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Technical Report

Portfolio Optimization by Means of Multiple Tandem Certainty-Uncertainty Searches

A Technical Description

Brian G. Chow

RAND Corporation

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1200 South Hayes Street, Arlington, VA 22202-5050
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Summary

American mathematician George Dantzig developed the simplex algorithm to solve linear programming problems, but he also pioneered solving such problems under uncertainty in 1955. To date, linear and nonlinear programming problems under uncertainty have been extensively studied. Those approaches that have found applications in businesses, whether in the public or private sector, have had to impose severe limitations on the numbers of decision variables, uncertain parameters, and uncertain scenarios that can be used. Otherwise, the combinatorial possible solutions would grow exponentially and prohibit even today's most powerful computers (or those in the foreseeable future) from exhausting all the possibilities in finding the optimal solution.

This paper introduces a new approach to allow these limitations to be greatly relaxed and describes a number of search algorithms or schemes that have been shown to have practical applications. This approach and its associated search algorithms have a key feature—they generate typically 10,000 uncertain scenarios or future states of the world according to their uncertainty distribution functions. While each of these scenarios is a point in the larger uncertainty space, the originally uncertain parameters are specified for the scenario and are, thereby, "determined" or "certain." Thus, the solvable mixed-integer linear programming model can be used "under certainty" (i.e., deterministically) to find the optimal solution for that scenario. Doing this for numerous scenarios provides a great deal of knowledge and facilitates the search for the optimal solution—or one close to it—for the larger problem under uncertainty. This approach allows one to decompose the problem under uncertainty into 10,000 solvable problems so that one can learn about the role each project plays in these 10,000 samples of the uncertainty space. Doing so allows one to avoid the impossible task of performing millions or trillions of searches to find the optimal solution for each scenario, yet enables one to gain just as much knowledge as if one were doing so.

The approach is to use transparent reasoning, as opposed to mathematical formulas, to design search schemes or algorithms to find the global optimum and not get trapped at one of the local optima. This approach relies on arguments from devil's advocates to uncover the shortcomings of an algorithm in terms of why under certain situations it will not lead to the global optimum. Once the weaknesses of a given algorithm are identified, hopefully the original algorithm can be modified to remove the shortcomings, or another algorithm can be designed to plug the reasoning hole of the original algorithm.

Experience with this approach has been good. However, if the shortcomings in these algorithms cannot be eliminated, this approach would have to rely on the simplicity of nonmathematical reasoning so that many analysts or even “crowd wisdom” can be used to devise completely different algorithms to do the job. Because all approaches, including this one, face the risk of potentially missing the global optimum, this approach based on reasoning can open a new way for drawing in talents from the nonmathematical world to devise search schemes to tackle this very difficult task of optimization under uncertainty.

These reasonable search algorithms are easy to understand. Implementing them amounts to creating a flow chart and does not require the use of complicated mathematics or formulas; as a result, the approach allows for wider adoption by analysts and organizations that possess different skill sets.

As described in two illustrative search schemes (SS-8 without replacement and SS-8 with replacement) discussed in the paper, the approach draws on the common-sense and commonly practiced ideas used in business decisions and daily activities.

The SS-8 without replacement search scheme is based on the idea of how to create a project team. Suppose a project sponsor has some “use-it-or-lose-it” money at the end of a fiscal year. While a project must be issued now within a broad study area, the sponsor will assign specific tasks over the course of the one-year project, but which tasks those will be is unclear. The company’s policy and the sponsor’s requirement, however, are such that the project leader must specify the team members at the project’s start, after which it will not be possible to change them. Under such circumstances, it makes sense to draw up a list of tasks that the sponsor *may* ask the project team to do and to start by then selecting the person (by analogy, the first project selected) suitable for the largest number of potential task combinations that can be anticipated. The next step would be to find the person (by analogy, the second project selected) to best complement the first in technical and managerial skills so that the pair is suitable for the largest number of possible task combinations. Similarly, the third person (by analogy, the third project selected) would be found to best complement the first two, and so on.

The second search scheme (SS-8 with replacement)[1] accounts for the possibility that if a different person were selected as the first person, the skill sets and personal

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[1] The terms *without replacement* and *with replacement* refer to whether search is conducted with a unique choice of the first team member (the first version) or whether the search is conducted with each of the possible people as the first team member.

chemistry to complement this different person might lead to a team composition different from the one based on SS-8 without replacement.

Mathematically, these approaches are very convenient. Let there be N persons to choose from in forming the project team. Instead of looking at 2^N—or easily millions or trillions of possible team compositions—the two search schemes together would generate only N possible project teams (by analogy, any project could be the first, but thereafter the choices would be determined by looking at results for the uncertain scenarios used), thus enormously reducing the complexity of the search.

This type of reasoning in designing and using complementary algorithms can give analysts a way—which may either be more transparent than mathematics or at the least a supplement to it—to uncover and mitigate logical lapses. Also, analysts using this approach may feel more confident that, even if these algorithms do not find the global optimum, the local optima they find should be near the global one, because the logic of these algorithms are used often and work well in the analysts' other daily activities.

Applications of this approach were developed in a series of studies called *Toward Affordable Systems* (*Toward Affordable Systems II* and *Toward Affordable Systems III)* that were sponsored by the Deputy Assistant Secretary of the Army (Cost and Economic Analysis), Office of Assistant Secretary of the Army (Financial Management and Comptroller). This paper discusses two of these applications. Applications of this approach that have appeared in *Toward Affordable Systems II* involved 75 projects or decision variables and 75 independent uncertain parameters. Each parameter corresponds to a project that has a 90 percent chance of successful completion and a 10 percent chance of failure.

The applications that appeared in *Toward Affordable Systems III* involve 26 or 183 projects; 26 uncertain costs in procuring a system derived from each project; and one more uncertain parameter corresponding to the budget for acquiring, operating, and maintaining the needed systems.

The applications with 26 or 183 projects are compared to two typical approaches: benefit/cost ratios and mixed-integer linear programming. The comparison shows that this new approach will save money for any given confidence level for meeting requirements or will yield higher confidence for equal cost.

Each of the algorithms developed in this paper takes minutes or hours to find the optimal solution, even for uncertainty problems involving substantially more decision variables (75 used here versus typically ten in other methods), uncertain parameters (75 used here versus typically a few in other methods), and uncertain scenarios (10,000 used here versus a few in other methods—or, alternatively, more than 10,000 used but then restricting variables and/or parameters to around ten) than other methods would have allowed. Thus, the relatively shorter run time offers the possibility of performing several complementary algorithms for the same problem, enhancing the chance of finding the global optimum.

The objective function is chosen to be the highest chance of meeting the requirements within a budget. This can be a way to introduce the idea of a confidence level in dealing with uncertainty. Then again, if analysts prefer using more conventional objective functions, such as minimizing expected cost or regret, this method can be modified to use such objective functions with other changes in the formulation and search algorithms, which may be akin to something as simple as moving from a simple average to weighted sums.

This approach is also suitable for parallel computing, because the 10,000 runs for each data point, the runs of different data points, and the runs for different algorithms can all be performed independently and simultaneously. The current advances in parallel computing and the rapid decline in cost of on-demand computer power favor this multiple search approach.

This paper proposes a common platform so that solutions derived from different approaches and search algorithms can be objectively compared to determine which gives the best solution. This implies that the platform is supported by a library of test problems with known solutions so that different algorithms can be tested and compared. As the platform and its database accumulate more and more comparisons, there will be better confidence about which algorithm works the best for which types of problems.

This paper also proposes to extend the applications of the approach and associated algorithms in several dimensions:

- Apply it to different types of problems beyond the current focus on project portfolio problems under uncertainty. One may start the expansion with other resource allocation problems, such as production planning.
- Use other objective functions for the uncertainty problem, such as the minimization of expected total cost or regret.
- Program the multiple search algorithms for parallel computing to shorten the run time.

Finally, this paper proposes a systematic examination of approaches and their search algorithms, with the goal of combining their individual strengths and mitigating their weaknesses to give users ways to better perform optimization under uncertainty.

Because uncertainties are inherent in input data and the future, better ways to factor uncertainties into consideration are critically important for any type of decisionmaking.