXC.NE.0) ATEMP(K)=-ATEMP(K)                                 00960
      A(KI,KJ)=ATEMP(K)                                                 00970
      END=1.0                                                           00980
  250 CONTINUE                                                          00990
      IF (END.NE.0.0) GO TO 200                                         01000
      PRINT 9992                                                        01010
      PRINT 9500,((S(K),SB(K)),K=1,L)                                   01020
 9500 FORMAT (14(3X,I4,A1))                                             01030
      PRINT 9992                                                        01040
      PRINT 9600,(C(J),J=1,N)                                           01050
      PRINT 9992                                                        01060
      PRINT 9600,(B(I),I=1,M)                                           01070
      PRINT 9992                                                        01080
      DO 251 I=1,M                                                      01090
      PRINT 9600,(A(I,J),J=1,N)                                         01100
      PRINT 9991                                                        01110
  251 CONTINUE                                                          01120
      PRINT 9992                                                        01130
C  ALL DATA READ FOR THIS RUN                                           01140
C  PERFORM CHANGE OF VARIABLES NOW IF NECESSARY                         01150
      DO 255 J=1,N                                                      01160
      CS(J)=C(J)                                                        01170
      IF (C(J).GE.0.0) GO TO 255                                        01180
```

```
      C(J)=-C(J)                                                        01190
      DO 253 I=1,M                                                      01200
      B(I)=B(I)+A(I,J)                                                  01210
  253 A(I,J)=-A(I,J)                                                    01220
  255 CONTINUE                                                          01230
 9600 FORMAT (6(2X,1PE15.8))                                            01240
      IF (ZBAR.GT.0.0) GO TO 300                                        01250
      ZBAR=0.0                                                          01260
      DO 275 J=1,N                                                      01270
  275 ZBAR=ZBAR+C(J)                                                    01280
  300 ZS=0.0                                                            01290
      DO 325 I=1,M                                                      01300
  325 BS(I)=B(I)                                                        01310
      DO 330 J=1,N                                                      01320
  330 NS(J)=J                                                           01330
      IF (L.EQ.0) GO TO 400                                             01340
      DO 375 K=1,L                                                      01350
      J1=S(K)                                                           01360
      K1=IABS(J1)                                                       01370
      NS(K1)=0                                                          01380
      IF (J1.LE.0) GO TO 375                                            01390
      ZS=ZS+C(J1)                                                       01400
      DO 350 I=1,M                                                      01410
  350 BS(I)=BS(I)+A(I,J1)                                               01420
  375 CONTINUE                                                          01430
  400 CONTINUE                                                          01440
      IF (MO+ISCMAX.GT.50) ISCMAX=50-MO                                 01450
      I1=MO+ISCMAX                                                      01460
      DO 425 I=M1,I1                                                    01470
      NF(I)=N                                                           01480
      DO 425 J=1,N                                                      01490
      JF(I,J)=J                                                         01500
  425 CONTINUE                                                          01510
      CALL DATIME (0,IT0)                                               01520
      IT1=IT0                                                           01530
      IT2=IT0                                                           01540
      IT3=IT0                                                           01550
C  INITIALIZATION COMPLETE NOW. START FIRST ITERATION                   01560
      GO TO 1910                                                        01570
C  PREPARE TO COMPUTE SURROGATE CONSTRAINT                              01580
 1000 CONTINUE                                                          01590
      IF (SC.EQ.0) GO TO 2400                                           01600
      JSCFR=JSCFR+1                                                     01610
      IF (ISCFR.GT.JSCFR) GO TO 2400                                    01620
      ML=N-L                                                            01630
      IF (ML.LE.1) GO TO 2400                                           01640
      JSCFR=0                                                           01650
 1050 DO 1060 J=1,N                                                     01660
 1060   MS(J)=0                                                         01670
      NSIMP=NSIMP+1                                                     01680
      IF(L.EQ.0) GO TO 1076                                             01690
      DO 1075 I=1,L                                                     01700
        J=IABS(S(I))                                                    01710
 1075   MS(J) =-S(I)                                                    01720
      IF (NOP.NE.0) GO TO 1076                                          01730
      PRINT 3600,((S(K),SB(K)),K=1,L)                                   01740
C  SOLVE THE IMBEDDED LINEAR PROGRAM                                    01750
 1076 CALL SIMPLE (II, N,MO,A,C,B,   KO,XL,D,JH,XX,Y,OBJ,E,NOP)        01760
      IF (NOP.NE.0) GO TO 1077                                          01770
```

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      PRINT 9600,OBJ,ZBAR                                                    01780
 1077 CONTINUE                                                               01790
      II=II+IPOST                                                            01800
C  KO(1) EQ 0 MEANS OBJ LESS THAN ZBAR, EQ 2 MEANS INFINITY, EQ 4 MEANS     01810
C  TROUBLE, EQ 6 MEANS OBJ GE ZBAR                                           01820
      IF (KO(1).EQ.2) GO TO 3400                                             01830
      IF (KO(1).EQ.4) GO TO 100                                              01840
      IF (KO(1).EQ.6) GO TO 1500                                             01850
      VLPS=-OBJ                                                              01860
      IF (VLPS.LE. (-ZBAR))GO TO 1499                                        01870
      DO 1350 I=1,N                                                          01880
      IF (D(I).NE.AINT(D(I)).AND.NS(I).NE.0)  GO TO 1500                     01890
 1350 CONTINUE                                                               01900
      DO 1450 J=1,N                                                          01910
      IF (NS(J).EQ.0) GO TO 1450                                             01920
      I=J                                                                    01930
      L=L+1                                                                  01940
      NS(J)=0                                                                01950
      SB(L)=BCIB                                                             01960
      IF (D(I).NE.0.0) GO TO 1400                                            01970
      S(L)=-J                                                                01980
      GO TO 1450                                                             01990
 1400 S(L)=J                                                                 02000
      ZS=ZS+C(J)                                                             02010
      DO 1425 I1=1,M                                                         02020
 1425 BS(I1)=BS(I1)+A(I1,J)                                                  02030
C  NATURAL DUAL INTEGER SOLUTION FOUND                                       02040
 1450 CONTINUE                                                               02050
      NID=NID+1                                                              02060
      GO TO 2320                                                             02070
 1499 KO(1)=6                                                                02080
C  COMPUTE NEW SURROGATE CONSTRAINT                                          02090
 1500 IF (ISCMAX.LE.0) GO TO 1599                                            02100
      BMP1=ZBAR                                                              02110
      DO 1505 I=1,MO                                                         02120
 1505 BMP1=BMP1+XL(I)*B(I)                                                   02130
      IF (ABS(BMP1-B(M)).LE.0.0005) GO TO 1599                               02140
      IF (M-MO.LT.ISCMAX) GO TO 1520                                         02150
      DO 1510 I=M1,M                                                         02160
      B(I)=B(I+1)                                                            02170
      BS(I)=BS(I+1)                                                          02180
      DO 1510 J=1,N                                                          02190
 1510 A(I,J)=A(I+1,J)                                                        02200
      M=M-1                                                                  02210
 1520 B(M+1)=BMP1                                                            02220
      DO 1550 J=1,N                                                          02230
      ZJH =XX(J)                                                             02240
      IF (JH(J).GE.(-N)) ZJH=-ZJH                                            02250
      IF (JH(J).GT.0) ZJH=0.                                                 02260
 1550 A(M+1,J)=ZJH                                                           02270
      M=M+1                                                                  02280
      BS(M)=B(M)                                                             02290
      DO 1575 K=1,L                                                          02300
      K1=S(K)                                                                02310
      IF (K1.LE.0) GO TO 1575                                                02320
      BS(M)=BS(M)+A(M,K1)                                                    02330
 1575 CONTINUE                                                               02340
      IF (NOP.NE.0) GO TO 1599                                               02350
      PRINT 1598,M                                                           02360
```

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      PRINT 9600,(A(M,J),J=1,N),B(M),BS(M)                              02370
 1598 FORMAT (22HOSURROGATE CONSTRAINTS,2X,I4)                          02380
 1599 IF (KO(1).EQ.6) GO TO 3400                                        02390
C  CHECK THE ROUNDED DUAL SOLUTION FOR FEASIBILITY                      02400
 1900 CONTINUE                                                          02410
      TD=.5                                                             02420
  900 F=ZS                                                              02430
      F1=BS(M)                                                          02440
  905 DO 910 J=1,N                                                      02450
      IF (NS(J).EQ.0) GO TO 910                                         02460
      IF (D(J).LT.TD) GO TO 910                                         02470
      F=F+C(J)                                                          02480
      F1=F1+A(M,J)                                                      02490
  910 CONTINUE                                                          02500
      IF (F.GE.ZBAR) GO TO 2400                                         02510
      IF (F1.GE.0.0) GO TO 920                                          02520
  915 GO TO 2400                                                        02530
  920 DO 930 I=1,MO                                                     02540
      F2=BS(I)                                                          02550
      DO 925 J=1,N                                                      02560
      IF (NS(J).EQ.0) GO TO 925                                         02570
      IF (D(J).LT.TD) GO TO 925                                         02580
      F2=F2+A(I,J)                                                      02590
  925 CONTINUE                                                          02600
      IF (F2.LT.0.0) GO TO 915                                          02610
  930 CONTINUE                                                          02620
C  ROUNDED DUAL SOLUTION FEASIBLE                                       02630
  931 NCID2=NCID2+1                                                     02640
  932 NOPT=NOPT+1                                                       02650
      CALL DATIME (0,IT3)                                               02660
      IF (M.EQ.MO) GO TO 940                                            02670
      DO 935 I=M1,M                                                     02680
      B(I)=B(I)+F-ZKBAR-ZBAR                                            02690
  935 BS(I)=BS(I)+F-ZKBAR-ZBAR                                          02700
  940 ZBAR=F-ZKBAR                                                      02710
      DO 945 J=1,L                                                      02720
      SMAXB(J)=SB(J)                                                    02730
  945 SMAX(J)=S(J)                                                      02740
      K=L                                                               02750
      DO 950 J=1,N                                                      02760
      IF (NS(J).EQ.0) GO TO 950                                         02770
      K=K+1                                                             02780
      SMAXB(K)=BLANK                                                    02790
      SMAX(K)=J                                                         02800
      IF (D(J).LT.TD) SMAX(K)=-J                                        02810
  950 CONTINUE                                                          02820
      IF (NOP.NE.0) GO TO 960                                           02830
      PRINT 3310,F                                                      02840
      PRINT 3600,((SMAX(J),SMAXB(J)),J=1,N)                             02850
  960 CONTINUE                                                          02860
      NOBJ=OBJ                                                          02870
      ZOBJ=NOBJ                                                         02880
      IF (OBJ.NE.ZOBJ) ZOBJ=ZOBJ+1.0                                    02890
      IF (F.EQ.ZOBJ) GO TO 3500                                         02900
      GO TO 2400                                                        02910
C  BEGINNING OF AN ITERATION. MAKE CHEAP ATTEMPT TO FATHOM              02920
 1910 IJK=0                                                             02930
 1920 CONTINUE                                                          02940
      IF (ZS.GE.ZBAR) GO TO 3100                                        02950
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      DO 1950 I1=1,MO                                                   02960
 1950 IF (BS(I1).LT.0.0) GO TO 1980                                     02970
      GO TO 2320                                                        02980
C  SEE IF ANY VARIABLES MUST BE 0                                       02990
 1980 CONTINUE                                                          03000
      DO 2000 J=1,N                                                     03010
      IF (NS(J).EQ.0) GO TO 2000                                        03020
      IF (ZS+C(J).LT.ZBAR) GO TO 2000                                   03030
      NS(J)=0                                                           03040
      L=L+1                                                             03050
      SB(L)=BCIB                                                        03060
      S(L)=-J                                                           03070
 2000 CONTINUE                                                          03080
      KINS=0                                                            03090
      IF (IJK.EQ.1) GO TO 2200                                          03100
      IF (IJK.EQ.2) GO TO 1000                                          03110
      IJK=1                                                             03120
      IF (M.LT.M1) GO TO 2025                                           03130
      MSC=0                                                             03140
      I1=M1                                                             03150
      I2=M                                                              03160
      GO TO 2050                                                        03170
 2025 MSC=1                                                             03180
      I1=1                                                              03190
      I2=MO                                                             03200
C  PERFORM BINARY FEASIBILITY TEST                                       03210
 2050 DO 2220 I=I1,I2                                                   03220
      Q=BS(I)                                                           03230
      DO 2100 J=1,N                                                     03240
      IF (NS(J).EQ.0) GO TO 2100                                        03250
      IF (A(I,J).GT.0.0) Q=Q+A(I,J)                                     03260
 2100 CONTINUE                                                          03270
 2110 IF (Q.LT.0.0) GO TO 3000                                          03280
C  SEE IF ANY FREE VARIABLE MUST BE 0 OR 1                               03290
      K=NF(I)                                                           03300
      DO 2200 K1=1,K                                                    03310
      J1=JF(I,K1)                                                       03320
      IF (NS(J1).EQ.0) GO TO 2200                                       03330
 2120 IF (Q.GE.ABS(A(I,J1))) GO TO 2200                                 03340
      NS(J1)=0                                                          03350
      L=L+1                                                             03360
      SB(L)=BCIB                                                        03370
      IF (A(I,J1).GT.0.0) GO TO 2150                                    03380
      S(L)=-J1                                                          03390
      GO TO 2200                                                        03400
 2150 S(L)=J1                                                           03410
      ZS=ZS+C(J1)                                                       03420
      DO 2175 I9=1,M                                                    03430
 2175 BS(I9)=BS(I9)+A(I9,J1)                                            03440
      KINS=KINS+1                                                       03450
 2200 CONTINUE                                                          03460
      IF (KINS.GE.IINS) GO TO 1920                                      03470
 2220 CONTINUE                                                          03480
      IF (MSC.EQ.0) GO TO 2025                                          03490
      IF (KINS.EQ.0) GO TO 1000                                         03500
      IJK=2                                                             03510
      GO TO 1920                                                        03520
C  A BETTER FEASIBLE SOLUTION HAS BEEN FOUND                             03530
 2320 CONTINUE                                                          03540
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      IF (M.EQ.MO) GO TO 2340                                          03550
C  REVISE B(I) AND BS(I) USING NEW ZS                                   03560
      DO 2325 I=M1,M                                                   03570
      B(I)=B(I)+ZS-ZKBAR-ZBAR                                          03580
 2325 BS(I)=BS(I)+ZS-ZKBAR-ZBAR                                        03590
 2340 ZBAR=ZS-ZKBAR                                                    03600
      DO 2350 J=1,N                                                    03610
 2350 SMAX(J)=S(J)                                                     03620
      GO TO 3300                                                       03630
C  AUGMENTATION STEP                                                   03640
 2400 K1=0                                                             03650
      IF (SC.EQ.0) GO TO 2415                                          03660
      IF (IFRST.NE.0) GO TO 2415                                       03670
      IFRST=1                                                          03680
      DO 2410 J=1,N                                                    03690
      IF (NS(J).EQ.0) GO TO 2410                                       03700
      J1=0                                                             03710
      IF (D(J).EQ.1.0) J1=J                                            03720
      IF (D(J).EQ.0.0) J1=-J                                           03730
      IF (J1.EQ.0) GO TO 2410                                          03740
      L=L+1                                                            03750
      NS(J)=0                                                          03760
      S(L)=J1                                                          03770
      IF (J1.LT.0) GO TO 2410                                          03780
      ZS=ZS+C(J)                                                       03790
      DO 2405 I=1,M                                                    03800
 2405 BS(I)=BS(I)+A(I,J)                                               03810
 2410 CONTINUE                                                         03820
 2415 CONTINUE                                                         03830
      IF (ITB.EQ.0) GO TO 2425                                         03840
      IF (JSCFR.NE.0) GO TO 2425                                       03850
      DO 2420 J=1,N                                                    03860
      IF (NS(J).EQ.0) GO TO 2420                                       03870
      IF (D(J).EQ.AINT(D(J))) GO TO 2420                               03880
      K1=K1+1                                                          03890
      T(K1)=J                                                          03900
 2420 CONTINUE                                                         03910
      GO TO 2505                                                       03920
 2425 CONTINUE                                                         03930
      DO 2500 J=1,N                                                    03940
      IF (NS(J).EQ.0) GO TO 2500                                       03950
      K1=K1+1                                                          03960
      T(K1)=J                                                          03970
      GO TO 2500                                                       03980
 2450 CONTINUE                                                         03990
 2500 CONTINUE                                                         04000
 2505 CONTINUE                                                         04010
      IF (K1.EQ.0) GO TO 3200                                          04020
      NAP=NAP+1                                                        04030
      P=-1.0E10                                                        04040
      DO 2575 K=1,K1                                                   04050
      J=T(K)                                                           04060
      P1=0.0                                                           04070
      DO 2550 I=1,M                                                    04080
      P2=BS(I)+A(I,J)                                                  04090
      IF (P2.GE.0.0) GO TO 2550                                        04100
      P1=P1+P2                                                         04110
 2550 CONTINUE                                                         04120
      IF (P1.LE.P) GO TO 2575                                          04130
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      P=P1                                                              04140
      J1=J                                                              04150
 2575 CONTINUE                                                          04160
      NS(J1)=0                                                          04170
      L=L+1                                                             04180
      S(L)=J1                                                           04190
      ZS=ZS+C(J1)                                                       04200
      DO 2600 I=1,M                                                     04210
 2600 BS(I)=BS(I)+A(I,J1)                                               04220
      GO TO 1910                                                        04230
C  FATHOMED DUE TO BINARY INFEASIBLE CONSTRAINT                         04240
 3000 NCON=NCON+1                                                       04250
      GO TO 3500                                                        04260
 3100 NRED=NRED+1                                                       04270
      GO TO 3500                                                        04280
C  FATHOMED DUE TO LACK OF FREE VARIABLES                               04290
 3200 NAUG=NAUG+1                                                       04300
      GO TO 3500                                                        04310
 3300 NOPT=NOPT+1                                                       04320
      CALL DATIME (0,IT3)                                               04330
      IF (NOP.NE.0) GO TO 3500                                          04340
      PRINT 3310,ZS                                                     04350
      PRINT 3600,((S(K),SB(K)),K=1,L)                                   04360
 3310 FORMAT (23H0 BETTER SOLUTION FOUND,5X,2HZ=,1PE15.8)               04370
      GO TO 3500                                                        04380
 3400 NLPF=NLPF+1                                                       04390
      GO TO 3500                                                        04400
C  BACKTRACK STEP                                                      04410
 3500 CONTINUE                                                          04420
      NENUM=NENUM+1                                                     04430
      IF (NENUM.LT.KENUM) GO TO 3530                                    04440
      NENUM=0                                                           04450
 3505 CONTINUE                                                          04460
      ENUM=0.0                                                          04470
      DO 3510 K=1,N                                                     04480
 3510 IF (SB(K).EQ.BCIB) ENUM=ENUM+.5**K                                04490
      CALL DATIME (0,IT2)                                               04500
      ELT1=IT2-IT0                                                      04510
      ELT2=IT2-IT1                                                      04520
      IT1=IT2                                                           04530
      ELT1=ELT1/1000.0                                                  04540
      ELT2=ELT2/1000.0                                                  04550
      IF (IT2-IT0.LT.MAXT) GO TO 3515                                   04560
      MAXT=-1                                                           04570
      GO TO 3517                                                        04580
 3515 CONTINUE                                                          04590
      IF (NOP.NE.0) GO TO 3700                                          04600
 3517 CONTINUE                                                          04610
      PRINT 3520,ENUM,ELT1,ELT2,L                                       04620
 3520 FORMAT (1H0,F10.5,38H OF THE SOLUTIONS HAVE BEEN ENUMERATED,5X,   04630
     *        15HTIME IN SECONDS,2X,5HTOTAL,F8.3,2X,7HELAPSED,F8.3,     04640
     *        5X,2HL=,I3)                                               04650
 3530 CONTINUE                                                          04660
      IF (MAXT.LT.0) PRINT 3600,((S(K),SB(K)),K=1,L)                    04670
 3600 FORMAT (15(2X,I4,A1))                                             04680
      IF (MAXT.LT.0) GO TO 3738                                         04690
 3700 NFATH=NFATH+1                                                     04700
 3710 IF (SB(L).EQ.BLANK) GO TO 3900                                    04710
      J=IABS(S(L))                                                      04720
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      NS(J)=J                                                          04730
      IF (S(L).LT.0) GO TO 3735                                        04740
      ZS=ZS-C(J)                                                       04750
      DO 3725 I=1,M                                                    04760
 3725 BS(I)=BS(I)-A(I,J)                                               04770
 3735 SB(L)=BLANK                                                      04780
      S(L)=0                                                           04790
      L=L-1                                                            04800
      IF (L.GT.0) GO TO 3710                                           04810
C  FINISHED NOW. PREPARE AND GIVE FINAL OUTPUT.                        04820
 3738 CONTINUE                                                         04830
      PRINT 3739,H1,H2                                                 04840
 3739 FORMAT (1H1,5X,2A6)                                              04850
      DO 3740 J=1,N                                                    04860
 3740 S(J)=0                                                           04870
      DO 3742 J=1,N                                                    04880
      K=IABS(SMAX(J))                                                  04890
      IF (K.EQ.0) GO TO 3744                                           04900
 3742 S(K)=1                                                           04910
 3744 DO 3746 K=1,N                                                    04920
      IF (S(K).NE.0) GO TO 3746                                        04930
      SMAX(J)=-K                                                       04940
      J=J+1                                                            04950
 3746 CONTINUE                                                         04960
      CALL DATIME (0,IT2)                                              04970
      ELT1=IT2-IT0                                                     04980
      ELT1=ELT1/1000.0                                                 04990
      IF (MAXT.LT.0) GO TO 3752                                        05000
      PRINT 3750,ELT1                                                  05010
 3750 FORMAT (30HOIMPLICIT ENUMERATION COMPLETE,5X,11HTOTAL TIME=,F8.3) 05020
      GO TO 3758                                                       05030
 3752 PRINT 3755,ELT1                                                  05040
 3755 FORMAT (14HOTIME EXCEEDED,5X,11HTOTAL TIME=,F8.3)                05050
 3758 CONTINUE                                                         05060
      ZBAR=0.0                                                         05070
      DO 3835 J=1,N                                                    05080
      K=IABS(SMAX(J))                                                  05090
      IF (CS(K).LT.0.0) SMAX(J)=-SMAX(J)                               05100
      IF (SMAX(J).GT.0) ZBAR=ZBAR+CS(K)                                05110
 3835 CONTINUE                                                         05120
      PRINT 3840,ZBAR                                                  05130
 3840 FORMAT (33HOLEAST Z BEFORE VARIABLE CHANGE =,1PE15.8)            05140
 3800 DO 3810 K=1,N                                                    05150
 3810 T(K)=0                                                           05160
      DO 3820 K=1,N                                                    05170
      K1=IABS(SMAX(K))                                                 05180
 3820 IF (SMAX(K).GT.0) T(K1)=K1                                       05190
      PRINT 3830,(T(K),K=1,N)                                          05200
 3830 FORMAT (15(4X,I3))                                               05210
      ELT3=IT3-IT0                                                     05220
      ELT3=ELT3/1000.0                                                 05230
      NITER=NFATH+NFATH-1                                              05240
      PRINT 3850,NOPT,NRED,NCON,NAUG,NAP,NID,NCID2                     05250
      PRINT 3851,NLPF,NSIMP,NITER,ELT3                                 05260
 3850 FORMAT (23HONO. FEASIBLE SOLUTIONS,I5/                           05270
     *        11H ZS GE ZBAR,I5,6H TIMES/                             05280
     *        22H CONSTRAINT INFEASIBLE,I5,6H TIMES/                  05290
     *        24H AUGMENTATION IMPOSSIBLE,I5,6H TIMES/                05300
     *        22H AUGMENTATION POSSIBLE,I5,6H TIMES/                  05310
```

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     *         14H INTEGER DUALS,I5,6H TIMES/                                  05320
     *         26H NO. OF ROUNDED INT. DUALS,I5)                               05330
 3851 FORMAT (12H LP FATHOMED,I5,6H TIMES/                                     05340
     *         10H LP CALLED,I5,6H TIMES/                                      05350
     *         15H NO. ITERATIONS,I5/                                          05360
     *         26H LAST FEASIBLE SOLUTION AT,F8.3,9H  SECONDS)                 05370
C  END OF FINAL OUTPUT. LOOK FOR ANOTHER PROBLEM NOW.                          05380
      GO TO 100                                                               05390
C  COMPLEMENT AND UNDERSCORE LAST REMAINING ENTRY IN S.                       05400
 3900 SB(L)=BCIB                                                              05410
      S(L)=-S(L)                                                              05420
      J=IABS(S(L))                                                            05430
      IF (S(L).GT.0) GO TO 3950                                               05440
      ZS=ZS-C(J)                                                              05450
      DO 3925 I=1,M                                                           05460
 3925 BS(I)=BS(I)-A(I,J)                                                      05470
      GO TO 1910                                                              05480
 3950 ZS=ZS+C(J)                                                              05490
      DO 3975 I=1,M                                                           05500
 3975 BS(I)=BS(I)+A(I,J)                                                      05510
      GO TO 1910                                                              05520
      END                                                                     05530
$IBFTC SIMPLE                                                                 05540
C  AUTOMATIC SIMPLEX       REDUNDANT EQUATIONS CAUSE INFEASIBILITY           05550
      SUBROUTINE SIMPLE(INFLAG,MX,NN,A,B,C,KO,KB,P,JH,X,Y,OBJ,E,NOP)          05560
      REAL B(1),C(1),P(1),X(1),Y(1) ,OBJ                                      05570
      REAL E(90,90)                                                           05580
      INTEGER INFLAG,MX,NN,KO(6),KB(1),JH(1)                                  05590
      EQUIVALENCE (XX,LL)                                                     05600
C THE FOLLOWING DIMENSION SHOULD BE THE SAME HERE AS IT IS IN CALLER.         05610
      REAL A(50,90)                                                           05620
      REAL AA,AIJT,BB,COST,DT,RCOST,TEXP,TPIV,TY,XOLD,XX,XY,YI,YMAX,EM        05630
      INTEGER I,IA,INVC,IR,ITER,J,JT,K,KBJ,LL,M,N,JT2                         05640
      INTEGER NCUT,     NUMVR,NVER,NUMPV                                      05650
      LOGICAL TRIG,VER                                                        05660
      LOGICAL FINV,FFRZ,SCH                                                   05670
      COMMON /BLS/MS(90),ZBAR                                                 05680
      DIMENSION NF(90)                                                        05690
C          SET INITIAL VALUES, SET CONSTANT VALUES                            05700
      FINV = .FALSE.                                                          05710
      TRIG = .FALSE.                                                          05720
      ITER = 0                                                                05730
      LPSEQ = LPSEQ+1                                                         05740
      NUMVR = 0                                                               05750
      NUMPV = 0                                                               05760
      M = MX                                                                  05770
      N = NN                                                                  05780
      TEXP  =  .5**16                                                         05790
      NVER  =  M/2  +  5                                                      05800
      NCUT  =  4*M + 10                                                       05810
      IF (INFLAG.EQ.0) GO TO 1410                                             05820
C        IMPOSE CORRECT TEMPERATURE ON ROWS                                   05830
      FFRZ =.TRUE.                                                            05840
      DO 1960 L=1,M                                                           05850
        IF (MS(L).EQ.NF(L)) GO TO 1955                                        05860
        IF(MS(L)*NF(L).GT.0.OR.(MS(L).EQ.0.AND.X(L).GE.0.)) GO TO 1950        05870
        I=L                                                                   05880
        IF (NF(L).NE.0) GO TO 1925                                            05890
 1920   IF (JH(I).GT.0) GO TO 1930                                            05900
```

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C          IF JH DISAGREES WITH MS DO SPECIAL PIVOT                      05910
         IF (MS(L).GT.0.AND.JH(L).GE.(-M)) GO TO 1950                    05920
         IF (MS(L).LT.0.AND.JH(L).LT.(-M)) GO TO 1950                    05930
C  SPECIAL PIVOT, SWITCH SINGLETONS                                       05940
 1925    DO 1926  J=1,M                                                  05950
           P(J) = P(J) + E(I,J)                                          05960
           E(I,J) = -E(I,J)                                              05970
 1926    CONTINUE                                                        05980
         OBJ = OBJ + X(I)                                                05990
         X(I) = - X(I)                                                   06000
         JHL = JH(L)                                                     06010
         IF (JHL.GE.(-M)) JH(L) = -L-M                                   06020
         IF (JHL.LT.(-M)) JH(L) = -L                                     06030
         GO TO 1950                                                      06040
C       DO FULL PIVOT ON SINGLETON                                       06050
 1930    JT = -I                                                         06060
         COST = P(I)                                                     06070
         IF (MS(I).GT.0) GO TO 1931                                      06080
         JT = JT-M                                                       06090
         COST = 1.-COST                                                  06100
 1931    EN = 1.                                                         06110
         GO TO 630                                                       06120
C          GET COLUMN(JT)                                                06130
 1932    SCH = .FALSE.                                                   06140
         IF (COST.GT.0.) GO TO 1938                                      06150
 1935    GO TO 1000                                                      06160
C          SELECT ROW(IR)                                                06170
 1936    IF (IR.NE.0.OR.SCH) GO TO 1940                                  06180
         SCH =.TRUE.                                                     06190
 1938    EN =-EN                                                         06200
         DO 1937 J=1,M                                                   06210
           Y(J) = -Y(J)                                                  06220
 1937    CONTINUE                                                        06230
         GO TO 1935                                                      06240
 1940    IF((SCH.AND.ABS(COST).GT.TPIV).OR.IR.EQ.0)  GO TO 1980          06250
 1941    IF (EN.GT.0.)  GO TO 1945                                       06260
         DO 1942 J =1,M                                                  06270
           Y(J) = -Y(J)                                                  06280
 1942    CONTINUE                                                        06290
 1945    GO TO 901                                                       06300
C          PIVOT(IR,JT)                                                  06310
 1950    NF(L) = MS(L)                                                   06320
 1955    IF(JH(L).LT.0) GO TO 1960                                       06330
         IA=JH(L)                                                        06340
         KB(IA)=L                                                        06350
 1960 CONTINUE                                                           06360
      FFRZ = .FALSE.                                                     06370
      GO TO 910                                                          06380
C*           START WITH SINGLETON BASIS                                  06390
 1410 DO 1402  J=1,N                                                     06400
        KB(J) = 0                                                        06410
 1402 CONTINUE                                                           06420
      FFRZ =.FALSE.                                                      06430
 1400 DO 1401  I = 1,M                                                   06440
        JH(I) =-I                                                        06450
        NF(I) = MS(I)                                                    06460
        IF (NF(I).LT.0.OR.(NF(I).EQ.0.AND.B(I).LT.0.)) JH(I)=-I-M        06470
 1401 CONTINUE                                                           06480
C*           CREATE INVERSE FROM 'KB' AND 'JH'      (STEP 7)             06490
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```
 1320 VER = .TRUE.                                                      06500
      INVC = 0                                                          06510
      NUMVR   =  NUMVR  +1                                              06520
      TRIG = .FALSE.                                                    06530
      OBJ = 0.                                                          06540
      DO 1113  I = 1,M                                                  06550
        DO 1151  J=1,M                                                  06560
          E(J,I) = 0.                                                   06570
 1151   CONTINUE                                                        06580
        IF (JH(I).LT.(-M)) GO TO 1111                                   06590
        IF (JH(I).GT.0) JH(I) = 0                                       06600
        E(I,I) = 1.                                                     06610
        P(I) = 0.                                                       06620
        X(I) =  B(I)                                                    06630
        GO TO 1113                                                      06640
 1111   E(I,I) = -1.                                                    06650
        P(I)   = +1.                                                    06660
        OBJ = OBJ  + B(I)                                               06670
        X(I)   = -B(I)                                                  06680
 1113 CONTINUE                                                          06690
      DO 1102  JT = 1,N                                                 06700
        IF (KB(JT).EQ.0)  GO TO 1102                                    06710
        GO TO 600                                                       06720
C       GET CCLUMN(JT)                                                  06730
 1114   TY = TPIV                                                       06740
        IR = 0                                                          06750
        COST = C(JT)                                                    06760
        DO 1104 I = 1,M                                                 06770
        COST = COST + A(JT,I)*P(I)                                      06780
          IF(JH(I).NE.0.OR.X(I).NE.0..OR.ABS(Y(I)).LE.TY) GO TO 1104    06790
          TY = ABS(Y(I))                                                06800
          IR = I                                                        06810
 1104   CONTINUE                                                        06820
        IF (IR.NE.0) GO TO 1119                                         06830
        TY = 0.                                                         06840
        DO 1105 I = 1,M                                                 06850
          IF(JH(I).NE.0.OR.X(I).EQ.0..OR.ABS(Y(I)).LE.TPIV) GO TO 1105  06860
          IF (ABS(Y(I)).LE.TY*ABS(X(I)))  GO TO 1105                    06870
          TY = ABS(Y(I)/X(I))                                           06880
          IR = I                                                        06890
 1105   CONTINUE                                                        06900
 1119   IF (IR.NE.0) GO TO 900                                          06910
C       PIVOT(IR,JT)                                                    06920
        FINV = .TRUE.                                                   06930
      IF (NOP.EQ.0) PRINT 1199,LPSEQ                                    06940
 1199   FORMAT(15HOINVERT FAIL LP,I4)                                   06950
        GO TO 1410                                                      06960
 1102 CONTINUE                                                          06970
C* PERFORM A SIMPLEX ITERATION                                          06980
 1200 VER = .FALSE.                                                     06990
  500 DO 503 I = 1,M                                                    07000
        IF (NF(I).EQ.0.AND.X(I).LT.0.) X(I)=0.                          07010
  503 CONTINUE                                                          07020
C*          FIND MINIMUM REDUCED COST              (STEP 3)             07030
  599 JT = 0                                                            07040
      BB  =  0.0                                                        07050
      DO 701  J =1,N                                                    07060
        IF (KB(J).NE.0)   GO TO 701                                     07070
        DT = C(J)                                                       07080
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      DO 303 I = 1,M                                                    07090
        DT = DT + A(J,I)*P(I)                                           07100
  303   CONTINUE                                                        07110
        IF (DT.GE.BB)  GO TO 701                                        07120
        BB = DT                                                         07130
        JT = J                                                          07140
  701 CONTINUE                                                          07150
      DO 702 I=1,M                                                      07160
        IF (JH(I).LT.0) GO TO 702                                       07170
        IF (P(I).LT.BB) GO TO 703                                       07180
        IF ((1.-P(I)).GE.BB) GO TO 702                                  07190
        BB = 1.-P(I)                                                    07200
        JT = -I-M                                                       07210
        GO TO 702                                                       07220
  703   BB=P(I)                                                         07230
        JT = -I                                                         07240
  702 CONTINUE                                                          07250
      COST = BB                                                         07260
      IF (JT.EQ.0)  GO TO 203                                           07270
      IF (ITER.GE.NCUT)  GO TO 160                                      07280
      ITER = ITER +1                                                    07290
C*          MULTIPLY INVERSE TIMES A(.,JT)          (STEP 4)          07300
      IF (JT.LT.0) GO TO 630                                            07310
C     BEGIN SUBROUTINE GET COLUMN(JT)                                   07320
  600 DO 610  I= 1,M                                                    07330
        Y(I) = 0.0                                                      07340
  610 CONTINUE                                                          07350
      DO 605   I= 1,M                                                   07360
        AIJT = A(JT,I)                                                  07370
        IF (AIJT.EQ.0.) GO TO 605                                       07380
        DO  606  J = 1,M                                                07390
          Y(J) = Y(J) + AIJT*E(J,I)                                     07400
  606   CONTINUE                                                        07410
  605 CONTINUE                                                          07420
      GO TO 640                                                         07430
  630 JT2 = -JT                                                         07440
      EM = 1.                                                           07450
      IF (JT2.LE.M) GO TO 631                                           07460
      JT2 = JT2 - M                                                     07470
      EM = -1.                                                          07480
  631 DO 632  I=1,M                                                     07490
        Y(I) = EM*E(I,JT2)                                              07500
  632 CONTINUE                                                          07510
  640 YMAX = 0.                                                         07520
      DO 620  I  =  1,M                                                 07530
        YMAX = AMAX1( ABS(Y(I)),YMAX )                                  07540
  620 CONTINUE                                                          07550
      TPIV  =  YMAX * TEXP                                              07560
C     END OF GET COLUMN                                                 07570
      IF (FFRZ) GO TO 1932                                              07580
      IF (VER)  GO TO 1114                                              07590
      RCOST = YMAX/BB                                                   07600
      IF (TRIG.AND.BB.GE.(-TPIV)) GO TO 203                             07610
      TRIG=BB.GE.(-TPIV)                                                07620
C*          SELECT PIVOT ROW                        (STEP 5)          07630
 1000 AA = TPIV                                                         07640
      IR = 0                                                            07650
 1002 DO 1003 I = 1,M                                                   07660
        IF (X(I).NE.0..OR.Y(I).LE.AA.OR.NF(I).NE.0) GO TO 1003          07670
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        AA = Y(I)                                                          07680
        IR = I                                                             07690
 1003 CONTINUE                                                             07700
      IF (IR.NE.0) GO TO 1020                                              07710
      AA = 0.                                                              07720
      DO 1010  I  = 1,M                                                    07730
        IF (NF(I).NE.C.OR.Y(I).LE.TPIV.OR.Y(I).LE.AA*X(I)) GO TO 1010      07740
        AA = Y(I)/X(I)                                                     07750
        IR = I                                                             07760
 1010 CONTINUE                                                             07770
 1020 IF (FFRZ) GO TO 1936                                                 07780
      IF  (IR.EQ.0)  GO TO 207                                             07790
C*          PIVOT ON (IR,JT)                          (STEP 6)             07800
  901 IA = JH(IR)                                                          07810
      IF (IA.GT.0)  KB(IA) = 0                                             07820
C     BEGIN SUBROUTINE PIVOT(IR,JT)                                        07830
  900 NUMPV  =  NUMPV  +  1                                                07840
      JH(IR) = JT                                                          07850
      IF (JT.GT.0)  KB(JT) = IR                                            07860
      YI = -Y(IR)                                                          07870
      Y(IR) = -1.0                                                         07880
      DO  904  J = 1,M                                                     07890
        XY = E(IR,J)/YI                                                    07900
        IF (XY.EQ.0.) GO TO 904                                            07910
        P (J) = P (J) + COST * XY                                          07920
        E(IR,J) = 0.                                                       07930
        DO 906  I = 1,M                                                    07940
          E(I,J) =  E(I,J) + XY * Y(I)                                     07950
  906   CONTINUE                                                           07960
  904 CONTINUE                                                             07970
      XY  =  X(IR) / YI                                                    07980
      DO 908  I  = 1, M                                                    07990
        XOLD = X(I)                                                        08000
        X(I) = XOLD + XY * Y(I)                                            08010
  908 CONTINUE                                                             08020
      Y(IR) = -YI                                                          08030
      X(IR) = -XY                                                          08040
C     END OF PIVOT                                                         08050
      OBJ =  OBJ + XY*COST                                                 08060
      IF (VER) GO TO 1102                                                  08070
C       EXCHANGE ROWS IF SLACK PIVOTED IN WRONG ROW                        08080
      IF  (JT.GT.0.OR.JT2.EQ.IR) GO TO 907                                 08090
      XY = X(IR)                                                           08100
      X(IR) = X(JT2)                                                       08110
      X(JT2) = XY                                                          08120
      DO 909 I =1,M                                                        08130
        XY =  E(IR,I)                                                      08140
        E(IR,I) =  E(JT2,I)                                                08150
        E(JT2,I) = XY                                                      08160
  909 CONTINUE                                                             08170
      IA = JH(JT2)                                                         08180
      JH(JT2) = JT                                                         08190
      JH(IR) = IA                                                          08200
      KB(IA) = IR                                                          08210
  907 INVC   = INVC   +1                                                   08220
C   TO STEP 1 IF NOT INVERTING, TO STEP 7 IF INVERTING                     08230
      IF (FFRZ) GO TO 1950                                                 08240
      IF (OBJ.GE.ZBAR) GO TO 180                                           08250
      IF (FINV) GO TO 1200                                                 08260
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  910 IF (INVC.GE.NVER) GO TO 1320                                          08270
      GO  TO  1200                                                          08280
C*  END OF ALGORITHM, SET EXIT VALUES                 ***                   08290
  207 IF                (RCOST.LE.(-1000.)) GO TO 203                       08300
C         INFINITE SOLUTION                                                 08310
      K  =  2                                                               08320
      GO TO 250                                                             08330
  180 K=6                                                                   08340
      GO TO 250                                                             08350
C         PROBLEM IS CYCLING PERHAPS                                        08360
  160 K  =  4                                                               08370
      PRINT 161,LPSEQ                                                       08380
  161 FORMAT(31HOITERATION LIMIT EXCEEDED ON LP,I4)                         08390
      GO TO 250                                                             08400
C         FEASIBLE OR INFEASIBLE SOLUTION                                   08410
  203 K  =  0                                                               08420
  250 DO 1399  J = 1,N                                                      08430
        XX  =  0.0                                                          08440
        KBJ  =  KB(J)                                                       08450
        IF (KBJ.NE.0)  XX = X(KBJ)                                          08460
        KB(J) = LL                                                          08470
 1399 CONTINUE                                                              08480
      KO(1) = K                                                             08490
      KO(2) = ITER                                                          08500
      KO(3) = INVC                                                          08510
      KO(4) = NUMVR                                                         08520
      KO(5) = NUMPV                                                         08530
      KO(6) = JT                                                            08540
      IF (NOP.NE.0) RETURN                                                  08550
      PRINT 162,LPSEQ,(KO(I),I=1,6)                                         08560
  162 FORMAT(3H LP,I5,6H  KO  ,6I6)                                         08570
C     PRINT 1982                                                            08580
 1982 FORMAT(21HOI  JH NF MS ,P,Y,X,B/1X)                                   08590
C     DO 1983 I=1,M                                                         08600
C       PRINT 1984,I,JH(I),NF(I),MS(I),P(I),Y(I),X(I),B(I)                  08610
 1983 CONTINUE                                                              08620
 1984 FORMAT(1X,4I3,4F12.6)                                                 08630
      RETURN                                                                08640
 1980 IF (NOP.EQ.0) PRINT 1981,LPSEQ,L,IR,SCH,COST                          08650
 1981 FORMAT( 3HOLP,I4,12H FAIL, SLACK,I3,4H IR=I3,5H SCH=L1,3H C=F19.6)    08660
      IF (IR.NE.0) GO TO 1941                                               08670
      GO TO 1410                                                            08680
       END                                                                  08690
                                                                  0869 CARD
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