MEMORANDUM
RM-3388-PR
JULY 1963

FLIGHT REGIMES IN THE ATMOSPHERES OF VENUS AND MARS

Peter P. Wegener

PREPARED FOR:
UNITED STATES AIR FORCE PROJECT RAND

The RAND Corporation
SANTA MONICA • CALIFORNIA

MEMORANDUM
RM-3388-PR
JULY 1963

FLIGHT REGIMES IN THE ATMOSPHERES OF VENUS AND MARS

Peter P. Wegener

This research is sponsored by the United States Air Force under Project RAND—contract No. AF 49(638)-700 monitored by the Directorate of Development Planning, Deputy Chief of Staff, Research and Development, Hq USAF. Views or conclusions contained in this Memorandum should not be interpreted as representing the official opinion or policy of the United States Air Force.

The RAND Corporation
1700 MAIN ST • SANTA MONICA • CALIFORNIA

PREFACE

The atmospheres of Mars and Venus have been traditionally important in astrophysical studies; however, space exploration has renewed the interest in these atmospheres because knowledge of their structures is required for the design of vehicles intended for planetary entry at high speeds. Accordingly, this study presents preliminary estimates of several characteristic aerodynamic flight parameters, Reynolds numbers, and Mach numbers that may be encountered during atmospheric entries. The viewpoint throughout is that of the aerodynamicist rather than that of the astrophysicist.

In the present study, the author introduces the flight regimes for models of the Mars atmosphere much as was done for those of Venus in RM-2946-PR, Estimates of Flight Regimes in the Venus Atmosphere. The Venus-atmosphere models given here review those of the previous study and include a model atmosphere (proposed by Spinrad and containing 95 per cent N_2 by mass), whose structure reflects more recent opinion.

SUMMARY

Preliminary estimates of the flight parameters to be encountered on Mars and Venus are desirable. Although considerable qualitative information on the two planets is available, there are few quantitative data that contribute to an understanding of their atmospheres. From the available quantitative data, limiting models of the two atmospheres have been constructed, and from these models it has been possible to estimate the approximate extremes of aerodynamic parameters likely to be encountered. It is seen that the uncertainties in aerodynamic flight conditions encompass orders of magnitude, particularly at high altitudes.

ACKNOWLEDGMENT

The author gratefully acknowledges frequent enlightening discussions with Rupert Wildt of Yale University on the astrophysical approach to our understanding of planetary atmospheres.

CONTENTS

LIST OF SYMBOLS

Å = Angstrom unit

a = speed of sound, defined by Eq. (3)

c = mean molecular speed, defined by Eq. (6)

c_p, c_v = specific heats at constant pressure and volume

g = acceleration due to gravity

H = scale height, defined by Eq. (14)

h = altitude, km

K = Knudsen number, defined by Eq. (4)

ℓ = characteristic length

M_∞ = free-stream Mach number, defined by Eq. (1)

p = pressure

R = universal gas constant

Re_∞ = free-stream Reynolds number, defined by Eq. (2)

T = absolute temperature

u = flight speed, km/sec

β = lapse rate, ^{o}K/km

Γ = dry adiabatic lapse rate, ^{o}K/km, defined by Eq. (22)

γ = ratio of the specific heats; c_p/c_v

η = dynamic viscosity

λ = mean free path of the molecules

μ = molecular weight

ρ = density

Subscripts

c = visible cloud layer of Venus

E = Earth

M = Mars

V = Venus

o = planetary surface

∞ = free-stream conditions

Conversion Factors

$1\text{Å} = 10^{-8}$ cm

1 atmosphere = 760mm Hq = 1.013 bar

1 bar = 10^6 dyne/cm^2 = 10^3 mb

1 micron = 10^{-3} mm

1 poise = 1 gm/cm sec (unit of viscosity)

R = 8.314×10^7 erg/$^\circ$K mole = 82.06 cm^3 atm/$^\circ$K mole

$\mu_{air} = 28.97$

$\mu_{N_2} = 28.02$

$\mu_{CO_2} = 44.01$

$\mu_A = 39.9$

I. STUDY OF PLANETARY ATMOSPHERES

From the time of the origin of the solar system, gaseous envelopes have been produced around the planets, and on planets of sufficient mass and radius (and, therefore, sufficient gravitational attraction) certain gases have been selectively retained. The processes that led to the present composition and volume of the atmospheres differed from planet to planet because conditions on the planets themselves varied. Among these differing conditions are the position of the planet in the solar system and the consequently different amounts of energy received from the sun, the planet's gravitational attraction, and the detailed sequence of chemical reactions of the atmosphere as with the crust or with water.*

Because of the complexity of these reactions, however, the present composition and structure of planetary atmospheres cannot be derived from "geochemical" estimates of the formation of atmospheres alone. The aerodynamicist who searches for detailed models of planetary atmospheres will have to turn to the quantitative results of astrophysical observations. Unfortunately, the many findings in the literature demand a wide latitude of interpretation for Venus and Mars, the planets which will be considered here.

Astrophysical observations rest on an interpretation of the radiation received from the planets. This radiation consists primarily of reflected and scattered sunlight. Second, radiation may be produced at some height in the planet's atmosphere or at its surface. The restrictions on radiation studies by an earthbound observer are that all of the radiation must pass once through the Earth's atmosphere and that the reflected light must, in addition, twice traverse the other planet's atmosphere. The Earth's atmosphere acts as a filter, and the absorption due to its constituents makes it transparent to radiation only in certain distinct regions of the spectrum. These transparent regions, or "windows," exist in parts of the microwave, the infrared, the visible, and the near-ultraviolet portions of the

---

*These concepts are discussed by Kuiper[2] and Urey.[3,4]

spectrum. Observations have been made in a great variety of ways. In conjunction with knowledge of molecular and atomic physics, they have led to such quantitative inferences on the composition and structure of planetary atmospheres as those of Kuiper[2] and Kellogg and Sagan (Chapter I of Ref. 5).

Some of the major techniques will be briefly mentioned here. Most quantitative results on the composition of planetary atmospheres are obtained by recording spectra of the far-red and the infrared radiation. The molecular species that have absorption bands in this region may be thus identified; in principle, the amounts in a column traversed by the radiation can be measured. Such spectra are investigated in conjunction with comparison spectra, usually taken of the Moon, whose atmosphere in this context is negligible. A comparison of planetary and lunar spectra permits elimination of the features of the planetary spectra that are due to absorption by the Earth's atmosphere. The net result must next be compared with spectra taken in the laboratory under conditions similar to those prevailing on the planet, i.e., with a long light path and with the respective gases at relatively low pressure. Unfortunately, only limited laboratory work of this kind is recorded. Also, nitrogen, which is assumed to be the major constituent of the atmospheres of Mars and Venus, in analogy to the Earth, is spectrally inaccessible from the surface of the Earth, as it is in the far ultraviolet region of the spectrum.

Depending on the relative speed or rate of rotation of a planet with respect to the Earth, the Doppler effect provides in some instances another aid for the separation of a molecular species from that same species in Earth's atmosphere. Further, inferences on temperature also may be obtained from the spectra by observing the rotational fine structure, i.e., the detail of the absorption pattern caused by molecular rotation and its interaction with the vibrational mode. However, such temperatures cannot be clearly assigned to any certain altitude of a planetary atmosphere.

In addition, it is possible to obtain temperatures by radiometry or thermocouple measurements through telescopes averaging over part or all of the planet's surface. The ratio of incident to reflected

energy, the albedo, can be determined at various wave lengths, and estimates of the energy balance may be made. Also, the conditions at the surface or the nature of "clouds" may be inferred from measurements of the polarization of scattered light received from the planet. Under certain conditions of assumed composition and scattering mechanisms, atmospheric pressures may be deduced.

If the light of a star of known position is shut out by a planet (although such an occultation of a bright star is a rare event), the bending of the star's light through the planet's atmosphere and its light curve to final disappearance may be evaluated to reveal such characteristics as density gradients and scale height.

Finally, in recent years the microwave (wave lengths ranging from 1 mm to 100 cm) emission of planets has been discovered, and the measurement of this emission is an additional important tool in the study of the atmospheres. Under certain circumstances the equivalent-brightness temperatures of this radiation may be interpreted as representing surface temperatures, a fact that has led to a revision of the thoughts about the atmosphere of Venus, as will be seen later.

Visual observations and photography have led to the discovery of a great variety of features relating to the understanding of planetary atmospheres. Daily and seasonal changes may be observed, deductions on atmospheric circulation are possible, and clouds or haze layers may be visible. However, these observations are ordinarily qualitative, and therefore they have little direct bearing on the quantitative description of discrete values of parameters required to construct a model atmosphere.

Collecting evidence on the presence of certain constituents, estimating the amounts of certain constituents, and estimating thermodynamic parameters at certain heights will lead to a variety of results, which must some day be pieced together to form a consistent picture of a planetary atmosphere. Such consistency cannot yet be achieved for either Mars or Venus, and possible limiting models should therefore be estimated. It will be seen that from the practical viewpoint of an aerodynamicist, these possible limits are still wide indeed. In fact, it appears that even such wide limits may not be found for Venus. Atmos-

pheric calculations are chiefly based on integration of the basic equation of hydrostatics with certain assumptions on variables in a given height interval. Some of the background of these techniques is given in Appendix A.*

It should be noted that R. H. Zimmerman and C. D. Jones[11] have also studied models of Mars and Venus. They, however, calculate specific aerodynamic parameters such as aerodynamic heating and deceleration for several types of entry, whereas the present Memorandum emphasizes atmospheric models and the resulting regions of aerodynamic flight. Their study appears to be complementary to this one.

---

*The results collected from the widely dispersed literature on planetary atmospheres may be found in several recent surveys. Results for Venus are contained in Refs. 5 (p. 37), 6, 7 (p. 113), and 8 (p. 94), and those for Mars are in Refs. 5 (p. 21), 8 (p. 94), 9 (p. 151), and 10. The orbital elements and elements of the globes of the two planets are given, for example, in Ref. 7 (p. 115 for Venus and p. 153 for Mars).

II. MODELS OF THE ATMOSPHERE OF VENUS

Venus is the planet whose size most closely approaches that of the Earth. Its mass and radius, and therefore its gravitational attraction, are near that of the Earth. However, these may be the only respects in which the two planets are similar.

This is particularly true with respect to their atmospheres. Currently, three atmospheric models and some variants exist, all of which are based on the available observations of Venus. This perplexing state of affairs led the *Ad Hoc* Panel on Planetary Atmospheres of the Space Science Board to state that "...the case of Venus in 1961 is an extraordinary and challenging example of a scientific riddle with a variety of partial answers."* This situation has not changed much as of mid-1962.(12) Several main observations underlie present theories of the Cytherean atmosphere.

CLOUD COVER

The planet is surrounded by a continuous cloud cover with some structure visible in the ultraviolet as seen, for example, in the photographs taken by Ross.(13) The nature of the cloud cover is unknown, since it cannot be reliably inferred from measurements(14) of the polarization of scattered sunlight. The cloud cover prohibits the determination of either the precise diameter of Venus or its rotational period by classical techniques. However, radar measurements offer hope of resolving this point.

There is spectroscopic evidence of the existence of a number of constituents, including water vapor,(2,4,5) above the "visible" cloud layer, which acts as a reflecting layer for radiation of about 8000 Å. Of these detectable constituents, only carbon dioxide is present in large amounts. According to the method of evaluating the spectrograms, the comparison with laboratory spectra, and the interpretation of the data, it is found that there may be anywhere from 0.1 to 2 km of carbon

---

*See p. 37 of Ref. 5.

dioxide at standard atmospheric temperature and pressure (see Appendix A) above the visible clouds.[2,4,15,16] Current evidence favors the latter figure. Also, depending on interpretation and omitting minor constituents, it has been estimated through the course of the years that carbon dioxide provides from nearly 100 (early estimates) to about 3 per cent (present consensus) by volume of the total upper atmosphere (see Abstract 25 of Ref. 12). In analogy to the Earth, it is assumed that nitrogen is the inert filler gas making up the remainder, although argon may be present in unknown amounts.

TEMPERATURE CONDITIONS

Temperatures on Venus have been found to change little from the day to the night side. Direct thermocouple measurements through telescopes by Pettit and Nicholson[17] are in general agreement with more recent results by Sinton and Strong,[18] obtained by using infrared techniques; a value of about 235°K may reasonably be assigned to the atmosphere in the vicinity of the cloud layer.[4,6] In addition, Chamberlain and Kuiper[19] derived a temperature of about 285°K from examining the distribution of intensities in the fine structure of the rotational part of the Cytherean spectrum near 8000 Å. This temperature cannot be assigned to any level; it may be "...characteristic of the average temperature above the clouds."[4] However, Spinrad's recent re-examination[16] of spectrograms of the rotation-vibration band of carbon dioxide at 7820 Å assigns values ranging from 214°K to 440°K to different depths in the atmosphere. From several observations, it appeared reasonable, until the advent of microwave measurements, either to assign to the unseen surface a temperature of about 350°K[2,4] or to choose a higher value of around 400°K.[20]

The discovery of microwave radiation from Venus by Mayer, McCullough, and Sloanaker[21] led to a further upheaval of the thoughts on the temperature structure of Venus. These original microwave results, in conjunction with much additional work by several groups,* may be reduced to obtain an equivalent brightness temperature. This is an average temperature value of about 580°K to 600°K for wave lengths of about 3

*See, for example, p. 55 of Ref. 22.

to 21 cm; the lower temperatures are found at the shorter wave lengths. Most recently, temperatures up to 750°K, as well as variations between the dark and the bright sides of the planet, have been inferred.[12] For many reasons, we shall assign this temperature to the surface of Venus,[23] and we shall search for a model of the atmosphere that can sustain this hot surface. However, we cannot yet definitely rule out the possibility that the microwave radiation is emitted at high altitude from an ionosphere[24] rather than from the surface. This would make the relatively low surface temperatures credible.

The astrophysical studies then, present us with alternative versions of atmospheric models, depending on the attitude adopted towards the microwave findings. There are two main justifications for assuming a high surface temperature. Sagan proposes, from considerations of the energy balance, a greenhouse model of the atmosphere, in which ice crystals make up a thin cloud layer.[6,23] Öpik[25] suggests a wind-driven, dust-laden troposphere with heat produced by wind shear--an atmospheric model called "aeolosphere" by him. In this model, the visible clouds are layers of haze, below which heavy dust clouds exist.

Conversely, belief in an ionosphere of high electron density (the ionosphere model) permits the assumption of an atmosphere somewhat more comparable with that of the Earth. The features of these models have been summarized in Ref. 5, and the existing variants have been discussed in Ref. 26. It appears that final observations from space probes will be necessary in order to confirm present theories or, by providing new data, to establish an entirely different picture of Venus.

Finally, the observations of the occultation of Regulus by Menzel and de Vaucouleurs* predicted certain parameters at an altitude of 60 ± 10 km above the cloud layer. These need to be evaluated in the light of the assumed composition.

Scrutiny of the observed facts in conjunction with the proposed models shows that, for the purposes of aerodynamic design, it is permissible in a consideration of the upper atmosphere to disregard the inconsistencies of the models from the surface up to the upper atmosphere.

---

*See p. 38 of Ref. 5.

The greenhouse and aeolosphere models, for example, describe rather similar structures above the cloud level, whereas the nature of the clouds currently has only secondary aerodynamic interest, in view of the serious uncertainties. The ionosphere model leads to a much smaller total mass from the surface up; however, the difference above the cloud layer, taken as a reference level, concerns primarily higher temperatures at high altitude. Therefore, it appears logical to truncate the models horizontally at the visible cloud layer and to discuss the tropospheres separately. This approach will be adopted in this Memorandum.

CONDITIONS ABOVE THE CLOUD LAYER

The many assumptions on which this discussion must be based are to some extent arbitrary and are drawn from various of the sources cited here.

To account for possible differences, we will assume as one limit of atmospheric composition a pure carbon-dioxide atmosphere, to be called the CO_2 model. Assuming that an amount equivalent to a column of 1 km of CO_2 (at standard pressure and temperature, STP) is present above the cloud layer,[2,4,6,23] we find a corresponding cloud-level pressure of $p_c = 0.170$ atm (see Appendix A). It is interesting to note that Dole[27] arrived at essentially a pure carbon-dioxide atmosphere from considerations of the historical development of outgassing and surface reactions on Venus.

Next, we assign two different temperature distributions to the CO_2 model. The simpler temperature structure is based on the assumption of an isothermal atmosphere at the rotational temperature of 285°K. We will term this combination Model CO_2-I of the Cytherean atmosphere.* We shall consider a second temperature distribution assigned to Model CO^2-II, whose composition and cloud pressure remain unchanged from Model CO_2-I. We choose 235°K as the cloud temperature and take the atmosphere

*Tabulations and derivations of values for these and the following models of the atmosphere of Venus (excepting those of the N_2-III model) are given by Wegener,[1] and the results are summarized here in Tables 1 and 2 and Figs. 1 and 2.

to be isothermal up to 85 km above the cloud layer. At higher altitudes, an arbitrary linear increase in temperature is assumed (Table 1). These models are identical to those previously assumed in Ref. 1.

Table 1

CO_2 MODELS I AND II OF THE ATMOSPHERE OF VENUS

Item	Model I	Model II
General Parameters		
Composition, per cent of CO_2	100	100
Molecular weight, μ	44	44
Acceleration due to gravity, g; cm/sec^2 (constant)	860	860
Low-frequency speed of sound, a[a]	(b)	(b)
Viscosity at 1 atm, η	(b)	(b)
Pressure at cloud layer, p_c; atm	0.170	0.170
Surface to Cloud Layer, h_c		
Structure	Dry adiabatic	Dry adiabatic
Dry-adiabatic lapse rate, Γ; ^oK/km	-10	-10
Surface pressure, p_o; atm	7.02	0.995
Surface temperature, T_o; ^oK	600	350
Ratio of specific heats, γ (constant)	1.25	1.30
Altitude of visible cloud layer, h_c; km	31.5	11
Cloud Layer to h_c + 85 km		
Structure	Isothermal	Isothermal
Temperature at visible cloud layer, T_c; ^oK	285	235
h_c + 85 km to h_c + 200 km		
Structure	Isothermal	Polytropic
Lapse rate, β; ^oK/km	...	6.14

[a]Function of pressure and time.
[b]See Ref. 28.

As another composition limit, we devise an atmospheric model (an N_2 model) primarily containing nitrogen, a condition currently considered to be most likely.[15,16] It will have three variants: N_2-I, N_2-II, and N_2-III. Of these, N_2-I and N_2-II (given previously in Ref. 1) are 85 per cent N_2 and 15 per cent CO_2 by volume, and they exhibit an identical structure above the cloud level, although, as we shall see later, the tropospheric conditions will be dissimilar (Table 2). This composition, which is 0.1 km CO_2 at STP above the cloud level, leads to a cloud-level pressure of 0.09 atm;[15] again, 235°K will be assigned to this point. There may first be a slight decrease in temperature above the visible clouds,* and an arbitrary linear increase of temperature is assumed for the upper atmosphere (see Fig. 1).

This upper atmosphere is in general accord with the ionosphere model, and it does not deviate appreciably from the previously mentioned model's description of the area above the cloud layer. All models discussed so far roughly agree[1] with the high-altitude findings made during the observations of the occultation of Regulus.

Finally, in accord with Spinrad's recent work,[16] we assume an N_2-III model atmosphere with only 5 per cent of CO_2 by mass (or about 3 per cent by volume). Spinrad obtains pressure values greater than 2 atm for layers that are probably below the top of the visible cloud level. Sagan (Abstract 28 of Ref. 12) recently estimated 0.6 atm of pressure for the cloud level, and as an estimate for a dense limit, we will take 1 atm for this point. Again, this cloud pressure will be combined in the N_2-III model with an isothermal temperature distribution of 285°K above the clouds. The corresponding densities for all models are shown in Fig. 2.

TROPOSPHERE

Speculations on conditions of flight in the troposphere, and in fact, on the location of the solid surface below the cloud level, vary greatly according to the surface temperature assumed. Since the varied temperatures result in equally varied aerodynamic conditions, we

---

*Private communication from L. D. Kaplan.

Table 2

N_2 MODELS I, II, AND III OF THE ATMOSPHERE OF VENUS

Item	Model I	Model II	Model III
General Parameters			
Composition, per cent	[redacted]	[redacted]	[redacted]
N_2 (by volume)	85	85	96.75
CO_2 (by volume)	15	15	3.25
N_2 (by mass)	...	...	95
CO_2 (by volume)	...	...	5
Molecular weight, μ, constant	30.4	30.4	28.5
Acceleration due to gravity, g; cm/sec^2	860	860	860
Speed of sound, a	$(\gamma RT/\mu)^{1/2}$	$(\gamma RT/\mu)^{1/2}$	$(\gamma RT/\mu)^{1/2}$
Viscosity of N_2 at 1 atm	(a)	(a)	(a)
Ratio of specific heats, γ	1.39 const.	1.39 const.	1.4[a]
Surface to Cloud Layer, h_c			
Altitude, h_c; km	38.2	13.1	49.4
Structure	Dry adiabatic	Dry adiabatic	Dry adiabatic
Dry-adiabatic lapse rate, Γ; ^oK/km	-8.81	-8.81	-8.4
Surface pressure, p_o; atm	2.21	0.367	23
Surface temperature, T_o; ^oK	580	350	700
Cloud-level pressure, p_c; atm	0.0888	0.0888	1
Cloud-level temperature, T_c; ^oK	235	235	285
Cloud Layer to h_c + 60 km			
Structure	Polytropic	Polytropic	Isothermal
Lapse rate, β; ^oK/km	-0.75	-0.75	...
h_c + 60 km to h_c + 200 km			
Structure	Polytropic	Polytropic	Isothermal
Lapse rate, β; ^oK/km	2.21	2.21	...

[a]See Ref. 28.

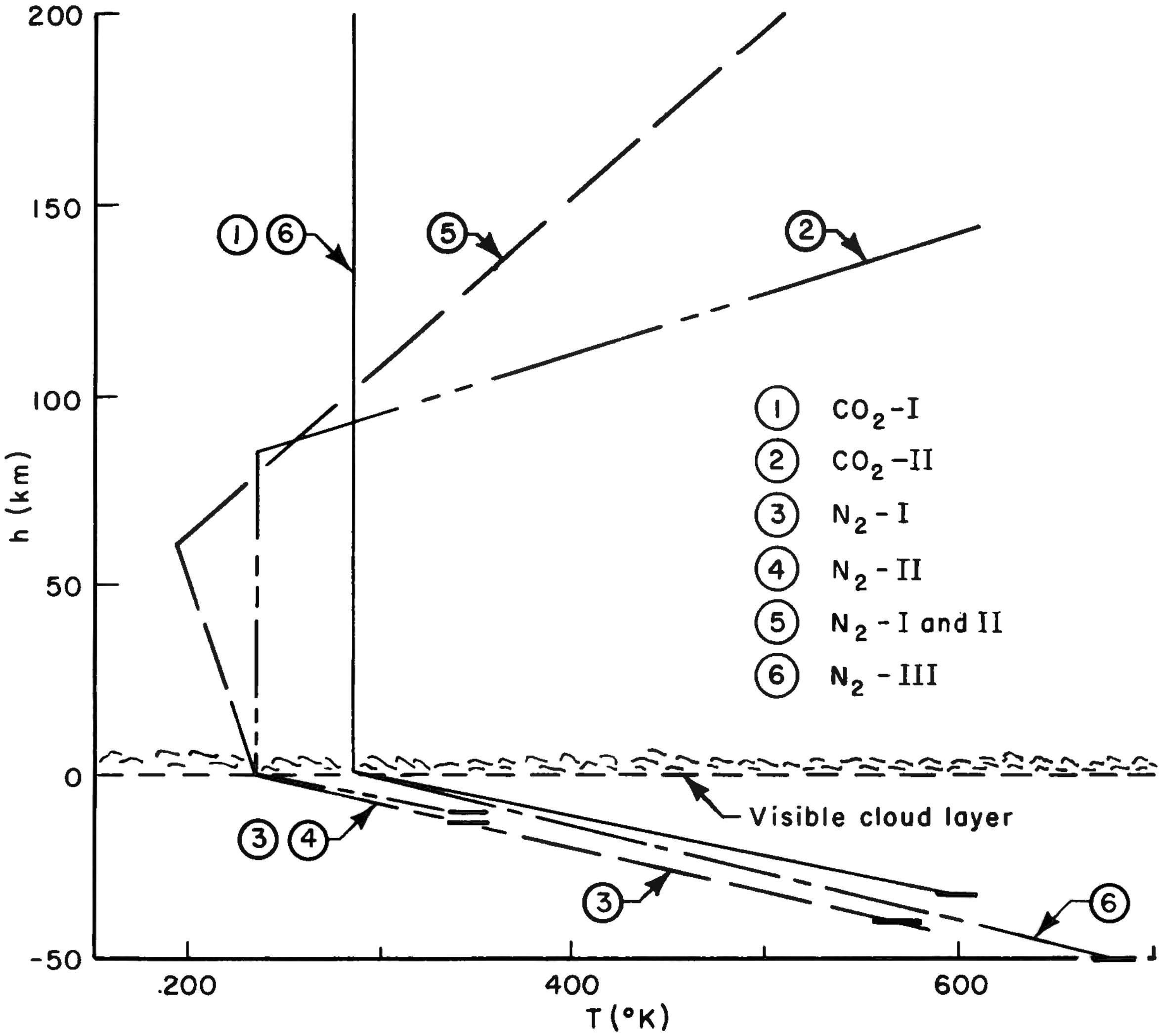

Fig. 1 — Temperature of the atmosphere of Venus as a function of altitude

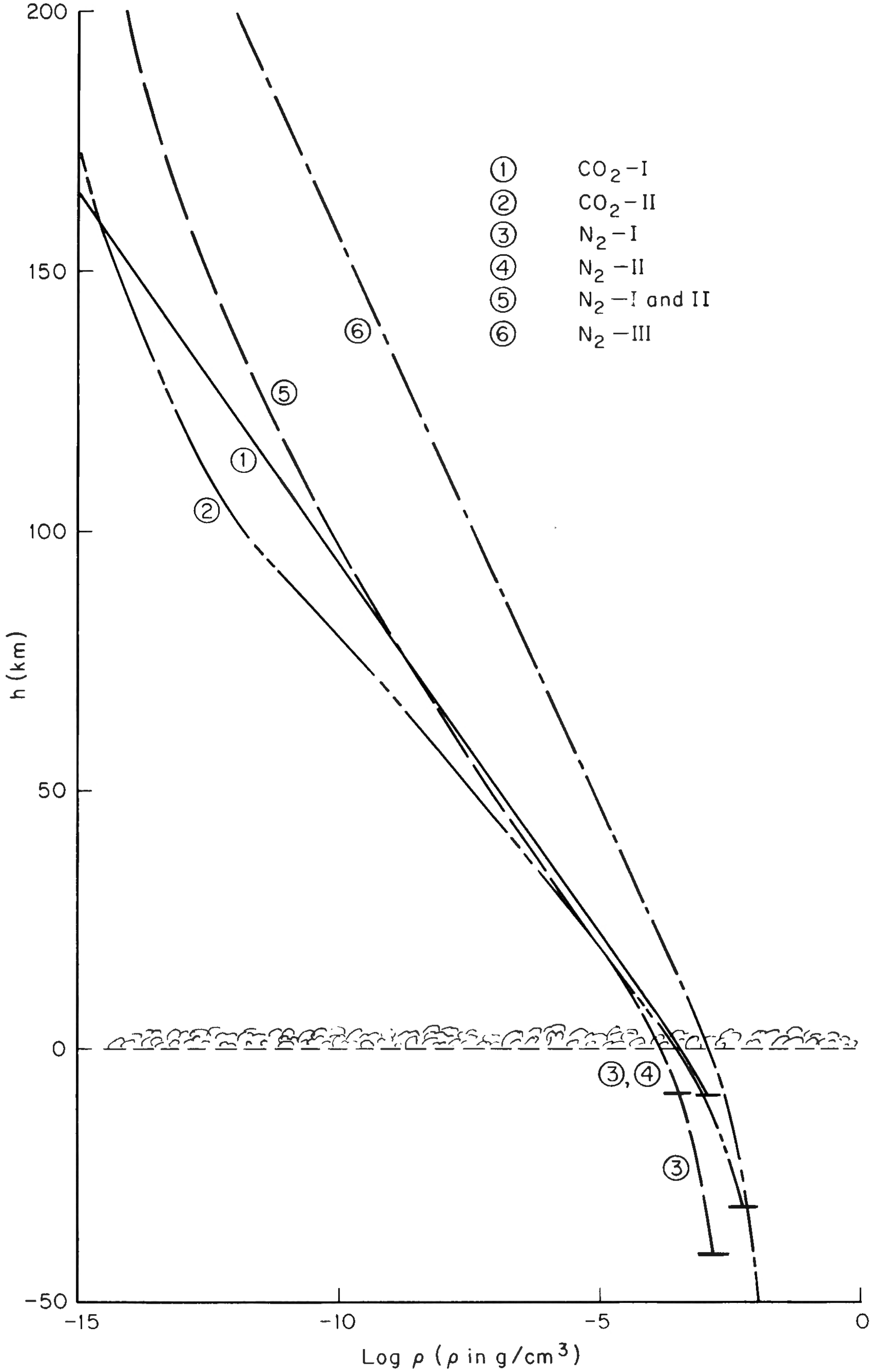

Fig. 2 — Density of the atmosphere of Venus as a function of altitude

have chosen models which assume very high and very low surface temperatures. In each instance, our assumption of cloud-level conditions is that the atmospheric structure is a dry adiabatic one (see Appendix A). This statement implies that atmospheric equilibrium has been established by convection in the troposphere. That strong convection must certainly be present on Venus, we deduce for a variety of reasons, among them the near-equality of the day and the night temperatures. As our extreme case, we choose an average surface temperature of $700^{\circ}K$ (Model N_2-III), in view of the suggestions of Sagan, Kuzmin and Salomonovich, and Bibinura, et al. (Abstracts 28, 31, and 32 of Ref. 12).

Correspondingly, the possible surface pressures range from about 1/3 to about 50 atm, and the possible height of the troposphere ranges from roughly 10 to 100 km. In other words, the surface of the planet may be at these distances below the visible clouds. A wide range of surface pressures has been proposed as compatible with the "hot" surface models; we chose p_o = 23 atm, which is consistent with the data given in Table 2. Estimates of limits or variations with altitude of the acceleration of gravity and other parameters are ignored, in view of the serious uncertainties. No claim is made that the given atmospheric models delimit the actual conditions on Venus with any degree of certainty.

III. MODELS OF THE ATMOSPHERE OF MARS

Since the surface of Mars is visible from the Earth, a wealth of quantitative and qualitative detail about it has been accumulated by astronomers.[10] For the aerodynamicist, such features as surface markings, seasonally changing polar caps, and haze layers are of little consequence in an atmospheric model. (However, since the rotational period of about one day is well established, attempts have been made to account for seasonal changes and other observational facts by estimating circulation and other meteorological processes.)[29,30] In our search for a mean atmospheric model and possible limits, we need make little use of these studies.

From the uncertainties of the values of radius and mass, Kirby, as quoted by Schilling,[31] arrived at a range of values of gravitational acceleration from $360 < g_o < 390$ cm/sec^2. From spectroscopic evidence, a number of constituents have been identified; of these, only carbon dioxide occurs in relatively large amounts.[2,4,5] About 40 m of CO_2 at STP are estimated, an amount appreciably greater than that in the Earth's atmosphere. The surface pressure was estimated by Dollfus[32] from his polarization measurements to be about 85 mb, and since this value has been corroborated by others (e.g., Ref. 10), a total composition estimate may be made.

Since the known columnar mass of carbon dioxide cannot account for the surface pressure, a major amount of nitrogen must again be postulated, as with Venus, although no direct identification of this molecule is possible from the surface of the Earth. Also, argon produced by the radioactive decay of potassium may be present in appreciable amounts. The amount of argon present is hard to ascertain; it may indeed be more plentiful than the amount quoted later (Brown, Chapter IX of Ref. 2). A calculation of the average molecular weights of the compositions assumed by different authors yields a range of about $28 < \mu < 31$, assuming there is no more than 4 per cent of argon present. This result is in accord with Schilling's estimates.[31] A summary of the current views[4,10] would indicate an assumed distribution of about 2.2 per cent (by volume) of carbon dioxide, 4 per cent

of argon, small amounts of oxygen, water vapor, and other gases; the major part consists of nitrogen, in analogy with the Earth.

The surface temperature, and daily and seasonal temperature variations have been found by infrared measurements, and in fortunate contrast to Venus, there is also general agreement with the values of temperature inferred from microwave observations. The aerodynamicist may ignore detailed variations and, following Schilling, allow a range of surface temperature of $200^{\circ}K < T_o < 300^{\circ}K$. From these results, density, adiabatic lapse rate, and scale height at the surface (see Appendix A) may be calculated, and they are given in Table 3 for data taken from Schilling's critical analysis.(31)

Table 3

RANGE OF SURFACE CONDITIONS OF MARS[a]

Parameter	Minimum	Mean	Maximum
Acceleration due to gravity, cm/sec^2	360	375	390
Molecular weight	30	29	28
Dry adiabatic lapse rate, $^{\circ}K/km$	-3.79	-3.76	-3.71
Pressure, atm	0.0405	0.0840	0.131
Temperature, $^{\circ}K$	200	250	300
Density, g/cm^3	7.40×10^{-5}	1.19×10^{-4}	1.49×10^{-4}
Scale height, km	15.4	19.1	22.8

[a]After Ref. 31.

In contrast to Venus, Mars will present a hospitable environment for flight. Because of the low gravitational acceleration, and in spite of the lower surface density, the atmosphere is denser at high altitude than that of the Earth (see Fig. 1 of Ref. 8). It is this fact that will primarily govern our later aerodynamic considerations.

ATMOSPHERE FROM THE SURFACE TO 200 KM

In our consideration of the Mars atmosphere below 200 km, we will follow Schilling's thoughts on limiting atmospheric models.(31) In

Fig. 3, we see the temperature distribution estimated by various authors, and we see that between the minimum and maximum temperatures of Schilling's Model II* and some part of his Model III, the range of possible temperatures may in fact be delimited. His Model II, originating at the surface from the range of values given in Table 3, implies a troposphere in convective equilibrium with its corresponding dry adiabatic lapse rate and an isothermal upper atmosphere. Equatorial and polar temperature distributions estimated by Urey[4] and others are within our range. Also included in Yanow's model, which assumes T_o = 235°K and a chemical composition of 95 per cent nitrogen and 5 per cent carbon dioxide.

Schilling's conjectural atmosphere, his Model III, is based on facts about Mars combined with information on the atmosphere of the Earth. The comparison with Earth, owing to the difference in gravitational attraction, deals with conditions that might exist here above a plateau about 11 km high and in an atmosphere with a low ozone concentration. However, no matter how little oxygen (and consequently ozone) there may be on Mars, the ozone may greatly influence the radiative heating. The corresponding density distributions of these models appear in Fig. 4; they have been tabulated partly in Ref. 31. Also shown is the density distribution for the Earth, as compiled from Refs. 33, 34, and 35. With the exception of the minimum figure in Schilling's Model II, above 20 to 25 km, densities on Mars for these models are higher than on Earth. However, we cannot assign different probabilities to any of the density distributions discussed. In fact, if very little oxygen is present and if the carbon dioxide is supercooled, the density may be lower than that on Earth at all levels.**

UPPER ATMOSPHERE

As we will find later, even at altitudes of about 200 km, the atmosphere of Mars is dense in the aerodynamic sense, i.e., a flight vehicle would still encounter the continuum regime. This forces us to

---

*Schilling terminates his Model II at h = 80 km, but we have simply extended the isothermal structure to h = 200 km. Correspondingly, density has been computed for this isothermal extension (see Appendix A) with these results shown in the following figures.

**Private communication from M. H. Davis of The RAND Corporation.

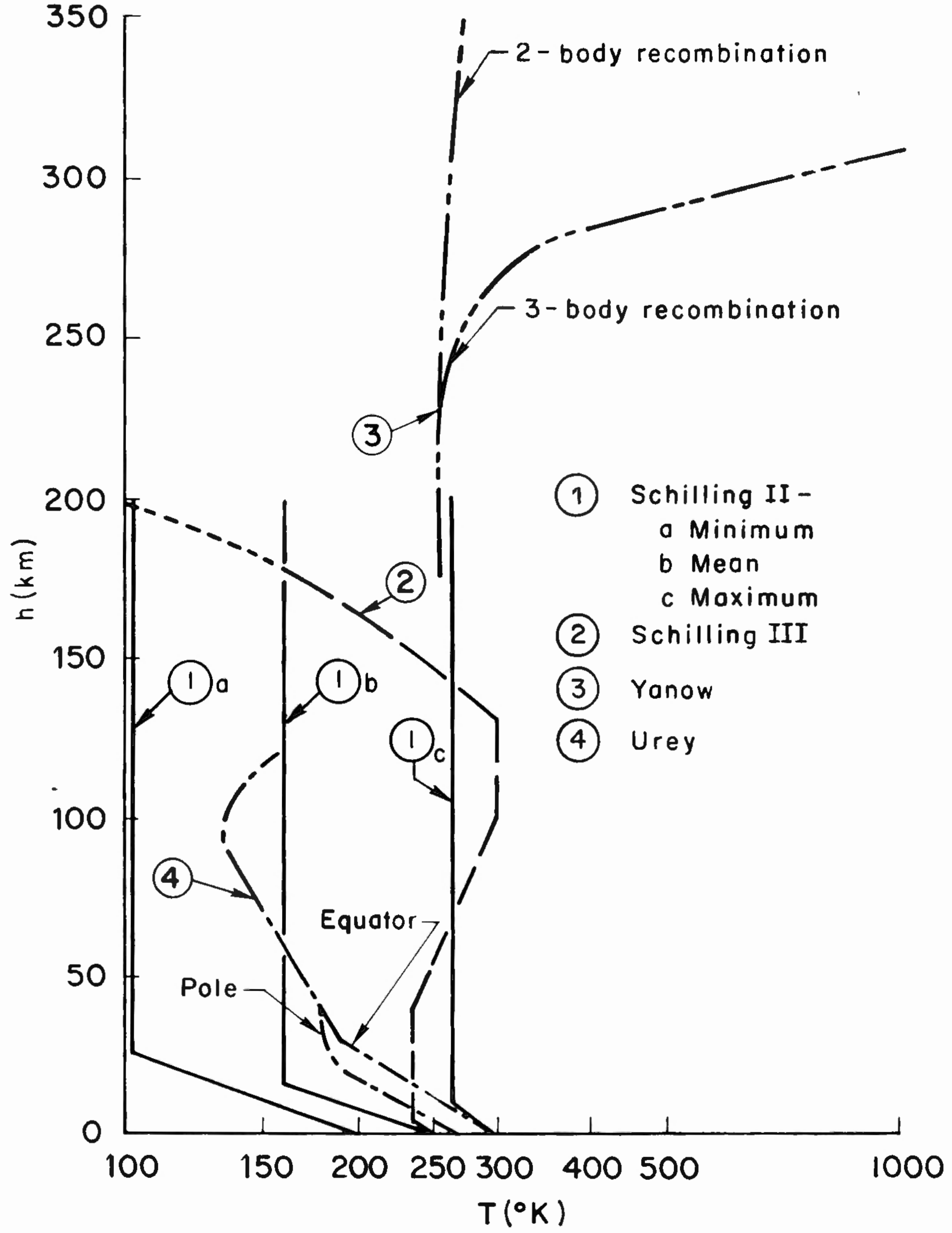

Fig. 3 — Temperature of the atmosphere of Mars as a function of altitude

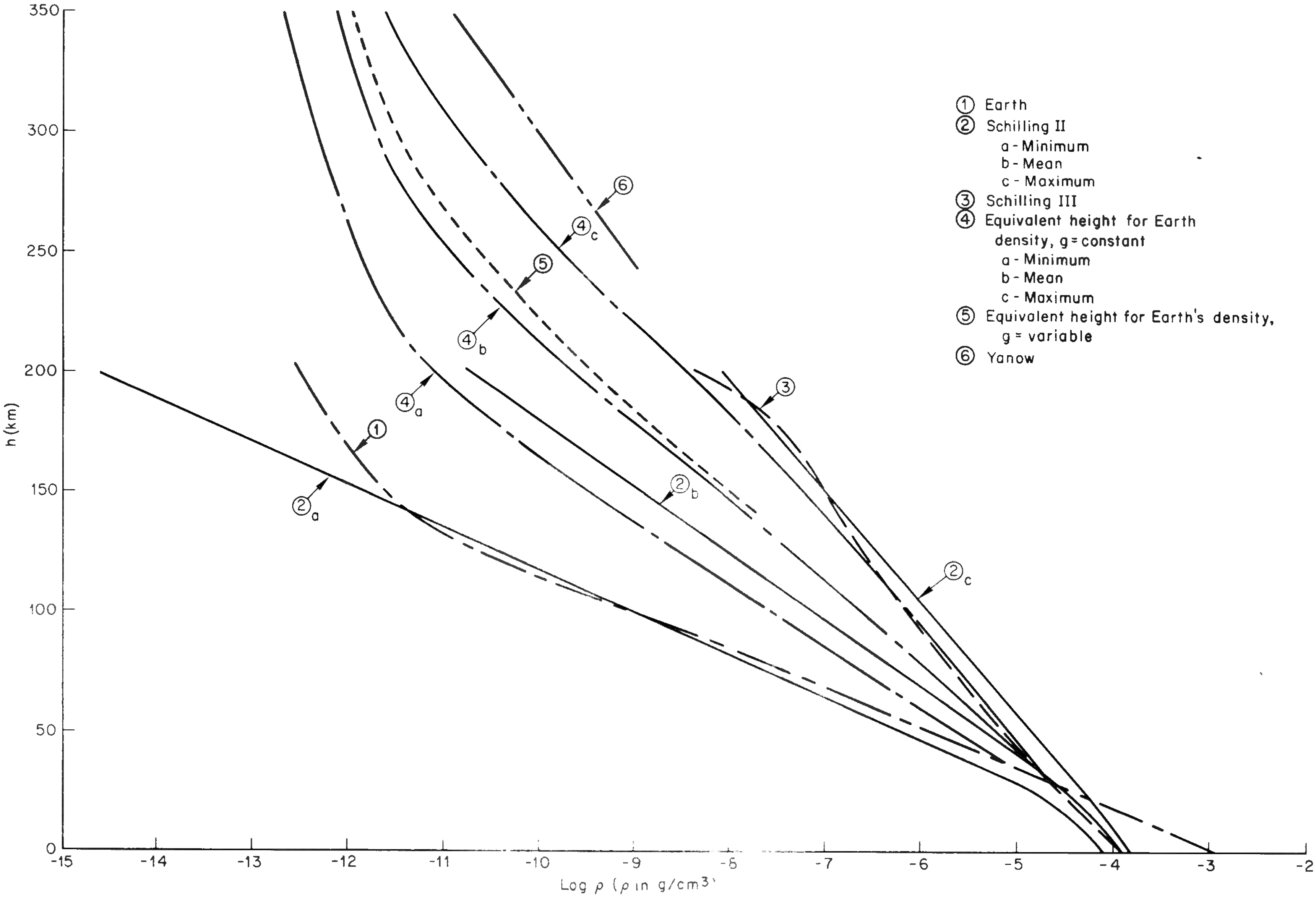

Fig. 4 — Density of the atmosphere of Mars as a function of altitude

speculate about the upper atmosphere of Mars. In order to estimate parameters for the structure of the upper atmosphere (while neglecting minor changes in molecular weight), we will follow two approaches: that proposed by Yanow,[36] which is based on photochemical studies and assumes a composition of 95 per cent nitrogen and 5 per cent carbon dioxide, and that proposed by Barth,[37] which is based on our knowledge of the Earth's density distribution.

Yanow's computation of the composition and structure of the upper atmosphere of Mars assumes an original unperturbed atmospheric composition of 95 per cent nitrogen and 5 per cent carbon dioxide. (This is his example, which we plan to utilize.) He calculates the density as a function of altitude for surface conditions of $T_o = 253^oK$ and $\rho_o = 10^{-4}$ g/cm^3, molecular weight, $\mu = 28$, but he includes variable gravitational attraction with $g_o = 372.5$ cm/sec^2 (see Appendix A). We note that all these values are near the mean values of Table 3. It is assumed in this calculation that Mars has been initially insulated from solar radiation. Next, Yanow assumes a chain of photochemistry applicable to the quoted constituents, and he chooses 12 possible reactions for which collision cross-sections and rate constants of recombination can either be found in the literature or be estimated. Finally, he estimates a power density of solar radiation in some important groups of wave lengths pertaining to the outside of the Mars atmosphere. With these tools established, and some boundary conditions determined, he performs a numerical calculation beginning with the initial density distribution. This calculation requires the simultaneous solution of the differential reaction equations. For each incremental layer from the top down, a new composition was found, and the increase of temperature over the initial isothermal value was calculated by assuming equipartition of energy in a given layer. The resulting temperature also appears in Fig. 3 with two alternative solutions, depending on the choice of two- or three-body recombination as the prevalent mechanism. Yanow's results indicate the possible presence of a multilayered ionosphere on Mars not unlike that of Earth, but with the layers stretched out at great height because of the low gravitational acceleration. The resulting values for density are shown in Fig. 4. The density itself

is close to that computed for the initial isothermal distribution. Also, for the altitudes shown, the mean molecular weight is practically constant to an altitude of about 400 km. The omission of an estimate of electron attachment* in this work has been suggested[5] as an explanation of the possibly unreasonably high values of electron density given by Yanow. Also, Chamberlain (Abstract 39 of Ref. 12) estimates lower electron densities. However, for our purposes, this would only result in overestimating the temperature at extreme altitude, whereas density is our prime parameter of interest, as will be shown later.

A second model of the upper atmosphere of Mars may be constructed using the known density distribution of Earth as an analogy. The underlying idea is to determine the "equivalent-density altitude," namely, an altitude on Earth and one on Mars at which the densities are the same. This approach may be reasonable in view of the fact that nitrogen is considered to be the major constituent in both atmospheres. Such an approach has been proposed by Barth[37] to obtain an initial density distribution for work on the Martian photochemistry.

For such scaling, we may assume conductive equilibrium on both planets for the integration of the hydrostatic equation. Furthermore, for a first approximation, the calculation may be performed by assuming the atmosphere on both planets to be an inert-gas mixture. Equating the resulting expression for density on each planet at a given altitude (with g being variable or constant) leads to an expression for the equivalent-density altitude (see Appendix A). It is seen that in this model comparison, only the density and scale-height ratios at the surface apply; these ratios may be computed for the surface scale-height and density limits proposed by Schilling. These limiting ratios, based on the values of Table 3, are given in Table 4. The resulting equivalent-density altitudes are shown in Fig. 5.

---

*This is the process by which an electron attaches itself to a neutral atom (or molecule) to form a negative ion. The energy required for this process may be very low; however, nitrogen does not exhibit this behavior and it is the major constituent of this model atmosphere.

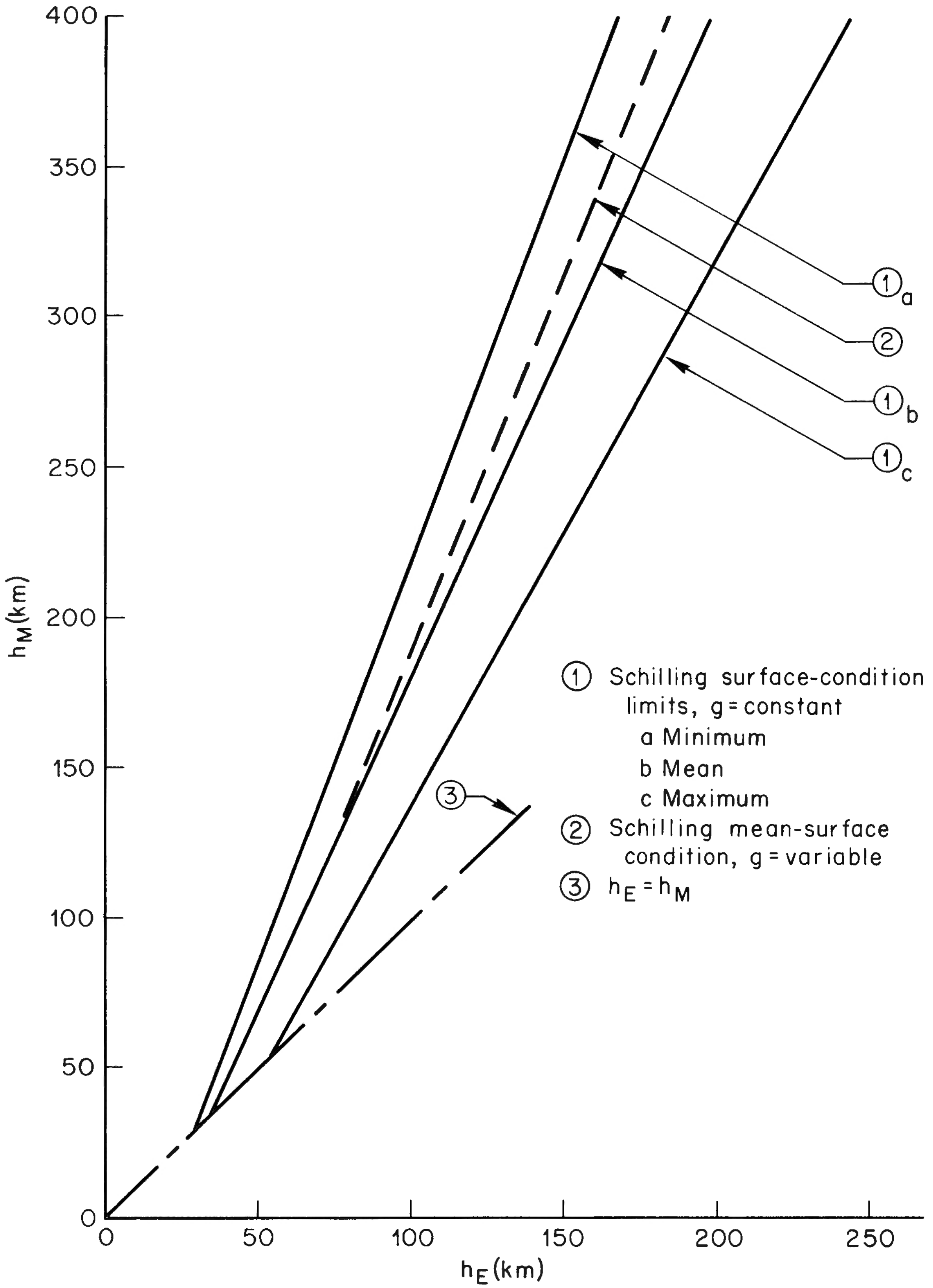

Fig. 5 — Equivalent-density altitudes in the atmospheres of Earth and Mars

Table 4

RATIOS OF SURFACE CONDITIONS FOR EARTH AND MARS[a]

Ratio	Minimum	Mean	Maximum
H_M/H_E	1.83	2.27	2.71
ρ_{oE}/ρ_{oM}	16.6	10.3	8.26

[a]Earth: $H_E = 8.4$ km, $\rho = 1.23 \times 10^{-3}$ g/cm^3

It is seen that taking a variable acceleration of gravity into account does not lead to an answer that is, in view of the uncertainties, significantly different. From this derivation, the altitude at which the densities are equal may be found (see Appendix A). Below this altitude, the atmosphere of Mars is less dense than that of the Earth; above it, the converse is true. Under Schilling's limits of surface conditions, this altitude varies from 28 to 52 km. The range of density distribution with altitude for Mars in the Earth-analogy model is also shown in Fig. 4. It is interesting to note that up to h = 200 km, this model is bracketed by a density range from about the mean value to the maximum value of Schilling's Model II. Also, at high altitude the density predicted by Yanow's analysis is not exceeded. However, even if we disregard the minimum, the density-sensing device of a space probe, to be useful, would have to have a sampling sensitivity extending through several orders of magnitude, as may be seen from Fig. 4.

IV. DISCUSSION OF FLIGHT REGIMES

The aerodynamicist requires a knowledge of similarity parameters for a vehicle trajectory of a given geometry. Ordinarily, the aerodynamic coefficients of drag, lift, etc., are known as functions of these parameters. The free-stream Mach and Reynolds numbers are first defined as

$$M_\infty \equiv u/a_\infty \tag{1}$$

and

$$Re_\infty \equiv u\rho_\infty \ell/\eta_\infty \tag{2}$$

that is, the gas-mixture properties are based on conditions undisturbed by the flow field. For atmospheres in which carbon dioxide is the major constituent, the values of the sound speed for carbon dioxide as a function of temperature and pressure are taken from tabulations.[28] This gives the low-frequency acoustic velocity where the vibrational modes of the molecule are in thermodynamic equilibrium. In those atmospheres in which nitrogen is dominant, the speed of sound is computed from

$$a_\infty = (\gamma R T_\infty/\mu)^{1/2} \tag{3}$$

where the molecular weight is determined for the mixture. The ratio of the specific heats may ordinarily be taken as independent of temperature because of the low atmospheric temperatures found at lower altitudes. The viscosity[28] is taken as that of the major constituent. The uncertainties of atmospheric properties do not warrant the calculation of viscosities of the gas mixtures in question. The characteristic dimension, ℓ, rests on an arbitrary choice; for example, it may be the nose radius of an entry body.

To determine whether the flow field of the vehicle, at a given velocity and altitude, pertains to the continuum, the free-molecule

flow (FMF), or the transitional regime, it is customary to find in addition the free-stream Knudsen number

$$K_\infty \equiv \lambda_\infty / \ell \tag{4}$$

The Knudsen number may be related simply to the other two given similarity parameters by remembering from kinetic theory that the viscosity may be expressed by

$$\eta = \frac{1}{2} \rho \; c \; \lambda \tag{5}$$

where

$$c = (3RT/\mu)^{1/2}$$ [redacted] (6)

is the mean molecular speed. From these relations we find

$$\frac{Re_\infty}{M_\infty} = \frac{a}{c} \; \frac{2}{K_\infty} = (\frac{4}{3} \gamma)^{1/2} \; K_\infty^{-1} \tag{7}$$

Assuming again that γ is constant, we obtain for the constant factor before K_∞^{-1} a numerical value of 1.37 for diatomic gases (γ = 1.40) and 1.33 for polyatomic gases (γ = 1.33).

If we know the composition and structure of a planetary atmosphere, we may plot for a given characteristic length the traditional flow-regime diagrams of $M_\infty = f(R_\infty, h)$, and we can insert the function $M_\infty = f(Re_\infty, K_\infty)$ for chosen fixed values of Knudsen number. Such diagrams are shown for Venus and Mars in Figs. 6, 7, 8, and 9 for the models described in the previous section as indicated in the graphs. If we can

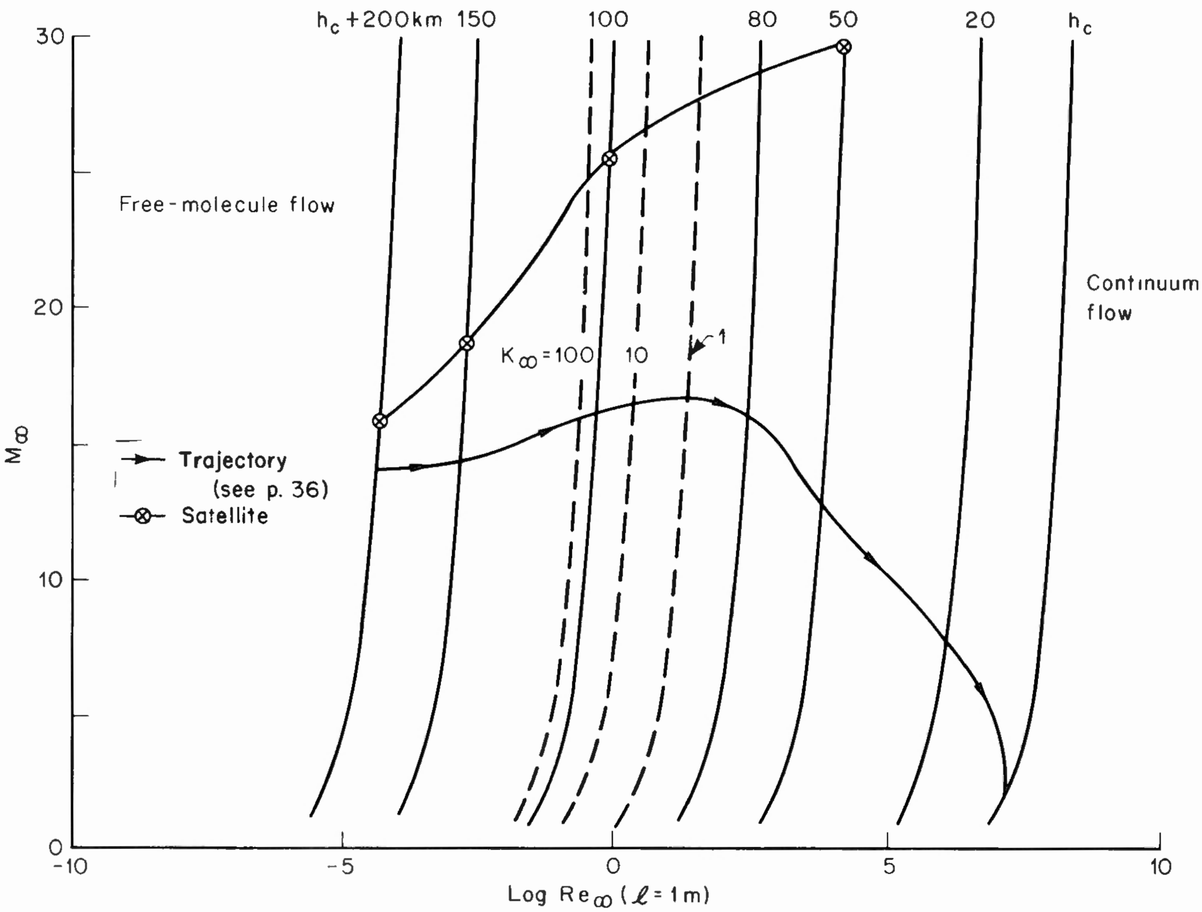

Fig. 6 — Free-stream Mach and Reynolds numbers in the CO_2-II model of Venus

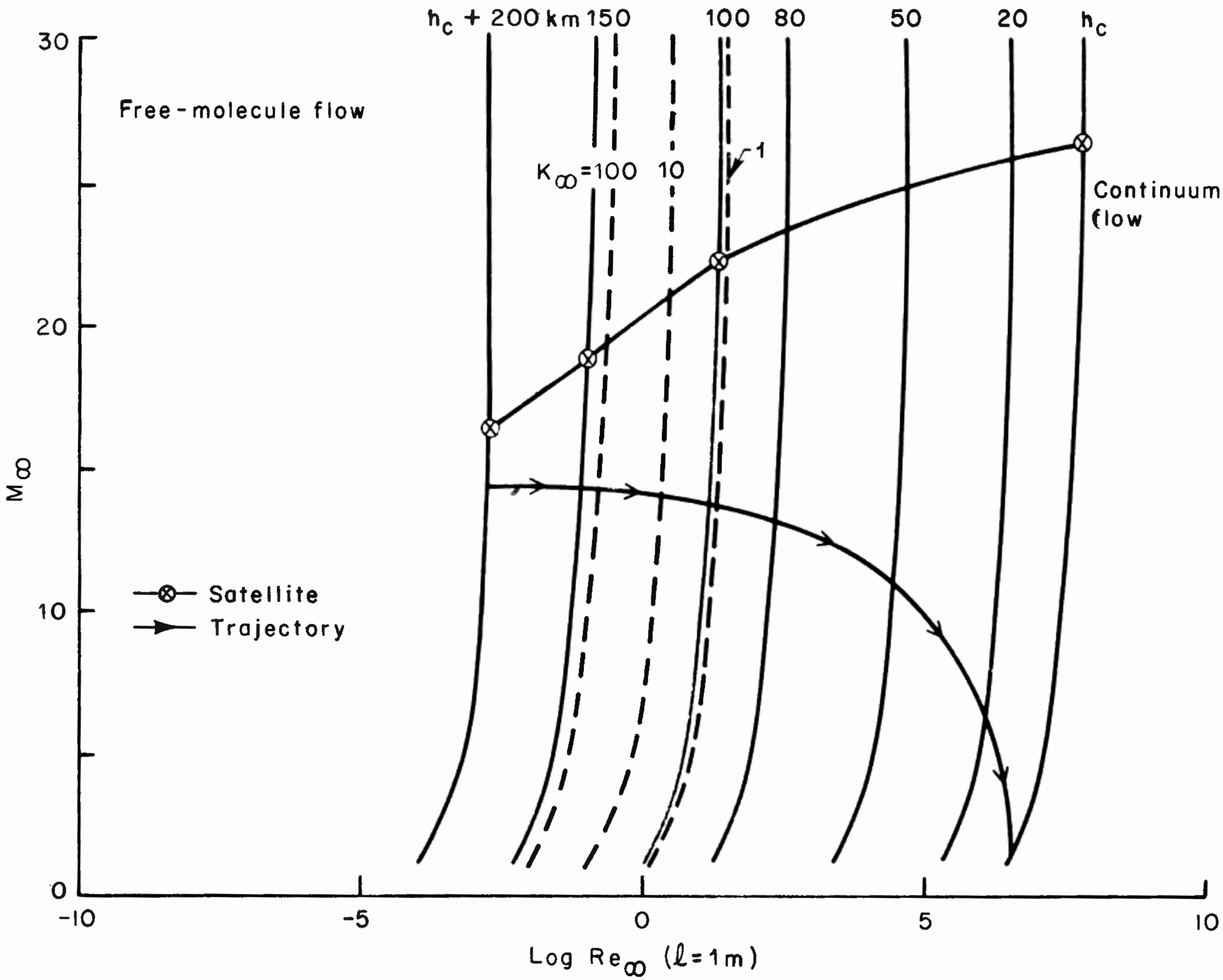

Fig. 7 — Free-stream Mach and Reynolds numbers in the N_2-I and N_2-II models of Venus

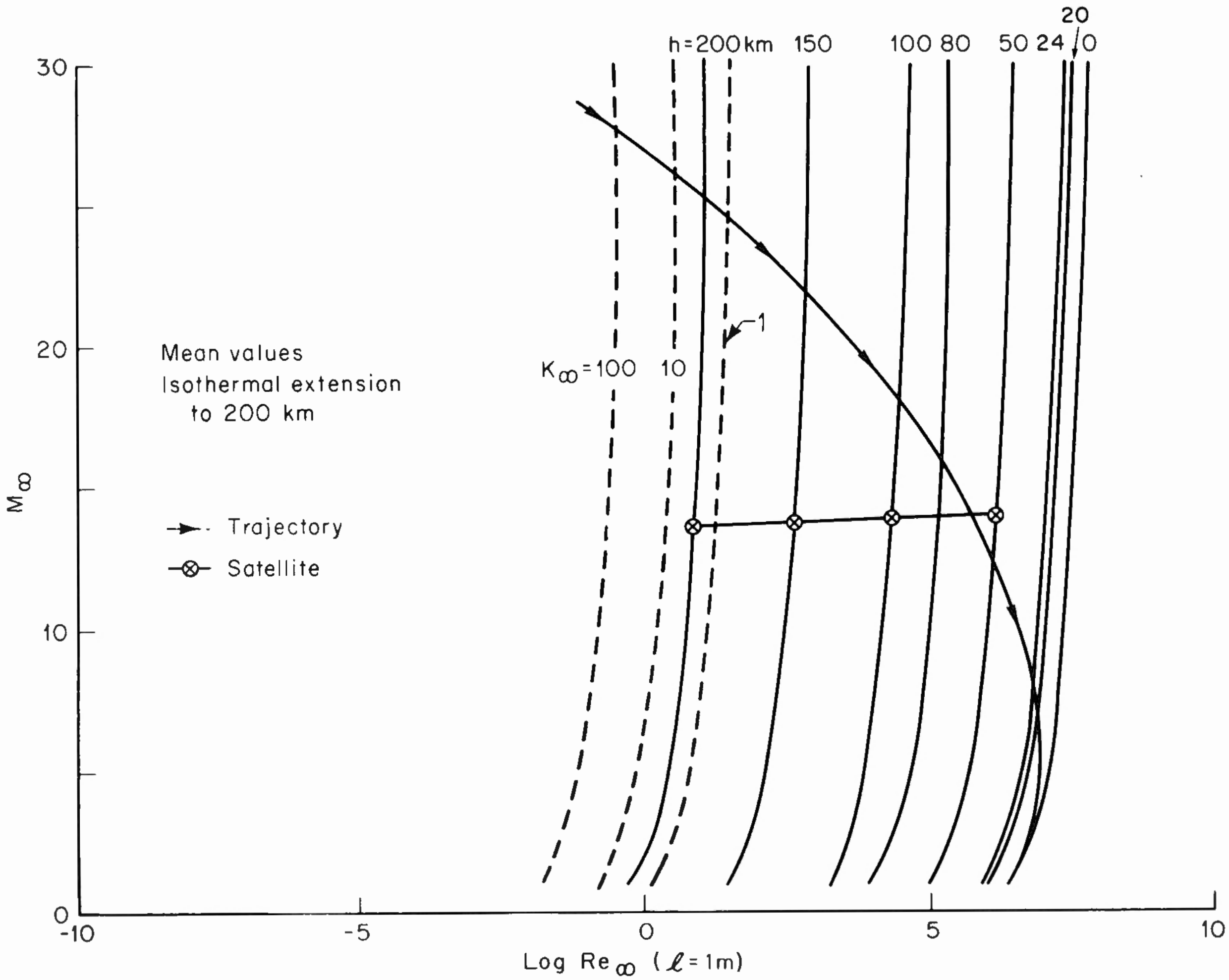

Fig. 8 — Free-stream Mach and Reynolds numbers in Schilling's Model II of the atmosphere of Mars

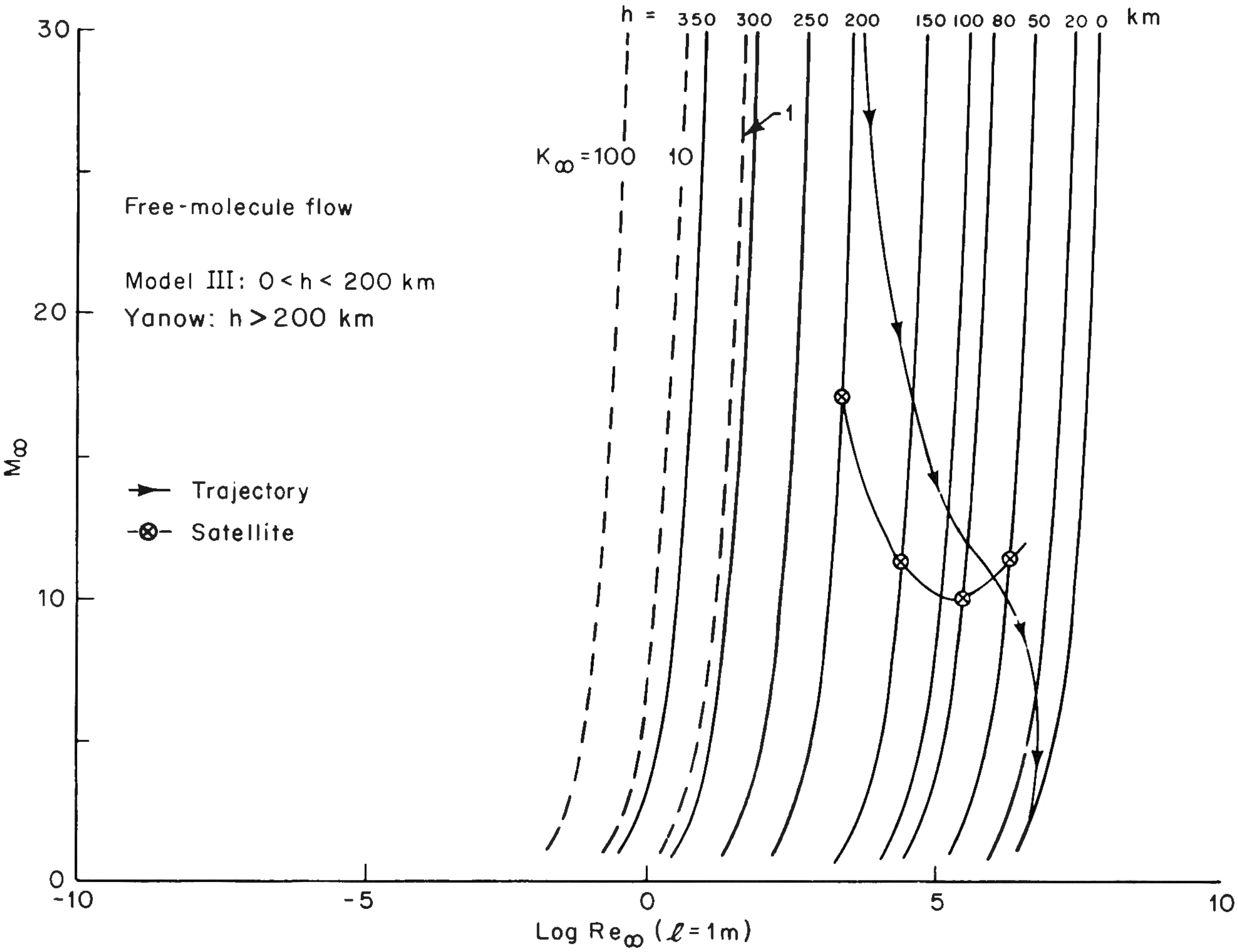

Fig. 9 — Free-stream Mach and Reynolds numbers for Schilling's Model III and the extension to Yanow's model of the atmosphere of Mars

relate certain values of free-stream Knudsen number (or, from Eq. (7), Reynolds number at fixed Mach number) to the character of the flow field, our orientation as to flow regimes will be complete. In order to compare conditions for several atmospheric models in one diagram, it is advantageous to write

$$\frac{Re_\infty}{M_\infty} = \frac{a_\infty \rho_\infty \ell}{\eta_\infty} \tag{8}$$

and plot $Re_\infty/M_\infty = f(h)$ for each model. These curves are immediately valid for $M_\infty = 1$, and after reading a value at given h for some model, the Reynolds number for any Mach number may be found immediately. This result is shown in Figs. 10 and 11 for all models of the atmospheres of Venus and Mars. From Eq. (7) we see, furthermore, that we have one value of the ratio Re_∞/M_∞ for a given Knudsen number.

Viewing the atmospheric parameters entering Eq. (8) in relation to our findings on the atmospheres of Venus and Mars, we see that the density at a given altitude will be the major atmospheric variable of aerodynamic importance. We found that density varies with altitude by many orders of magnitude for the atmospheric models of Venus and Mars. At a given altitude, the models proposed show, unfortunately, major ranges of uncertainty of density.

Temperatures, in contrast, vary less than one order of magnitude in the atmospheres at the altitudes considered. Inspecting Eq. (8), we recall that the speed of sound is proportional to $T^{1/2}$. The viscosity at low and high temperatures is proportional to $T^{3/4}$ and $T^{1/2}$, respectively. From this, we find that $Re_\infty/M_\infty \sim \rho_\infty$ for a fixed geometry.

High speeds, such as initially superorbital velocity, will be characteristic of atmospheric-entry problems. It is clear that under such conditions the flow regime of the vehicle may not be described sufficiently by a statement of free-stream parameters only. More information about the flow field is required in order to utilize such methods of entry-trajectory calculation as those proposed by Chapman[38] and Peterson.[39]

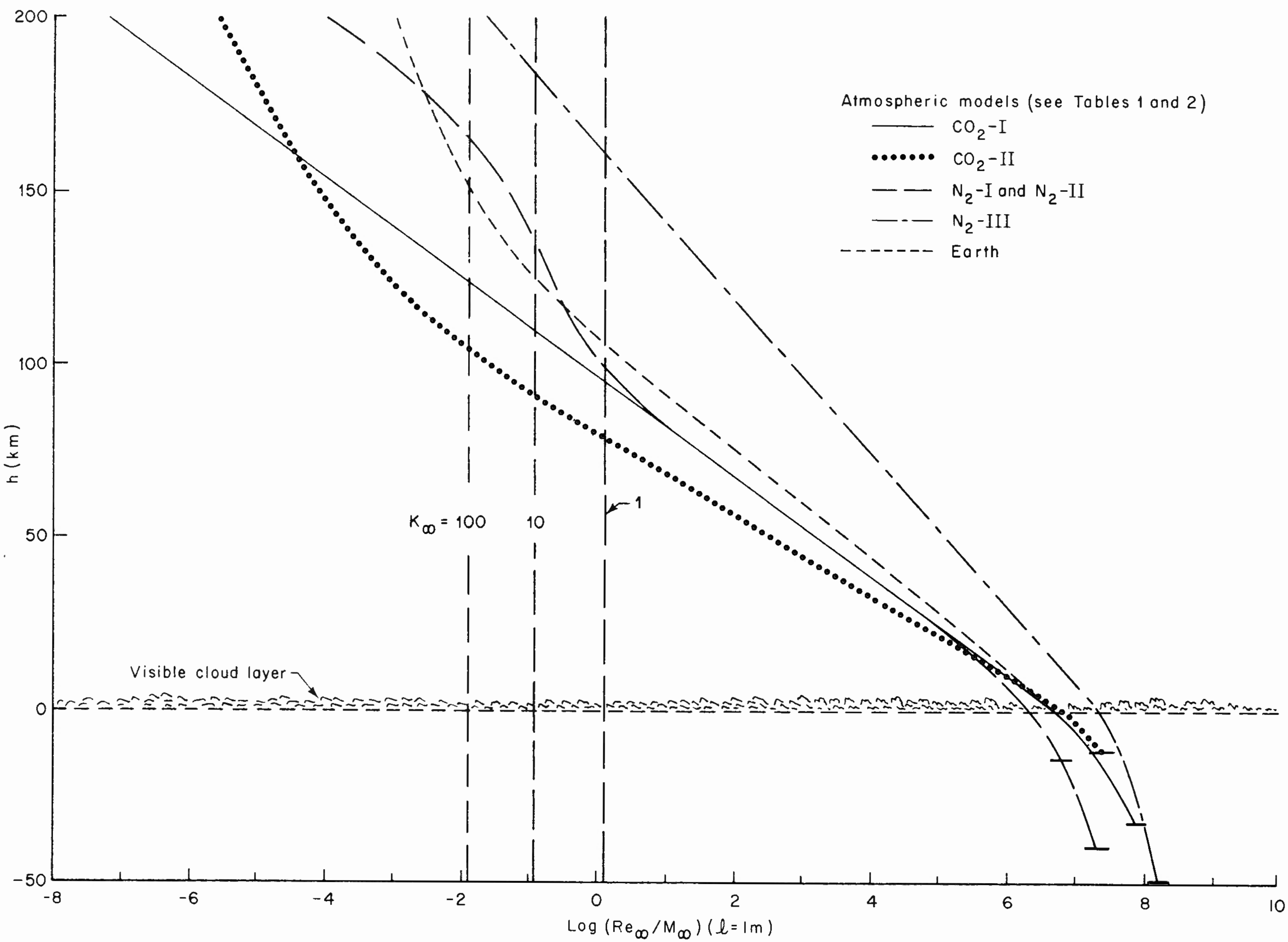

Fig. 10 — Summary of flight conditions in the atmosphere of Venus

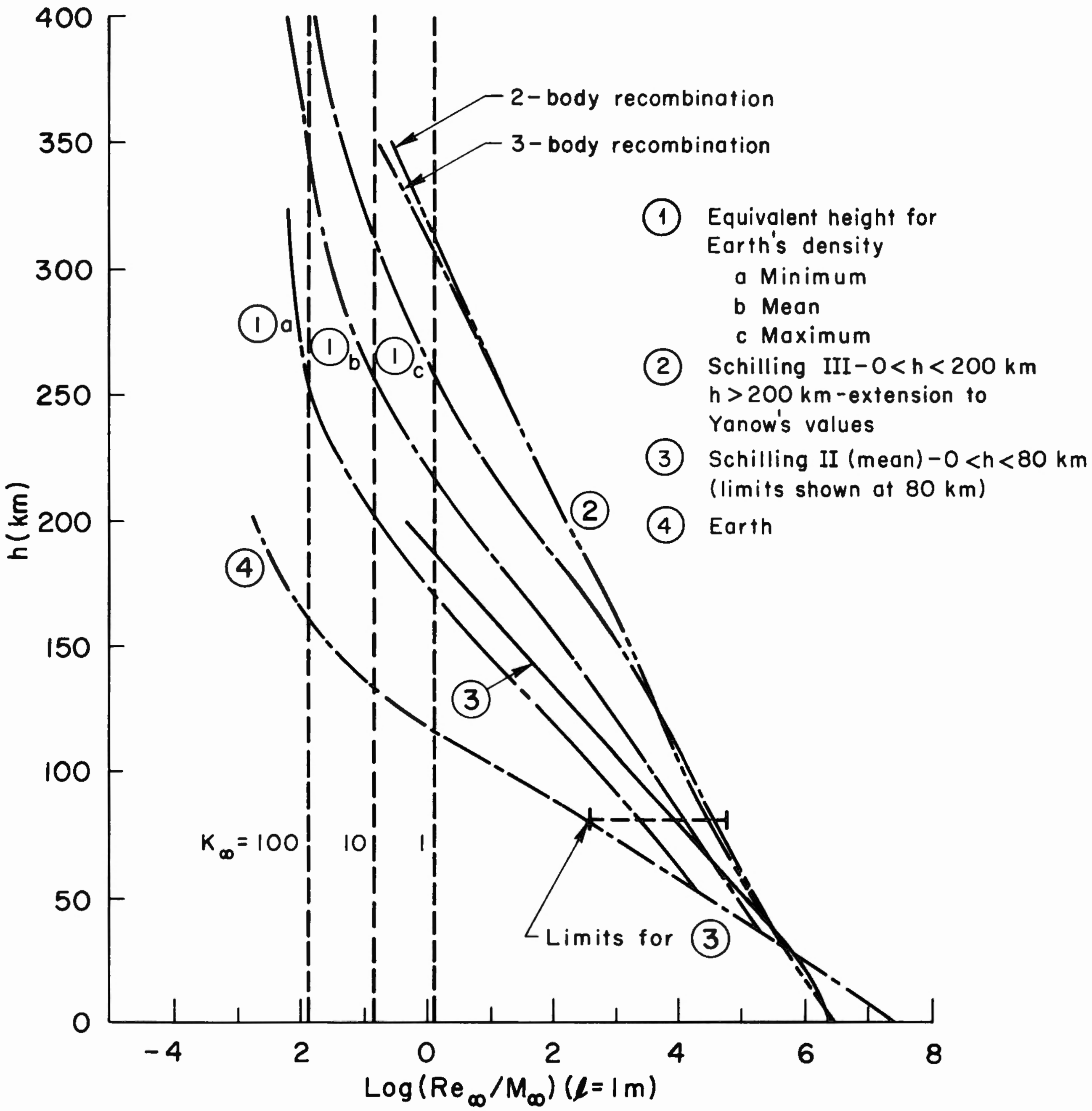

Fig. 11 — Summary of flight conditions in the atmosphere of Mars

There is no clear-cut dividing line between the initial free-molecule-flow field and the later continuum flow. A variety of conditions exists, including the single impact of molecules on the surface of the vehicle, repeated impact, the formation of a shock wave with a single shock layer to the surface, the establishment of an inviscid region between shock wave and a boundary layer on the surface, and changes in character of the boundary layer. The detailed thermodynamic conditions depending on the gas composition (chemical reactions, ionization, radiation, etc.) are related to these flow fields. A sequence of the fluid-dynamic events mentioned has been mapped by Probstein.[40] In general, the smallest mean free path in the flow field in relation to the vehicle geometry governs the treatment of the aerodynamics. It is relatively simple to convert the free-stream Reynolds number or Knudsen number to that in the shock layer, across which the mass flow (see Eq. (2)) remains constant and where viscosity must be assigned to the stagnation temperature. However, for complicated geometries, the situation is far from clear. To simplify the issue, it is reasonable to assume for initial entry that the vehicle surface is highly cooled. If, furthermore, the flight speed is high with respect to the thermal motion of molecules in the free stream, and also if the molecules emitted from the cooled surface are slow, such a condition may be termed hyperthermal.[41]

If we superimpose free-stream Knudsen-number values computed from the values on the graph of flow regimes shown by Probstein (see Fig. 1 of Ref. 40) for hyperthermal flow on Earth we find the following result: According to Eq. (4), for a characteristic length of 1 m, $K_\infty = 100$ may be assigned to free-molecule flow. For $K_\infty = 10$ we expect the first collision regime,[40,41] and finally for $K_\infty = 1$ we see the transitional layer characterized by shock formation. Incorporating these facts with our knowledge of planetary-atmosphere flight regimes, and considering our general uncertainties, we may state that transition to continuum takes place somewhere in the range $100 > K_\infty > 1$. Specifically, conditions of shock formation for blunt-body flows are expected at the relatively low value of about $K_\infty = 1$.

In Fig. 10 we observe that flight above the visible cloud layer on Venus (assuming the N_2-I and N_2-II models) is similar to that on Earth for the free-stream conditions. However, the N_2-III exhibits Reynolds numbers that are much higher. Generally, the carbon dioxide models lead to equal or lower Reynolds numbers at a given height. The continuum regime is entered somewhere between the altitudes of 80 and 160 km above the cloud layer. However, if the Cytherean atmosphere proves to be even deeper than shown,[16] all Reynolds numbers will have been underestimated, and therefore no limiting conditions can rationally be estimated at present.

In Fig. 12, we show the differences expected for free-stream conditions in the nitrogen models of Venus, depending on whether the high or the low surface temperature is the correct one. For the N_2-I and N_2-III models, the expected Reynolds numbers at a given flight speed in the troposphere appreciably exceed all values familiar to us on Earth. Since we can also expect high stagnation temperatures, the design of suitable vehicles will be difficult indeed.

The composite flight picture for Mars is shown in Fig. 11. The major difference from Earth is the high altitude at which high Reynolds numbers must be expected; this is due to the effect of the acceleration of gravity on density. We may have been somewhat more fortunate in defining the range of uncertainty on Mars than in the case of Venus. However, we find it large indeed with the 1-m blunt body entering the continuum flow at an altitude somewhere between 170 and 315 km. Conversely, in the lowest 25 km, the Reynolds numbers will be as low as one order of magnitude (surface) less than those on Earth. (Appendix A suggests practical uses for the given graphs.)

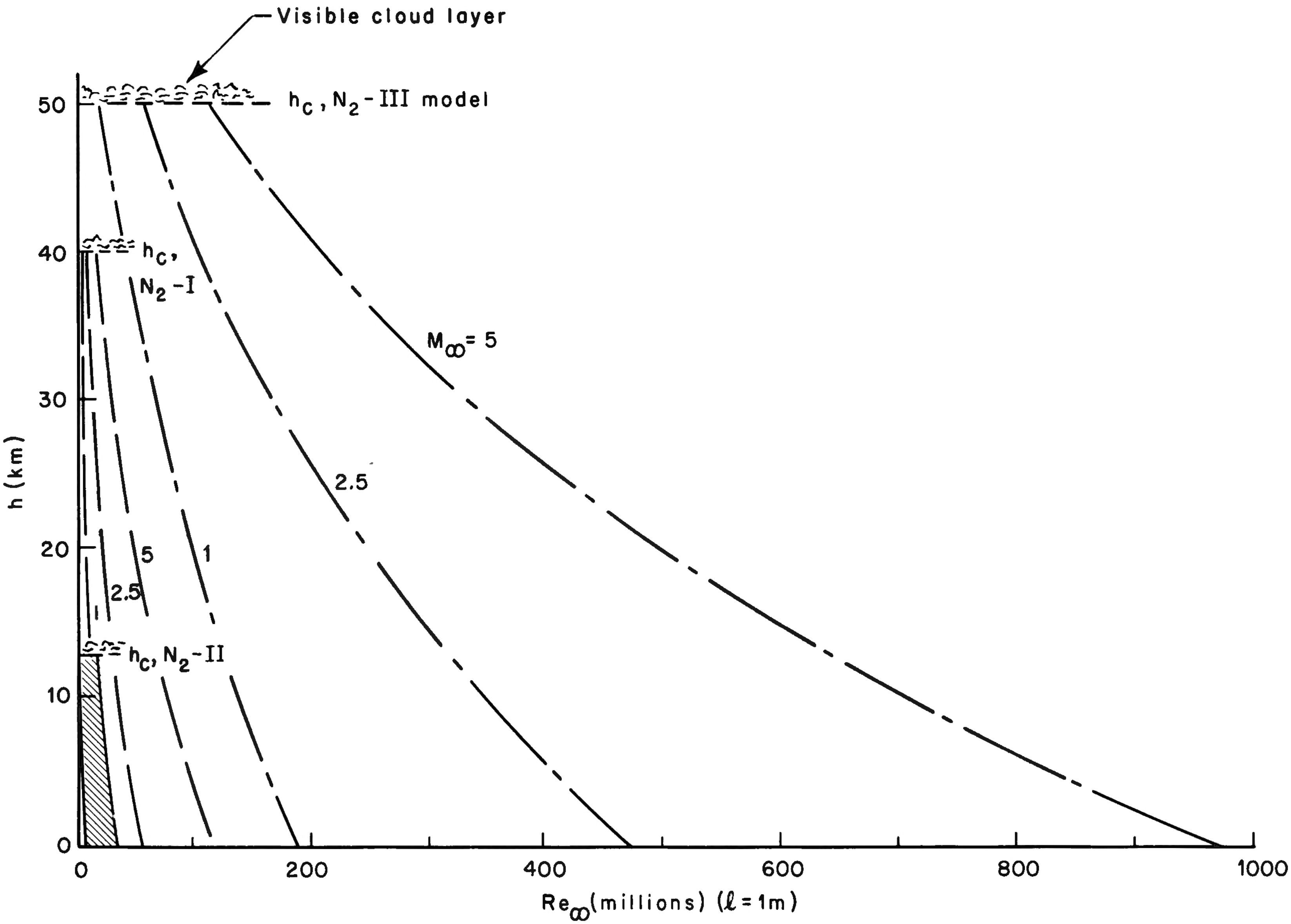

Fig. 12 — Flight conditions in the troposphere of Venus

V. AERODYNAMIC EXAMPLES

Two elementary aerodynamic examples (satellite and entry) were chosen to illuminate the differences in flight conditions on the two planets and between the various atmospheric models.

First, circular-orbit speeds for a satellite were computed for different altitudes from

$$u = \left[g(r + h)\right]^{1/2} = r\left[\frac{g_o}{r + h}\right]^{1/2} \qquad (9)$$

with g_o chosen as the mean surface value. For Venus, $r_o = r_c$, and h indicates height above h_c. On Venus, the average circular-orbit speed is about 7.15 km/sec; this value is only about 3.5 km/sec for Mars. The aerodynamic similarity parameters for satellites with ℓ = 1 m are shown in Figs. 6, 7, 8, and 9. The strong Mach-number variation for some models results from the differences in local temperature. At some altitudes, high drag will appear at the relatively high Reynolds numbers.

Secondly, a simple entry-trajectory condition is indicated in the same figures. It was assumed that a body with a 1-m characteristic dimention enters the atmosphere vertically with a velocity of 6.4 km/sec at 200 km above the cloud layer of Venus or above the surface of Mars. By unspecified means, this body is decelerated uniformly at 0.1 km/sec^2, or approximately 10 Earth g. The total flight time to the surface or cloud layer is about 1 min. The corresponding speeds are

Flight speed (km/sec)	Height above surface or cloud layer (km)
6.4	200
5.5	150
4.5	100
4.1	80
3.2	50
2.5	30
1.5	10
0.53	0

This example of planetary entry shows a speed near the surface (or cloud layer) which results in Mach numbers between about 1 and 2. The extreme aerodynamic differences between the atmospheric models can also be observed in Figs. 6 - 9.

Appendix A

CALCULATION OF ATMOSPHERIC STRUCTURE

Calculations of the structure of stable atmospheres are based on the fundamental equation of hydrostatics

$$dp = - g\rho dh \tag{10}$$

Owing to the nearly spherical shape of the planets, the acceleration of gravity decreases with altitude approximately by

$$g = g_o \left(\frac{r}{r + h} \right)^2 \tag{11}$$

where r is the planet's radius and g_o pertains to the gravity at the planet's surface. In view of the uncertainty as to the properties of the atmospheres of other planets, it is generally permissible to set $g = g_o$ = constant, because $h \ll r$ where aerodynamic interest enters.

Planetary atmospheres may be treated as thermally perfect gas mixtures with

$$\rho = \frac{p\mu}{RT} \tag{12}$$

as long as no dissociation occurs, i.e., as long as the molecular weight, μ, calculated for the gas mixture remains constant. Assuming the density of an atmosphere to be constant with altitude, Eq. (10) may be integrated to give

$$\Delta h = h_2 - h_1 = \frac{1}{g\rho} (p_1 - p_2) \tag{13}$$

Setting $p_2 = 0$, $p_1 = p_o$, and $h_1 = 0$ (subscript o for surface), we find with $H_o = h_2$, Eqs. (12) and (13)

$$H_o = \frac{p_o}{g_o \rho_o} = \frac{RT_o}{\mu g_o} \tag{14}$$

H_o is a distance equal to the height of a homogeneous atmosphere, and it may be found if only molecular weight, surface temperature, and gravitational acceleration are known. The presence of certain gases in a planetary atmosphere can be deduced from spectra; the amount of these gases is usually measured by using Eq. (14) as length at a given standard pressure and temperature, as this length may be related to the height of a uniform atmosphere from which the columnar mass may be derived.

Assuming an atmosphere to be at constant temperature, Eq. (10) may be integrated to give the well-known barometric-height formula

$$\frac{p}{p_o} = \frac{\rho}{\rho_o} = \exp\left[-\frac{g_o\mu}{RT_o}h\right] = \exp\left[-\frac{h}{H_o}\right] \qquad (15)$$

The symbol h represents the height from the surface. In such an isothermal atmosphere, equilibrium has been established by heat conduction. Pressure and density decrease exponentially with altitude. In this context, H_o, given by Eq. (14), is termed the scale height. This is constant with altitude, and it gives immediately an altitude interval in which pressure (or density) drops to 1/e times the initial value.

It is noted that the acceleration of gravity occurs in the exponent of Eq. (15) and, in particular, if we deal with the upper atmosphere or exosphere, errors may be introduced by assuming g = constant. For integration of Eqs. (10) (including the variation of g, with T = constant), (11), and (12) we obtain

$$\frac{dp}{p} = \frac{d\rho}{\rho} = -\frac{1}{H_o}\left(\frac{r}{r+h}\right)^2 dh \qquad (16)$$

Upon integration we find

$$\frac{p}{p_o} = \frac{\rho}{\rho_o} = \exp\left[-\frac{1}{H_o}\left(\frac{rh}{r+h}\right)\right] \qquad (17)$$

The scale height is now a function of altitude, and H_o pertains to the surface.

On the other hand, if we assume that a stable atmosphere is established by convection, it follows that pressure, temperature, and density at two altitude levels (denoted by 1 and 2) are related by Poisson's equation

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma-1} \tag{18}$$

Assuming that the ratio of the specific heats $\gamma = c_p/c_v$ is constant, and again assuming constant molecular weight and gravitational acceleration, Eq. (10) may be integrated with Eqs. (12) and (18) to give

$$\Delta h = h_2 - h_1 = H_1 \frac{\gamma}{\gamma - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{19}$$

Solving for pressure we find

$$\frac{p_2}{p_1} = \left(1 - \frac{\gamma - 1}{\gamma} \frac{\Delta h}{H_1}\right)^{\frac{\gamma}{\gamma-1}} \tag{20}$$

or writing

$$\frac{p_2}{p_1} = \left(1 + \Gamma \frac{\Delta h}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \tag{21}$$

where with Eq. (14) and $g = g_o$

$$\Gamma \equiv \left(\frac{\partial T}{\partial h}\right)_{ad} = -\frac{\gamma - 1}{\gamma} \frac{g\mu}{R} = -\frac{g}{c_p} \left(\frac{\text{temperature}}{\text{length}}\right) \tag{22}$$

is defined as the adiabatic lapse rate. Solving with Eq. (18) for density and temperature respectively, we have

$$\frac{\rho_2}{\rho_1} = \left(1 + \Gamma \frac{\Delta h}{T_1}\right)^{\frac{1}{\gamma - 1}} \tag{23}$$

and

$$\frac{T_2}{T_1} = 1 + \Gamma \frac{\Delta h}{T_1} \tag{24}$$

It is seen from these results that in a convective atmosphere of constant composition and specific heats in a uniform gravitational field, the temperature drops linearly with altitude and that the adiabatic lapse rate, Eq. (22), may be found with a minimum of information about the planet's atmosphere.

The preceding formulas may be rearranged in varying ways to estimate the structure of planetary atmospheres. Also, Eqs. (21), (23), and (24) may be used for any assumed value of the lapse rate, or in turn for any assumed value for the exponents in Eq. (18), and such an atmosphere might be termed polytropic or one of constant lapse rate, β. Finally, it is to be remembered that condensation processes have been excluded in the foregoing discussion and that Γ is therefore often called the dry adiabatic lapse rate.

In view of the fact that atmospheres in convective equilibrium are expected primarily at lower altitudes it is generally not necessary to include variable gravitational acceleration in the integration of Eq. (10) as in the instance of conductive equilibrium.

Appendix B

CALCULATION OF EQUIVALENT HEIGHT FOR EQUAL DENSITY ON TWO PLANETS

We shall assume that the distribution of density with altitude is known for the atmosphere of a given planet, A. If there is a planet B of different size, mass, etc., whose atmospheric composition might be grossly comparable, corresponding density distributions can be estimated by finding equivalent altitudes for equal densities of planets A and B. To each equivalent height for planet B, the known density of planet A may next be assigned. This comparison for the upper atmosphere may be made, for example, by assuming that both planets have isothermal atmospheres.

Taking the acceleration of gravity to be different but constant for both planets, we may write from Eq. (15) for planet A

$$\ell n \rho_A = \ell n \rho_{oA} - \frac{h_A}{H_A} \qquad (25)$$

and for planet B

$$\ell n \rho_B = \ell n \rho_{oB} - \frac{h_B}{H_B} \qquad (26)$$

For $g = g_o$ = constant, the respective scale heights are constant. Setting

$$\ell n \rho_A = \ell n \rho_B \qquad (27)$$

we find from Eqs. (25) and (26)

$$h_B = h_A \frac{H_B}{H_A} - H_B \, \ell n \left(\frac{\rho_{oA}}{\rho_{oB}} \right) \qquad (28)$$

giving that altitude on planet B at which the density is equal to that found at a known altitude on planet A. The two planetary scale heights and the surface densities need not necessarily be known individually. For purposes of estimation, it may often be possible to obtain at least the ratios of these quantities, in fact, we may compute a set of results for a range of these ratios.

At higher altitudes it may again not be permissible to neglect the variation of gravitational attraction. To estimate this effect we treat Eq. (17) in a similar manner, and after some rearrangement we find

$$h_B = \frac{r_B}{\dfrac{r_B}{\dfrac{H_{oB}}{H_{oA}} \dfrac{r_A h_A}{r_a + h_A} - H_{oB}\, \ell n \dfrac{\rho_{oA}}{\rho_{oB}}} - 1} \tag{29}$$

Obviously the planets' radii enter this equation, and the scale-height ratios are those applicable to the surface. From Eq. (28) or Eq. (29) we may now find $\rho_B(h_B)$ from the known function $\rho_A(h_A)$.

Finally, there is an equal altitude for equal densities, h_{AB}, on both planets. This is the altitude at which the individual density functions intersect each other. On Mars, for example, the density above this layer will be higher, and below, it will be lower, than that of Earth. If g is constant, we set $h_A = h_B = h_{AB}$ in Eq. (28) and find

$$h_{AB} = \frac{H_B\, \ell n \dfrac{\rho_{oA}}{\rho_{oB}}}{\dfrac{H_B}{H_A} - 1} \tag{30}$$

for the altitude of equal densities.

Appendix C

SUGGESTIONS FOR AERODYNAMIC USE OF RESULTS

1. Assume that a vacuum trajectory for entry of a planetary atmosphere has been established. Find the velocity u at some high altitude h above the planet's surface.

2. Choose an atmospheric model (or limiting models) for a given planet from the data and graphs given. Remember that the values on the graphs may be read off with higher accuracy than our knowledge of the models justifies. Find the temperature for the altitude of interest. Assume a corresponding composition to obtain the molecular weight and compute the speed of sound (Eq. (3)) and free-stream Mach number (Eq. (1)) at the altitude in question.

3. From the graphs $M_\infty = f(Re_\infty, h)$, or $M_\infty/Re_\infty = f(h)$ it can be seen immediately if the trajectory lies in the free-molecule, transition, or continuum-flow regime at the starting altitude. This statement is true for a characteristic length of $\ell = 1$ m. Remember that the length enters the Reynolds number linearly; therefore, Re_∞ may easily be ascertained if $\ell \neq 1$ m.

4. Low Mach-number flight. With M_∞ and Re_∞ known for the trajectory at some h, and presumably with coefficients of drag, lift, etc., known for the vehicle as function of free-stream similarity parameters, aerodynamic calculations may be performed for flight to the surface.

5. High Mach-number flight. Ascertain the atmospheric composition, or range of possible compositions, for the trajectory at some h. Determine whether real gas effects are to be expected by computing the stagnation enthalpy for the velocity and the composition in question. Using the techniques developed for flight in the Earth's atmosphere, compute heat transfer rate to the stagnation point, etc., taking real gas effects into account.

REFERENCES

1. Wegener, P. P., Estimates of Flight Regimes in the Venus Atmosphere, The RAND Corporation, RM-2946-PR, January 1963.

2. Kuiper, Gerard P. (ed.), The Atmospheres of the Earth and Planets, rev. ed., University of Chicago Press, Chicago, 1952.

3. Urey, Harold C., The Planets, Yale University Press, New Haven, Connecticut, 1952.

4. Urey, Harold C., "The Atmospheres of the Planets," in S. Fluegge (ed.), Encyclopedia of Physics, Vol. 52, Springer-Verlag, Berlin, 1959.

5. Kellogg, W. W., and Carl Sagan, The Atmospheres of Mars and Venus, Publication 944, National Academy of Sciences--National Research Council, Washington, D. C., 1961.

6. Sagan, Carl, "The Planet Venus," Sci., Vol. 133, No. 3456, March 24, 1961, pp. 849-858.

7. Moore, P., and S. W. Greenwood, "Venus as an Astronautical Objective," in F. I. Ordway, III, (ed.), Advances in Space Science and Technology, Vol. 3, Academic Press, New York, 1961.

8. De Vaucouleurs, G., "Remarks on Mars and Venus," in R. Jastrow (ed.), The Exploration of Space, The Macmillan Company, New York, 1960.

9. Hess, S. L., "Mars as an Astronautical Objective," in F. I. Ordway, III, (ed.), Advances in Space Science and Technology, Vol. 3, Academic Press, New York, 1961.

10. De Vaucouleurs, G., Physics of the Planet Mars, The Macmillan Company, New York, 1954.

11. Zimmerman, R. H., and C. C. Jones, Flight Environment Design Parameters for Mars and Venus, Ohio State University Research Foundation, ASD-TDR-62-805, September 1962.

12. "Summary of Communications," Eleventh International Colloquium of Astrophysics, Physics of the Planets, Prof. M. Minnaert, Chairman, organized by the Institute of Astrophysics, University of Liege, Cointe-Sclessin, Belgium, July 9-11, 1962.

13. Ross, F. E., Astrophys. J., Vol. 68, 1928, p. 57.

14. Lyot, B., "Récherches sur la Polarisation de la Lumière des Planètes et de Quelques Substances," Ann. Obs. Paris (Meudon), Vol. 8, 1929; or Ref. 4, Fig. 1.

15. Kaplan, L. D., "A New Interpretation of the Structure and CO_2 Content of the Venus Atmosphere," Planet. Space Sci., Vol. 8, No. 1, October 1961.

16. Spinrad, H., "Spectroscopic Temperature and Pressure Measurements in the Venus Atmosphere," Publ. Astron. Soc. Pacific, Vol. 74, No. 438, June 1962, pp. 187-201.

17. Pettit, E., and S. B. Nicholson, Publ. Astron. Soc. Pacific, Vol. 83, 1936, p. 84.

18. Sinton, W. M., and J. Strong, "Radiometric Observations of Venus," Astrophys. J., Vol. 131, No. 2, March 1960, pp. 470-490.

19. Chamberlain, J. W., and Gerard P. Kuiper, "Rotational Temperature and Phase Variation of the Carbon Dioxide Bands of Venus," Astrophys. J., Vol. 124, No. 2, September 1956, pp. 399-405.

20. Wildt, R., Astrophys. J., Vol. 91, 1940, p. 266.

21. Mayer, C. H., T. P. McCullough, and R. M. Sloanaker, "Observations of Venus at 3.15-cm Wave Length," Astrophys. J., Vol. 127, No. 1, January 1958, pp. 1-10.

22. Wyckoff, R. C., "Venus Mission--1962," Astronautics, Vol. 7, No. 7, July 1962, pp. 54-59.

23. Sagan, C., The Radiation Balance of Venus, Jet Propulsion Laboratory, California Institute of Technology, Technical Report 32-34, September 15, 1960.

24. Jones, D. E., "The Microwave Temperature of Venus," Planet. Space Sci., Vol. 5, No. 1, January 1961, pp. 166-167.

25. Öpik, E. J., "The Aerolosphere and Atmosphere of Venus," J. Geophys. Res., Vol. 66, No. 9, September 1961, pp. 2807-2819.

26. Mintz, Y., "Temperature and Circulation of the Venus Atmosphere," Planet. Space Sci., Vol. 5, No. 1, January 1961, pp. 141-152.

27. Dole, S. H., The Atmosphere of Venus, The RAND Corporation, P-978, October 12, 1956.

28. Hilsenrath, J., et al., Tables of Thermal Properties of Gases, National Bureau of Standards Circular 564, U.S. Government Printing Office, Washington 25, D. C., November 1955.

29. Hess, S. L., "Some Meteorology of Mars," Sky and Telescope, Vol. 9, No. 7, May 1950, pp. 155-157.

30. Goody, R. M., Weather, Vol. 12, 1957, p. 3.

31. Schilling, G. F., Extreme Model Atmospheres of Mars, The RAND Corporation, RM-2782-JPL, June 1961.

32. Dollfuss, A., "Etude des Planètes par la Polarisation de Leur Lumière," Ann. d'Astrophys., Supplement 4, 1957.

33. Minzner, R. A., V. S. W. Chamption, and H. L. Pond, The ARDC Model Atmosphere, 1959, AFCRC TR-59-267, Geophysics Research Directorate, AFCRL, Bedford, Massachusetts.

34. Mikhnevich, V. V., et al., Some Results of the Determination of the Structural Parameters of the Atmosphere Using the Third Soviet Artificial Earth Satellite, National Aeronautics and Space Administration, TT F-13, May 1960.

35. Paetzold, H. K., Die Naturwissenschaften, Vol. 48, 1961, p. 39.

36. Yanow, G., J. Astron. Sci., Vol. 8, No. 4, Winter 1961, p. 103.

37. Barth, C. A., "Atomic Reactions in the Upper Atmospheres of the Earth and Mars (Abstract)," J. Geophys. Res., Vol. 66, No. 5, May 1961, p. 1551.

38. Chapman, D. R., An Analysis of the Corridor and Guidance Requirements for Supercircular Entry into Planetary Atmospheres, National Aeronautics and Space Administration, Technical Report R-55, Washington, D. C., 1959.

39. Peterson, V. L., Motions of a Short 10° Blunted Cone Entering a Martian Atmosphere at Arbitrary Angles of Attack and Arbitrary Pitching Rates, National Aeronautics and Space Administration, TN D-1326, Washington, D. C., May 1962.

40. Probstein, R. F., "Shock Wave and Flow Field Development in Hypersonic Re-Entry," J. Amer. Rocket Soc., Vol. 31, No. 2, February 1961, pp. 185-194.

41. Schamberg, R., "Analytic Representation of Surface Interaction for Free Molecule Flow with Application to Drag of Various Bodies," in D. J. Masson (comp.), Aerodynamics of the Upper Atmosphere, The RAND Corporation, R-339, June 8-10, 1959.