RESEARCH SUMMARY

Graph Theoretic Algorithms for the Ground Based Strategic Deterrent Program

Prioritization and Scheduling

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ISSUE

As programs grow larger and the interdependencies of the program activities get complex, tools are needed to introduce more rigor into program management to reduce schedule risk. In this report, we present the underlying rationale behind algorithms already delivered to the Ground Based Strategic Deterrent (GBSD) program office that were designed to reduce the likelihood of rework in program execution, provide better insight into schedule risk, and provide insights into how to restructure task dependencies to manage schedule risk.

The algorithms help project managers consider "what if" scenarios and visualize potential pitfalls as a project progresses. Project managers can also use these algorithms to perform sensitivity analyses on individual activities to understand which activities have the greatest potential to derail a project. Although these algorithms were developed for the GBSD program office, specifically for the Toolbox for the Unified Certification Strategy Dashboard, the methods are general and applicable to any program.

APPROACH

This report builds on the project management methods of Program Evaluation Review Technique (PERT) and Critical Path Method in two novel ways. The first is to show how the critical path for execution depends on the overall structure of how the tasks are scheduled. The mathematical representation is called a *graph*, and the structure is called the *graph topology*. Monte Carlo simulations have been used for decades to find the critical path for schedule completion. Unfortunately, simulation-based methods become impractical for large graphs because of computation times. The second novelty is to present semi-analytical methods that are orders of magnitude faster than Monte Carlo simulations and can be used efficiently on very large graphs. We also explore how the reallocation of resources can change the risk of meeting expected schedules.

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FINDINGS AND CONCLUSIONS

Novel Topological Insights

The most probable critical path and degree of slack for project completion depend on graph topology. When managers have the opportunity to change the structure of activities, they can better manage where schedule risk lies in program execution. We examined three classes of graph topology: a simple topology defined by the number of parallel chains in the graph, random graphs that display Erdős-Rényi topology, and scale-free graphs.

The simple chain topology is elementary but contains many structural elements common to many PERT graphs. It is intuitive and contains the basic structural forms that a manager can readily adjust. The random and scale-free graph cases are common topologies in naturally occurring graphs. Many large graphs present one of these two topologies; therefore, they are useful baselines for analysis. We find the following:

- For the simple chain topology, although parallelizing tasks can theoretically reduce the overall time, it does so less than might be expected, because the likelihood of having at least one bad chain increases with the number of chains. There is a trade-off: Parallelizing development reduces the variance more than expected but increases the end-to-end time more than expected. Thus, although working tasks in parallel remains desirable, the project manager's expectations for time savings should be tempered.
- For Erdős-Rényi topology (random graphs), our analysis shows the same trade-off between variance and average completion time as seen in the simple chain topology, though the effect is more subtle. In this case, the presence of interlinking paths makes it less likely that a single chain will dominate, because the chains are not independent.
- For the scale-free graph topology, our analysis shows that graphs with a single, dominant longest path are much more likely to complete on time than graphs with several parallel paths that may each be the critical path. This result means that, for scale-free graphs, the project manager can reduce the overall completion time with only a marginal increase in the corresponding risk by placing important tasks in series with one another.

Novel Numerical Methods

Although Monte Carlo techniques are useful for finding the critical path in a small PERT graph, as graph size increases, these methods are too computationally slow to be of practical use. To overcome this limitation, we present two numerical approaches that are many orders of magnitude faster:

- The fastest—and preferred—method transforms the problem into a new space with Chebyshev polynomials as a basis. The relevant calculations are performed in that space; then, the results are inverted back to the original space.
- The other method uses trapezoidal integration but is preferred only when software packages limit the ability to manipulate Chebyshev polynomials.

Resource Constraints and Future Analysis

We found three main resource constraints and opportunities for future analysis:

- We describe how slack in a task network can be manipulated to reduce project completion time; to reduce the overall need for resources; or to respond to other needs, such as start-up constraints, changes in workforce availability or capacity, or cash flows.
- Current methods rely on either expected-value algorithms, which may not sufficiently capture uncertainty, or Monte Carlo simulations, which require excessive amounts of runtime for large, complex projects.

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- Future work in this area could extend the novel numerical methods described in this report to closely approximate solutions that could prove invaluable to the program manager.

The algorithms that have been delivered to the GBSD program office and this explanation of those approaches should aid in the management of complex programs, especially those following digital engineering practices.

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