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Defining and Evaluating Reliable Options for Overseas Combat Support Basing

Thomas Lang

This document was submitted as a dissertation in August 2009 in partial fulfillment of the requirements of the doctoral degree in public policy analysis at the Pardee RAND Graduate School. The faculty committee that supervised and approved the dissertation consisted of Ronald McGarvey (Chair), Mahyar Amouzegar, Don Snyder, and Susan Marquis.
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Abstract

To meet the Air Force’s goals of global strike and persistent dominance, it is vital that the support for the warfighter be efficient in all aspects of deployment, employment, and redeployment. In order for rapid deployments to succeed, the Air Force must determine where combat support assets should be forward positioned. Previously, much of the focus has been on allocating resources to different regions of the world; now the focus is on finding a more efficient and effective global allocation that is not regionally constrained.

The objective of this dissertation is to identify a robust set of facility locations for the Air Force to place combat support basing materiel that will cover a broad range of potential missions (e.g., training, humanitarian, and major combat operations) that may occur around the world. We model these decisions using mixed integer programming models. Because the Air Force faces risks associated with the loss of access to such storage sites, this dissertation addresses the ability of the network to perform well even when parts of it fail, a concept we refer to as reliability. We will use these models to identify the additional costs necessary to build varying levels of reliability into the solutions. These solutions will take into account risk and uncertainties, while meeting time constraints associated with the delivery of materiel.
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Abbreviations

AB       Air Base
ACL      allowable cabin load
AFB      Air Force Base
APF      afloat preposition fleet
APS      afloat preposition ship
APT      airport
ARM      alternative reliability model
BEAR     Basic Expeditionary Airfield Resources
CDC      Centers for Disease Control and Prevention
CONUS    Continental United States
DHHS     Department of Health and Human Services
DoD      Department of Defense
DSNS     Division of Strategic National Stockpile
FOC      full operational capability
FOL      forward operating location
FSL      forward support location
FYDP     Future Years Defense Program
GAMS     General Algebraic Modeling System
IAP      International Airport
IOC      initial operating capability
JFAST    Joint Flow and Analysis System for Transportation
JHSV     joint high speed vessel
JIT      just-in-time
MCO      major combat operation
MIP      mixed integer program
MNRFRM   multiple node failure reliability model
MOG      maximum on ground
MOOTW    Military Operations Other Than Warfare
MPMS     Multi-Period-Multi-Scenario
MPT      military airport
MSC      Military Sealift Command
NATO     North Atlantic Treaty Organization
NEO      Noncombatant Evacuation Operation
NRM      non-reliable model
O&M      operations and maintenance
OAF      Operation Allied Force
OEF      Operation Enduring Freedom
OIF      Operation Iraqi Freedom
POM      Programmed Objectives Memorandum
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<td>PPBE</td>
<td>planning, programming, budgeting, and execution</td>
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<td>RM</td>
<td>reliability model</td>
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<td>RMIP</td>
<td>relaxed mixed integer program</td>
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<td>RO</td>
<td>robust optimization</td>
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<td>RO/RO</td>
<td>roll-on/roll-off</td>
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<td>RSS</td>
<td>receiving, storing, and staging facility</td>
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<td>SECDEF</td>
<td>Secretary Of Defense</td>
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<td>SLP</td>
<td>stochastic linear programming</td>
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<td>SNS</td>
<td>Strategic National Stockpile</td>
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<td>SOF</td>
<td>Special Operations Forces</td>
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<td>START</td>
<td>Strategic Tool for the Analysis of Required Transportation</td>
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<td>SWA</td>
<td>Southwest Asia</td>
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<td>TBM</td>
<td>theatre ballistic missile</td>
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<td>United States Air Force</td>
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<td>USTRANSCOM</td>
<td>United States Transportation Command</td>
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<td>war reserve materiel</td>
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Chapter 1: Introduction

Objective

The objective of this dissertation is to identify a robust set of facility locations for the Air Force to place combat support basing materiel that will cover a broad range of potential missions (e.g., training, humanitarian, and major combat operation) that may occur around the world using mixed integer programming models. Because the Air Force faces risks associated with the loss of access to such storage sites, this dissertation addresses the ability of the network to perform well even when parts of it fail, a concept we refer to as reliability. We will use these models to identify additional costs necessary to build varying levels of reliability into the solutions. These solutions will take into account risk and uncertainties, while meeting time constraints associated with the delivery of the materiel.

The overall policy question is: How should the United States Air Force structure and locate war reserve materiel (WRM) in order to cover a broad set of potential missions around the globe?

Four questions will be addressed with this research:

1. How do policy makers determine the network demand requirements that a WRM posture needs to support?

2. What is a suitable method to model and measure the reliability of a supply network?

3. How much materiel is necessary to support a desired risk level?

4. What are the associated costs of varying levels of reliability?

This methodology can be used by policymakers who need to select resource locations in the presence of uncertainty. Specifically, this dissertation should be of interest to logisticians, operators, and mobility planners throughout the Department of Defense (DoD), especially those in the Air Force. It should also be of interest to Homeland Security officials and industries who may be concerned about prepositioning of emergency response materiel, etc.
Background

For more than 50 years, U.S. deterrent strategy was based on assured destruction, informing potential adversaries that it had overwhelming nuclear capabilities and that it could assure the destruction of state actors should they launch a first strike against the United States. The intent of the strategy was to assured deterrence by making the thought of a first strike inconceivable (Hitch, 1960). This nuclear deterrent strategy was accompanied by the creation of a large standing conventional force that could be employed to win conventional wars with the Soviet Union and North Korea (even if supported by the People’s Republic of China) (Naval Studies Board, 1997). Other contingencies were deemed to be a lower intensity version of the major theater war scenarios. This strategy resulted in the development of large “standing capabilities” that could be augmented quickly by reserve components.

Over the last 15 to 20 years, the focus has shifted to the post-Cold War paradigm of building capabilities in order to avoid a nuclear war by preparing for nonrecurring major regional conflicts. The threat facing U.S. interests is now different, and so are the necessary deterrent capabilities. As it did in the past, nuclear deterrence continues to be vital against possible state actors, but a different conventional deterrent strategy is essential for the foreseeable future. While still preparing to engage and prevail in major theater wars, the U.S. is shifting its attention to a continuous and rapid projection of forces in ongoing and successive deployments, engagements, and reconstitutions in order to deter aggression and coercion from state and non-state actors throughout the world. This concept has the dual objectives of promoting stability and demonstrating that the U.S. can project power and destroy or diminish the capability of terrorist groups or state actors should they threaten U.S. or allied interests in the region. The rapid and recurring global force projection capability is needed to deter aggression, and if that fails, to take quick action to defeat state and non-state actors (U.S. DoD, 2006).

The United States has established and maintained a large number of overseas military bases, presently numbering more than 700 locations around the globe.¹ This massive presence has enabled the U.S. military to operate in every part of the world and respond to crises quickly. It is important to note that the end of the Cold War did not reduce the burden on U.S. forces. In fact, in the last decade of the twentieth century the U.S. carried a significant portion of the

¹ “The forces of the United States military are located in nearly 130 countries around the world performing a variety of duties from combat operations, to peacekeeping, to training foreign militaries.” (globalsecurity.org, 2005) For more information see Eyal (2003), U.S. Congressional Budget Study (2004) and DoD (2004).
security and peacekeeping responsibilities around the globe. The Air Force has been called upon to make numerous overseas deployments, many on short notice--using downsized Cold War legacy force and support structures--to meet a wide range of mission requirements associated with peacekeeping and humanitarian relief, while maintaining the capability to engage in major combat operations such as those associated with operations over Iraq, Serbia, and Afghanistan. A recurring challenge facing the post- Cold War Air Force has been its increasing frequency of deployments to increasingly austere locations. Figure 1.1 illustrates the geographic diversity of some of these deployments.

Based on the unpredictability of the nature and location of recent conflicts, it is growing more apparent that U.S. defense policymakers can no longer just plan for one particular deployment in a specific region of the world, as the geopolitical divide of the last century has been replaced with a security environment that is more volatile. In the conflict in Serbia, the U.S. and coalition Air Forces played a major role in driving the Serbian forces from Kosovo. The common thought of the day was that all future conflicts would be air dominated. The events of September 11, 2001 and the consequent U.S. reprisal against the Al-Qaeda in Afghanistan, Operation Enduring Freedom (OEF), reemphasized the importance of asymmetric warfare and the fundamental role of Special Forces. These events, however, have not lessened the need for a powerful and agile aerospace force; in Afghanistan, the United States Air Force (USAF) flew long-range bombers to provide close air support to the Special Operations Forces (SOF) working with the indigenous resistance ground force, far from existing U.S. bases. In Operation Iraqi Freedom (OIF), the USAF played a substantial role throughout the conflict, from its initial role to suppress and disable the Iraqi command and control and air defense system, to providing close air support in urban environments (Tripp et al., 2004; Lynch et al., 2005). During both of these operations, the USAF flew continual intelligence, surveillance, and reconnaissance missions.

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2 For example, in fiscal year 1999, USAF operations included 38,000 sorties associated with Allied Force, 19,000 sorties to enforce the no-fly zones in Iraq, and about 70,000 mobility missions to over 140 countries (see Sweetman, 2000). As of August 2003, of the Army’s 33 combat brigades, 16 are operating in Iraq and only about 7 percent of the approximately 160,000 coalition soldiers in Iraq are non-American.
Although the past conflicts and engagements may not be repeated in the same manner in the future, we can leverage our understanding of those events to help shape our planning for the future. Moreover, focus can be placed on the characteristics of the past events to create a broad set of alternative potential future environments.

**Air Force Combat Support Network**

To begin with a simple overview, the Air Force performs a large number of missions throughout the world each year. To support these missions, the Air Force prepositions supplies at various locations around the globe. Decisions need to be made on where to locate the supplies and how to allocate supplies across sites. Optimization models can be developed to assist policy makers with these decisions.

In order to support the new paradigm, it is essential that the Air Force has a reliable combat support network. In its simplest form, the combat support network is a series of demand nodes (locations from which forward-deployed forces operate), supply nodes (locations from which support resources are located and sent to the demand nodes), and the network routes
connecting the two sets of locations. This section will briefly explain four of the elements of the combat support network that we more frequently refer to throughout this dissertation.

1. **War Reserve Materiel (WRM)** is the equipment and supplies needed to support forward-deployed units. The materiel is prepositioned in order to reduce reaction time and to sustain forces. Air Force war reserve materiel is comprised primarily of ammunition, Bare Base assets, medical equipment, vehicles, and aircraft-related support equipment. We will be concentrating on Basic Expeditionary Airfield Resources (BEAR), munitions, and rolling stocks (e.g. trucks) because they comprise the bulk of the items in the WRM package. BEAR items consist of housekeeping and industrial operations required for an austere or semi-austere airfield to reach operational capability. Figures 1.2 and 1.3 show examples of some of the different types of WRM.

2. **Forward Operating Locations (FOLs)** are the demand nodes in the network. These are locations forward-deployed, out of which tactical forces operate. FOLs can have differing levels of demands for combat support resources to support a variety of employment timelines. These FOLs might be augmented by other, more austere FOLs that would take longer to spin up. In parts of the world where conflict is less likely or humanitarian missions are the norm, all FOLs might be austere.

3. **Forward Support Locations (FSLs)** are the supply nodes in the network. These are sites near or within the theater of operation for storage of heavy combat support resources, such as munitions or war reserve materiel. The sites are also used for consolidated maintenance and other support activities. The configuration and specific functions of FSLs depend on their geographic location, the threat level, steady state and potential wartime requirements, and the costs and benefits associated with using these facilities. Some FOLs might be collocated with FSLs if extremely rapid response is necessary.

4. **A Transportation Network** connects the FOLs and FSLs with each other, including locations providing en route tanker support. This is an essential part of the combat support network in which FSLs need assured transportation links to support expeditionary forces. FSLs themselves might be sites with transportation infrastructure if

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3 By “austere” we’re referring to a lack of permanent infrastructure that drives a higher requirement for combat support.
4 FSL is the terminology that we are using in this dissertation. The Air Force uses the term contingency support location to refer to the same type of location.
WRM is not collocated with FOLs. Links also exist between the FSLs and Continental United States Locations (CONUS).

Figure 1.2
Vehicles and Shelters

Figure 1.3
Materiel-Handling Equipment, Specialized Equipment, and Munitions
Network Risk

All types of networks run the risk of disruptions when parts of the system fail. These failures may occur along the arc (e.g., an obstacle along a transportation route), or at a node itself. The duration of the disruption can vary between temporary and more long-term. The magnitude of the disruption also can vary between a temporary affect on a small portion of the network, and causing the entire network to fail.

First, we take a brief look at arc failures. The media is filled with reports of transportation network failures occurring when convoy movements are attacked in Iraq and Afghanistan. For example, in June 2009, a convoy of 40 trucks carrying supplies for North Atlantic Treaty Organization (NATO) troops in Afghanistan was attacked by a suicide bomber in Southwestern Pakistan, killing four people and injuring ten others (BBC News, 2009).

An example of an arc failure causing an entire network to fail was the Northeast Blackout of 2003. On August 14, 2003, a widespread power outage occurred affecting the Northeast and Midwest United States and Ontario, Canada. All in all, the blackout affected 45 million people in the United States and 10 million people in Canada. It was later determined that the cause of the massive power failure were power lines that came in contact with trees in Northeastern Ohio causing a cascading effect that shutdown generating units at 265 power plants (CBC News Online, 2003).

In this dissertation, we will focus on node failures; that is, failures occurring at the FSL. The following section will look at several real world examples of these occurring.

Node Failure

One of the difficult problems when selecting FSL locations is the risk and uncertainty associated with node failure. In the USAF context, node failure might take the form of lack of access to a base or limited capabilities at a base. Node failure can occur for many reasons, such as denial of access by host countries; this type of failure occurred in 2003, when the United States was denied access to use bases in Turkey for staging operations in Iraq (Pan, 2003). Access to Turkey would have allowed the U.S. to more easily open a northern front in Operation Iraqi Freedom. For another, more recent example, in February 2009, Kyrgyzstan announced that

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Arc refers to the line connecting two nodes in the network.
it would be closing the U.S. air base at Manas (Priks, 2009). Manas Air Base is the only U.S. base in Central Asia, and is used to transport military personnel and cargo to Afghanistan and refuel aircraft.

It is not feasible, either financially or politically, for the U.S. to obtain a guarantee for potential base access from every nation in regions where problems may occur. Given the difficulty in knowing where all potential conflicts will be located in the future, policymakers can benefit from understanding where possible political access problems may occur. It is up to the analyst to provide policymakers with tools for minimizing the disruptive effects of such problems.

A second potential cause of node failure is an attack by an adversary; overseas bases are vulnerable to theater ballistic missiles (TBMs), long-range fixed wing aircraft, special operations forces, and also non-state actors. An example of this type of failure is the attack of the USS Cole while at port in Yemen on October 12, 2000 (Ricks, 2000). A more recent example was the rocket attack on Bagram Air Base, Afghanistan on June 21, 2009 (Dallasnews.com, 2009). Of these threats, TBMs may be the easiest and least expensive for enemies to develop and deploy, and the most difficult for the Air Force to defend against. The TBM threat is also the threat that is most sensitive to support location selection, because of the limited range of the majority of the world’s ballistic missiles. Short-range (less than 600 nautical miles [nmi]) ballistic missiles are the most plentiful of the missile threats; there are tens of thousands of short-range ballistic missiles around the world, they are produced by more than 15 different countries, and they are openly sold through weapons dealers (Amouzegar, et al., 2006).

Natural disasters are a third possible cause of node failure. Natural disasters can occur on a scale large enough to shut down operations at a location entirely, or can occur on a smaller scale that still disrupts normal operations. Two examples of large scale natural disasters are the 2004 Indian Ocean Tsunami and Hurricane Katrina in 2005. For a more extreme example, one only needs to look at Clark Air Base in the Philippines, which was once the largest U.S. overseas military base in the world. The volcanic eruption of Mount Pinatubo in April 1991 forced the complete evacuation of the base and contributed in part to its eventual closure.

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6 The two countries have since reached an agreement with the U.S. paying $60 million a year for the use of the location through 2011 (Miles, 2009).
7 The sensitivity comes from the geographical relationship of the base to that of potential adversaries. The closer the base is to potential adversaries, the greater the risk of TBM attacks. For support locations that are fairly remote or afloat (ships serving as support locations), the threat is minimal.
It is very difficult, if not impossible, to predict these events; but it is possible to plan and prepare for them. Analytical methods should be able to optimize against and minimize the costs and effects associated with potential disturbances.

Currently, limited work has been conducted on node failure with respect to Air Force combat support networks. The following sections will explain the difference between a robust network model and a reliable network model, describe the research in this area that has already been completed, and present an outline for how we will address this issue.

**Robustness vs. Reliability**

While the terms “robustness” and “reliability” are often used interchangeably, several authors have used these terms to distinguish two different concepts. Supply chain robustness refers to the extent to which a supply network is able to handle an uncertain future demand (Snyder, 2003; Daskin, et al., 2005). Robust facility location models hedge against uncertainty in the problem data; it is a demand-side issue. These uncertainties occur in future demand, costs, and so forth. It is important to point out that while the robust solution is one that performs well under every realization of uncertain parameters, the solution may not necessarily be the minimum cost solution for any specific set of input parameters.

In contrast, supply chain reliability refers to the ability of a system to perform well even when parts of the system fail (Daskin, et al., 2005). Reliability models hedge against uncertainty in the network itself; it is a supply-side issue. These uncertainties are focused on the availability of facilities (nodes) that are in the solution. A reliable solution is one where even if one or more facilities become unavailable, the remaining system is still adequate to meet demand.

Snyder (2003) points out that the distinction between robustness and reliability in modeling supply chains is just a framework to approach a problem, it is not meant to suggest that a network design problem cannot use a combination of robustness and reliability techniques.

In addition to the definitional difference, the robustness and reliability of a network must be evaluated differently. To a large extent, sensitivity analysis can usually evaluate the robustness of a system; however, evaluating the reliability of a system requires more advanced modeling techniques (Snyder, 2003; Daskin, et al., 2005).
Two previous reports have attempted to address robustness and reliability with regards to the Air Force combat support network. The following section will briefly summarize these reports with a more thorough review provided in Chapter 3.

**Previous Work**

*Global Basing Analysis*

Amouzegar, et al., 2006 focused on building an analytic framework for evaluating options for overseas combat support basing. The framework is based on the notion that U.S. interests are not only global but dynamic as well, particularly when the United States is confronted with emerging anti-access and area denial threats. The study considered the various costs (e.g., operations, maintenance, and transportation) associated with training and deterrent exercises. The study also ensured the necessary storage capacity and system throughput to engage in major combat operations should deterrence fail. This analysis captured robustness considerations by identifying a basing posture that performed well across a broad range of alternative future deployment requirements.

*WRM Prepositioning Analysis*

McGarvey, et al., 2009 expanded on the work from Amouzegar, et al. This report revisited the global prepositioning of WRM, evaluating the concept of prepositioning assets not only in permanent facilities, but also in shipping containers at locations sited at or near sea ports, that could easily be transported by sealift to support deployment requirements. The study also analyzed the costs and benefits associated with building “reliability in the event of disruption” considerations into the prepositioning posture.

**This Dissertation**

This dissertation builds upon the previous work by performing a reliability analysis of the Air Force combat support network. More specifically, we are building a reliability model with a goal of identifying the lowest-cost reliable FSL posture. We expand upon previous approaches to model FSL disruption by considering multiple FSL failures occurring simultaneously and take into account the failure of the FSL over the entire time horizon.
**Organization of this Dissertation**

Chapter Two explores some of the key methodological issues underlying the analysis in subsequent chapters. The chapter begins by reducing the overseas basing problem to the general class of location-allocation-flow problems in which it falls. We will then discuss the prior literature on modeling network uncertainty and conclude with how this dissertation will be expanding on the work.\(^8\)

In Chapter Three, we will begin with a detailed discussion of the two studies discussed in the Chapter One (Amouzegar, et al., 2006 and McGarvey, et al., 2009). Following this discussion, we map the Air Force combat support network to the general location-allocation-flow problem discussed in Chapter 2. We will then move into a description of the demand scenarios we designed, and all other relevant data used in the model runs.

Chapter Four starts with a discussion of the multiple posture model we first developed for designing a combat support network. The model returns multiple posture solutions which we use to gain insights into developing the reliable model which will identify the single posture solution. The chapter starts with a discussion about the mathematical model we created and then presents the results of the computational runs returned by the model using the scenarios and data described in Chapter Three.

In Chapter Five, we will introduce the single node failure reliability model that we developed in order to identify a single posture reliable combat support network. We will compare the solution returned by this model to the solutions returned from the multiple posture model. The chapter will also explore how the model results demonstrate the various options available to policy makers when designing a reliable combat support network.

Chapter Six presents the multiple node failure reliability model. We will explore how the solutions returned by the model compare to the multiple posture and single node failure reliability models. We will also demonstrate how this model can be expanded to take on larger problems than the previous models.

In Chapter Seven, we will present a network problem from a different policy arena, that can be solved using the techniques designed in this dissertation. While not military related, the network possesses elements similar to those found in the combat support problem.

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\(^8\) By network uncertainty, we are referring to the probability that some part of the network will fail in the future.
Chapter Eight presents the conclusions and recommendations to policy makers for designing a joint reliable and robust supply chain network, as well as how this analytic framework can be expanded in future research.
Chapter 2: Methodological Background

In this chapter, we explore some of the key methodological issues underlying the analysis in subsequent chapters. We will begin by reducing the overseas basing problem to the general class of problems in to which it falls. By reducing the problem down to its simplest form, we are able to show the key elements that are common across all types of problems falling into this class. We will then discuss the prior literature on modeling network uncertainty. The chapter concludes with a discussion of how this dissertation will be expanding on this previous work.

Reduction of the General Problem

The design of combat support networks falls under the general class of problems known as location-allocation-flow problems. There are three elements to location-allocation-flow problems; where should facilities be located, how should commodities be allocated across them, and how should commodities be moved between facilities. In these problems, three factors are generally known: the location of each demand node, the demand at each node, and the costs of shipping from the supply node to the demand node. Four factors must then be determined: the number of supply nodes, where to locate the supply nodes, how to allocate commodities across the supply nodes (their capacity), and the methods for transporting commodities between facilities (Cooper, 1963). This class of problems can be complicated due to the large number of differing components; therefore, it is beneficial to break these networks down to their simplest form to identify the key elements they have in common. This section will identify those key elements (not tied to any specific type of network) and show how they fit into the model (e.g., constraints or objectives).

What is Being Shipped

To begin with, an “item” is needed at a location. We will call this “item” a commodity. Commodities can be physical goods such as tents, vehicles, and ammunition. Commodities can also be non-physical goods such as data and electricity. The physical characteristics of the commodities are important characteristics that need to be considered when modeling the network. We can think of this as short tons or square feet with respect to physical goods, and megabytes or kilowatt hours with respect to non-physical goods. Operations researchers develop
mathematical models to determine how these commodities will be allocated across storage sites, and how they will be shipped from one location to another. Commodities will appear in constraint functions where there are limitations in the ability to store or transport them. Commodities may also appear in objective functions when the cost associated with shipping these commodities from the supply to the demand nodes is to be minimized.

**Demand Nodes**

Demand nodes, often called sink nodes, are the locations where the item is ultimately needed. We can think of demand nodes as retail stores, military units, residential homes, and even computers. Usually demand nodes are known in advance when designing the network. If the exact location is not known, then likely locations are used. For example, when designing a combat support network, the location of all future deployments are not known with certainty in advance. Instead, decisions may be dictated by current political situations or forecasts and planned training missions.

**Supply Nodes**

Supply nodes, often called source nodes, represent where the commodity is stored until it is needed and then shipped to the point of demand. Supply nodes vary widely from geographical locations containing plants and/or warehouses to network servers. Three decisions that need to be made for a supply node are where to locate the node, how much capacity to endow it with, and how much of the endowed capacity to utilize. Depending on the design of the network, a supply node may service a single location, or multiple locations.

Depending on the type of network being designed, supply nodes may be located at any point in the plane, or at any point along a fixed network of arcs, these are considered continuous location models. The other possibility is discrete location models which only locate supply nodes at the end points of the arcs. The decision of node location is based on several factors. First, how close should the node be located to possible points of demand? Sometimes supply nodes are placed in the geographical center of all demand points. Other times, if demand is not equally dispersed across points, the nodes are placed closer to the points with higher demand. Other issues to consider are the risks associated with each site location (e.g., attacks from adversaries, harsh weather conditions) and costs (e.g., construction and operating).
Once the decision on the placement of supply nodes is complete, the decision on how to allocate commodities across the nodes needs to be made. The amount of commodities being placed at the node is constrained by the physical size of the location. For example, a location can only fit a certain number of warehouses and the warehouse itself can only fit so many square feet of commodities. Other important considerations may involve hedging against possible node failure by evenly dispersing commodities across several sites, evaluating whether future demand is known with certainty or whether it might fluctuate greatly, and whether upper or lower bounds should be placed on the number of facilities that are opened.

Transportation Options

Unless the demand and supply nodes are in the same location, the commodities will need to be transported from the supply node to the demand node. The method of transportation will depend on the type of commodity being transported. For example, physical commodities can be transported via land, sea, and/or air vehicles. In contrast, non-physical commodities can be transported via cables or even satellites.

The model may be constrained by the number of transportation “vehicles” that are in the network to begin with and also the ability to procure new “vehicles” over time. In the latter case, this would factor into the objective function for a minimum cost problem.

Costs

Differing costs have to be taken into consideration when designing a network. These costs will vary depending on the network, but some costs that are common across the different types are: facility opening costs, operating and maintenance, and transportation/infrastructure. There may be costs associated with opening a supply node. For example, a purchase or other kind of lease contract may have to be made to obtain the land or space where the supply node is to be placed. Then, once the physical space for the supply node is obtained, additional costs may exist if storage facilities need to be leased or constructed in order to store commodities before shipping to demand nodes.

Operating and maintenance costs exist to keep the supply node functioning. These costs include utilities and general maintenance or upkeep. They are also a function of what is stored there, and the distance from other facilities.
Transportation/infrastructure costs will also be a consideration. If commodities need to be shipped to the demand nodes via vehicles, vehicle procurement or lease costs will exist, in addition to fuel expenses. For commodities such as data, electricity, or water, wires and pipes will need to be put in place before materials can be shipped.

Costs may fall into the objective function, constraints, or both. For minimum cost objective functions, all of the costs are included in the objective function. For problems that involve budgetary considerations, costs act as a constraint on the ability of the network to function.

**Time**

There is often a time component for this class of problems. One time element is the time it takes for the commodity to be sent from the supply to the demand node. In the case of data being transmitted over networks, this can be virtually instantaneous. In the case of physical goods, this can take several days or weeks. The other time element is when the commodity is needed at the supply node. This can be very important for systems such as just-in-time (JIT) inventory management. The JIT strategy aims to improve the return on investment of a company by reducing inventory levels and the associated carrying costs. JIT relies on a series of signals that occur at predetermined points in the manufacture process which identify when more items need to be ordered. JIT focuses on having the right items, at the right place, at the right time, in the exact amount needed (Hillier and Lieberman, 2005). However, JIT systems involve a certain level of risk. Without the safety net of inventory, delays in production may occur if the items do not arrive on time.

**Objective Function**

“Objective functions in optimization models quantify the decision consequences to be maximized or minimized” (Rardin, 1998). Deciding on the objective function is often times not a straight forward process. Classic facility location problems often fall under one of two categories, $p$-median and $p$-center. For $p$-median problems, we consider the set of $p$ facilities that supply the same service in a region. Users select their nearest facility among the $p$ facilities and the cost of using this facility is a function of the distance between the user and their nearest
facility. The objective function is to determine the location of the \( p \) facilities such that the total cost (total weighted distance) of all users is minimized (Drezner, 1995).

For \( p \)-center problems, as in the \( p \)-median problems, we assume that users use the nearest facility among the \( p \) facilities in the region. The objective function is to determine the location of the \( p \) facilities such that the maximum distance from any user to its nearest facility is minimized (Drezner, 1995).

When we move away from the classic facility location problems into more specific problems, such as location-allocation-flow problems, alternative objective functions may be needed. Different objectives define different versions of the location-allocation-flow problem. Some examples are minimizing: cost, distance, time, and number of facilities. The objective function may also seek to maximize the same alternatives when trying to identify worst case possibilities.

Sometimes the objective function is dictated by the problem description. For network design problems that have budgetary constraints, minimizing cost (transportation, operations and maintenance, etc.) would be an appropriate objective. For JIT inventory problems, minimizing delivery time would be an appropriate objective.

Another way to decide on the correct objective function is to eliminate possibilities that the policy-maker would definitely not want to consider. In the case of a network that is highly susceptible to node failure, it may not be appropriate to model the minimum number of facilities because if that small number of nodes were to fail, then the network would not remain viable.

The following section will discuss the prior literature on modeling network uncertainty.

**Literature Review**

There are numerous techniques for modeling network robustness and reliability. Each technique can be categorized under three broad classes of literature (Snyder and Daskin 2005). The first class, network reliability models, is concerned with the chances of a network remaining connected even in the face of random failures. Optimization models within this class seek to maximize the probability that the networks will remain connected. These models appear in areas such as telecommunications and computer networks (Shooman, 2002). The only costs that are considered in these models are those of constructing the network; transportation, maintenance, and similar costs are not included. Traditionally, prior studies have focused on failures occurring
along the network routes and not at the nodes. The failures may occur when the network is congested from too much “traffic.”\textsuperscript{9} In other words, such models may not account for the proper time-phasing of demands.

The second class, backup covering models, focuses on assigning customers to multiple facilities with each facility serving a pre-specified fraction of demand. The fractions of demand covered by each facility are decided upon and input before running the model. Optimization models within this class seek to maximize expected coverage when facility availability is uncertain. These problems are dependent on probabilities that are input by the modeler (i.e., the probability that a facility is not available for various reasons). Much of the work done in this area has looked at where to locate emergency service vehicles and other similar emergency placement problems (Church and Revelle 1974).

The final class, supply chain disruption models, provides techniques for designing and operating supply chains that are resilient to disruptions of all sorts. These disruptions may occur along network routes or at the nodes. This is a more recent class of literature, appearing only in the last decade. Optimization models within this class seek to minimize expected cost of initial network design as well as transportation and ongoing operational costs. The majority of the research on supply chain disruption models has been qualitative.

Snyder developed this further by categorizing each model based on the existing supply network (Snyder, et al., 2006). The first step in this categorization is to determine the underlying network design (i.e. does the network already exist or is it being designed from scratch?) The development of the network at the time of modeling will determine the flexibility of the methods that can be used. If the network exists, we would want to focus on fortifying this network against disruptions. If the network is being designed from scratch, reliability can be built into the design. The second step is to determine the underlying model (i.e., is it a facility location problem or a network design problem?) Facility-location problems focus on locating facilities in order to best serve customers at a minimum total cost. Network design problems focus on how to route goods from the supply nodes to the demand nodes, sometimes through intermediate shipment nodes, in order to minimize total cost. Finally, the risk measure, or the method for

\textsuperscript{9} The term “traffic” refers to the number of users on a computer network trying to send and receive data. Alternatively, it can refer to the number of cell phone users trying to place calls.
quantifying risk, should be determined. Two examples of risk measures are minimum expected cost and worst-case cost (min-max).

**Literature on Modeling Network Uncertainty**

This section of the chapter will cover the major developments in literature for modeling network uncertainty. While a large number of variations of modeling techniques have been used, most of them fall under these overall categories.

**Sensitivity Analysis**

The first step in the evolution of network modeling was the use of sensitivity analysis. The introduction of sensitivity analysis allowed researchers to study and account for uncertainty in supply chains. Sensitivity analysis assumes that no disruptions occur and once the deterministic model is run and the network established, it examines how uncertainties in the data affect how the network operates (e.g., costs or time). For example, a network model is solved using fuel costs that are decided on before running the model. After the model returns a solution, the fuel costs can be increased or decreased to see how the changes affect this solution (i.e., is it still the optimal solution). A limitation of using sensitivity analysis is that it is a reactive approach that only discovers the impact of data uncertainties on the model’s recommendations.

**Stochastic Linear Programming**

Further developments in network modeling occurred with the use of stochastic linear programming. In 2002, Morton, Salmeron, and Wood addressed the vulnerability of military sealifts to attacks. They began by discussing the use of deterministic mixed-integer programming models that provided an exact assignment and routing of ships to deliver cargo as efficiently as possible. While it is a useful form of modeling, they noted that it ignores the fact that an enemy may disrupt the deployment by attacking the network in some forward node. To correct for this issue, the authors used a stochastic-programming model (Morton, et al., 2002).

Stochastic linear programming (SLP) uses discrete scenarios, each with a given probability of occurring, where the objective is to minimize expected cost. SLP has several limitations. First, it only easily handles small problems. For example, stochastic programs for large-scale military mobility systems require a large amount of computing power and therefore
are expensive to solve because they grow exponentially as the number of time stages and scenarios increase. Also, the simulated attacks that the authors use follow a probability distribution, and since the future of an attack is so uncertain, it is difficult to validate the model.

**Robust Optimization**

A complaint some analysts have against stochastic programming methods is determining the correct probabilities to use. In addressing this issue, a new method was developed called robust optimization (RO). RO is used when probabilities are unknown and the objective is to minimize cost or regret. In 1995, Mulvey, Venderbei, and Zenios conducted work on robust optimization of large-scale systems, observing that many “mathematical programming models are assumed to be deterministic and are typically formulated by solving a worst-case scenario.” However, solving these worst-case problems returns solutions that are very conservative and potentially expensive. To address this issue, they formulate a model that by design yields solutions that are less sensitive to the model data.

RO generates a series of solutions that are progressively less sensitive to realizations of the model data from a set of scenarios that are created by the analyst. The solution is determined to be robust if it remains “close to optimal for any possible scenario and remains almost feasible to any possible scenario.” One of the difficulties with using RO models is that they are complex and computationally expensive to solve with regards to both time and money. Also, the scenarios need to be defined by the user in advance; RO provides no means to specify them (Mulvey, et al., 2005).

Snyder, et al., 2009 also looked at RO. The authors created a RO model which optimized across multiple-scenario sets rather than within a single-scenario set. The work addressed the following question: How should spending be distributed over programs to achieve the maximum, balanced capabilities across these programs under a number of possible futures? Like Mulvey, et al., the authors also address the idea of models which solve a worst-case scenario. They note that it can be tempting to think that solutions which satisfy the worst-case scenario will also prepare better for other, less challenging scenarios. This is not usually the case as the proportions of resources needed for each scenario set is generally different. In other words, the worst-case scenario may require a different mix of resources than less demanding scenarios.
Worst-Case Models

In some cases, policymakers are concerned with planning for the worst-case possibility. They argue that by designing a supply network that can handle the worst-case, they will be prepared for anything. Worst-case models fall under the general class of models known as fortification models. Fortification models make the current supply network more resilient to attacks by identifying the weak components to the network so that resources can be concentrated on fortifying these components and identifying the best defense plan.

In a 2004 study, Salmeron, et al., analyzed the security and resilience of electrical power grids against disruptions caused by terrorist attacks. Their approach began with studying how to attack power grids, searching for the set of attacks that causes the largest possible disruption, and therefore designing appropriately conservative protection plans. Their model, as a result, “identifies grid components (nodes) which, when ‘hardened,’ yield the best improvement in system security.”10 A weakness of their approach is that actual terrorist ‘resources’ are uncertain so the model solutions are highly dependent upon these assumptions. The technique also requires the modeler to weight the relative importance of each component (node) which means that they have some prior knowledge as to which nodes would be the most important in the future.

Bilevel Programming

In a 2006 study, Brown, et al., take a more general approach, the bilevel programming model, using an attacker-defender (and therefore max-min) type method. This method is widely used in game theory problems. The attacker is the non-state actor and the defender is the U.S. In the model the attacker leads with an attack and the defender follows with a response. The key assumption is that the attacker has a perfect model of how the defender will optimally operate his system, and the attacker will manipulate that system to the best of its advantage. When setting up the model every node is considered vulnerable to attack unless changes have already been made to harden it. Threat is addressed by allowing different levels of offensive resources for the terrorists. At the end of the analysis, criticality of system components can be determined. The model is able to determine the value of protecting them, hardening them, or of adding new

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10 Hardening refers to fortifying the node to make it more resilient to attacks.
components into the system for purposes of redundancy. These options allow the policymaker to identify the best possible defense plan given a limited defense budget.

**Primary and Secondary Assignment Models**

Alternatives to the techniques discussed above for modeling network uncertainty have been developed using the concept of primary and secondary node assignments. The models assign each demand node in the network to a primary supply node. Additionally, each demand node is assigned to a secondary supply node at an additional cost and is only used if the primary supply node becomes unavailable. The model solution returns a trade-off curve to assist policy makers by showing the different combinations of levels of reliability and cost that are possible (Snyder and Daskin, 2005; Meepetchdee, et al., 2007). Policy makers are presented with the list of primary and potential secondary supply node locations, they can then look at the cost associated with establishing each secondary node to determine if it is feasible to do. They may find that for budgetary reasons, they can only establish a few secondary nodes and will want to concentrate on the facilities that will be the most useful.\(^{11}\)

**This Dissertation**

This dissertation builds upon the previous work by performing a reliability analysis of the Air Force combat support network. More specifically, we are building a reliability model with a goal of identifying a single reliable FSL posture.

To follow-up on the categories used in the previous literature review section, this dissertation will add to the literature on supply chain disruption models. The underlying network falls under the following categories: the network is being designed from scratch and it is a network design problem.

The next chapter begins with a discussion of some of the previous work on combat support networks. The following section maps the Air Force combat support network to the general location-allocation-flow problem discussed in this chapter. We will then move into a detailed description of the demand scenarios we designed and all other relevant data used in the model.

\(^{11}\) Perhaps some locations serve as secondary nodes for more than one demand node so it may be worth establishing these locations first.
Chapter 3: Scenarios and Data

The first section of this chapter discusses some of the previous work on combat support networks. The next section maps the Air Force combat support network to the general location-allocation-flow problem discussed in the previous chapter. We will then give an explanation of the demand scenarios we designed and all other data elements used in the model.

Combat Support Network Studies

The two studies in this section use a combination of methods discussed in the previous literature review section. While not strictly modeling a reliable or robust network, these models combine techniques from both areas to design an Air Force combat support network that is both reliable against node disruptions and robust against problems or uncertainties in the data. This section will discuss the previous work.

Global Basing Analysis

Amouzegar, et al., 2006 focused on building an analytic framework for evaluating options for overseas combat support basing. The framework is based on the notion that U.S. interests are not only global but dynamic as well, particularly when the United States is confronted with emerging anti-access and area denial threats (e.g., the recent denial of U.S. military access to the countries of Turkey and Kyrgyzstan12). The model minimized the operation, maintenance, construction, and transportation costs of meeting the training and deterrent exercises needed to demonstrate U.S. global power projection capability and thereby deter aggression, while maintaining the necessary storage capacity and system throughput to engage in major combat operations should deterrence fail.

The study attempted to answer two questions: How capable is the Air Force’s current overseas combat support bases of managing the future environment? And what are the costs and benefits of using additional or alternative overseas combat support bases for storing combat support materiel?

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12 As mentioned previously, an agreement has now been reached between the two countries for continued U.S. access.
In order to hedge against uncertainty and ensure that the solutions returned by the model were robust, several different sets of the deployment timelines were developed to identify a robust posture. Given each scenario set, the optimal FSL posture was determined. A portfolio analysis was then performed to identify the postures that performed well across every timeline, thus providing “robust” solutions.

The analyses showed the costs and deployment timelines for various FSL options under different degrees of stress on combat support while taking into account existing infrastructure, basing characteristics, deployment distances, strategic warning, transportation constraints, dynamic requirements, and reconstitution conditions. The authors developed several sets of deployment scenarios using the Multi-Period-Multi-Scenario (MPMS)\(^\text{13}\) concept, with each including training exercises, deterrent missions, and major combat operations. These sets of deployment “timelines” allow the model to measure the effect of timing, location, and the intensity of operational requirements on combat support—and vice versa.

The optimization model assessed the cost and capability of various portfolios of overseas combat support bases or FSLs for meeting a wide variety of global force projections. The model attempts to minimize the overall system cost while meeting operational requirements.

After determining the desired requirements in terms of combat support resources, the optimization model selects a set of FSL locations that would minimize the costs of supporting these various deterrence and training exercises while maintaining the capability to support the major regional conflicts should deterrence fail. The model essentially allows for the analysis of various “what-if” questions and assesses the solution set in terms of resource costs for differing levels of combat support capability. The end result of the analysis is a portfolio containing alternative sets of FSL postures, including allocations of WRM to the FSLs, which can then be presented to decision makers. This portfolio allows policymakers to assess the merits of various options from a global perspective.

Several policy recommendations were made based on the analysis performed. Some of those recommendations were:

- using a global approach to select combat support basing locations is more effective and efficient than allocating resources on a regional basis;

\(^\text{13}\) The MPMS concept is a framework that relies on a sequenced, potentially simultaneous set of deployment scenarios.
• political and other concerns need to be addressed in any decision about potential overseas basing locations; and
• closer attention should be paid to Africa and South America both as a source of instability and as a possible location for combat support bases.

**WRM Prepositioning Analysis**

McGarvey, et al., 2009 expanded on the work of Amouzegar, et al (2004, 2006). The study revisited the global prepositioning of WRM, this time evaluating the concept of prepositioning assets in shipping containers at locations sited at or near sea ports and then moving the containers to vessels to be transported by sealift to support deployment requirements. The study also analyzed the costs and benefits associated with building “reliability in the event of disruption” considerations into the prepositioning posture.

The research addressed two key questions: To what extent can a WRM posture balance the cost of building and operating storage sites with potential reductions in the transport costs in support of deployments other than major combat operations (MCOs)? And what are the characteristics of the tradeoffs between WRM system efficiency and redundancy?

In addressing uncertainty, a robust model and a separate reliability model were built, but they did not perform a joint reliability and robustness analysis. The optimization models in the analysis extended the previous approach by constructing a model that could simultaneously address multiple sets of future deployment requirements and also consider unplanned network disruptions caused by short-term disruptions limiting or preventing U.S. access to prepositioned materiel.

To simultaneously account for multiple potential futures, a set of alternative futures was defined, where each future consists of a set of time-phased deployment demands for WRM at FOLs. Multiple versions of all decision variables were created and constraints were given a temporal dimension, to track the state of the system under each alternative future.

The model identified a single WRM posture, defined as a set of FSLs and an allocation of WRM assets to those FSLs, which would be capable of satisfying the demands across all futures. Because the model addresses performance across multiple futures, the meaning of the objective function that “minimizes cost” is somewhat ambiguous, depending upon the decision maker’s level of risk-tolerance and the weight associated with each potential future.
The analysis also altered the model in order to identify a prepositioning posture that is able to satisfy WRM delivery requirements in the event of loss of access to any prepositioning facility. The analysis assumed that only one set of future deployment scenarios were addressed for each model run (i.e. non-robust), access was only lost to a single FSL, and that access was only temporarily lost to assets located at FSLs where disruptions occur.

The model is solved by finding values for a set of decision variables that minimize the desired objective, which is a function of the costs of building a WRM prepositioning posture and conducting a set of exercises and lesser contingencies, while constraining the solution set to have an infrastructure capable of supporting MCO requirements if deterrence should fail. This single WRM posture is capable of satisfying all deployment demands either across all considered futures, or in the event of loss of access to any individual FSL.

Several policy recommendations were made based on the analysis performed. Some of those recommendations were:

- a set of geographically dispersed WRM FSLs is an attractive option when considering the balance between predictable and unpredictable costs;
- alternative packaging configurations can allow for dispersed WRM prepositioning postures without incurring significant investments in infrastructure construction; and
- substantial robustness and reliability can be designed into WRM prepositioning postures at relatively little cost.

**Air Force Mapping**

In Chapter 1, we explored the different elements of the combat support network. The elements of the combat support network that we are modeling in this dissertation can be mapped to the different categories of the location-allocation-flow problem discussed in Chapter 2. The following list illustrates the mapping.

- **What is Being Shipped**- The items being shipped in our network include BEAR (i.e., billeting, electrical systems, etc.), munitions, and rolling stocks (e.g., trucks).\(^{14}\)
- **Demand Nodes**- The demand nodes in our network are FOLs (i.e., locations forward-deployed, out of which tactical forces operate).

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\(^{14}\) BEAR provides the required airfield operational capability, such as housekeeping or industrial operations, to open an austere or semi-austere airbase.
• **Supply Nodes** - The supply nodes in our network are FSLs (locations around the world where materiel is stored and shipped to FOLs).

• **Transportation Modes** - We will consider three classes of vehicles: air, ground, and sea. Each class is simplified to consider a single vehicle type (i.e., C-17’s, trucks, and High Speed Sealift).\(^\text{15}\)

• **Costs** - We have several costs that we take into account. Facility construction costs include the costs associated with building additional storage facilities at an FSL. Facility operations and maintenance costs include all costs associated with the operations and general upkeep of the FSL. Transportation costs are the costs of transporting commodities from the FSL to the FOL. Commodity procurement costs are the costs of procuring additional commodities that are not already in the system at time zero. Commodity reallocation costs are the costs of moving existing commodities between FSLs.

• **Time** - Depending on which constraint we choose to enforce, commodities are required to move from FSLs to FOLs within two time limits (i.e., 10 and 30 days).\(^\text{16}\) We also use time elements when designing demand scenarios and vehicle constraints (e.g. loading and unloading, traveling from one location to another, etc.).

• **Objective Function** - Our object function minimizes the costs of conducting training and deterrent exercises over a given time horizon.

• **Recourse** - Some decisions are less permanent than others meaning that changes can be made with a relatively short notice. We will differentiate between recourse and non-recourse decisions.

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\(^{15}\) Joint high speed vessels (JHSV) are a class of ships with three attributes: light weight, high performance, and large payload. Some examples of JHSV are the INCAT 046 “Devil Cat,” which is capable of carrying 500 metric tons and reaching speeds up to 43 knots (ship-technology.com).

\(^{16}\) We require our model to meet a 10 day initial operating capability (IOC) or 30 day full operational capability (FOC), meaning the units at the forward location have what they need to begin operations within 10 days (or 30 days for FOC) of the contingency start day. IOC is defined as the state achieved when a capability is available in its minimum usefully deployable form, while FOC is defined as the state when one hundred percent capabilities have been reached. As of August 10, 2009: [http://www.aof.mod.uk/aofcontent/tactical/randa/content/glossary.htm](http://www.aof.mod.uk/aofcontent/tactical/randa/content/glossary.htm)
Data Section

This dissertation develops an analytic framework to identify a reliable forward support basing architecture. It is designed to evaluate alternative FSL configurations in an effort to identify a reliable set of FSLs that would perform well with respect to various measures of interest, such as facility and operating costs, deployment time, and transportation requirements, across a broad range of potential scenarios. Figure 3.1 describes our methodology for evaluating alternative FSL sites.

Figure 3.1
Overview of Analytic Process

Our methodology begins with the selection of sample scenarios (the scenarios will be explained in detail in the following section). These scenarios drive requirements for materiel such as base operating support equipment, vehicles, and munitions at FOLs. We estimate these requirements using a RAND model, the Strategic Tool for the Analysis of Required Transportation (START) (Snyder and Mills, 2004). The START model builds requirements at the unit level and, with the exception of munitions, does not estimate consumables (e.g., food and fuel). START combines the output list of the various unit types with the Manpower and Equipment Force Packaging movement characteristics for each unit. It converts the operational

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17 For the purpose of this dissertation, we will use the term scenario to refer to a series of time-phased deployments occurring in various locations around the world, each requiring different levels of combat support requirements.
18 The Air Force uses five-digit codes known as Unit Type Codes (UTCs) to distinguish between each unit type.
capability desired at a deployed location into a list of materiel and manpower needed to generate that capability.

These requirements, combined with the set of potential FSLs and FOLs that are derived from the scenarios, serve as the inputs for the optimization model. This set of deployments is then scheduled over a planning horizon to determine timelines that represent potential future deployment schedules. We then use the model to determine an optimal set of FSL locations for a given timeline, along with their inventory allocations, inventory requirements, and transportation requirements.

Regarding the input data for our model, it should be apparent that the solutions returned will be sensitive to the set of scenarios provided. A different set of input scenarios will quite possibly return a different solutions set of FSLs. Therefore, it is important to consider a broad range of potential future deployments and engagements in order to identify a robust set of facility locations.

It should be noted that the purpose of these scenarios is to demonstrate the methods developed in this dissertation and we are not advocating any of the base placements returned by the model. We can, however, make broader recommendations such as the benefits and drawbacks to strategies such as consolidating commodities to a few large sites versus disperse across many small sites. It would not be possible to illustrate the methods without putting data into the model. Therefore, we are using these future scenarios in order to utilize data that is realistic.

**FOL Demand Scenarios**

An effective combat support system should be responsive to various types of demands and stresses. Indeed, the unpredictability of the future security environment requires the evaluation of support concepts across a broad range of combat and noncombat scenarios with varying degrees of intensity. We made an assessment of capability needs associated with differing types of deployments by surveying a range of scenarios and generating a list of capability requirements. These scenarios include strategic factors such as deployment distance, likely amount of strategic warning, deployment duration, and current Air Force reconstitution requirements. The scenarios have varying degrees of infrastructure richness, such as availability of fuel, communications, and transportation.
To perform this analysis, it is necessary to create a list of scenarios that stress the combat support network. Using historical data to estimate what future requirements might look like, we considered a broad range of potential future engagements to identify a robust set of facility locations. Table 3.1 lists the regions we investigated, classified into three categories: major conflicts, exercises and other deterrent missions, and humanitarian and military operations other than warfare (MOOTW). Careful attention needs to be given to the exercise and other deterrent missions category. This includes specifying the force packages and associated support forces that will need to deploy to deter aggression in the most likely deployments.

The goal in creating this list was to develop a set of scenarios in different regions of the world that stress the system across a range of demands on the combat support network, with respect to both location and quantity demand. These scenarios include potential military and nonmilitary operations in the Near East, the Asia-Pacific region, Central Asia, South America, and northern and sub-Saharan Africa.

<table>
<thead>
<tr>
<th>Demand Scenarios Considered</th>
<th>Southeast Asia</th>
<th>South America</th>
<th>Near East</th>
<th>Central Asia</th>
<th>Indian subcontinent</th>
<th>Horn of Africa</th>
<th>Southern Africa</th>
<th>Central America and the Caribbean</th>
<th>South Pacific</th>
<th>Central and North Africa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercises and Other Deterrent Missions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Humanitarian and MOOTW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We computed Air Force beddown using historical data and expert judgment. Given an aircraft beddown for each scenario, we estimated the combat support requirements needed at

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19 We will not list the locations assumed for MCOs. In developing the characteristics of deployments in these regions of the world, we have relied on lessons learned from recent military activities. We acknowledge that past conflicts are merely indicators and not predictors of future requirements.

20 What we mean here is the number of aircraft by type and by location, also known as the Air Order of Battle.
each FOL using the START model. Although combat support is comprised of many consumable and repairable items, we will be concentrating on BEAR, munitions, and rolling stocks (e.g. trucks) because they comprise the bulk of the items in the WRM package.

Prepositioned munitions are a special case because they are heavy and require special handling. To simplify things, for the remainder of this dissertation we will use the term WRM to refer to BEAR, munitions, and rolling stock unless otherwise noted. Because WRM is designed for use at austere locations, we are only concerned with deployments to Categories 3 and 4 FOLs for nonmunitions WRM and Categories 2, 3, and 4 FOLs for munitions using the following classification scheme (Galway et al., 2000):

- **Category 1**: Main Operating Base; fuel, infrastructure, munitions are available; FOC within 24 hours of arrival
  
- **Category 2**: Standby Base; some facility/support plus-ups required; fuel available; FOC within 48 hours of arrival
  
- **Category 3**: Limited Base; minimal infrastructure available; access to fuel; FOC within 48-96 hours of arrival
  
- **Category 4**: Bare Base; runway/taxiway/aircraft parking available; FOC within 72-120 hours of arrival.

**Scenario Construction (Determining Demand)**

To select a set of reliable overseas combat support locations, we developed the following system for the construction of deployment scenarios:

- Although it is impossible to select combat support bases without specific operational deployments, the selection process should not be dependent upon a particular deployment. For that reason, we do not seek to optimize the system for a handful of deployments alone.
- Combat support requirements should be dynamic and deployment scenarios should cast a wide geographical net in order to stress the combat support and transportation requirements.

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21 Time estimates for full operational capability are general and are dependent on closure of base operations capability packages.
• Deployments should be sequenced in time and space in order to evaluate physical reach and test long-term effect of location and allocation of assets.
• To hedge against the uncertainty of the future security environment, multiple series of possible scenarios should be developed to test the reliability of the overseas combat support bases.

Figure 3.3 illustrates the variability of combat support requirements across the set of regions presented in Table 3.1 (excluding the major conflicts), in terms of the relative WRM combat support requirements. The vertical bars indicate the range of requirements for various potential deployments. On the Y-axis, the relative scale of recent deployments, in terms of WRM requirements, is noted to allow for comparison.

**Figure 3.2**
Relative Size of Combat Support Requirements Across Regions of Interest

In certain instances, deterrence operations (e.g., South America) require greater combat support than a traditional major regional conflict such as Operation Allied Force. This is due to the fact that the FOLs considered in OAF are primarily well-developed locations with significant existing infrastructure, while the South American contingency requires deployment to more

22 In this instance, the term dynamic refers to the fact that deployments occur at differing intervals throughout the future timeline, so combat support requirements will change over the future timeline as various deployments start and finish.
austere locations. Therefore, although the OAF deployment may be much larger in terms of forces deployed, its requirements on WRM are less than those of the “smaller” deployment.

In each region there may be several deployments, exercises, or deterrent missions, each with its own unique logistical characteristics. Table 3.2 lists some of the deployment missions we considered, along with their combat support requirements and number of forward operating locations. Note that the MCOs are labeled as MCO1 and MCO2. These represent full-scale conventional warfare requiring large numbers of fighters and bombers; as such, they do not belong in the category of exercises and other deterrent missions. The actual

Table 3.2
Deployment Characteristics for Different Missions

<table>
<thead>
<tr>
<th>Contingency</th>
<th># FOLs</th>
<th>BEAR ($/Tons)</th>
<th>Vehicles ($/Tons)</th>
<th>Munitions ($/Tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCO1</td>
<td>8</td>
<td>15,520</td>
<td>6,954</td>
<td>11,147</td>
</tr>
<tr>
<td>MCO2</td>
<td>14</td>
<td>21,727</td>
<td>11,896</td>
<td>43,742</td>
</tr>
<tr>
<td>Taiwan</td>
<td>4</td>
<td>5,194</td>
<td>2,189</td>
<td>12,778</td>
</tr>
<tr>
<td>SWA1</td>
<td>4</td>
<td>16,200</td>
<td>6,104</td>
<td>19,470</td>
</tr>
<tr>
<td>SWA2</td>
<td>1</td>
<td>5,843</td>
<td>2,348</td>
<td>10,044</td>
</tr>
<tr>
<td>SWA3</td>
<td>4</td>
<td>7,098</td>
<td>3,307</td>
<td>3,509</td>
</tr>
<tr>
<td>S. America 1</td>
<td>4</td>
<td>14,069</td>
<td>5,634</td>
<td>28,077</td>
</tr>
<tr>
<td>C. Asia</td>
<td>3</td>
<td>1,807</td>
<td>1,016</td>
<td>283</td>
</tr>
<tr>
<td>Spratley Islands</td>
<td>2</td>
<td>4,937</td>
<td>1,943</td>
<td>12,404</td>
</tr>
<tr>
<td>Thailand</td>
<td>2</td>
<td>4,411</td>
<td>1,057</td>
<td>1,855</td>
</tr>
<tr>
<td>Singapore</td>
<td>1</td>
<td>4,031</td>
<td>1,539</td>
<td>6,394</td>
</tr>
<tr>
<td>Egypt</td>
<td>2</td>
<td>4,411</td>
<td>1,057</td>
<td>1,855</td>
</tr>
<tr>
<td>India</td>
<td>2</td>
<td>4,411</td>
<td>1,057</td>
<td>1,855</td>
</tr>
<tr>
<td>Southern Africa</td>
<td>2</td>
<td>1,807</td>
<td>1,119</td>
<td>0</td>
</tr>
<tr>
<td>Cameroon</td>
<td>2</td>
<td>1,807</td>
<td>1,119</td>
<td>0</td>
</tr>
<tr>
<td>Liberia</td>
<td>2</td>
<td>1,807</td>
<td>1,119</td>
<td>0</td>
</tr>
<tr>
<td>Sierra Leone</td>
<td>2</td>
<td>1,807</td>
<td>1,119</td>
<td>0</td>
</tr>
<tr>
<td>Haiti</td>
<td>1</td>
<td>1,681</td>
<td>1,119</td>
<td>0</td>
</tr>
<tr>
<td>Chad</td>
<td>2</td>
<td>1,807</td>
<td>1,119</td>
<td>0</td>
</tr>
<tr>
<td>Rwanda</td>
<td>1</td>
<td>1,681</td>
<td>1,119</td>
<td>0</td>
</tr>
<tr>
<td>East Timor</td>
<td>2</td>
<td>1,807</td>
<td>1,119</td>
<td>0</td>
</tr>
<tr>
<td>Northern Africa</td>
<td>3</td>
<td>3,811</td>
<td>2,050</td>
<td>0</td>
</tr>
<tr>
<td>Horn of Africa</td>
<td>2</td>
<td>3,686</td>
<td>2,050</td>
<td>0</td>
</tr>
<tr>
<td>S. America 2</td>
<td>2</td>
<td>4,010</td>
<td>2,639</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: SWA = Southwest Asia; MCO = Major Combat Operation
cost of engaging in MCOs is not programmed because they are sourced through a different funding mechanism (i.e., supplemental).\textsuperscript{23}

With the list of deployments defined, we can now outline the sequencing and recurrence of those deployments. We chose to schedule the deployments and contingencies into a scenario comprising a six-year time frame to align with the PPBE’s future years defense program (FYDP).\textsuperscript{24} Two major conflicts are included in order to sufficiently size the facilities to support regional conflicts specified in the planning guidance. However, as mentioned earlier, since the operational cost for wartime execution is not included in the costs, it can be assumed that these conflicts occur at the end of the six-year time period. We are assuming that non-MCOs would be halted in the event of an MCO. This allows us to model the MCOs at the end of the six-year time horizon without the loss of fidelity.

Figure 3.4 shows a notional set of deterrence, exercises, and MCOs. Any combat support storage location that is selected must be able to support those deployments, including the possible MCOs. The deployments vary in size, in terms of combat support requirements, as shown on the y-axis. Such a set of deployments, considered in unison, provides an integrated view of deterrence. Commanders need to consider a range of possibilities; therefore, this integrated, simultaneous assessment of deterrence and preparedness will address those needs.

\textsuperscript{23} Resources necessary to support the routine deployment of forces to exercise sites is included in the Programmed Objectives Memorandum (POM) process. POM is the critical tool of the programming phase in the Planning, Programming, Budgeting, and Execution (PPBE) System. The PPBE process is the current system for creating the Department of Defense’s (including all the services’) contribution to the presidential budget. The actual costs of engaging in contingencies should deterrence fail is not part of deterrence, and the supplemental funding to engage in contingency activities would need to be funded on a case-by-case basis, as it is today. This analysis will focus on programmed costs.

\textsuperscript{24} The time horizon of six years is in keeping with the FYDP convention. The FYDP is the “official program that summarizes Secretary of Defense (SECDEF) approved plans and Department of Defense (DoD) programs. Published at least annually and updated to reflect budget decisions and re-programming action six years out.” As of August 10, 2009: http://web.nps.navy.mil/~kishore/mpt/glossary.htm
An Example of Variations in Combat Support Requirements Across Time

Note: SWA = Southwest Asia; MCO = Major Combat Operation; OIF = Operation Iraqi Freedom; OEF = Operation Enduring Freedom; OAF = Operation Allied Force.

Remaining Data Elements

The previous section discussed how FOL demand is determined for modeling. This section will cover the remaining data elements being used in the model. These elements include airfield throughput capacity, vehicle alternatives and their characteristics, materiel storage, and candidate FSL sites.

Airfield Throughput Capacity

An important data element that shows the capacity of an airfield is the maximum-on-ground (MOG) capability. MOG generally refers to the maximum number of parking spaces an airfield can provide (parking MOG), but it can be specialized to include the maximum number of aircraft that can be served by maintenance, aerial port, or other facilities (working MOG). MOG can also refer to the maximum number of aircraft that can be refueled simultaneously (fuel

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25 The MOG typically refers to parking space, maintenance capacity, and the ramp space for storing and assembling the support equipment at an airbase (see Stucker et al., 1998). MOG and other factors determine the throughput of a base. In this report, we use the number of aircraft that can land, unload, be serviced, and take off per hour as a more effective measure of throughput constraint.
MOG), In our analysis, we used both working MOG and parking MOG to compute the airfield capability or throughput with the following equation:

\[
Throughput = \frac{MOG \times WorkDay}{ServiceTime},
\]

Where \( MOG \) is the smaller of the working or parking MOG numbers, \( WorkDay \) is the number of working hours in a day, and \( ServiceTime \) is the required hours to load, unload, and service a particular aircraft.\(^{26}\) Thus, \( Throughput \) is the maximum number of aircraft that can be processed through an airfield in one working day.

A MOG of 2 was used for most of the FOLs. The MOG of the FSLs, for the most part, range between 1 and 5, although a few of the FSLs had a substantially higher MOG. One of the FSLs had a MOG of 24.

Although this section has focused on air vehicles, constraining factors—such as throughput, fleet size, and load capacity—apply to all modes of transportation and must be taken into consideration when there is an option to select an alternative mode.

**Modes of Transportation**

In the past, the Air Force assumed that shipments would be made by air only. We considered three classes of vehicles in the model: air, ground, and sea. There are several advantages to using sealift or ground transportation in place of, or in addition to, airlift. Allowing for alternative modes of transportation might bring some FSLs into the solution set that otherwise may have been deemed infeasible or too costly.

Trucks are, of course, cheaper to operate than aircraft or ships and are readily available in most locations through local contract. They do not require specialized airfields and, although they travel more slowly than aircraft, under certain circumstances they could contribute greatly to the delivery of materiel, especially when they are used in conjunction with airlift. For example, as noted in Amouzegar, et al., 2004, with only 200 trucks, 10 Harvest Falcon sets (or about 11,000 pallets) can be delivered to various locations in Southwest Asia within 75 days. The same amount of materiel can be delivered in about 58 days using 24 C-17s, or in 85 days using 47 C-130s. The best single-mode result, 40 days, is attained using approximately 400

\(^{26}\) Lack of access to fuel MOG data prevented us from incorporating the fuel MOG in the equation.
trucks. However, a mixed strategy of 200 trucks and seven C-17s can achieve the same goal in only 12 days.

Ships have a higher hauling capacity than do aircraft and can carry outsized and super-heavy equipment. In addition, ships do not require over-flight rights from any foreign government. However, ships are slow relative to airplanes, and may require specialized ports and equipment for loading and offloading. Nevertheless, sealift can be an effective alternative to airlift. For example, a recent study (GlobalSecurity.org, 2005) found that in a notional 4,000-nautical-mile scenario comparing C-17s and the new large medium-speed roll-on/roll-off (RO/RO) ships, assuming no prepositioned ships in the theater, airlift could deliver only 72,000 tons of cargo in 36 days, whereas a sealift could deliver 3,960,000 tons in the same number of days.

Vehicle Characteristics

In our model, vehicles are defined by their capacity (tons and square feet), loading and unloading time, a space conversion factor, and a utilization rate. Airlift information comes from AFPAM 10-1043 (U.S. Air Force, 2003). Air vehicles have a one-way cost. For the return trip we assume a retrograde cost based on the lower of the air, sea, and land transit costs. Air distances are determined using the Joint Flow and Analysis System for Transportation (JFAST) (USTRANSCOM, 2002).\(^{27}\) From the air distances, transit time and costs are computed as well. Air cost is given as the cost per ton over the air link; distance is in nautical miles.

A standard truck has a capacity of about 20 tons or 40 passengers. The distance required for land transport is based on output from JFAST. The transportation time is based on the road conditions of the country. For FSLs located in countries where the road conditions are “good,” the daily distance covered by land transport is 194 miles per day. For FSLs located in countries where road conditions are “poor,” the daily distance covered by land transport is 70 miles per day.\(^{28}\) Cost for land transport is set at $0.25/(ton mile), adjusted for the country cost factor.\(^{29}\) Each FSL-FOL pair has a land transport distance. The time for transit is determined by using the

\(^{27}\) JFAST is a multimodal transportation analysis model used to determine transportation requirements for delivering troops and equipment by air, land, and sea (USTRANSCOM, 2002).

\(^{28}\) Here, “poor” refers to roads which may be dirt or gravel, and may be filled with large potholes, etc. Daily land transport speeds are taken from Headquarters, Department of the Army (1998).

\(^{29}\) The country cost factors are a set of values that we computed to take into account the effect of differences in regional cost of living.
distance and the road conditions of the FSL. In addition, each border crossing (outside of Europe) results in a delay of one day.

The standard ship type is JHSV. The sustained speed is 30 knots with a capacity of 400 tons of equipment. The transit time is based on JFAST port-to-port sailing distances of the ports associated with the FSL and FOL in question. Costs are based on Military Sealift Command (MSC) rates.\(^{30}\) Retrograde for sea transport is assumed to be over sea links and carries the same cost. Each FSL and FOL is assigned to a nearby port, if available. In addition to the sea transit time, when the port-to-FSL/FOL distance is more than 25 miles, land travel time is also added—based on the road conditions of the country that hosts the FOL. We also consider the number of canal crossings required, referring to the Panama or the Suez canals. For each canal crossing required, a delay of one day’s transit time is added to the total transportation time.

**Candidate FSLs**

We generated an initial list of over 300 potential FSL sites worldwide. Vulnerability, physical constraints, and political restrictions were taken into consideration in an attempt to reduce this list to a more manageable size.

In selecting regions and locations for forward support locations, the vulnerability of the candidate locations to attacks from adversaries in future conflicts must be considered. Forward support locations could be primary targets for adversaries with long-range fixed-wing aircraft, cruise missiles, TBMs, special operations forces, or primary targets of an attack by non-state actors.

Another major factor in selecting a forward support location is its transport capability and capacity. The parking space, the runway length and width, the fueling capability, and the capacity to load and offload equipment are all important factors in selecting an airfield to support an expeditionary operation.\(^{31}\) Runway length and width are key planning factors and are commonly used as first criteria in assessing whether an airfield can be selected. All these factors combined dictate the type of aircraft that can be used at a base and the load capacity it can handle. The selection of each FSL will be based heavily on the airfield restriction.

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\(^{30}\) Rates are as shown in COMSCINST 7600.3J CH-10 MSC Billing Rates, October 4, 2002.

\(^{31}\) In our analysis, some of these factors are computed parametrically in order to assess a minimum requirement of a potential field in order to meet a certain capability.
We also examined the potential for storing combat support resources aboard an afloat preposition fleet (APF). Although afloat prepositioning does offer additional flexibility and reduced vulnerability versus land-based storage, the APF is much more expensive than land-based storage and presents a serious risk with regard to deployment time. Even if a generous advance warning is assumed to allow for steaming toward a scenario’s geographic region, it can be difficult to find a port that is capable of handling these large cargo ships. The requirements placed on the port, including preemption of other cargo movement, also restrict the available ports that can be used by an APF.

Each APF ship has a predetermined homeport and, for each given contingency, a port of call. We have assumed there are seven days of strategic warning prior to the contingency start day in which an APF ship can begin steaming toward its designated port. After arriving at its port, the combat support equipment is delivered to FOLs using land transportation.

Additionally, we considered political restrictions. We touch on this in the first chapter when discussing node failures and disturbances. In general, the U.S. military has had an excellent record of maintaining working relationships with other host nations. However, these relationships vary greatly, and in our assessment of potential forward support locations we must evaluate the possibility of denial of access and its effects on combat capability, as was demonstrated during Operation Iraqi Freedom.

Arguably, one of the most important regions for potential forward support locations is Europe. European countries have been host to U.S. forces for more than 50 years. European forward bases have been used not only for local conflicts but also for operations in the Near East and Africa. The rich infrastructure, modern economies, stable democracies, and historical and cultural ties to the United States have made Europe an obvious choice for forward support and operating locations. Although there has been some political discontent regarding the resistance of France and Germany to support Operation Iraqi Freedom, such disagreements have by no means lessened the importance of European nations as hosts to U.S. forces.

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32 There is one exception. Because the South American scenario is not located near any potential FSLs, 19 days strategic warning for steam time is assumed for that scenario.

33 Such disagreements, though disconcerting, should be expected even from the closest U.S. allies, as was demonstrated by the resistance of the United Kingdom, Spain, Italy, Greece, and Turkey to allow even over-flight rights during operation Nickel Grass, the airlift to Israel during the 1974 Israeli-Arab conflict (see Shlapak et al., 2002).
NATO has been expanded to include many of the former Soviet Bloc countries of Eastern and Central Europe. In 1999, NATO admitted Poland, Hungary, and the Czech Republic, and since then has admitted Bulgaria, Estonia, Latvia, Lithuania, Romania, Slovakia, and Slovenia. Romania and Bulgaria are of particular interest in this study, as they are situated in proximity to regions of potential conflicts and have shown great interest in supporting U.S. forces in recent conflicts.

The United States continues to maintain a strong and sizeable presence in Asia. Bilateral defense agreements with South Korea, Japan, Australia, Thailand, and the Philippines, along with other security commitments to some of the islands in the Pacific, ensure a continued presence of U.S. forces in the region. Nevertheless, many countries in the region may be wary of openly supporting a large permanent U.S. presence in their territories, and others may not want to increase tension by taking sides in a potential conflict in which the United States, for example, aids Taiwan against the People’s Republic of China.

One of the most important regions in terms of security is the Near East, yet this region may be the most problematic in terms of base access. The United States military kept a sizeable presence in Saudi Arabia after Operation Desert Storm, but that decade-long arrangement was fraught with political and social issues. Eventually, the DoD decided to withdraw its troops from the Kingdom. The United States has been successful in negotiating formal defense arrangements with Kuwait, Bahrain, Qatar, Oman, and the United Arab Emirates. However, as in the Asia-Pacific region, the granting of base and facility access does not necessarily mean guaranteed access for use of these facilities.

Given the large number of recent U.S. military operations and exercises in Africa, it may be of interest to consider potential facility locations in Africa.

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34 For the current list of 28 NATO members along with 20 other nations belonging to the Euro-Atlantic Partnership Council, see [http://www.nato.int/cps/en/natolive/index.htm](http://www.nato.int/cps/en/natolive/index.htm) (as of August 10, 2009).

35 Bulgaria allowed overflight rights during OAF despite domestic opposition. This was in contrast to Greece, a NATO member, who refused access to its airspace or airfields (Shlapak et al., 2002).

36 Korea is included in the analysis, but the resources allocated to Korea have been excluded from the model. We assume that the resources needed to support exercises and operations in Korea have been quarantined from reallocation. Future analysis may allow for reallocation of Korean resources if the threat from North Korea is deemed to have diminished.

37 In 2004 the senior U.S. military commander for the European Command, General Charles Wald, stated that Sao Tome is a location of particular interest.
Ultimately, we settled on 38 potential sites, including an Afloat Preposition Ship (APS), MUN2 at Andersen AFB, which we felt were sufficient for conducting the study.\textsuperscript{38} The final list is presented in Table 3.3.

<table>
<thead>
<tr>
<th>Table 3.3 Potential Forward Support Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bagram, Afghanistan</td>
</tr>
<tr>
<td>Darwin, Australia</td>
</tr>
<tr>
<td>Baku, Azerbaijan</td>
</tr>
<tr>
<td>Shaikh Isa, Bahrain</td>
</tr>
<tr>
<td>Burgas, Bulgaria</td>
</tr>
<tr>
<td>Djibouti Ambouli, Djibouti</td>
</tr>
<tr>
<td>Cotopaxi, Ecuador</td>
</tr>
<tr>
<td>Beni Suef, Egypt</td>
</tr>
<tr>
<td>Spangdahlem AB, Germany</td>
</tr>
<tr>
<td>Souda Bay, Greece</td>
</tr>
<tr>
<td>Andersen AFB, Guam</td>
</tr>
<tr>
<td>Chennai, India</td>
</tr>
<tr>
<td>Chhatrapati Shivaji IAP, India</td>
</tr>
<tr>
<td>Balad, Iraq</td>
</tr>
<tr>
<td>Aviano AB, Italy</td>
</tr>
<tr>
<td>Kadena AB, Japan</td>
</tr>
<tr>
<td>Bishkek-Manas, Kyrgyzstan</td>
</tr>
<tr>
<td>Kaduna Airport, Nigeria</td>
</tr>
<tr>
<td>Masirah Island, Oman</td>
</tr>
</tbody>
</table>

Note: IAP = International Airport; APT = Airport; AFB = Air Force Base; AB = Air Base

Materiel Storage

Some storage warehouses are already existing at FSLs and additional warehouses may be constructed. Warehouses are 20,000 sq ft and cost $72/sq ft times the country cost factor to build. Operating costs for warehouses are $9.2/sq ft per year.

In the next chapter, we will explore the multiple posture model we first developed for designing a combat support network. The chapter starts with a discussion about the mathematical model we created and then presents the results of the computational runs returned by the model using the scenarios and data described in this chapter.

\textsuperscript{38} The list of FSLs is identical to the one used in Amouzegar, et al. 2006. The scenario deployment requirements are drawn from one of the timelines examined in Amouzegar, et al., 2006.
Chapter 4: Multiple Posture Model

This chapter takes a “naïve” approach to reliability modeling. We will begin by creating an optimization model that will identify a set of FSL locations and allocate commodities across those locations, assuming that all of the facilities are available (zero probability of node failure). Once the location and allocation of commodities has been identified, we will march through sequentially, and remove each FSL one by one from the solution set, to identify the different costs associated with each FSL failure. We are using this model to first, identify a baseline case to compare the solutions of other model variations against; and second, to show why the more advanced modeling techniques that we have developed in later chapters are needed.

The following sections present the multiple posture optimization model we developed in order to design a reliable combat support network. We will begin with a detailed discussion of the mathematical model we created. We will then explore the runs that were conducted as well as the results of those runs. In doing this, we will present the different policy options that are available to decision makers. In building a reliable combat support network, there are four options for policy makers to consider:

- placing additional inventory at existing FSLs;
- opening additional FSLs; or,
- using a combination of opening additional FSLs and placing additional inventory at existing FSLs; and,
- greater dispersion of commodity allocations across FSLs.

The Mathematical Model

In this section, we present the Mixed Integer Programming (MIP) model we developed to design a reliable combat support network. The model was developed using the General Algebraic Modeling System (GAMS).39 The MIP explicitly models transportation constraints (number of transport vehicles, vehicle utilization rates, vehicle capacity, and vehicle throughput), time-phased demand for commodities at FOLs, procurement of commodities, and facility constraints (storage space constraints within a warehouse and number of warehouses that may be

39 Brooke et al., 2003. The complete GAMS model coding is presented in Appendix B.
placed at an FSL). The optimization model then selects the minimum-cost combat-support bases from the list of candidate locations, allocates resources among the selected combat-support locations, and determines the feasible transportation routings. The following section presents a simplified version of the mathematical programming formulation of the optimization model.\cite{40} Notation is summarized in Table 4.1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>commodity index</td>
</tr>
<tr>
<td>$j$</td>
<td>FSL index</td>
</tr>
<tr>
<td>$k$</td>
<td>FOL index</td>
</tr>
<tr>
<td>$m$</td>
<td>transportation mode index</td>
</tr>
<tr>
<td>$t$</td>
<td>time index</td>
</tr>
<tr>
<td>$\Delta_j$</td>
<td>fixed cost incurred to open FSL $j$</td>
</tr>
<tr>
<td>$\Theta_m$</td>
<td>cost of obtaining an additional vehicle of mode $m$</td>
</tr>
<tr>
<td>$E_j$</td>
<td>construction cost per unit of storage at FSL $j$</td>
</tr>
<tr>
<td>$Y_{jk}$</td>
<td>operating cost per unit of storage at FSL $j$</td>
</tr>
<tr>
<td>$\gamma_{ik}$</td>
<td>shortage cost of commodity $i$ not fulfilled at FOL $k$</td>
</tr>
<tr>
<td>$\Omega_{ikm}$</td>
<td>cost of commodity $i$ transported from FSL $j$ to FOL $k$ via mode $m$</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>number of time periods necessary to load a mode $m$ vehicle</td>
</tr>
<tr>
<td>$J_m$</td>
<td>maximum load in tons per mode $m$ vehicle</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>utilization rate for mode $m$</td>
</tr>
<tr>
<td>$\tau_{jkm}$</td>
<td>one-way transportation time from FSL $j$ to FOL $k$ via mode $m$</td>
</tr>
<tr>
<td>$F$</td>
<td>cost of procuring additional tons of commodity $i$</td>
</tr>
<tr>
<td>$B_{ij}$</td>
<td>initial allocation of commodity $i$ at FSL $j$</td>
</tr>
<tr>
<td>$L_0$</td>
<td>cost of reallocation of tons of commodities</td>
</tr>
<tr>
<td>$A_{jk}$</td>
<td>max on ground at FSL $j$</td>
</tr>
<tr>
<td>$C_m$</td>
<td>inventory of mode $m$ vehicles</td>
</tr>
<tr>
<td>$D_{it}$</td>
<td>demand for commodity $i$ at FOL $k$ at time $t$</td>
</tr>
<tr>
<td>$E_j$</td>
<td>minimum units of storage needed for an economically feasible FSL at location $j$</td>
</tr>
<tr>
<td>$E_{jk}$</td>
<td>maximum potential units of storage at FSL $j$</td>
</tr>
<tr>
<td>$\omega_{jkm}$</td>
<td>time it takes to load vehicle $m$ at FSL $j$, move from FSL $j$ to FOL $k$, and unload at FOL $k$</td>
</tr>
<tr>
<td>$\nu_{jkm}$</td>
<td>additional units of storage needed beyond $E_{jk}$ at FSL $j$</td>
</tr>
<tr>
<td>$\theta_{jkm}$</td>
<td>number of mode $m$ vehicles available at FSL $j$ at the start of time period $t$</td>
</tr>
<tr>
<td>$\eta_{jkm}$</td>
<td>additional mode $m$ vehicles obtained</td>
</tr>
<tr>
<td>$\varepsilon_{jkm}$</td>
<td>shortfall below demand for commodity $i$ at FOL $k$ not fulfilled by time $t$</td>
</tr>
<tr>
<td>$\psi_{jkm}$</td>
<td>number of mode $m$ vehicles available at FSL $j$ at the end of time $t$</td>
</tr>
<tr>
<td>$w_j$</td>
<td>binary variable indicating status of FSL $j$</td>
</tr>
<tr>
<td>$\xi_{jkmi}$</td>
<td>commodity $i$ sent from FSL $j$ to FOL $k$ via mode $m$, beginning loading on time $t$</td>
</tr>
<tr>
<td>$\kappa_{jkm}$</td>
<td>number of mode $m$ vehicles tasked to transport commodities from FSL $j$ to FOL $k$, beginning loading on time $t$</td>
</tr>
<tr>
<td>$\zeta_{jkm}$</td>
<td>number of mode $m$ vehicles tasked to make the return trip from FOL $k$ to FSL $j$, departing on time $t$</td>
</tr>
<tr>
<td>$\psi_{jkm}$</td>
<td>additional units of commodity $i$ procured at FSL $j$</td>
</tr>
<tr>
<td>$E_{j}$</td>
<td>inventory of commodity $i$ at FSL $j$ that is reallocated to other FSLs</td>
</tr>
<tr>
<td>$A_I$</td>
<td>the sum of the additional amount of commodity $i$ procured across the entire system</td>
</tr>
</tbody>
</table>

\cite{40} The complete mathematical model along with a detailed discussion appears in Appendix A.
**Constraints**

We begin by defining the following variables:

- \( w_j \): Binary variable indicating status of FSL \( j \), \( w_j = 1 \) when FSL is open and, \( w_j = 0 \) otherwise
- \( x_{ijkmt} \): Quantity of commodity \( i \) sent from FSL \( j \) to FOL \( k \) via mode \( m \) at time \( t \)
- \( s_{ikt} \): Shortfall below demand for commodity \( i \) at FOL \( k \) not fulfilled by time \( t \)
- \( nn_j \): Additional units of storage needed beyond \( E_j \) at FSL \( j \)\(^{41}\)

The demand constraint requires cumulative arrivals at FOL \( k \) by time \( t \), to satisfy the cumulative demand by time \( t \). This constraint requires the declaration of parameter \( \omega_{jm} \), which is equal to the number of time periods necessary to load a mode \( m \) vehicle at FSL \( j \), the transit time from FSL \( j \) to FOL \( k \), the number of time periods necessary to unload the vehicle at FOL \( k \), and any additional time needed after unloading to transport the commodities. The demand constraint is defined as:

\[
\sum_{jm,n\leq t} x_{ijkm(n-\omega_{jm})} \geq D_{ikt} - s_{ikt} \quad \forall i, k, t
\]  

(4.1)

where \( D_{ikt} \) is the cumulative demand, in tons, for commodity \( i \) at FOL \( k \) by time \( t \).

FSL storage constraints limit the space available for commodities. The storage capacity is satisfied by the following constraints:

\[
\sum_{ikmt} x_{ijkm} \leq E_j w_j + nn_j \quad \forall j
\]  

(4.2)

\[
nn_j \leq (F_j - E_j) w_j \quad \forall j
\]  

(4.3)

where \( E_j \) is the minimum units of storage needed for an economically feasible FSL at location \( j \), and \( F_j \) is the maximum potential units of storage space at FSL \( j \). These constraints control the opening and closing of FSLs.

---

\(^{41}\) Units of storage are based upon area (e.g., square feet) of storage space.
Each FSL $j$ starts out with an initial allocation of commodity $i$, $IA_{ij}$, and is able to procure additional units of commodity $i$ beyond this initial allocation. Additionally, FSLs may reallocate units of commodity $i$ to other FSLs if they do not need to ship these commodities to FOLs.

Next we define the following variables:

$AU_{ij}$ Additional units of commodity $i$ that is procured at FSL $j$ (this is a gain of inventory at FSL $j$)

$EI_{ij}$ Inventory of commodity $i$ at FSL $j$ that is reallocated to other FSLs (this is a loss of inventory at FSL $j$)

$AP_i$ The sum of the additional amount of commodity $i$ procured across the entire system

The allocation of commodities is controlled through the following constraints:

$$\sum_{kmt} x_{ijkmt} + EI_{ij} \leq IA_{ij} + AU_{ij} \quad \forall i, j \quad (4.4)$$

$$\sum_{j} AU_{ij} \leq \sum_{j} EI_{ij} + AP_i \quad \forall i \quad (4.5)$$

We also need to constrain the amount of commodities that can be shipped on each individual mode $m$ vehicle. The vehicle capacity constraint is defined as:

$$\sum_{i} x_{ijkmt} \leq \gamma_m y_{jkmt} \quad \forall j, k, m, t \quad (4.6)$$

where $\gamma_m$ is the maximum load in tons per mode $m$ vehicle and the variable $y_{jkmt}$ is the number of node $m$ vehicles tasked to transport commodities form FSL $j$ to FOL $k$, beginning loading on time $t$.

Some final necessary variables are:

$q_{jm}$ Number of mode $m$ vehicles available at FSL $j$ at start of time period $t=1$

$v_{jmt}$ Number of mode $m$ vehicles available at FSL $j$ at the end of time $t$

$z_{jkmt}$ Number of mode $m$ vehicles tasked to make the return trip from FOL $k$ to FSL $j$, departing on time $t$
Constraints placing limits on the total number of vehicles available system-wide, equal to the initial number of available vehicles \( C_m \) plus the variable \( r_m \) which denotes the additional number of vehicles procured\(^{42} \), and the total vehicles available for loading at each FSL are defined as:

\[
\sum_j q_{jm} \leq (C_m + r_m) \quad \forall m
\]  

(4.7)

The following flow balance equations for the number of available vehicles, at time periods \( t = 1 \) and \( t \geq 2 \) respectively are needed in order to translate \( q_{jm} \) into \( v_{jm1} \) and \( v_{jm1} \) into \( v_{jm(t-1)} \), etc.

\[
\sum_k y_{jkmt} \leq v_{jm(t-1)} \quad \forall j, m; t \geq 2
\]  

(4.8)

\[
v_{jm1} = q_{jm} - \sum_k y_{jkm1} \quad \forall j, m
\]  

(4.9)

\[
v_{jmt} = v_{jm(t-1)} + \sum_k [z_{jkmt} - y_{jkmt}] \quad \forall j, m; t \geq 2
\]  

(4.10)

The third constraint requires the declaration of parameter \( \tau_{jkm} \), which is equal to the one-way transportation time from FSL \( j \) to FOL \( k \) (or in the opposite direction) via mode \( m \).

FSL MOG constraints are defined in such a way as to account for both the amount of physical space the vehicle takes up on the ground and the amount of time the vehicle is on the ground. The MOG at each FSL is modeled separately for each of the three classes of vehicles. Air, ground, and sea vehicles each use different loading equipment and also have different physical space requirements (e.g. water ports versus airstrips). Each FSL is assumed to have a maximum number of vehicle spaces allowed for loading each class of vehicle at any one time. Within each class of vehicle, different modes are assumed to consume different fractions of this loading space. Also, each of these different modes of transport is assumed to consume the loading space for different lengths of time.

---

\(^{42}\) We are using procured to mean vehicles purchased, not contracted.
Let $A_j$ be the MOG capacity for mode $m$ vehicles at FSL $j$. Then, defining $\alpha_m$ as the number of time periods necessary to load a mode $m$ vehicle, the MOG constraints are defined over all modes $m$ as:\(^{43}\):

$$\sum_{k} \sum_{n=0}^{\alpha_m-1} y_{jkm(t-n)} \leq A_j \quad \forall j, t, m$$  \hspace{1cm} (4.11)

The FOL maximum-on-ground constraints similarly restrict the FOLs based on the available unload space at each FOL.

Once vehicles $y$ finish unloading at FOL $k$, the following constraint reassigns those vehicles to return trips to FSLs:

$$\sum_{j} y_{jkm(t-n)} = \sum_{j} y_{jkm(t-n)} \quad \forall k, m, t$$  \hspace{1cm} (4.12)

Notice that this model formulation does not assign an individual transport vehicle to a single FSL, to a single FOL, or to a single commodity type. Instead, a given C-17 may transport commodities from FSL A to FOL B. After unloading at FOL B, the aircraft then makes the return trip to FSL C where it is loaded with a completely different commodity type. Also, individual FOLs do not necessarily have all of their demand shipped from one FSL. Instead, multiple FSLs may send commodities to a given FOL.

The following constraint limits the average fleetwide utilization for each transport mode over the entire scenario duration, to be less than the planning factor $\sigma_m$:\(^{45}\):

$$\sum_{jkt} \left[ \sum_{jkm(t-n)} \left( y_{jkm(t-n)} + z_{jkm(t-n)} \right) \right] \leq (C_m + r_m) \sigma_m \quad \forall m$$  \hspace{1cm} (4.13)

\(^{43}\) In order to simplify the model, we converted each mode of vehicle within each class into equivalent vehicles of the same type. For example, all modes of air vehicles were converted into C-17 equivalents so that we only have one mode of air vehicle. Similarly, we only have one mode of land vehicle (Truck) and sea vehicle (JHSV).

\(^{44}\) A total of three FSL MOG constraints exist, one for each air, ground, and sea.

\(^{45}\) The utilization rate is expressed as the average flying hour goal per day divided by 24 hours for mode $m$. The average flying hour goal was obtained from USAF 2003. Utilization rate only applies to aircraft, all other modes of transport have a utilization rate of one.
**Objective Function**

The model is solved by finding the set of $q_{jms}$, $v_{jmts}$, $w_j$, $r_m$, $m_j$, $x_{jikmts}$, $y_{jkm}$, $z_{jkms}$, $A_P$, $A_{Uij}$, and $E_{Iij}$ that satisfies the set of contingency requirements and also minimizes the costs of conducting training and deterrent exercises over a given time horizon. That is, the peacetime costs of conducting training and deterrent missions are minimized while the solution set is constrained to have the storage and throughput required to meet “planned” contingency scenarios should deterrence fail. This is accomplished through scheduling major combat operations (MCOs) within our sequence of deployment timelines. The time-phased demands associated with these large contingencies ensure that the FSL network is capable of supporting these large demands. Specifically, the model formulation minimizes the net present value of opening and operating facilities, along with the non-MCO transportation costs, over a specific time horizon. The model outputs a transportation plan and reports the time needed for FOLs to achieve initial and final operational capabilities.

Defining the following cost parameters:

- $\Delta_j$: Fixed cost incurred to open FSL $j$ with $E_j$ square feet of storage space
- $\Xi_j$: Construction cost per unit of storage needed beyond $E_j$ at FSL $j$
- $\Upsilon_j$: Operating cost (discounted over the time horizon) per unit of storage at FSL $j$
- $\Omega_{ijkm}$: Cost per ton of commodity $i$ transported from FSL $j$ to FOL $k$ via mode $m$
- $\Theta_m$: Cost of obtaining an additional vehicle of mode $m$
- $P_i$: Cost of procuring additional tons of commodity $i$
- $L_i$: Cost of reallocating tons of commodity $i$
- $\Psi_{ik}$: Shortfall cost per time unit per ton of commodity $i$ not fulfilled at FOL $k$

and the variable:

- $w_{wj}$: units of storage utilized at FSL $j$

---

46 The cost of reallocating tons of commodity $i$ does not vary as a function of which FSL is reallocating commodities and which FSL is receiving them.

47 $w_{wj}$ is only the square feet of warehouse space utilized at the FSL. It is possible that an FSL has more warehouse space than is being utilized to store commodities so we distinguish between the two (utilized and non-utilized) because operating and maintenance costs will differ between the two.
We attain the following objective function:

\[
\min \sum_j \left( \Delta_j w_j + \Xi_j n_j + \Upsilon_j v_j \right) + \sum_{ijkmt} \Omega_{ijkm} x_{ijkmt} + \sum_m \Theta_m r_m + \sum_i P_i A_P_i \\
+ \sum_{ij} L_{ij} E_{ij} + \sum_{ikt} \Psi_{ikt} s_{ikt}
\]  

(4.14)

There are approximately 80,000 variables and 46,000 constraints for the model. Using an average desktop computer, the model takes between three minutes to fifteen minutes to return a solution depending on the particular model run. The following section explains the different runs that were conducted using this model.

**Overview of the Model Runs**

Our desired end result of the modeling methods developed in this dissertation is to construct a model which will identify a single reliable FSL posture. In order to identify the posture, we perform several alternative sets of runs. The sets are designed to stress the system components in order to identify the single reliable posture. The following sections will explain the details of each of the sets of runs we performed.\(^{48}\)

**Baseline Case**

The first step was to establish a baseline case to compare all future runs against. To determine the baseline, we ran the model using what we call the “global deterrent scenario.” The scenario, presented in Figure 4.1, focuses on supporting a number of deployments in the Persian Gulf region, Asian Littoral, South America, and Africa over a time horizon of six years. We have scheduled the MCOs in the scenario for execution at the end of the six-year period. This approach focuses attention on providing resources to support deterrent deployments to ensure their funding while also placing MCO requirements in the planning, programming, budgeting, and execution (PPBE) process. The exercises vary in sizes of combat support requirements, as shown on the y-axis. The sizes of recent deployments are also given as a reference.

\(^{48}\) Note that for all modeling runs, no additional vehicles or shortfall was allowed (i.e., we set \(r_m=0\) and \(s_{ikt}=0\)).
We solved the problem finding the least-cost solution that has the capability and capacity to meet the operational requirements given all of the constraining factors. The model selected ten FSLs from the list of thirty-eight possible FSL locations (Table 4.2). Figure 4.2 illustrates the geographical dispersion of the selected FSLs; the red dots on the map represent the facilities that were selected. As the figure shows, the FSLs are spread out around the world, with no individual region containing a majority of the facilities.

Table 4.2

<table>
<thead>
<tr>
<th>Baseline Set of Forward Support Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al Udeid AB, Qatar</td>
</tr>
<tr>
<td>Aviano AB, Italy</td>
</tr>
<tr>
<td>Bagram, Afghanistan</td>
</tr>
<tr>
<td>Clark AB, Philippines</td>
</tr>
<tr>
<td>Cotopaxi IAP, Ecuador</td>
</tr>
</tbody>
</table>

Note: IAP = International Airport; AB = Air Base
There are no commodity procurement or reallocation costs associated with this run for a few reasons. We had limited prior knowledge of what commodities to place as an initial allocation at FSLs in the model, so we used the baseline run to determine the initial allocation for all future runs.49 We ran the model assuming that the initial allocation at all FSLs for all commodities was zero and that there was no cost to procure additional commodities. The model chose the amount of commodities at each FSL that would be needed to meet the demands of the scenario, and we used these amounts to determine the initial allocation for all future runs. For all future runs, the initial allocation of commodities at the ten FSLs selected in the baseline case equaled the amount of inventory allocated to that FSL during the baseline run, and all other twenty-eight FSLs had an initial allocation of zero for all commodities. For these future runs,

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49 The limited knowledge we had came from Amouzegar, et. al., 2006 and McGarvey, et. al., 2009. We acknowledge that this "soft" data might play a part in which bases the model chooses to open, and as more concrete values become available, they should be incorporated into the model.
commodity procurement and reallocation costs were included. Figure 4.3 presents the allocation of total commodities at each of the selected FSLs.

**Figure 4.3**

*Initial Allocation of Commodities from Baseline Case*

The total costs, shown in Figure 4.4, were approximately $73 million with construction costs totaling $22 million ($\Xi_j$ in the mathematical model), transportation costs totaling $16 million ($\Omega_{ijkm}$), and operations and maintenance (O&M) costs totaling $35 million ($\Upsilon_j$).

**Figure 4.4**

*Costs Associated with Baseline Case*

---

50 We again acknowledge that this data is “soft” and should be updated as more concrete values become available.

51 These are also the initial allocation of total commodities at each of the FSLs for all future runs in this dissertation.

52 Costs are amortized over ten years.
Remaining Runs

As previously stated, we are interested in identifying a combat support network which performs well even if access is lost to any single FSL location. Table 4.3 gives an overview of the various sets of modeling runs that were conducted in order to identify the reliable posture. We used each of the sets to explore the various policy options that are available to decision makers to build a reliable combat support network. As stated earlier, the three options are:

- placing additional inventory at existing FSLs;
- opening additional FSLs; or,
- using a combination of opening additional FSLs and placing additional inventory at existing FSLs; and,
- greater dispersion of commodity allocations across FSLs.

<table>
<thead>
<tr>
<th>Table 4.3</th>
<th>Summary of Runs Conducted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Set 1</td>
</tr>
<tr>
<td>Allow new FSLs on runs following baseline case</td>
<td>No</td>
</tr>
<tr>
<td>Allow additional inventory at existing FSLs on runs following baseline case</td>
<td>No</td>
</tr>
</tbody>
</table>

The remainder of this section will explain the details of the different sets of modeling runs, with the following section covering the actual results of the runs. The underlying process for each set of runs was similar. For the initial run (the baseline case), we ran the model, allowing the model to select from the list of all possible FSL locations. For all subsequent runs in the sets, we forced locations in and out of the solution in order to examine the different policy options.

During the baseline run, the model selected ten FSLs (Table 4.2). For the first set of runs, we forced each of the ten FSLs out of the solution set one at a time while forcing the other nine FSLs into the solution set.\(^{53}\) We did not allow the option to open any additional FSLs in addition to the nine that were forced open and we placed an upper bound on the amount of

---

\(^{53}\) There were a total of eleven runs for this and all subsequent sets, the baseline run plus ten additional runs. For example, for the first run following the baseline run, Al Udeid AB, Qatar was forced out of the solution while the remaining nine FSLs, Aviano AB, Italy through Tocumen IAP, Panama, were forced into the solution set. A similar process is followed for the remaining nine runs.
inventory at each of the nine FSLs. The upper bound is equal to the initial allocation of
commodities at each site as determined from the baseline run. We were expecting that there was
a high probability of each of the runs returning a result of an infeasible solution and this is what
happened.\textsuperscript{54}

For the second set of runs, we followed the same initial procedure as in the previous set
of runs. We continued to disallow the option to open any additional FSLs besides the nine that
were forced open (excluding the “failed” FSL), but this time we did not place an upper bound on
the amount of inventory at each of these nine FSLs. In other words, the opened FSLs were
permitted to procure additional amounts of commodities to make up for the loss of the one
facility. This set of runs allowed us to explore the first policy option of building a reliable
combat support network by placing additional inventory at the facilities that were previously
opened.

Similarly, for the third set of runs, we used the same initial process as in the previous sets
of runs. For these runs, we allowed the option to open additional FSLs in addition to the nine
that were forced open. However, we re-established an upper bound on the amount of inventory
at each of the original nine FSLs that was equal to the initial amount of inventory at each of the
nine FSLs that were forced open.\textsuperscript{55} This set of runs allowed us to explore the second policy
option of building a reliable combat support network by opening additional facilities to account
for the loss of any one facility.

For the fourth set of modeling runs, we used the same initial process as all of the
previous sets of runs. For the subsequent runs, we allowed the option to open new FSLs in
addition to the nine that were forced open, and did not place an upper bound on the amount of
inventory at each of the open FSLs. This set of runs allowed us to explore the third policy option
of using a combination of opening additional FSLs and placing additional inventory at existing
facilities. The results of all four of the sets of runs are presented in the following section.

\textsuperscript{54} Infeasibility means that no solution exists for the model that satisfies all of the constraints.
\textsuperscript{55} Any additional FSLs that were opened beyond the nine that were forced open did not have an upper bound on
inventory and had an initial allocation of zero. Of course, all of the constraints on storage, shipping, etc. were still
required to be met.
Results of the Four Sets of Runs

For each of the four sets of runs, we have obtained multiple FSL postures. Within each set, ten runs were conducted (in addition to the baseline run), each time eliminating one of the FSLs from the initial set of ten that were selected by the baseline case. Before going through the complete results from each of the ten FSLs being removed for each of the different sets (i.e., all 40 runs), we will use one example case and walk through the results of the four runs. Following this discussion, we will give a summary of the complete results from all of the runs.

For the example case, we will explore what happens when Aviano Air Base, Italy is removed from the solution set. For the first set of runs—removing Aviano Air Base from the set and not permitting any new FSLs to be opened nor any additional inventory—the model could not find a feasible solution. We can interpret this result to mean the model was not able to make up for the loss of access to the commodities at Aviano Air Base with just the nine FSLs that were forced open and existing commodities at the nine locations.

For the second set of runs, allowing additional inventory at existing FSLs, Figure 4.5 shows that procurement costs increased substantially while all other costs remained close to the initial run. The increase in procurement costs is expected because the nine FSLs that are forced open must procure additional commodities to make up for what was previously being supplied by Aviano Air Base (6053 tons of commodities). Transportation costs increased slightly due to the greater distances that some commodities, which were previously shipped from Aviano Air Base, now need to be shipped. Reallocation costs were zero. That is to say, the model did not compensate for the loss of access at Aviano by moving commodities from an FSL that was a far distance from Aviano to an FSL that was nearer to Aviano (e.g., from Clark AB to Al Udeid AB).

The initial allocation is not based in observations of the current system. As explained in the previous section, to determine the initial allocation of commodities at each FSL, we ran the baseline case assuming that the initial allocation at all FSLs was equal to zero. The model run results identified the amount of each type of commodity that was sent out of each FSL. We used these values as the amount of initial allocation for all future runs that were performed. This is important because there should be little excess inventory beyond demand levels at FSLs in the baseline due to this assumption.
Also note that the procured commodities are spread out across the nine FSLs. No one facility received a substantially larger portion of the procured commodities than any other facility (Figure 4.6). Also, no FSL was allocated a lower amount of commodities than they were in the baseline run. The model formulation did not require this to be so, instead the optimal solution did not make use of such a strategy.
For the third set of runs, allowing additional FSLs to be opened but not additional inventory at existing sites, Figure 4.7 shows that the model accounted for the loss of access to Aviano Air Base by opening three additional FSLs: Shaikh Isa, Bahrain; Spangdahlem Air Base, Germany; and Incirlik Air Base, Turkey. The yellow dots on the map represent the three new facilities that were selected. The figure shows the three FSL locations are all geographically relatively close to Aviano Air Base.

Observe that, it was a lower cost solution for the model to open three new facilities and allocate commodities to the three locations instead of opening one additional facility and placing all the procured commodities there.

The procurement costs from this run increased by the same amount as in the previous run while all other costs remained close to the initial run. The procurement costs were the same because the model chose to procure a total of 6053 tons of commodities to replace those commodities lost at Aviano Air Base. The transportation costs are less than for the second set of
runs because commodities do not need to be shipped as far since the new facility locations are located geographically close to Aviano Air Base. In fact, taking out commodity procurement and reallocation costs (which equal “Additional Inventory” costs) which total $62.96 million, this set of runs was only $1.45 million more than the baseline case (Figure 4.8).

Figure 4.8
 Costs Associated with Third Set of Runs

![Costs Associated with Third Set of Runs](image)

The procured commodities are spread out across the three FSLs that were opened with Incirlik Air Base, Turkey receiving the largest portion.

Figure 4.9
 Allocation of Commodities from Third Set of Runs

![Allocation of Commodities from Third Set of Runs](image)
For the fourth set of runs, allowing additional FSLs to be opened and additional inventory at existing FSLs, Figure 4.10 shows that the model accounted for the loss of access to Aviano Air Base by opening two additional FSLs: Spangdahlem Air Base, Germany and Incirlik Air Base, Turkey.

The results show it is a lower cost solution for the model to open two new facilities and allocate commodities in both the locations instead of opening one additional facility and placing all procured commodities there.

Figure 4.10
FSL Locations from Fourth Set of Runs

The procurement costs from this run increased by the same amount as in the previous two runs, again because the model chose to procure a total of 6053 tons of commodities to replace those commodities lost at Aviano Air Base, while all other costs remained close to the base case. The transportation costs are slightly less than the initial run, again because the facility locations that were opened are located geographically close to Aviano Air Base, however some commodities are now traveling a shorter distance (Figure 4.11). As before, taking out
commodity procurement and reallocation costs which total $62.96 million, this set of runs was only $360,000 more than the base case.

**Figure 4.11**
Costs Associated with Fourth Set of Runs

The procured commodities are mostly spread out across the two FSLs that were opened with Incirlik Air Base, Turkey again getting the largest portion. A small amount of additional commodities were placed at six of the original set of existing FSLs from the initial run.

**Figure 4.12**
Allocation of Commodities from Fourth Set of Runs
A Summary of the Results from the Four Sets of Eleven Runs

The previous section went through an example of one facility (Aviano Air Base, Italy) being removed from the solution set. We followed this same procedure for each FSL in the baseline posture (Table 4.2) for a total of forty-one runs (baseline case plus four sets of ten runs each). This section will present the complete results of the forty-one runs. Table 4.4 maps the run number to the name of the FSL that was removed from the solution set for that particular run.

Table 4.4
Details of Runs for Each Set

<table>
<thead>
<tr>
<th>Run Number</th>
<th>FSL Removed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 0</td>
<td>None (Baseline Case)</td>
</tr>
<tr>
<td>Run 1</td>
<td>Al Udeid AB, Qatar</td>
</tr>
<tr>
<td>Run 2</td>
<td>Aviano AB, Italy</td>
</tr>
<tr>
<td>Run 3</td>
<td>Bagram, Afghanistan</td>
</tr>
<tr>
<td>Run 4</td>
<td>Clark AB, Philippines</td>
</tr>
<tr>
<td>Run 5</td>
<td>Cotopaxi IAP, Ecuador</td>
</tr>
<tr>
<td>Run 6</td>
<td>Louis Botha, South Africa</td>
</tr>
<tr>
<td>Run 7</td>
<td>Masirah Island, Oman</td>
</tr>
<tr>
<td>Run 8</td>
<td>Roosevelt Roads, Puerto Rico</td>
</tr>
<tr>
<td>Run 9</td>
<td>Paya Lebar, Singapore</td>
</tr>
<tr>
<td>Run 10</td>
<td>Tocumen IAP, Panama</td>
</tr>
</tbody>
</table>

As with the Aviano AB example, the first set of runs, not permitting any new FSLs to be opened nor any additional inventory, the model could not find a feasible solution for any of the runs. This was expected and we can interpret this result to mean the min-cost non-reliability model is not able to account for the loss of any one of the facilities with the determined location-allocation of the remaining network.

For the second set of runs, allowing additional inventory at existing FSLs, the results are similar to those described for the Aviano AB example. The results show that procurement costs increased substantially while all other costs remained close to the initial run (Figure 4.13). There are several things to point out about this set of runs. While procurement costs increased across the board, some runs had a larger increase than others. The main reason this occurs is that the FSL that has been removed for that particular run was allocated a larger amount of inventory, so when the inventory was lost, the model needed to procure a larger amount of commodities. Also, the model returned an infeasible solution when Cotopaxi International Airport, Ecuador was removed from the set. This is because, to a large extent, Cotopaxi supported the FOLs in
South America and when it is removed, none of the remaining FSLs are able to supply the FOLs in South America under the existing time constraints.

**Figure 4.13**

*Complete Cost Results for the Second Set of Runs*

In most cases, the procured commodities are spread out across the nine remaining FSLs (Table 4.5). There were a few cases where an existing FSL received a relatively larger amount than the other remaining FSLs. For example, when Al Udeid Air Base, Qatar was removed from the solution set (Run 1), most of the commodities were procured at Masirah Island, Oman. This is not surprising, as the two facilities are located geographically close to each other and therefore it makes sense that the FOLs previously supported by Al Udeid now receive commodities from Masirah Island. As long as the Masirah Island can handle the increase in capacity, aircraft throughput, etc.; transportation costs will not show a large increase since it is located geographically close to Al Udeid.
### Table 4.5
Complete Commodity Allocation Results for the Second Set of Runs

<table>
<thead>
<tr>
<th>FSL</th>
<th>Run 0</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
<th>Run 6</th>
<th>Run 7</th>
<th>Run 8</th>
<th>Run 9</th>
<th>Run 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tons</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al Udeid Ab, Qatar</td>
<td>9348</td>
<td>17%</td>
<td>8%</td>
<td>0%</td>
<td>17%</td>
<td>20%</td>
<td>2%</td>
<td>5%</td>
<td>2%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aviano AB, Italy</td>
<td>6053</td>
<td>3%</td>
<td>2%</td>
<td>5%</td>
<td>2%</td>
<td>0%</td>
<td>0%</td>
<td>3%</td>
<td>5%</td>
<td></td>
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<tr>
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<td>0%</td>
<td>66%</td>
<td>6%</td>
<td>1%</td>
<td>4%</td>
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</tr>
<tr>
<td>Clark AB, Philippines</td>
<td>5554</td>
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<tr>
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<td>3%</td>
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<td>1%</td>
<td>7%</td>
<td>3%</td>
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</tr>
<tr>
<td>Louis Botha, South Africa</td>
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<td>2%</td>
<td>7%</td>
<td>2%</td>
<td>0%</td>
<td>6%</td>
<td>9%</td>
<td></td>
<td></td>
</tr>
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<td>39%</td>
<td>6%</td>
<td>-1%</td>
<td>-2%</td>
<td>-1%</td>
<td>-1%</td>
<td>-2%</td>
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<td></td>
</tr>
<tr>
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<td>1%</td>
<td>2%</td>
<td>51%</td>
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<td></td>
</tr>
<tr>
<td>Paya Lebar, Singapore</td>
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<td>10%</td>
<td>3%</td>
<td>129%</td>
<td>9%</td>
<td>6%</td>
<td>0%</td>
<td>0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tocumen IAP, Panama</td>
<td>4241</td>
<td>0%</td>
<td>5%</td>
<td>8%</td>
<td>1%</td>
<td>0%</td>
<td>3%</td>
<td>22%</td>
<td>2%</td>
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</tr>
</tbody>
</table>

For the third set of runs, allowing additional FSLs to be opened but not additional inventory at existing sites, the results show that in each instance the model opened additional FSLs to compensate for the one that was removed from the solution set. The number of new FSLs ranged from one to three per run. In a few cases, some facilities were opened in support of loss of access to more than one FSL, such as Dakar, Senegal which was opened both when Roosevelt Roads, Puerto Rico and Tocumen International Airport, Panama were dropped from the solution set.

The procurement costs for this set of runs increased by similar amounts to those of the second set of runs, and all other costs remained relatively close to the base case except in one instance (Figure 4.14). Unlike the second set of runs, the model found a feasible solution when Cotopaxi International Airport, Ecuador was removed from the set. Two new FSLs were opened as replacements (APS at Andersen Air Base, Guam and Thumrait, Oman). For this run, the costs increased by a large amount compared to the other runs because of the opening of the APS at Andersen Air Force Base. For the math formulation, we folded the lease cost of the APS, approximately $99 million, under construction costs.
The procured commodities were placed at the newly opened FSLs. When more than one facility was opened, the commodities were dispersed among them. Table 4.6 shows the results of the set of runs. In particular, it shows which new FSLs were opened to replace the one that was removed from the solution set, and also shows the allocation of commodities across the runs.

Table 4.6
Complete Commodity Allocation Results for the Third Set of Runs

<table>
<thead>
<tr>
<th>FSL</th>
<th>Run 0</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
<th>Run 6</th>
<th>Run 7</th>
<th>Run 8</th>
<th>Run 9</th>
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</thead>
<tbody>
<tr>
<td>Tons</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>Percentage Change vs.</td>
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<td></td>
<td></td>
<td></td>
<td>Run 0</td>
</tr>
<tr>
<td>Run 0</td>
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<td></td>
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</tr>
</tbody>
</table>

<table>
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<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
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<td>0%</td>
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</tr>
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<td>0%</td>
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<td>0%</td>
</tr>
<tr>
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</tr>
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</tr>
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</tr>
<tr>
<td>Masirah Island MPT, Oman</td>
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<td>-2%</td>
<td>0%</td>
<td>-2%</td>
<td>0%</td>
<td>-1%</td>
<td>-2%</td>
</tr>
<tr>
<td>Roosevelt Roads, Puerto Rico</td>
<td>2069</td>
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<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
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<td>0%</td>
</tr>
<tr>
<td>Paya Lebar, Singapore</td>
<td>3452</td>
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<td>0%</td>
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<td>0%</td>
</tr>
<tr>
<td>Tocumen IAP, Panama</td>
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<td>0%</td>
<td>0%</td>
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<tr>
<td>Total Tons</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The percentage change is calculated compared to Run 0.
For the fourth set of runs, allowing additional FSLs to be opened and additional inventory at existing FSLs, each run used a different combination of placing additional inventory at existing FSLs and opening new FSLs. The number of new FSLs ranged from zero to two per run. The only new facility that was opened across multiple FSL failures was Seeb, Oman, which was opened on two occasions.

The model returned a feasible solution for each run with all costs remaining relatively close to the previous runs (Figure 4.15). As in the previous set of runs, the large increase in costs when Cotopaxi International Air Port, Ecuador was removed is a result of the lease cost of the APS at Andersen Air Force Base.

**Figure 4.15**
**Complete Cost Results for the Fourth Set of Runs**

![Complete Cost Results for the Fourth Set of Runs](image)

Table 4.7 shows the commodity allocation for the set of runs. It also shows which new FSLs were opened to replace the one that was removed from the solution set.
Table 4.7
Complete Commodity Allocation Results for the Fourth Set of Runs

<table>
<thead>
<tr>
<th>FSL</th>
<th>Run 0</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
<th>Run 6</th>
<th>Run 7</th>
<th>Run 8</th>
<th>Run 9</th>
<th>Run 10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tons</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al Udeid AB, Qatar</td>
<td>9348</td>
<td>0%</td>
<td>2%</td>
<td>5%</td>
<td>0%</td>
<td>0%</td>
<td>2%</td>
<td>7%</td>
<td>0%</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>Aviano AB, Italy</td>
<td>6053</td>
<td>2%</td>
<td>0%</td>
<td>2%</td>
<td>1%</td>
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<td>0%</td>
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<td>5%</td>
</tr>
<tr>
<td>Bagram, Afghanistan</td>
<td>1919</td>
<td>11%</td>
<td>29%</td>
<td>2%</td>
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<td>0%</td>
<td>7%</td>
<td>9%</td>
<td>4%</td>
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</tr>
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The results from the modeling runs are not very useful to a policy maker. Because we cannot assume that the policy maker knows in advance for which facility access may be lost, there is no way for them to know which of these forty-one postures to implement. It’s possible that they could take aspects from each posture, but it’s not even clear how they would do this. Further modeling techniques are needed to assist the policy maker in identifying a single reliable posture.

The model developed in this chapter will be the base of all additional model formulations in this dissertation. Chapter 5 will introduce the single node failure reliability model that we developed to identify a single posture reliable combat support network. The second half of the chapter will compare the single posture solution to the multiple posture solution returned from the model in this chapter.
Chapter 5: Single Node Failure Reliability Model

In the previous chapter, we took the “naïve” approach to reliability modeling. The model assumes that the policy maker knows in advance which facility access would be lost to, and can make decisions to fortify the network against this loss. In reality, the sequencing is reversed and the policy maker should design a network which satisfies demand, irregardless of which facility access is lost too. This chapter identifies a method that accomplishes this task.

In the previous chapter, we described the non-reliable optimization model (NRM) that we constructed, explained the different runs that were performed, and displayed the results of those runs.57 As noted, those runs returned multiple solutions, with a different posture for each loss-of-access scenario. In this chapter, we will discuss the changes that we made to the NRM in order to return a single reliable posture. We will begin by presenting the mathematical details of the new reliability model (RM). Then we will explain the runs that were conducted as well as the results of those runs. Later on in the chapter, we will show that the solution returned from the model ensures that if any one FSL is removed from the solution set, demand will still be met and all constraints will be satisfied. We will conclude the chapter by comparing the single posture solution returned from the RM with the multiple posture solutions previously returned by the NRM.

The Mathematical Model

In this section we present the mathematical modifications we made to the NRM in order to construct the RM. For the most part, the same sets of variables and constraints remain in the model as before; however, some additional variables and constraints were needed in order to require that the model satisfy the demand constraint even in the event of a single facility failure.58 We will not repeat the variable and data parameter definitions that were defined in the previous chapter.

In order to facilitate the changes to the model, we first have to update the data set. From set $J$, the complete set of all thirty-eight FSLs presented in Table 3.4, we made a series of

---

57 In the previous chapter we called this model the multiple posture model. Henceforth, we will refer to this model as the non-reliable model (NRM).
58 Appendix D presents the complete GAMS model coding for the RM. Appendix C presents the complete mathematical model along with a detailed discussion when differences occur from the NRM.
subsets, $J_1$ through $J_{38}$. Each subset contains thirty-seven FSLs with a different FSL having been removed from the complete set of thirty-eight. For example, subset $J_1$ contains all of the FSLs except for Andersen AFB, Guam. Subset $J_2$ includes all of the FSLs (including Andersen AFB, Guam) but now has Aviano AB, Italy removed. This process continues for all thirty-eight subsets so that each of the FSLs has been removed from one, and only one, subset.

**Recourse**

In order to construct the RM model, we need to introduce the concept of “recourse.” Recourse refers to constraints where decisions are dependent on the loss of access scenario, and applies primarily to the variables or constraints with a “$t$” subscript. Non-recourse refers to a constraint where a single decision is made that applies across all loss-of-access scenarios. The reason we do not allow recourse for all constraints is that some decisions cannot be made at the spur of the moment while others can be made more quickly. Construction is an example of a non-recourse decision. Once a decision is made on the number and size of facilities needed, it takes time to build these facilities. If we determine later, when access is lost to an FSL, that more or less facility space is needed in the network, it would take even more time to make these changes. Therefore, we will determine the maximum amount of storage space that may be needed at a facility and ensure that this amount is available. In contrast, shipping decisions can vary across scenario and are therefore a recourse decision. Whether or not a vehicle should be routed from one facility to another can be decided upon and the change made with very little time needed.

An important example of a constraint allowing for recourse is the demand constraint. The demand constraint requires that the quantity of commodity $i$ sent from FSL $j$ to FOL $k$ be greater than or equal to the cumulative demand of commodity $i$ at FOL $k$ by time $t$. Since what is sent from FSL $j$ depends on if FSL $j$ is available (i.e., access is lost or not), we will allow for recourse. The constraint on storage capacity at each FSL is an example of a constraint not allowing for recourse. We will ensure that there is enough storage capacity at an FSL to support the maximum commodity allocation across all scenarios. We will use these concepts in the following section.
Variables

In the previous chapter, we defined the variables $x_{ijkm}$ and $s_{ikt}$. In the RM, we replace them with the following definitions:\textsuperscript{59}

\[ x_{ijkm} \] Quantity of commodity $i$ sent from \textit{FSL} $j$ \textbf{which is part of subset} $J_1$ to \textit{FOL} $k$ via mode $m$, beginning loading on time $t$, allowing the model to capture movement decisions in the event of loss of access to FSL 1

\[ s_{ikt} \] Shortfall below demand for commodity $i$ at FOL $k$ not fulfilled by time $t$ \textbf{when access is lost to FSL} 1

Because our data set has thirty-eight elements in set $J$, we need to create thirty-eight versions of each variable ($x_1, \ldots, x_{38}$ and $s_1, \ldots, s_{38}$) to account for a potential loss of access at each FSL. We will eventually use these variables in a series of demand constraints, each associated with a different subset of FSLs, requiring that demand is satisfied in the event of loss of access at each FSL. We will explain this in further detail in the following section on constraints.

We need to follow a similar procedure for the three variables related to vehicles. We redefine the variables:

\[ y_{ijkm} \] Number of mode $m$ vehicles tasked to transport commodities from \textit{FSL} $j$ \textbf{which is part of subset} $J_1$ to \textit{FOL} $k$, beginning loading at time $t$

\[ v_{ijmt} \] Number of mode $m$ vehicles available at \textit{FSL} $j$ \textbf{which is part of subset} $J_1$ at the end of time $t$

\[ z_{ijkm} \] Number of mode $m$ vehicles tasked to make the return trip from \textit{FOL} $k$ to \textit{FSL} $j$ \textbf{which is part of subset} $J_1$, departing on time $t$

We will also create thirty-eight versions of each of these variables to account for a potential loss of access at each FSL. These new variables will permit us to create a series of additional constraints for each of the previous vehicle constraints, allowing for the full set $J$ of

\textsuperscript{59} In the variable definitions, we will emphasize the part of the definition that changed from how it was defined in Chapter 4, in bold text.
FSLs, and each of the FSL subsets $J_1, \ldots, J_{38}$. As with the additional demand constraints added, we will explain this in more detail in the following section.

Finally, we need to create the variable:

\[ xx_{ij} \]

The maximum total tons of commodity $i$ sent from FSL $j$ across the full set, $J$, of FSLs, and all thirty-eight subsets of FSLs, $J_1, \ldots, J_{38}$.

**Constraints**

Once we identify the variables to add to the model, we need to examine which constraints need to be added to the model, and which existing constraints need to be modified. As we stated at the beginning of the chapter, all of the constraints that were in the NRM are also in the RM, although some are in a slightly modified form. Therefore, as with the section on variables, this section will only discuss those constraints that have been modified or added.

We will next discuss the constraints that allow for recourse. For each of these constraints, there will be thirty-nine versions for our example problem (one for the full set of FSLs, $J$, and one for each subset of FSLs, $J_1, \ldots, J_{38}$). The first constraint in the set of thirty-nine is identical to the constraint that appeared in the NRM.\(^60\) There are an additional thirty-eight similar versions, one for each subset of FSLs.

The easiest place to start is the demand constraint. The demand constraint requires cumulative arrivals at FOL $k$ by time $t$ to satisfy cumulative demand by time $t$. As in the NRM, the demand constraint is defined as:

\[
\sum_{jm,n \leq t} x_{ijkm(n-\omega jkm)} \geq D_{ikt} - s_{ikt} \quad \forall i, k, t
\] (5.1)

We will create an additional thirty-eight versions of the demand constraints, one for each subset of FSLs. The previous constraint ensures that demand is satisfied when all FSLs are available to ship commodities. Now, with the new additional constraints, demand must also be satisfied when commodities cannot be shipped from each individual FSL, as defined across the thirty-eight subsets of FSLs. The new demand constraints are defined as:

---

\(^60\) This constraint is over the full set, $J$, of FSLs.
Later on in the chapter, we will provide an example from our sample problem demonstrating that the solution returned from the RM ensures that if any one FSL is removed from the solution set, demand will still be met and all constraints will be satisfied.

The various constraints involving vehicles also require that additional constraints be added for each subset of FSLs, to allow for recourse. The constraint on the total number of vehicles available for transporting commodities at time \( t \) must be less than or equal to the total number of vehicles available at the end of the previous time period and is defined as:

\[
\sum_k v_{jkmt} \leq v_{jm(t-1)} \quad \forall j, m; t \geq 2
\]  

(5.3)

Thirty-eight additional constraints will be added to the model, one for the decisions made in the event of loss of access at each potential FSL. The constraints are defined as:

\[
\sum_k y_{1jkmt} \leq v_{1jm(t-1)} \quad \forall m; \ j \in J_1, t \geq 2
\]  

\[
\sum_k y_{38jkmt} \leq v_{38jm(t-1)} \quad \forall m; \ j \in J_{38}, t \geq 2
\]  

(5.4)

Multiple versions of the following flow balance equations are also needed. The flow balance equations for the number of available vehicles, at time periods \( t = 1 \) and \( t \geq 2 \) respectively translate \( q_{jm} \) into \( v_{jm1} \) and \( v_{jm1} \) into \( v_{jm(t-1)} \) etc.

\[
v_{jm \prime} = q_{jm} - \sum_k y_{jkm \prime}\quad \forall j, m
\]  

(5.5)

\[
v_{jmt} = v_{jm(t-1)} + \sum_k [z_{jkmt} - y_{jkmt}] \quad \forall j, m; t \geq 2
\]  

(5.6)

The flow balance constraints that have been added to account for each subset of FSLs are defined as:
\[
\begin{align*}
\nu^{1}_{jm} = & \ q_{jm} - \sum_{k} y^{1}_{jkm} & \forall m; j \in J_{1} \\
\vdots & & \\
\nu^{38}_{jm} = & \ q_{jm} - \sum_{k} y^{38}_{jkm} & \forall m; j \in J_{38}
\end{align*}
\] (5.7)

\[
\begin{align*}
\nu^{1}_{jmt} = & \ v^{1}_{jm(t-1)} + \sum_{k} [z^{1}_{jkm(t-\tau_{jkm})} - y^{1}_{jkm}] & \forall m; j \in J_{1}, t \geq 2 \\
\vdots & & \\
\nu^{38}_{jmt} = & \ v^{38}_{jm(t-1)} + \sum_{k} [z^{38}_{jkm(t-\tau_{jkm})} - y^{38}_{jkm}] & \forall m; j \in J_{38}, t \geq 2
\end{align*}
\] (5.8)

In the NRM, there are a total of six MOG constraints, one for each mode of vehicle at the FSLs and FOLs. In the RM, the same six constraints remain and an additional set of constraints are added for each subset of FSLs. For example, the MOG for air vehicles at FSL \(j\) is defined as:

\[
\sum_{k} \sum_{n=0}^{\alpha_{m-1}} y_{jkm(t-n)} \leq A_{jm} \quad \forall j, t, m
\] (5.9)

The additional MOG for air vehicles at FSL \(j\) constraints added to the reliability model are defined as:

\[
\sum_{k} \sum_{n=0}^{\alpha_{m-1}} y^{1}_{jkm(t-n)} \leq A_{jm} \quad \forall t, m; j \in J_{1}
\] (5.10)

The constraints controlling for how many vehicles are needed to transport commodities from FSL \(j\) to FOL \(k\) are defined as:

\[
\sum_{i} x_{ijkmt} \leq y_{m} v_{jkm} \quad \forall j, k, m, t
\] (5.11)

\(^{61}\) A total of three types of FSL and FOL MOG constraints exist, one each for air, ground, and sea.
The additional constraints to capture decisions under recourse are defined as:

\[ \sum_{i} x_{ijkmt} \leq \gamma_{m} y_{jkmt} \quad \forall k, m, t; j \in J_1 \]

\[ \vdots \]

\[ \sum_{i} x_{38,ijkmt} \leq \gamma_{m} y_{38, jkmt} \quad \forall k, m, t; j \in J_{38} \]  

(5.12)

Similarly, the constraint that reassigns vehicles for return trips to FSLs once they complete unloading at FOL \( k \) is defined as:

\[ \sum_{j} z_{jkmt} = \sum_{j} y_{jkmt(t-\omega_{jk})} \quad \forall k, m, t \]  

(5.13)

The constraints added to the model are defined as:

\[ \sum_{j \in J_1} z_{1,jkmt} = \sum_{j \in J_1} y_{1,jkmt(t-\omega_{jk})} \quad \forall k, m, t \]

\[ \vdots \]

\[ \sum_{j \in J_{38}} z_{38,jkmt} = \sum_{j \in J_{38}} y_{38,jkmt(t-\omega_{jk})} \quad \forall k, m, t \]  

(5.14)

The final vehicle constraint that must be modified to account for decision making under recourse is the constraint limiting average fleetwide utilization for each transport mode. The constraint is defined as:

\[ \sum_{jkt} [r_{jk}(y_{jkmt} + z_{jkmt})] \leq (C_{m} + r_{m}) \sigma_{m} \quad \forall m \]  

(5.15)

The additional constraints that have been added are defined as:
The following constraints are used to compute the value of $x_{ij}$, which does not allow for recourse. By enforcing the set of constraints, the model ensures that $x_{ij}$ is the maximum total tons of commodity $i$, sent from FSL $j$, across the complete set of FSLs and all 38 subsets of FSLs. The set of equations are defined as:

$$
\sum_{k} \sum_{t} \left[ \tau_{jkm} \left( y_{jkm}^{1} + z_{jkm} \right) \right] \leq (C_{m} + r_{m}) \sigma_{m} \quad \forall m
$$

(5.16)

$$
\vdots
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$$
\sum_{k} \sum_{t} \left[ \tau_{jkm} \left( y_{jkm}^{38} + z_{jkm}^{38} \right) \right] \leq (C_{m} + r_{m}) \sigma_{m} \quad \forall m
$$

The following constraints are used to compute the value of $x_{ij}$, which does not allow for recourse. By enforcing the set of constraints, the model ensures that $x_{ij}$ is the maximum total tons of commodity $i$, sent from FSL $j$, across the complete set of FSLs and all 38 subsets of FSLs. The set of equations are defined as:

$$
\sum_{kmt} x_{ijkmt} \leq x_{ij} \quad \forall i, j
$$

(5.17)

$$
\sum_{kmt} x_{1ijkmt} \leq x_{ij} \quad \forall i; j \in J_{1}
$$

(5.18)

We will now discuss the constraints that were modified that do not allow for recourse. As a reminder, non-recourse refers to constraints where decisions are not dependent on the different loss of access scenarios, the decisions are made only once. For these constraints we will utilize the new variable we computed $x_{ij}$, which equals the maximum total tons of commodity $i$ sent from FSL $j$ across the full set, $J$, of FSLs, and all thirty-eight subsets of FSLs, $J_{1}, \ldots, J_{38}$. There is no recourse with this variable because the variable does not differ over time (it has no “$t$” subscript). Once the value is determined, it remains the same over the entire time period. The first non-recourse constraint controls for FSL storage capacity. The only difference between the way the constraint appears in the RM and the way it appeared in the NRM is the use of the $x_{ij}$ variable. Using this variable ensures that enough storage is available for the maximum amount of commodities that the FSL may have on hand across all loss of access scenarios.\footnote{As explained on page 69, once a decision is made on the number and size of facilities needed, it takes time to build these facilities. If we determine later, when access is lost to an FSL, that more or less facility space is needed in the network, it would take even more time to make these changes. Therefore, we will determine the maximum amount of storage space that may be needed at a facility and ensure that this amount is available.} The constraint is defined as:

$$
\sum_{kmt} x_{ijkmt} \leq x_{ij} \quad \forall i; j \in J_{1}
$$
\[ \sum_i x_{ij} \leq E_j w_j + n_{nj} \quad \forall j \] (5.19)

The other non-recourse constraint that we will modify controls for the total amount of commodities in the entire system. The following constraint says that the maximum amount of commodity \( i \) sent out of FSL \( j \) to all FOLs plus the amount of commodity \( i \) reallocated from FSL \( j \) to other FSLs, must be less than the initial allocation of commodity \( i \) at FSL \( j \) plus any additional amount of commodity \( i \) procured at FSL \( j \).

\[ xx_{ij} + EI_{ij} \leq IA_{ij} + AU_{ij} \quad \forall i, j \] (5.20)

All of the remaining constraints that were not discussed in this section remain the same as in the NRM.

**Objective Function**

Similar to the various constraints, the objective function also needs to be modified in the RM to take into account all of the subsets of FSLs. The model is solved by finding the set of \( q_{jm} \), \( v_{jmt} \), \( w_j \), \( r_m \), \( nn_j \), \( x_{ijkm} \), \( y_{jkmt} \), \( z_{jkmt} \), \( xx_{ij} \), \( x_{1ijkmt} \), \( x_{38ijkm} \), \( y_{1jkmt} \), \( y_{38jkmt} \), \( y_{1jkmt} \), \( z_{1jkmt} \), \( z_{38jkmt} \), \( AP_i \), \( AU_{ij} \), and \( EI_{ij} \) that satisfies the set of contingency requirements and also minimizes the costs of conducting training and deterrent exercises over a given time horizon. The difference between the objective function in the RM and that in the NRM is that this time instead of a minimum cost solution, the RM determines finding a minimum average cost solution. Because the transportation and shortfall costs vary across the different loss-of-access scenarios, in order to keep these costs in the appropriate relative magnitude to the other cost components, we average the transportation cost and shortfall cost across the full set, \( J \), of FSLs, and all thirty-eight subsets of FSLs, \( J_1, \ldots, J_{38} \)\.63

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63 For our test problem, we thus divide both the transportation cost and shortfall cost in the objective function by thirty-nine.
We attain the following objective function:

\[
\min \sum_{j} \left( \Delta_j w_j + \Xi_j m_j + \Psi_j w_j \right) + \left[ \sum_{i,j,k,m} \frac{\Omega_{ijkm}}{39} x_{ijkm} + \sum_{i,k,m} \sum_{j \in J} \frac{\Omega_{ijkm}}{39} x_{ijkm} + \cdots + \sum_{i,k,m} \sum_{j \in J} \frac{\Omega_{ijkm}}{39} x_{ijkm} \right] + \sum_{m} \Theta_{m} r_{m} + \sum_{i} \sum_{j} \sum_{l} \sum_{k} \sum_{m} \varphi_{ik} s_{k} + \sum_{i} \sum_{k} \sum_{m} \varphi_{ik} s_{k} + \cdots + \sum_{i} \sum_{k} \sum_{m} \varphi_{ik} s_{k} \right] \tag{5.21}
\]

With all of the modifications, we have dramatically increased the numbers of variables and constraints in the model; and therefore, the computing power necessary to solve the model. Where the NRM has approximately 80,000 variables and 46,000 constraints, the RM has approximately 1,000,000 variables and 570,000 constraints. Due to the size of the RM, it was necessary for us to go through an iterative process to solve the model.\(^{64}\)

**Running the Model**

We first attempted to solve the entire MIP model; however, the computer memory was quickly consumed and the model was not able to obtain a solution. Even though the model did not return an optimal MIP solution, it did get far enough in the process to return a root relaxation solution\(^{65}\) of $301,984,777. Given how the GAMS user interface works, we were unable to observe the variable values that accompany this solution. Since we needed to reduce the size of the model in some way to get the model to run, we wanted to identify the variable values from this root relaxation solution. The strategy for doing this is that if we can identify variables with a value of zero in the relaxed solution, we could force those variables to zero prior to solving the MIP in order to reduce the overall size of the problem that the program has to solve.\(^{66}\)

In order to identify the variable values, we changed the GAMS coding from a MIP to a relaxed mixed integer programming problem (RMIP) and ran the model.\(^{67}\) One would expect that the RMIP would complete the run and that the solution returned would be close to the root

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\(^{64}\) Note that for all modeling runs, no additional vehicles or shortfall was allowed (i.e., we set \(r_m = 0\) and \(s_{ikl} = 0\), \(s_{l1} = 0\), ..., \(s_{l38} = 0\)).

\(^{65}\) When GAMS begins solving the mixed integer programming model, one of the first steps in the process is that it tries to find a “relaxed” solution. That is, it relaxes the integrality of the variables and solves the model.

\(^{66}\) Note that such an approach cannot be guaranteed to return a truly optimal solution. However, we can compare the objective function value obtained by this approach to the root relaxation solution and identify an upper bound on our solution’s optimality gap.

\(^{67}\) GAMS allows the user to solve a relaxed mixed integer programming problem (RMIP) in which the program relaxes the integrality of the variables and solves the model.
relaxation solution of the MIP. This in fact did occur; the RMIP completed the run and returned an optimal value of $301,984,778. This solution is within $3 \times 10^{-7}$ percent of the root relaxation solution. We were now able to observe which $w_j$ variables had a nonzero value in the optimal RMIP solution. Table 5.1 lists the twelve FSLs thus identified by the RMIP.

<table>
<thead>
<tr>
<th>FSLs Used in the RMIP Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al Udeid AB, Qatar</td>
</tr>
<tr>
<td>Aviano AB, Italy</td>
</tr>
<tr>
<td>Bagram, Afghanistan</td>
</tr>
<tr>
<td>Clark APT, Philippines</td>
</tr>
<tr>
<td>Cotopaxi, Ecuador</td>
</tr>
<tr>
<td>Masirah Island, Oman</td>
</tr>
</tbody>
</table>

Note: IAP = International Airport; APT = Airport

Using the set of twelve FSLs identified by the RMIP and forcing the remaining twenty-six closed, we then reran the MIP problem. Even though we reduced the overall size of the model, all available computer memory was consumed again and the model was not able to obtain an integer solution. However, we were again able to observe the root relaxation solution and its objective function value was within $4 \times 10^{-7}$ percent of that identified previously in the “full” MIP, as would be expected. In reviewing the details of the modeling run, we observed that the model ran out of memory while trying to satisfy the various vehicle constraints.

As a final attempt at solving the MIP problem, we relaxed the integrality of the vehicle variables, $y_{jkm}$ and $z_{jkm}$ variables. Even though the objective value returned from this model run will not correspond to an integer solution for $y_{jkm}$ and $z_{jkm}$, we retain integrality on the FSL opening variable, $w_j$. The posture definition provides location ($w_j$) and allocation ($xx_{ij}$) variable values (i.e., non-recourse definitions). Therefore, the values of $w_j$ and $xx_{ij}$ that the model identifies will be used to compare the RM model to the posture identified by the NRM.

We ran the MIP model treating $y_{jkm}$ and $z_{jkm}$ as continuous variables and the model returned the objective value of $302,265,654$ which was within one percent of the original root relaxation solution. The model opened all twelve remaining FSLs and allocated between 3,000 and 6,000 tons of commodities to each location. The commodity allocations from the run are presented in Figure 5.1. Note however, that these costs cannot be contrasted directly with the cost obtained by the NRM in Chapter 4, since the vehicle variables were restricted to integer
values in the NRM, but allowed to take continuous values in the RM (this would lead to the RM solution achieving a smaller cost than would be otherwise possible).

**Figure 5.1**
Allocation of Commodities from the RM

![Total Allocation of Commodities (in tons)](chart)

**Reliability vs. Non-Reliability Model**

In order to compare the results returned by the RM to those returned by the NRM, we will test the posture identified by each (the FSLs selected and the commodity allocation to each of the FSLs) against a set of loss-of-access scenarios. To make any comparisons between the RM and NRM, a decision needs to be made on which set of NRM runs to use for comparison (i.e., runs allowing no additional inventory and no additional FSLs, runs allowing additional inventory, runs allowing additional FSLs, and runs allowing both).

We will first compare the results returned by the RM to those returned from the baseline case of the NRM. The baseline case of the NRM is the best possible outlook for the future, the model identified an FSL posture assuming there was zero probability that access would be lost to an FSL in the network. We will then use the fourth set of runs from the NRM, the set allowing for additional inventory at existing sites and additional FSLs to be opened, to compare against the RM results. We should point out that this is not exactly a fair comparison for any individual loss-of-access scenario, as the fourth set of runs from the NRM is the “best possible
performance.” That is, it assumes that the decision makers know in advance which FSL they are
going to lose access to and are able to open additional FSLs and procure additional inventory for
existing sites in response. However, we wish to see how the results of the RM stack up against
this “best possible case”.

Validity of Reliability Model

We will evaluate the performance of the RM solution by running this posture through the
NRM. We begin by forcing on (fix the \( w_j \) values to one) the twelve FSLs selected by the RM
and force off (fix the \( w_j \) values to zero) all of the remaining twenty-six FSLs. We also set a
lower bound on the available inventory at each of the twelve opened FSLs equal to the allocation
of commodities at the FSL identified from the RM solution. This will ensure that the FSLs and
commodity allocations are the same as those returned by the RM. Finally we set additional
procurement beyond this allocation of resources at each FSL, equal to zero.68 In other words, the
same set of FSLs and resource allocations will be used for this run as were returned from the
solution of the RM run. The integer restrictions now apply to the vehicle variables. Note that
the RM could potentially be infeasible when you apply integer restrictions to the vehicle
variables.

We first ensured that we accomplished what we wanted to in the RM, i.e., ensure that if
access is lost to any one facility along with the commodities located there, the remaining posture
will continue to satisfy demand without the procurement or reallocation of commodities nor the
opening of any additional FSLs. As was done for the NRM, we ran the model twelve times, each
time forcing one of the twelve FSLs out of the solution set one at a time while forcing the other
eleven FSLs into the solution set. We did not allow the procurement of any additional
commodities by the FSL beyond what was identified by the RM nor did we permit reallocation
across the eleven remaining FSLs. We also did not permit the model to open any additional
FSLs. For each run the model returned a feasible solution.

---

68 In Chapter 4, we used the baseline case to determine the initial allocation at the ten FSLs selected by the NRM.
Those ten FSLs begin with the same initial allocation of commodities for this run.
Comparison to Baseline Case

We want to begin by comparing the results of the RM against those of the baseline case of the NRM. With the changes made, we ran the model. The objective value was $226,492,790. The complete results are shown in Figures 5.2 and 5.3.

Figure 5.2
Afloat Prepositioned Ship Costs Separated Out

Figure 5.3
Allocation of Commodities from the NRM and RM
Figure 5.2 shows an estimate of the cost of risk reduction. At first glance, it appears that the objective value has increased substantially from $73,511,805 for the NRM to $226,492,790 for the RM, an approximate 208% increase. Note the large APS cost of $98,771,623, associated with the lease and operations of the MUN2 ship based out of Andersen AFB. This APS cost accounts for 65% of the difference in cost between the NRM and RM solution.

Excluding the APS cost, the remaining cost difference between the two runs is approximately $54 million. The other large cost difference between the two runs is the procurement cost. As with the construction cost, this cost difference is easily explained. Total demand in the set of deployment contingencies is equal to 47,429 tons. The total amount of commodity allocations across all FSLs in the RM is 53,247 tons. The largest allocation at any one individual FSL is 5,818 tons. The RM returns a solution that will satisfy all demand even if any one individual facility fails. Therefore, there needs to be enough commodities in excess of total demand across all FSLs such that if access is lost to any FSL (along with the commodities at that FSL), demand is still satisfied. More specifically, since the largest allocation at any one individual FSL is 5,818 tons, there needs to be at least 5,818 tons of commodities in excess of demand. This is exactly what occurs in the RM, with this additional procurement causing the increase in procurement cost.

Also, note that the range of costs increases if one were to assume full recourse over all decisions based on the NRM solution. If we allow for full recourse, the total costs range from a low of $96 million to a high of $269 million. However, one cannot make location/allocation decisions this quickly; and since we do not actually have full recourse, these solutions are infeasible if access is lost to an unplanned site. The RM solution is feasible with partial recourse across all loss of access scenarios.

Figure 5.3 shows that the commodities are allocated more evenly across FSLs for the RM than the NRM. The average commodity allocation per FSL for the NRM is 4743 tons with a standard deviation of 2672. The average commodity allocation per FSL for the RM is 4437 tons with a standard deviation of 1063.

Figure 5.4 shows a summary of the geographical locations of the FSLs.
Comparison to Fourth Set of Runs

Figure 5.5 and Table 5.2 show how the RM stacks up against the complete fourth set of runs from the NRM. We cannot compare the transport costs between the RM and the fourth set of runs. The fourth set of runs is really a best case scenario and if access is lost to any other single facility in the network, the model will return an infeasible solution and won’t satisfy demand. The NRM baseline case and the complete fourth set of runs may look less expensive and therefore attractive, but they do not satisfy demand in the event of any facility loss, only the one facility that was determined in advance (for the fourth set of runs).
For Figure 5.5, as with Figure 5.3, the RM cost lies between the lowest-cost and highest-cost result from set four, and yet the RM does not require perfect information in advance of a facility loss.

**Table 5.2**

<table>
<thead>
<tr>
<th>FSL</th>
<th>Total Amount of Commodities Shipped (in tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability Model Run 1 Run 2 Run 3 Run 4 Run 5 Run 6 Run 7 Run 8 Run 9 Run 10</td>
<td></td>
</tr>
<tr>
<td>Al Udeid, Qatar</td>
<td>5818 9322 9567 9793 9322 9346 9581 10048 9394 9322</td>
</tr>
<tr>
<td>Aviano, Italy</td>
<td>2939 6191 6053 6168 6123 6053 6053 6053 6053</td>
</tr>
<tr>
<td>Bagram, Afghanistan</td>
<td>2971 2133 2483 1956 1919 1919 2051 2101 1986 2070</td>
</tr>
<tr>
<td>Clark AB, Philippines</td>
<td>5818 5530 5755 5573 5530 5569 5825 5639 8067 5721</td>
</tr>
<tr>
<td>Cotopaxi, Ecuador</td>
<td>4919 9357 8922 8945 9153 8908 8714 8973 8714 12222</td>
</tr>
<tr>
<td>Louis Botha, South Africa</td>
<td>2939 2157 2026 2026 2026 2026 2026 2026 2026 2026</td>
</tr>
<tr>
<td>Macirah Island MPT, Oman</td>
<td>5237 4442 4358 3987 3987 3987 4007 3987 4121 4544</td>
</tr>
<tr>
<td>Roosevelt Roads, Puerto Rico</td>
<td>4502 2291 2240 2075 2114 2193 2069 2069 2069 2069</td>
</tr>
<tr>
<td>Paya Lebar, Singapore</td>
<td>2517 3568 3510 3452 3608 3460 3460 3595 3438 3572</td>
</tr>
<tr>
<td>Tocumen IAP, Panama</td>
<td>4702 4241 4279 4365 4241 4365 4241 4471 5139 4464</td>
</tr>
<tr>
<td>Shaikh Isa, Bahrain</td>
<td>7594 904</td>
</tr>
<tr>
<td>Incirlik, Turkey</td>
<td>1386 3043</td>
</tr>
<tr>
<td>Spangdahlem AB, Germany</td>
<td>3551 887</td>
</tr>
<tr>
<td>Seeb MPT, Oman</td>
<td>4150 2180</td>
</tr>
<tr>
<td>Kadana AB, Japan</td>
<td>2215</td>
</tr>
<tr>
<td>U Tapao Internat, Thailand</td>
<td>5288 7583</td>
</tr>
<tr>
<td>Sao Tome-salazar, Sao Tome</td>
<td>1958</td>
</tr>
<tr>
<td>Thumrait MPT, Oman</td>
<td>3967</td>
</tr>
</tbody>
</table>

The total cost for the RM is larger than all of the fourth set of runs of the NRM except for run five of the non-reliability model. Much of the explanation for the cost difference is similar to
the discussion in the previous section. When looking at the total cost components separately, we see that there is not that great of a difference in procurement costs. Most of the differences in costs appear in operating and constructions costs. This is understandable as the RM opened twelve FSLs compared to at most eleven FSLs in the NRM, with one of those twelve FSLs being the APS. The greater number of FSLs leads to larger operating costs. When we look at the fifth run of the NRM, we see that the NRM also opened an APS and in this case, the construction costs are actually greater than those of the RM. There are also no reallocation costs for the NRM runs as each FSL used all of their commodities.

Also of interest is the commodity allocation. It appears that the RM allocates commodities across the FSLs much more evenly than those of the NRM. In all ten runs of the NRM, there are a few significantly larger FSLs, in terms of commodity allocation, than the other FSLs that were opened.

**Policy Insights**

Some interesting policy insights have emerged from the results of the RM, which relate back to the options that are available to policy makers in building a reliable combat support network that we introduced back in Chapter 4. The policy options are: opening additional FSLs, placing additional inventory of commodities at existing FSLs, and more equal dispersion of commodities across FSLs. This section will examine these different policy options by making comparisons between the RM results and NRM results.

**Number of FSLs**

The RM opened twelve FSLs compared to the ten from the baseline NRM (Figure 5.6). All of the FSLs opened by the NRM were also opened by the RM with the exception of Louis Botha, South Africa. The geographical dispersion of the FSLs was similar between the two models with the exception of the South Africa location. No FSLs were opened in Africa for the RM run. It appears that demand requirements in Africa are being supplied for the most part from FSL locations in the Middle East.

We also need to look a little further at the three FSLs that were opened by the RM but not opened by the NRM. The RM opened two additional FSLs at Seeb MPT and Thumrait MPT, Oman. The opening of these FSLs emphasizes the importance of the geographic area. We
should point out that some of this result was driven by the scenarios we used to generate demand for the model runs. However, will the Middle East remain a strategically important area in the foreseeable future.

Also apparent in the results is the importance of a presence in South America. Since only one FSL at Cotopaxi IAP, Ecuador, supports the majority of operations in the area, if access is lost to Cotopaxi, the model chooses to open the APS. If more options existed for potential FSL locations in and around South America, the model would likely lower cost alternatives to choose from in addition to the APS. As mentioned in Chapter 4, it is not that viable sites do not exist on the continent, but there was a lack of data on these potential locations at the time of this dissertation.69

Amount of Commodities

For the most part, when access was lost to a facility, set four of the NRM chose to open additional facilities and place the majority of the procured commodities at the newly opened facilities. The amount of these commodities varies greatly depending on the size of the facility to which access was lost.

The RM also chooses to increase the overall amount of commodities throughout the entire system. More specifically, there are additional commodities in the system in an amount equal to the largest allocation of commodities for any one FSL. This ensures that if the access is lost to the largest facility, there are enough additional commodities in the system to satisfy demand.

Allocation and Dispersion of Commodities

The allocation of commodities across the FSLs in the combat support network varies greatly between the RM and NRM. The RM chose to disperse commodities more evenly across opened FSLs, while the NRM chose to place a relatively large portion of commodities at a few FSLs.

69 Some recent developments may help with this problem. On August 14, 2009, it was announced that the United States and Columbia have reached an agreement where the U.S. will gain access to three Colombian air force bases, two naval bases, and two army installations which, if included in the model when the data becomes available, will increase the number of candidate FSLs (U.S. Department of State, 2009).
For the RM, the average commodity allocation across FSLs is 4437 tons with a standard deviation of 1063 tons. The maximum allocation at any one facility is 5818 tons. For the NRM baseline case, the average commodity allocation across FSLs is 4743 tons with a standard deviation of 2672 tons. The maximum allocation at any one facility is 9348 tons. From a policy and reliability standpoint, the combat support network from the RM is not as dependent on any one individual facility. If any individual facility has a relatively larger share of commodities, it will be more detrimental to lose access to this facility and the commodities located there. These statistics are summarized in Table 5.3.

Table 5.3
Comparison of Statistics from the RM and NRM

<table>
<thead>
<tr>
<th>Allocation of Commodities Across FSLs (in tons)</th>
<th>Reliability Model</th>
<th>Non-Reliability Model</th>
<th>% Change (NRM/RM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>4437</td>
<td>4743</td>
<td>7%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1063</td>
<td>2672</td>
<td>151%</td>
</tr>
<tr>
<td>Minimum (at any 1 FSL)</td>
<td>2937</td>
<td>1919</td>
<td>-35%</td>
</tr>
<tr>
<td>Maximum (at any 1 FSL)</td>
<td>5818</td>
<td>9348</td>
<td>61%</td>
</tr>
</tbody>
</table>

Problems with Expanding to More than One Facility Failure

Theoretically speaking, we can expand the RM to account for a large number of simultaneous possible facility failures. However, the size of the problem as it exists now is too large and complex for most desktop computers to solve. The size of the computing power needed to solve the model has grown exponentially since we have added 920,000 new variables and 524,000 new constraints to the model in comparison to the NRM (a roughly 1248% and 1239% increase, respectively. The following chapter presents a new reliability modeling method that, while less accurate than the current RM, has been developed with the goal of reducing the size of the problem so that it will not consume as much computing power, allowing us to potentially solve problems addressing loss of access to multiple facilities.
Chapter 6: Multiple Node Failure Reliability Model

In the previous chapter, we described the reliability optimization model, RM, we developed to design a reliable combat support network. The model returned a single reliable posture which satisfied all constraints even when access was lost to any individual FSL within the posture. We noted the difficulty with using the RM to examine the case of simultaneous loss of access to multiple facilities due to the large amount of computer memory necessary to solve the model. In this chapter, we will construct an alternative version of the reliability model which takes considerably less computer memory to solve and therefore, can be expanded to consider simultaneous loss of access to multiple facilities. We will begin by presenting the mathematical details of the model, and then explore the runs that were conducted to test this model, as well as the results of those runs. We will conclude by comparing the results of the modeling runs against those of the NRM and the single node failure RM presented in the previous chapter.

The Mathematical Model

The Single Node Failure Case

In this section, we present the mathematical details of the new, alternative reliability model (ARM). We will start by showing the single node failure case, and then expand it into the multiple node failure case in a later section of the chapter. To begin, we will need to go back to the NRM presented in Chapter 4. The same basic sets of variables and constraints remain in the ARM as did in the NRM. A few new constraints and variables were constructed and some modifications were made to existing constraints, we will not repeat the variable and data parameter definitions that were previously defined in Chapter 4.70

We make one addition to the original data set for the ARM. We created the data parameter \( D'_{ik} \) which is equal to the cumulative demand of commodity \( i \) at FOL \( k \) over the entire time period. The mathematical equation used to compute the value is:

\[
D'_{ik} = \sum_i D_{ikt} \quad \forall i, k
\]  

(6.1)

70 Since there is such a small number of additional variables and constraints that were added to the NRM to create this ARM, we will not present the complete mathematical model and GAMS coding in the Appendix. Instead, one simply needs to make the additions and modifications presented in this section with the details of the NRM presented in Appendices A and B.
All other data elements remain unchanged. Most significantly, unlike for the RM presented in the previous chapter, multiple subsets of FSLs are not needed for this model; the ARM uses only the original set $J$.

**Variables**

First, we need to define the new variable:

$$u_{ik}$$  The maximum amount of commodity $i$ sent to FOL $k$ from any single FSL

**Constraints**

Once the additional data parameter and variable are defined, we need to examine which constraints should be added to the model, and which existing constraints need to be modified. For the ARM, we will only be adding two sets of additional constraints.

First, to compute the variable $u_{ik}$, we set the sum of commodity $i$ sent from FSL $j$ to FOL $k$ shipped via all modes of transportation $m$ over the entire time period to be less than or equal to the variable $u_{ik}$. The constraint is defined as:

$$\sum_{mt} x_{jkmt} \leq u_{ik} \quad \forall i, j, k$$  (6.2)

We next create a transportation bound constraint to ensure that the ratio of $u_{ik}$ to $D_{ik}$ is less than or equal to some predetermined percentage. How we decided upon this percentage will be explained in the next section. The constraint is defined as:

$$\frac{u_{ik}}{D_{ik}} \leq \text{predetermined} \% \quad \forall i, k$$  (6.3)

The constraint places an upper bound on the maximum amount of commodity $i$ sent to FOL $k$ from any FSL as a fraction of total cumulative demand of commodity $i$ at FOL $k$.

We will need to modify the demand constraint. As before, the demand constraint requires cumulative arrivals at FOL $k$ by time $t$, to satisfy cumulative demand by time $t$. But now, in addition to satisfying cumulative demand, cumulative arrivals must also satisfy an
additional amount of demand equal to the maximum amount of commodity $i$ sent to FOL $k$ from any FSL. The modified demand constraint is defined as:

$$\sum_{jm, n, t} x_{ijmlm(n-\eta, j, m)} \geq D_{ikt} \left(1 + \frac{u_{ik}}{D_{ik}}\right) - s_{ikt} \quad \forall i, k, t$$ (6.4)

This modified demand constraint ensures that if access is lost to any individual FSL, and that FSL was the maximum supplier of commodity $i$ to FOL $k$, the FOL will still receive enough commodity $i$ to satisfy demand from the remaining FSLs in the combat support network.

All of the remaining constraints that were not discussed in this section remain the same as in the NRM.\(^{71}\)

**Model Runs**

As was first mentioned in the previous section, we constrained the maximum amount of commodity $i$ sent to FOL $k$ from any FSL ($u_{ik}$) as a fraction of total cumulative demand of commodity $i$ at FOL $k$ ($D_{ik}$). This fraction must be less than or equal to some predetermined percentage. We included this constraint to identify multiple feasible solutions for the model. First, we need to determine the minimum percentage that we can use while still returning a feasible solution. In order to determine this value, we made a slight alteration to the model by changing the objective function from the current minimum cost configuration, to instead minimize the ratio of $u_{ik}$ to $D_{ik}$. We performed the run and the model returned a minimum percentage value of 61%.

Now that we determined the minimum percentage that can be used and still get a feasible solution, we returned to the minimum cost formulation of the model. We ran the model using various percentage values starting with 61% going all the way up to 100%.\(^{72}\)

We performed the series of runs using percentage values of 61%, 70%, and 100%. We would expect that as the percentage value increases, the objective value will decrease. This is

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\(^{71}\) Note that for all modeling runs, no additional vehicles or shortfall was allowed (i.e., we set $r_m=0$ and $s_{ikt}=0$).

\(^{72}\) Before starting these runs, we performed a “quality assurance” run using 60% to ensure that the model returned an infeasible solution. If the model did not return an infeasible solution, then 61% is not the true minimum percentage that can be used. As expected, 60% returned an infeasible solution.
because relaxing the right hand side of the transportation bound constraint allows the model to potentially ship more using least costly alternatives (transportation costs) and open a fewer number of facilities (operating and construction costs).

Figure 6.1 presents the costs associated with the three different runs.

Figure 6.1
ARM Runs Costs

Costs are fairly consistent across the three runs. As we relaxed the upper bound on the percentage of demands sourced from any single FSL, total cost decreased. Transportation, construction, and operating costs decreased while the other costs were fairly consistent. Note that the objective is not meaningful in the sense that the values in Figure 6.1 are not directly comparable to any of the results from the previous chapters. We will run this posture (FSLs and allocations) through the NRM model and we will compare the values obtained from those runs to the results from the previous models (these values are shown in Figure 6.3).

Table 6.1 shows the FSLs that were selected for each run.
### Table 6.1
ARM Runs FSL Sets

<table>
<thead>
<tr>
<th></th>
<th>61% Run</th>
<th>70% Run</th>
<th>100% Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andersen AB, Guam</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al Udeid Ab, Qatar</td>
<td>Al Udeid Ab, Qatar</td>
<td>Al Udeid Ab, Qatar</td>
<td></td>
</tr>
<tr>
<td>Aviano AB, Italy</td>
<td>Aviano AB, Italy</td>
<td>Aviano AB, Italy</td>
<td></td>
</tr>
<tr>
<td>Bagram, Afghanistan</td>
<td>Bagram, Afghanistan</td>
<td>Bagram, Afghanistan</td>
<td></td>
</tr>
<tr>
<td>Baku, Azerbaijan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clark AB, Philippines</td>
<td>Clark AB, Philippines</td>
<td>Clark AB, Philippines</td>
<td></td>
</tr>
<tr>
<td>Cotopaxi IAP, Ecuador</td>
<td>Cotopaxi IAP, Ecuador</td>
<td>Cotopaxi IAP, Ecuador</td>
<td></td>
</tr>
<tr>
<td>Dakar, Senegal</td>
<td>Dakar, Senegal</td>
<td>Dakar, Senegal</td>
<td></td>
</tr>
<tr>
<td>Andersen AB, Guam</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eielson, Alaska</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incirlik AB, Turkey</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kaduna MIL, Nigeria</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RAF Lakenheath, United Kingdom</td>
<td>RAF Lakenheath, United Kingdom</td>
<td>RAF Lakenheath, United Kingdom</td>
<td></td>
</tr>
<tr>
<td>MUN2, Andersen AFB</td>
<td>MUN2, Andersen AFB</td>
<td>MUN2, Andersen AFB</td>
<td></td>
</tr>
<tr>
<td>Louis Botha, South Africa</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Masroor, Pakistan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Masirah Island MPT, Oman</td>
<td>Masirah Island MPT, Oman</td>
<td>Masirah Island MPT, Oman</td>
<td></td>
</tr>
<tr>
<td>Thumrait MPT, Oman</td>
<td>Thumrait MPT, Oman</td>
<td>Thumrait MPT, Oman</td>
<td></td>
</tr>
<tr>
<td>Moron AB, Spain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roosevelt Roads, Puerto Rico</td>
<td>Roosevelt Roads, Puerto Rico</td>
<td>Roosevelt Roads, Puerto Rico</td>
<td></td>
</tr>
<tr>
<td>U Tapao Internat, Thailand</td>
<td>U Tapao Internat, Thailand</td>
<td>U Tapao Internat, Thailand</td>
<td></td>
</tr>
<tr>
<td>Seeb MPT, Oman</td>
<td>Seeb MPT, Oman</td>
<td>Seeb MPT, Oman</td>
<td></td>
</tr>
<tr>
<td>Shaikh Isa, Bahrain</td>
<td>Shaikh Isa, Bahrain</td>
<td>Shaikh Isa, Bahrain</td>
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<tr>
<td>Paya Lebar, Singapore</td>
<td>Paya Lebar, Singapore</td>
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<tr>
<td>Spangdahlem AB, Germany</td>
<td>Spangdahlem AB, Germany</td>
<td>Spangdahlem AB, Germany</td>
<td></td>
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<tr>
<td>Tocumen IAP, Panama</td>
<td>Tocumen IAP, Panama</td>
<td>Tocumen IAP, Panama</td>
<td></td>
</tr>
</tbody>
</table>

The 61% run opened 26 FSLs while the 70% and 100% runs opened 17 FSLs each. The allocation of commodities is presented in Figure 6.2.
The allocation of commodities amongst FSLs varies between each of the model runs, with some FSLs receiving a relatively larger allocation of assets than other FSLs. The results are fairly consistent across the model runs with respect to which FSLs receive the larger portion of commodities. Table 6.2 presents the complete descriptive statistics.

<table>
<thead>
<tr>
<th>Allocation of Commodities Across FSLs (in tons)</th>
<th>61% Run</th>
<th>70% Run</th>
<th>100% Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>2213</td>
<td>3502</td>
<td>3460</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1473</td>
<td>1421</td>
<td>1404</td>
</tr>
<tr>
<td>Minimum (at any 1 FSL)</td>
<td>208</td>
<td>1770</td>
<td>1767</td>
</tr>
<tr>
<td>Maximum (at any 1 FSL)</td>
<td>6152</td>
<td>7228</td>
<td>7075</td>
</tr>
</tbody>
</table>
Interpreting the Upper Bound

Before comparing the results of this model versus those of the NRM and RM, we should pause to interpret what a 61% upper bound implies. The constraint places an upper bound on the maximum amount of each commodity type that can be sent to each FOL from any single FSL as a fraction of total cumulative demand of each commodity type at the FOL.

Increasing this percentage value allows the model to potentially ship more resources via the various alternatives. If this percentage value gets too low, the constraint on the maximum amount of commodities that can be shipped out of each FSL will eventually return an infeasible solution, because for certain FOLs, a portion of their demands can only be satisfied from a particular FSL.

More specifically, some FOLs only receive a fraction of demand from a specific FSL because they can’t get everything airlifted in. An FOL with big demand and limited airlift capability must use sealift or truck. An APS is one way to do this; however, a certain amount of truck lift may also be needed. For FOLs in places like South America with a requirement for some truck movement, only South American FSLs can be used for this truck based support. There only needs to be one instance of not being able to be below the bound for the problem to be infeasible.

In our example problem, the FOL at Mariscal, Ecuador has a high demand and only limited airlift capability so must receive the majority of their demand via sealift or truck. When examining the output from the model run, it it appears that the 61% of Mariscal’s commodity demand is received from the FSL at Cotopaxi IAP, Ecuador and the APS MUN2 located at Andersen AFB with 100% of these commodities being transported via truck. When we tried to restrict this percentage to a value lower than 61%, the model returned an infeasible solution. This was the only instance of an FOL bumping up against the 61% bound in our example.

Comparison of Models

We next compare the results returned by the NRM, RM, and this ARM using the same procedure that was outlined in the previous chapter. We will run the ARM posture (set of FSLs used and allocation of assets across those FSLs) through the NRM model.

To recap the procedures we will follow, returning to the NRM, we begin by forcing on (fix the $w_j$ values to one) the FSLs selected by the ARM and forcing off (fix the $w_j$ values to
zero) all of the remaining FSLs. We also set a lower bound on the available inventory at each of the selected FSLs equal to the allocation of commodities at the FSL identified by the ARM, allowing for no further procurement or reallocation of assets.

For each of the three variations of the ARM (61%, 70%, and 100%), we ran the NRM multiple times, forcing a single FSL out of the solution set one at a time, to ensure that the solutions could indeed satisfy all demands even in the event of loss of access to any single FSL. For each run the model returned a feasible solution.

**Comparison of Solutions**

We want to compare the results of running the ARM values through the NRM assuming no loss of access, against those of the RM. The complete results of the runs are shown in Figures 6.3 and 6.4.73

---

73 Note that the values are lower than the amounts returned from the ARM model runs (Figure 6.1). As we explained, the results from the ARM model runs are not comparable to any previous model runs, while the results in Figure 6.3 are directly comparable.
Figure 6.4  
Allocation of Commodities from the RM, and ARM

As Figure 6.3 shows, the large APS costs of $98,771,623, is again associated with the lease and operations of the MUN2 ship based out of Andersen AFB. The largest cost category difference between the runs is procurement cost. The procurement cost increase in the ARM is caused from the additional amount of commodities that must be procured in the amount equal to the maximum amount of each commodity type sent to the FOL from any FSL ($u_k$). Part of the explanation for procurement difference is that ARM procures an amount equal to the max sent from any FSL, even though the “most important FSL” will vary across FOLs and commodities. The RM only needs to procure enough to account for the loss of “any single FSL”. This additional amount, $u_k$, must be procured, thus increasing procurement costs. Operating and construction costs are also higher for the 61% ARM run. This is because the 61% ARM opened twenty-six FSLs compared to seventeen for the 70% and 100% ARM runs and twelve opened by the RM, thus increasing operating and construction costs.

Figure 6.5 shows a summary of the geographical locations of the FSLs for the RM and 100% ARM.
Policy Insights

Some interesting policy insights can be made from the results of the ARM. These insights relate back to the three options available to policy makers in building a reliable combat support network that have been discussed throughout this dissertation. These policy options are: opening additional FSLs, placing additional inventory of commodities at existing FSLs, and more equal dispersion of commodities across FSLs. This section will examine these different policy options by making comparisons between the results of this ARM and those of the NRM and RM.
Number of FSLs

The NRM opened ten FSLs and the RM opened twelve FSLs. In contrast, the ARM presented in this chapter opened twenty-six FSLs for the 61% bound run and seventeen FSLs for the 70% and 100% bound runs. The geographical dispersion of the FSLs is consistent across the runs with the exception of Africa. The 61% bound run opened four FSLs in Africa and the 100% bound run opened one FSL in Africa. For the 70% bound run, it appears that demand requirements in Africa are being supplied for the most part from FSL locations in the Middle East. This is similar to what we observed in the previous models. The NRM opened an FSL in Africa while the RM did not. Again, for the RM, most of the demand requirements in Africa are being supplied from FSL locations in the Middle East.

If policy makers are only concerned about overall costs, the much more simplified ARM presented in this chapter stands up pretty well versus the RM presented in the previous chapter. When comparing the RM and the ARM presented in this chapter, the total cost for the RM is $226 million and the total cost of the ARM, using the 100% bound run, is $278 million. However, costs are often not the only concern. The ARM in this chapter chose to open a larger number of FSLs than the RM opened. Looking at the 100% bound run, the model opened seventeen FSLs, compared to the twelve opened by the RM, again showing the ARM stands up well.

Amount of Commodities

For the NRM, when access was lost to a facility, set four of the model runs opened additional facilities placing the commodities procured to replace the commodities that were located at the FSL where access was lost, typically at the newly opened facilities. For the RM, the model increased the overall amount of commodities throughout the entire system by an amount equal to the largest allocation of commodities for any one FSL. The additional commodities were dispersed across several FSLs. For the ARM, the model again increased the overall amount of commodities throughout the entire system, this time by an amount equal to the total sum of the maximum amount of each type of commodity that each FOL receives from any one FSL. The additional commodities were dispersed across all of the FSLs in the system.

The total amount of additional commodities procured in the NRM entirely depends on the size of the facility where access was lost. As demonstrated in previous chapters, this amount
varies greatly. When looking at the RM and ARM, the ARM procures a greater number of additional commodities. This is true for all of the runs independent of what value was used for the percentage bound. The RM procured an additional amount of commodities equal to 5818 tons. The ARM procured additional commodities in the amount of 10,098 for the 61% bound run, 12,109 for the 70% bound run, and 8,609 for the 100% bound run. This is an average increase of 42% for the ARM over the RM.

This is another area for policy makers to consider. Increasing the overall system inventory of commodities will increase other costs including operations and construction costs. If access is lost to a facility and these additional commodities are needed, then the additional costs are worth it. However, a policy maker will not know in advance when designing the combat support network if these additional commodities will be needed. Instead, they will have to weigh the various pros and cons as well as consider the current budgets in making decisions on what amount of additional commodities to procure.

**Allocation and Dispersion of Commodities**

The allocation of commodities across FSLs in the combat support network varies between the NRM, RM, and the ARM runs. The NRM chose to place a relatively large portion of commodities at a few FSLs, while the RM chose to disperse commodities more evenly across opened FSLs. The variation of commodity allocation for the ARM was greater than the RM but less than the NRM.

For the NRM, the average commodity allocation across FSLs is 4743 tons with a standard deviation of 2672 tons. The difference between the maximum allocation at any one facility and the minimum is 7429 tons. For the RM, the average commodity allocation across FSLs is 4437 tons with a standard deviation of 1063 tons. The difference between the maximum allocation at any one facility and the minimum is 2881 tons. For the ARM, averaging across all three runs, the average commodity allocation across FSLs is 3004 tons with a standard deviation of 1524 tons. The difference between the maximum allocation at any one facility and the minimum is 5853 tons.

From a policy and reliability standpoint, the combat support network for the RM is the least dependent on any one individual facility. However, while the ARM has a larger disparity between the largest and smallest facility size in terms of the amount of commodities stored there,
the standard deviation is close to that of the RM showing that the variation is not as large as it appears when only looking at the maximum and minimum values. These statistics are summarized in Table 6.3.

### Table 6.3
Comparison of Statistics from all Model Variations

<table>
<thead>
<tr>
<th>Allocation of Commodities Across FSLs (in tons)</th>
<th>Non-Reliability Model</th>
<th>Reliability Model</th>
<th>Alternative Reliability Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>61% Run</td>
<td>70% Run</td>
<td>100% Run</td>
</tr>
<tr>
<td>Average</td>
<td>4743</td>
<td>4437</td>
<td>2213</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2672</td>
<td>1063</td>
<td>1476</td>
</tr>
<tr>
<td>Minimum (at any 1 FSL)</td>
<td>1919</td>
<td>2937</td>
<td>208</td>
</tr>
<tr>
<td>Maximum (at any 1 FSL)</td>
<td>9348</td>
<td>5818</td>
<td>6110</td>
</tr>
</tbody>
</table>

**Expanding to Multiple Facility Failures**

The difficulty with expanding the RM from the single facility failure to the simultaneous multiple facility failure case is the amount of computing power (primarily memory) needed to solve the model. With the ARM, we no longer have this problem. Where the RM has 570,000 constraints and 1,000,000 variables, the ARM has 52,000 constraints and 80,400 variables. In comparison, the NRM has 46,000 constraints and 80,000 variables. The following section will demonstrate how the ARM can be expanded with only minor changes to consider simultaneous multiple facility failures.

**Multiple Node Failure**

**Mathematical Model**

With only a few minor modifications to the ARM, we can use the model to examine simultaneous multiple facility losses. We call this new model the multiple node failure reliability model (MNFRM). Similar to the single facility failure process we followed, the policy maker may want to consider procuring additional commodities equal to the maximum amount of commodity $i$ sent to FOL $k$ from any individual FSL plus an additional amount equal to the
second largest amount of commodity $i$ sent to FOL $k$ from any individual FSL. This would ensure that—ceteris paribus—if access was lost to the two most-heavily used FSLs, enough excess commodities exist in the system to satisfy demand.

**Redefine Variable**

For the MNFRM, we only need to make one modification to an existing variable in the ARM. We will redefine the following variable:

$$u_{ik}$$ The maximum amount of commodity $i$ sent to FOL $k$ from any two FSLs

Previously the $u_{ik}$ variable equaled the maximum amount drawn from any single facility, now the variable equals the maximum amount drawn from any pair of facilities. To compute this variable, we set the sum of commodity $i$ sent from FSL $j_1$ to FOL $k$ shipped via all modes of transportation $m$ over the entire time period and the sum of commodity $i$ sent from FSL $j_2$ to FOL $k$ via all modes of transportation $m$ over the entire time period (where $j_1$ does not equal $j_2$) to be less than or equal to the variable $u_{ik}$. The constraint is defined as:

$$
\sum_{mt} (x_{ij1km} + x_{ij2km}) \leq u_{ik} \quad \forall i, j_1, j_2, k; j_1 \neq j_2
$$

(6.5)

Elements $j_1$ and $j_2$ both belong to the same set $J$ of FSLs.

**Constraints**

The two constraints containing that variable $u_{ik}$, while not being modified, have different interpretations since $u_{ik}$ goes from representing the maximum amount of commodity $i$ from any single facility to the maximum amount of commodity $i$ from any two facilities.\(^{74}\) First is the transportation bound constraint, which ensures that the ratio of $u_{ik}$ to $D_{ik}$ is less than or equal to some predetermined percentage. As with the ARM, we are again using this constraint to identify multiple feasible solutions. We will obtain the value for the predetermined percentage following the same procedure as with the ARM. The interpretation of the demand constraint also

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\(^{74}\) The mathematical notation remains the same for these two constraints as in the ARM so will not be redefined here.
changes. The demand constraint ensures that if access is lost to any two FSLs simultaneously, and that those FSLs were the two largest suppliers of commodity $i$ to FOL $k$, the FOL will still receive enough commodity $i$ to satisfy demand from the remaining FSLs in the combat support network.

In moving from a reliability model designed to handle single node failures (i.e., the ARM) to the reliability model designed to handle simultaneous multiple node failures (i.e., the MNFRM), the overall number of constraints has increased while the number of variables remains the same. We have gone from 52,000 constraints for the ARM to 159,000 constraints for the MNFRM. The following section describes the model runs we conducted as well as the results of those runs.

**Model Runs**

We only made a minor change to one constraint to go from the ARM to the MNFRM. The reason we made this modification is because when we initially ran the model, it returned an infeasible solution. We identified this constraint as the one driving the infeasibility so we made the modification. The constraint on the total number of vehicles available system wide needs to be increased by some factor, which we call the “vehicle multiplier”, in order to accommodate the increase in the amount of commodities that must be shipped within the network. We simply multiply the right hand side of the constraint by the “vehicle multiplier”. In the following section, we will explain how we determined the value we used for the multiplier. The constraint is defined as:

$$\sum_j q_{jm} \leq (c_m + r_m) \times \text{(vehicle multiplier)} \quad \forall m$$  \hspace{1cm} (6.6)

Thus, this constraint modification is specific to our example problem (i.e., it wouldn’t necessarily be needed for another problem instance).

As mentioned in the previous section, we initially ran the model and it returned an infeasible solution. We identified the constraint in equation 6.8 as the one driving the infeasibility so we made the modification we described. The first step in the modeling process was to identify the minimum value we could use for the “vehicle multiplier” and still get an optimal solution. This was done through a trial and error process. We ran the model with no
“vehicle multiplier” (i.e., a multiplier equal to one) and the model returned an infeasible solution. Using an increment of 0.05, we started increasing the multiplier from one until the model returned a feasible solution at the value of 1.60. We will use this value for our multiplier.

Following the same procedures we used for the ARM to determine the minimum percentage that can be used in the transportation bound constraint while still having the model return a feasible solution; we ran the model returning a minimum percentage value of 74%. This is in line with what we would expect. For the single facility failure case, the ARM, where we were placing a bound on the maximum amount of commodity \(i\) \(FOL_k\) receives from any one facility, we got a minimum percentage of 61%. Since we are now placing a bound on the maximum amount of commodity \(i\) \(FOL_k\) receives from any two facilities, we would expect this percentage to be greater than 61% which is what we observe.

Now that we determined the minimum percentage that can be used, we returned to the minimum cost formulation of the model. We ran the model using the percentage values of 74% and 100%. We performed the two runs using percentage values of 74% and 100%. As previously noted, we would expect that as the percentage value increases, the objective value will decrease. This is because relaxing the right hand side of the transportation bound constraint allows the model to potentially ship more using the least costly alternatives.

Figure 6.6 presents the costs associated with the two runs.

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75 Before starting these runs, we performed the usual “quality assurance” run using 73% to ensure that the model returned an infeasible solution. If the model did not return an infeasible solution, then 74% is not the true minimum percentage that can be used. As expected, 73% returned an infeasible solution.
As expected, total costs decrease from the 74% run to the 100% run. The reallocation, construction, and operating costs are lower and the transportation costs are higher for the 100% run. To better understand these cost differences, we will have to look at the FSLs opened for each run. Note that the objective is not meaningful in the sense that the values in Figure 6.6 are not directly comparable to any of the results from the previous chapters. We will run this posture (FSLs and allocations) through the NRM model and we will compare the values obtained from those runs to the results from the previous models (these values are shown in Figure 6.8). Table 6.4 shows which FSLs were selected.
Table 6.4
MNFRM Runs FSL Sets

<table>
<thead>
<tr>
<th>Run</th>
<th>FSL Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>74% Run</td>
<td>Andersen AB, Guam</td>
</tr>
<tr>
<td></td>
<td>Al Udeid Ab, Qatar</td>
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<tr>
<td></td>
<td>Aviano AB, Italy</td>
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<tr>
<td></td>
<td>Bagram, Afghanistan</td>
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<td></td>
<td>Baku, Azerbaijan</td>
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<td></td>
<td>Balad, Iraq</td>
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<td>BeniSuef, Egypt</td>
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<tr>
<td></td>
<td>Bishkek-Manas, Kyrgyzstan</td>
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<tr>
<td></td>
<td>Chhatrapati Shivaji IAP, India</td>
</tr>
<tr>
<td></td>
<td>Burgas, Bulgaria</td>
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<tr>
<td></td>
<td>Clark AB, Philippines</td>
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<tr>
<td></td>
<td>Cotopaxi IAP, Ecuador</td>
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<td></td>
<td>Dakar, Senegal</td>
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<tr>
<td></td>
<td>Darwin, Australia</td>
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<tr>
<td></td>
<td>Diego Garcia</td>
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<tr>
<td></td>
<td>Djibouti Ambouli, Djibouti</td>
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<tr>
<td></td>
<td>Eielson, Alaska</td>
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<tr>
<td></td>
<td>Incirlik AB, Turkey</td>
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<tr>
<td></td>
<td>Kadena AB, Japan</td>
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<tr>
<td></td>
<td>Kaduna MIL, Nigeria</td>
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<tr>
<td></td>
<td>RAF Lakenheath, United Kingdom</td>
</tr>
<tr>
<td></td>
<td>MUN2, Andersen AFB</td>
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<td></td>
<td>Louis Botha, South Africa</td>
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<td></td>
<td>Chennai IAP, India</td>
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<td></td>
<td>Masroor, Pakistan</td>
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<td></td>
<td>Masirah Island MPT, Oman</td>
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<td>Thumrait MPT, Oman</td>
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<td></td>
<td>Moron AB, Spain</td>
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<td></td>
<td>Sao Tome-salazar, Sao Tome</td>
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<td></td>
<td>Okecie, Poland</td>
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<tr>
<td></td>
<td>Roosevelt Roads, Puerto Rico</td>
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<td>U Tapao Internat, Thailand</td>
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<td>Seeb MPT, Oman</td>
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<td>Shaikh Isa, Bahrain</td>
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<td>Paya Lebar, Singapore</td>
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<td>Souda Bay, Greece</td>
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<td></td>
<td>Spangdahlem AB, Germany</td>
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<tr>
<td>100% Run</td>
<td>Andersen AB, Guam</td>
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<td>Al Udeid Ab, Qatar</td>
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<td>Aviano AB, Italy</td>
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<td>Souda Bay, Greece</td>
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<tr>
<td></td>
<td>Spangdahlem AB, Germany</td>
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<tr>
<td></td>
<td>Tocumen IAP, Panama</td>
</tr>
</tbody>
</table>

There is a dramatic difference between the number of FSLs that were opened for the two runs. The 74% run opened all thirty-eight FSLs (out of a total of thirty-eight possible FSLs) while the 100% run only opened eighteen FSLs. This will account for the construction and operating costs being higher for the 74% run, since the larger number of facilities require more construction and operating costs. This will also account for the transportation costs being higher for the 100% run. Since a smaller number of FSLs were opened in the 100% run, the distance
commodities must travel is greater; that is, the larger number of facilities opened for the 74% run cover a larger geographical area. There is a greater likelihood an FOL will receive commodities from an FSL in the same region.

The obvious question is why did the 74% run open a larger number of facilities than the 100% run. One possible reason is that there may be multiple feasible solutions that return the same objective value but open a different number of FSLs. To test this theory, we reran the model using the 74% value and placed an upper bound on the number of facilities that the model could open, starting with twenty-five facilities and going up to thirty-seven facilities. Each time the model returned an infeasible solution.

In order to find a possible answer to the question, we needed to dig deeper into the model output. When looking at the output from the 74% run, it appears that the FOL at Mariscal, Ecuador bumps up against the 74% upper value for munitions delivery. The output shows that Mariscal is receiving 74% of their demand for munitions from the FSL at Cotopaxi IAP, Ecuador and the APS MUN2 based at Andersen AFB. The remainder of their munitions demand is met from three additional FSLs. When we increased the upper bound to 100%, we see that Mariscal is now receiving 100% of their munitions demand from Cotopaxi and MUN2, and the other three FSLs were not opened. We also observed this in a few other cases leading to more FSLs being opened in the 74% run.

The allocation of commodities is presented in Figure 6.7.
The allocation of commodities amongst FSLs varies between the two model runs. The average allocation of commodities across FSLs is higher for the 100% run with an average of 3474 tons of commodities per facility compared to an average of 1529 tons of commodities per facility for the 74%. The standard deviation is also higher for the 100% run; 1052 tons of commodities for the 100% run compared to 768 tons for the 74% run showing that there is a greater amount of variation in facility size for the 100% run in terms of total commodity allocation. In summary, it appears that the 74% run opens a larger number of facilities with a smaller average commodity allocation at each location than the 100% run. Table 6.5 presents the complete descriptive statistics.

Table 6.5
Comparison of Statistics across Runs

<table>
<thead>
<tr>
<th>Allocation of Commodities Across FSLs (in tons)</th>
<th>74% Run</th>
<th>100% Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>1529</td>
<td>3474</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>768</td>
<td>1052</td>
</tr>
<tr>
<td>Minimum (at any 1 FSL)</td>
<td>559</td>
<td>1404</td>
</tr>
<tr>
<td>Maximum (at any 1 FSL)</td>
<td>3398</td>
<td>5215</td>
</tr>
</tbody>
</table>
Comparison of Models

We will compare the results returned by the NRM, the 100% run of the ARM, and the 100% run of the MNFRM using the same procedure we have followed throughout this dissertation. We will use the FSLs selected by the 100% run of the MNFRM and the commodity allocation at each of the FSLs.

As before, returning to the NRM, we begin by forcing on (fix the \( w_j \) values to one) the FSLs selected by the MNFRM and forcing off (fix the \( w_j \) values to zero) all of the remaining FSLs. We also set a lower bound on the available inventory at each of the selected FSLs equal to the allocation of commodities at the FSL identified by the MNFRM, and disallow further procurement or reallocation.

For both the 74% and 100% variations of the MNFRM, we ran the model forcing various combinations of two different FSLs out of the solution set, while forcing the other FSLs into the solution set. We should note that we did not run every combination of two FSLs through the model because of the sheer number of possible combinations.\(^{76}\) Instead we performed around twenty runs for each variation making sure to run the combinations of two FSLs which would stress the system the most (e.g., for one run we forced the FSLs with the largest and second largest total allocation of commodities out of the solution set). All of the runs returned feasible solutions suggesting that we accomplished our goal; i.e., if access is lost to any two facilities simultaneously along with the commodities located at the facilities, the remaining posture will continue to satisfy all demand. However, we encountered one problem.

The NRM does not have the “vehicle multiplier” in the constraint on the total number of vehicles available system wide that we included in the MNFRM. Therefore, feeding the respective FSLs and commodity allocations through the NRM might possibly return an infeasible solution if there are not enough vehicles available in the network to ship all of the commodities. We only encountered this problem when we forced the combination of MUN2, Andersen AFB and Cotopaxi IAP, Ecuador closed. As we have seen previously through our other model formulations and runs, the FOLs in South America receive their commodities for the most part from Cotopaxi IAP. When Cotopaxi IAP is not available, the model opens the APS MUN2. If MUN2 is not available either, then there are not alternative FSLs allowing for trucking of assets.

\(^{76}\) The number of possible combinations for the 100% variation is 153 and the number of possible combinations for the 74% variation is 703.
and the model returns an infeasible solution since there is a limit on the number of aircraft in the system and without being able to fly a large portion of the commodities in from other FSLs, the time constraint is not met. Therefore, since we are looking at the multiple facility failure case, when we lose access to both Cotopaxi IAP and MUN2 simultaneously, the model will not return a feasible solution unless we increase the available aircraft in the system so more commodities can be flown in. For this run and only this run of the NRM, we included the multiplier value in the constraint on the total number of vehicles available system wide. We previously determined this value to be 1.60. Making this adjustment to the model returned a feasible solution for the run.

**Comparison of Solutions**

We will compare the results from running the 100% MNFRM values through the NRM against those of running the 100% ARM values through the NRM, and the baseline case of the NRM. The various costs of the runs are presented in Figure 6.8.\(^{77}\)

![Figure 6.8 Costs Associated with the NRM, 100% ARM, and the 100% MNFRM](image)

\(^{77}\) Note that the values for the MNFRM are lower than the amounts returned from the MNFRM model runs (Figure 6.6). As we explained, the results from the MNFRM model runs are not comparable to any previous model runs, while the results in Figure 6.8 are directly comparable.
Transportation costs are actually lower for the ARM and MNFRM runs than for the NRM run. The procurement costs increase in the latter two model runs is from the additional amount of commodities that must be procured in both the ARM and MNFRM runs to satisfy the additional amount of commodities that must be supplied to each FOL in excess of demand. Between the two runs, the procurement costs are higher for the MNFRM runs since the additional amount is equal to the maximum amount of commodity $i$ sent to FOL $k$ from any two FSLs compared to the maximum amount of commodity $i$ sent to FOL $k$ from any one FSL for the ARM.

Note that the costs presented for the NRM show the performance under “single facility loss”, these solutions are not capable of supporting loss of two facilities. If we allow for full recourse to overcome the loss of only any single site, the total cost of the NRM varies between $90$ and $265$ million (depending upon which FSL is lost). By contrast, the MNFRM solution allows for all demands to be met in the event of loss of access to any two FSLs, and it does not assume that location/ allocation decisions can be changed instantaneously, at a total cost of $327$ million.

From a policy standpoint, when designing a reliable combat support network, the total costs to go from a network reliable against a single facility failure to a network reliable against two simultaneous facility failures only increases by $48$ million. They can achieve a much greater level of reliability for a cost increase of only 17%. In addition to costs, the policy maker will also have to decide on what level of inventory in excess of demand that they are willing to maintain in the network. Figure 6.9 shows the commodity allocations across the different runs.

Figure 6.9
Allocation of Commodities from the NRM, 100% ARM, and the 100% MNFRM

![Figure 6.9](image)
The total amount of additional commodities procured in the NRM depends on the size of the facility where access is lost. For the ARM, commodities in excess of demand in an amount equal to 8,609 tons were procured. For the MNFRM, commodities in excess of demand in an amount equal to 15,099 tons were procured. The MNFRM procured almost double the additional amount of commodities which can be interpreted to mean that the sum of the maximum amount of commodity \( i \) sent to FOL \( k \) from the largest FSL is approximately equal to the sum of the maximum amount of commodity \( i \) sent to FOL \( k \) from the second largest FSL.

When going through the details of the model runs, this is often what we observed; the maximum amount sent from the largest FSL is equal to the maximum amount sent from the second largest FSL.

For example, Mariscal, Ecuador receives 258 tons of munitions from their largest supplier, Cotopaxi IAP, Ecuador. Mariscal also receives 258 tons of munitions from their second largest supplier, MUN2 at Andersen AFB. For the ARM, additional munitions equal to the amount sent from their largest supplier, 258 tons are procured. For the MNFRM, additional munitions equal to the amount sent from their two largest suppliers, 516 tons are procured. Thus, the MNFRM procures double the additional amount of munitions than the ARM.

The allocation of commodities across the FSLs for the model runs varies. While the NRM places a relatively larger portion of commodities at a few FSLs, the ARM and MNFRM spread commodities out more evenly across FSLs. The network established by the MNFRM is less dependent on any one facility and also has less variation between facilities. The difference between the maximum allocation at any one facility and the minimum is 3732 tons compared to 7429 tons for the NRM and 6284 tons for the ARM. The complete descriptive statistics are summarized in Table 6.6.
Table 6.6
Comparison of Statistics Across Model Variations

<table>
<thead>
<tr>
<th>Allocation of Commodities Across FSLs (in tons)</th>
<th>Non-Reliability Model</th>
<th>Alternative Reliability Model 100%</th>
<th>Multiple Node Failure Reliability Model 100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>4743</td>
<td>3296</td>
<td>3474</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2672</td>
<td>1617</td>
<td>1137</td>
</tr>
<tr>
<td>Minimum (at any 1 FSL)</td>
<td>1919</td>
<td>450</td>
<td>1395</td>
</tr>
<tr>
<td>Maximum (at any 1 FSL)</td>
<td>9348</td>
<td>6734</td>
<td>5127</td>
</tr>
</tbody>
</table>

**Expanding the Model**

While we will not demonstrate it in this dissertation, the model can be expanded very easily to account for three or more simultaneous facility failures. All one would have to do is redefine the $u_{ik}$ variable as we did when moving from the ARM (single facility failure case) to the MNFRM (two facility failure case). For example, to model the three simultaneous facility failure case, $u_{ik}$ would now be defined as the maximum amount of commodity $i$ sent to FOL $k$ from any three FSLs. Further expanding this model to consider three or more simultaneous facility failures will allow the policy maker to build a trade-off curve showing how total costs differ with varying levels of reliability.
Chapter 7: Disaster Preparedness

The reliability modeling methods developed in this dissertation are general enough to be used in many areas of policy analysis. While our example focused on overseas basing options for the Air Force, one could also use the methods for problems in other areas including supply chains, electric power grids, telecommunications, and homeland security. To demonstrate how this would work, we will take a problem from the disaster preparedness policy arena, the Strategic National Stockpile (SNS), and map the problem’s characteristics to the different elements of the location-allocation-flow problem described in Chapter 2.\textsuperscript{78} We will begin with a brief explanation of the problem followed by the mapping.

Background

In the event of a terrorist attack or natural disaster in the United States, large quantities of medicine and other medical supplies may be needed at the site of the attack or disaster. No one can predict exactly where an attack or disaster will occur, and few state and local governments have the resources to create sufficient stockpiles on their own. Therefore, the SNS was created to provide rapid access to these medical supplies when needed. The SNS maintains large quantities of prepackaged medical supplies that are meant to supplement state and local supplies when a public health emergency occurs. The SNS has stockpiled enough medical supplies to simultaneously protect the population of several large cities.\textsuperscript{79} Some examples of the supplies that are stored are: medicines, vaccines, bandages, and equipment (Thompson, 2001).

The medical supplies, equipment, and pharmaceuticals are stored in prepacked air cargo containers, called 12-Hour Push Packages, that are ready for immediate shipment. As the name implies, the 12-Hour Push Packages can be deployed anywhere in the United States within twelve hours after a request is made and the federal decision made to deploy SNS assets. The Push Packages have been configured to fit on either commercial cargo aircraft or trucks (Figure 7.1). They are shipped by commercial vendors through the coordination of the Division of Strategic National Stockpile (DSNS). The movement of a single Push Package requires at least eight 53-foot-tractor-trailer trucks carrying 16 to 18 cargo containers each. Each Push-Package

\textsuperscript{78} We followed a similar procedure for mapping the Air Force combat support network in Chapter 3.

\textsuperscript{79} The exact quantities of SNS assets are not available to the public.
consists of 130 containers. One 12-Hour-Push-Package will take up the entire cargo hold of a large cargo aircraft (Figure 5.2) (Piester, 2008).

Figure 7.1
12-Hour Push Packages in a Storage Facility

Source: http://www.cdc.gov

Figure 7.2
12-Hour Push Packages Being Loaded for Transit

Source: http://www.cdc.gov

While the exact locations are not available to the general public, these medical supplies are strategically located at twelve secure warehouses near major transportation hubs in the United States. The warehouses are located in such a way as to ensure that the supplies will reach the designated site within the twelve hour time requirement.
The decision to deploy SNS assets begins at the local level. When the local officials identify a situation they believe has the potential to threaten the health of their community, they notify the state emergency management agency or health department who in turn, notifies the governor’s office. If the governor’s office determines that the state does not have the resources necessary for the situation, they notify the Centers for Disease Control and Prevention (CDC) or Department of Health and Human Services (DHHS). The CDC and DHHS evaluate the situation and determine if assets are needed, in what quantity, and from where they should ship. They then issue the orders to ship the needed supplies. The Push Packages will arrive at a local receiving, storing, and staging facility (RSS), usually a warehouse (CDC, 2006). The quantity of assets shipped depends on the scale of the emergency (e.g., 1000 versus 1 million people affected), the type of the emergency (contagious versus non-contagious), and the location of the affected area (contained versus open).

Network Mapping

Using the different elements of the location-allocation-flow problem described in Chapter 2, the following list illustrates the mapping.

- **What is Being Shipped**- The items being shipped in the network are the 12-Hour Push Packages consisting of various medical supplies.

- **Demand Nodes**- The demand nodes in our network are local areas where the medical supplies are needed. More specifically, the demand nodes are the local RSS facility.

- **Supply Nodes**- The supply nodes in our network are the storage warehouses located around the country.80

- **Transportation Modes**- There are two modes of commercial transportation that are used, 53-foot-tractor-trailer trucks and cargo aircraft.

- **Costs**- Costs in the network include rental and/or facility construction costs, transportation, procurement costs when assets need to be replenished or replaced because of expiration dates, and other operation and maintenance costs.

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80 Currently, there are twelve locations around the country, but there is no reason to think that if we were modeling this, that twelve is the “right” number of supply nodes or that the current inventory is the “right” level.
- **Time**: From the time the request and the decision to deploy SNS assets is made, the assets need to arrive at the RSS facility within twelve hours. Time also factors into planning decisions.

- **Objective Function**: Depending on policy goals determined by government officials, the objective function may seek to minimize overall costs or transportation time.

Figure 7.3 shows a hypothetical example of what this network could look like.

![Hypothetical Example of Network](image)

We will not go through the process of running the problem through the reliability models we developed in this dissertation, but one can see how the elements of this problem mirror the elements of the Air Force basing problem we focused on in this dissertation and would be solved in a similar manner. By mapping this disaster preparedness problem to the general elements of the location-allocation-flow problem, we have demonstrated that the reliability models we have designed in this dissertation could be useful to decision makers in other types of policy arenas.
Chapter 8: Conclusion

The purpose of this dissertation was to develop analytic tools to assist policy makers in identifying a reliable set of facility locations for the Air Force to place combat support basing materiel that will cover a broad range of potential missions that may occur around the world and allow for potential disruptions to the combat support network. We developed mixed integer programming models to evaluate alternative postures, which are defined as a set of FSLs and an allocation of materiel across those locations. This chapter briefly reviews the models that were developed, the results of the model runs, and their policy implications. The chapter concludes with some suggestions on how this analytic framework can be expanded in future research.

Review of Models and Policy Implications

Multiple Posture Model

We began by developing an optimization model that does not account for the potential loss of access to FSLs, the NRM. We then evaluated the performance of the NRM when we removed single FSLs from the solution, in order to get a better understanding of how the network operated. For each loss-of-access scenario, we identified alternative postures that could meet the demand requirements when a particular FSL failed. The difficulty with trying to identify a reliable network using this model is that policy makers would have to know in advance to which facility location access would be lost, something that is highly unlikely.

The solutions returned from this iterative NRM approach used a combination of policy options in order to satisfy demand when access to any one individual facility, along with the commodities located there, were lost. In some cases, when restrictions were placed on which policy options were available (i.e., opening additional facilities and procuring additional inventory at existing sites), the model returned infeasible solutions; meaning, the model was not able to satisfy demand and all other constraints with the policy options that were available. When the model was free to choose between all policy options, the model chose to use a combination of opening additional facilities and procuring additional commodities at existing facilities; which options were used depended on which facility access was lost to. The total cost of the baseline run of the NRM was $74 million. For the subsequent loss-of-access runs, the total costs of the alternative postures ranged from $96 million to $247 million.
Two issues emerge from the results of these modeling runs. First, several large facilities existed in the solution set (meaning they possess a relatively larger amount of commodity allocation than other facilities). This places a stress on the system if access is lost to these particular facilities and the associated commodities allocated there. Second, the model returned multiple postures, meaning that if a policy maker knows in advance to which facility access will be lost, they can choose from the portfolio of results. However, since knowledge of node failure is most likely not known in advance, if any one individual facility were to fail in the network, the remaining network would not necessarily be able to satisfy demand and meet all other constraints with the components which remain in the network.81

**Single Node Failure Reliability Model**

In order to address these two issues, we designed the RM we presented in Chapter 5. We introduced a series of demand constraints meant to ensure that demand is met if access is lost, along with the allocated commodities, at any one individual FSL. The model returns a reliable solution in the sense that it is a single posture which will satisfy demand in the event of loss of access to any FSL.

Some interesting policy insights emerged from the results of the RM. First, the model opened twelve FSLs compared to the ten opened from the baseline run of the NRM. While the NRM opened FSLs in Africa, the RM solution did not utilize any African FSLs. The RM instead chose to support most demand from African FOLs from FSLs located in the Middle East. This emphasizes the strategic importance of locations in the Middle East beyond just supplying commodities to locations in the Middle East itself and also emphasizes the global view of a combat support network as compared to a theatre view.

Second, the model chose to increase the amount of overall commodities in the system in an amount equal to the largest allocation of commodities for any one individual FSL. This ensures that if the access is lost to the largest facility, there are enough additional commodities in the system to satisfy demand, without the need for any procurement of commodities beyond what exists in the system.

Third, unlike the results of the NRM, the RM chose to disperse commodities more evenly across the opened FSLs. No individual FSL has a relatively larger share of commodities than the

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81 This is what is demonstrated in the first set of model runs in Chapter 4.
other FSLs in the network. From a policy standpoint, based solely on commodity allocation, no individual facility is relatively much more important than the other facilities. Relative costs increased from $74 million for the NRM with no loss of access to $96 to $247 million for each of the individual loss-of-access full recourse postures for the NRM to $226 million for the RM.

**Multiple Node Failure Reliability Model**

Due to the amount of computer memory required to solve the RM when only considering one facility failure, we were unable to examine cases of multiple facility failures. To address this issue, the ARM we designed and presented in Chapter 6 reduces the size of the problem and therefore, the amount of computing power needed to solve multiple facility failure problems.

Going back to the NRM, we added and made a few minor modifications to existing constraints and the objective function. After a bit of a trial and error process, we found a model that returns a network which will satisfy demand and meet all other constraints when access is lost to a single facility in the network, with just the remaining network facilities and the commodities allocated to each of them. We then demonstrated how this model can be expanded to account for simultaneous multiple facility failures.

As with the RM from Chapter 5, some interesting policy insights emerged from the results of the model runs. The model opened a larger number of FSLs than either the NRM or the RM, and overall costs were slightly higher. However, the increases were not substantial in either case and when we take into account the size of the model in terms of computing power, compared to the RM the ARM model is an attractive alternative modeling method.

Similar to the RM from Chapter 5, from a policy standpoint, when only considering commodity allocation, no individual facility is relatively much more important than the other facilities in the network. Total relative costs increased from $226 million for the RM to $278 million for the ARM.

We then demonstrated how the model can be expanded to consider simultaneous multiple facility failures. The MNFRM increased the amount of overall commodities in the system by an amount equal to the total sum of the maximum amount of each type of commodity that each FOL receives from any two FSLs and again dispersed the commodities fairly evenly across the opened FSLs. The results showed that for only a slightly higher cost, we move from a network reliable
against a single facility failure, to a network which is reliable against two simultaneous facility failures.

**Future Analysis**

Extending this research to consider network designs from other policy arenas beyond the Air Force combat support network that we focused on in this dissertation would demonstrate the flexibility of the modeling methods which were presented in this dissertation, as well as provide additional insights into reliable network design options for policy makers. We touched on this briefly in the Chapter 7 when we looked at an example from the disaster preparedness arena.

Should the Air Force wish to continue this specific line of research, the data elements used in this dissertation should be updated as data from more recent Air Force deployments becomes available. Also, the data should be expanded to include more potential FSL locations in South America. Due to the limitations on our data for South American locations, the model is forced to use expensive Afloat Prepositioned Ships (APSs) when considering potential loss of access to the FSL in Ecuador. A greater number of alternative sites in this region could allow the model to choose amongst more budget friendly alternatives.  

Finally, we demonstrated the single node failure case with the RM presented in Chapter 5. With the ARM model presented in Chapter 6, we increased this to consider two facilities failing simultaneously. Future research should continue to expand on these methods in looking at multiple facility loss as well as considering disruptions in the network which may occur along the transportation routes (the arcs), not just at the FSL locations.

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82 As mentioned in Chapter 5, the recent agreement between the United States and Columbia will increase the number of possible locations to consider in the future.
Appendix A

FSL Site Selection and Transportation Non-Reliability Model Formulation

The Forward Support Location Site Selection and Transportation Model has been developed for this dissertation as a tool for optimally allocating WRM resources to FSLs. In this appendix, we present the mathematical formulation of the model. For a further discussion, see Chapter 3.

Sets and Set Indices

\( i \in I \) commodities; \( I = \{PAX, BOS, VEH, JDM, MIS, \ldots\} \)

\( j \in J \) FSL index; \( J = \{FSL1, FSL2, \ldots, APF1, APF2, \ldots\} \)

\( \text{AFL}(J) \) afloat FSLs; \( \text{AFL}(J) \subseteq J \); \( \text{AFL}(J) = \{APF1, APF2, \ldots\} \)

\( k \in K \) FOL index; \( K = \{FOL1, FOL2, \ldots\} \)

\( m \in M \) modes of transport; \( M = \{C-130, C-17, C-5, B747, TRUCK, HSS, \ldots\} \)

\( \text{AIR}(M) \) aircraft; \( \text{AIR}(M) \subseteq M \); \( \text{AIR}(M) = \{C-130, C-17, C-5, B747, \ldots\} \)

\( \text{LAN}(M) \) land vehicles; \( \text{LAN}(M) \subseteq M \); \( \text{LAN}(M) = \{TRUCK, \ldots\} \)

\( \text{SEA}(M) \) sea vehicles; \( \text{SEA}(M) \subseteq M \); \( \text{SEA}(M) = \{HSS, \ldots\} \)

\( h \in H \) phases; \( H = \{1, 2, \ldots\} \)

\( o \in \text{SCN} \) deployment scenarios; \( \text{SCN} = \{1, 2, \ldots\} \)

\( t \in T \) time periods which divide up each phase \( h \); \( T = \{1, 2, \ldots\} \)

Data Parameters

\( \Delta_j \) fixed cost incurred to open FSL \( j \) with \( E_j \) square feet of storage space

\( \Theta_{mh} \) cost of obtaining an additional vehicle of mode \( m \) at the beginning of phase \( h \)

\( \Xi_j \) construction cost per unit of storage needed beyond \( E_j \) at FSL \( j \)

\( \Upsilon_j \) operating cost (discounted over the time horizon) per unit of storage at FSL \( j \)

\( \Psi_{ik} \) shortfall cost per time unit per ton of commodity \( i \) not fulfilled at FOL \( k \)

\( \Omega_{ijkm} \) cost per ton of commodity \( i \) transported from FSL \( j \) to FOL \( k \) via mode \( m \)

\( \alpha_m \) number of time periods necessary to load a mode \( m \) vehicle

\( \beta_m \) number of time periods necessary to unload a mode \( m \) vehicle

\( \gamma_m \) maximum load in tons per mode \( m \) vehicle

\( \varepsilon_m \) maximum load in square feet per mode \( m \) vehicle

\( \zeta_k \) contingency start period at FOL \( k \)
\( \eta_k \) contingency finish period at FOL \( k \)

\( \mu_k \) phase of scenario associated with FOL \( k \)

\( \pi_{1jm} \) additional time needed prior to loading for commodities departing FSL \( j \) via mode \( m \)

\( \pi_{2km} \) additional time needed following unloading for commodities to reach FOL \( k \) via mode \( m \)

\( \rho_m \) conversion factor for parking space for mode \( m \)

\( \sigma_m \) utilization rate, expressed (for airlift) as the average flying hour goal per day divided by 24 hours, for mode \( m \)

\( \tau_{jkm} \) one-way transportation time from FSL \( j \) to FOL \( k \) (or in opposite direction) via mode \( m \)

\( \phi_i \) conversion factor for commodity \( i \) from tons to square feet of storage space

\( g \) maximum square feet per commodity unit of storage

\( P_i \) cost of procuring additional tons of commodity \( i \)

\( IA_j \) initial allocation (in tons) of commodity \( i \) at FSL \( j \)

\( L_i \) cost of reallocating tons of commodity \( i \) independent of FSL locations

\( STEAM_{jk} \) additional time needed for steaming to port before afloat FSL \( j \) can begin offloading at port associated with FOL \( k \)

\( FOLSCEN_k \) scenario associated with FOL \( k \)

\( SCENPHASE_o \) phase associated with scenario \( o \)

\( A_{Nj} \) max on ground, in class \( N \) \([\text{AIR} (M ) = 1, \text{LAN} (M ) = 2, \text{SEA} (M ) = 3]\) equivalent vehicles, at FSL \( j \)

\( B_{Nk} \) max on ground, in class \( N \) \([\text{AIR} (M ) = 1, \text{LAN} (M ) = 2, \text{SEA} (M ) = 3]\) equivalent vehicles, at FOL \( k \)

\( C_{mh} \) planned systemwide inventory of mode \( m \) vehicles at the beginning of phase \( h \)

\( D_{ikt} \) incremental demand, in tons, for commodity \( i \) at FOL \( k \) at time \( t \)

\( E_j \) minimum units of storage needed for an economically feasible FSL at location \( j \)

\( F_j \) maximum potential units of storage at FSL \( j \)

**Variables**

\( nn_j \) additional units of storage needed beyond \( E_j \) at FSL \( j \)

\( q_{jmh} \) number of mode \( m \) vehicles available at FSL \( j \) at the start of time period \( t = 1 \) during phase \( h \)

\( r_{mh} \) additional mode \( m \) vehicles obtained at the beginning of phase \( h \)

\( s_{ikt} \) shortfall below demand, in tons, for commodity \( i \) at FOL \( k \) not fulfilled by time \( t \)

\( v_{jmth} \) number of mode \( m \) vehicles available at FSL \( j \) at the end of time \( t \) during phase \( h \)

\( w_j \) binary variable indicating status of FSL \( j \), \( = 1 \) if opened, \( = 0 \) otherwise
\( x_{ijkmt} \) tons of commodity \( i \) sent from FSL \( j \) to FOL \( k \) via mode \( m \), beginning loading on time \( t \)

\( y_{jkmt} \) number of mode \( m \) vehicles tasked to transport commodities from FSL \( j \) to FOL \( k \), beginning loading on time \( t \). Integer

\( z_{jkmt} \) number of mode \( m \) vehicles tasked to make the return trip from FOL \( k \) to FSL \( j \), departing on time \( t \). Integer

\( \text{ww}_j \) units of storage utilized at FSL \( j \)

\( \text{xx}_j \) total tons of commodity \( i \) sent out of FSL \( j \)

\( \text{pp}_{jho} \) binary variable indicating if afloat FSL \( j \) supports scenario \( o \) during phase \( h \)

\( A\text{U}_{ij} \) additional units of commodity \( i \) (in tons) that is procured at FSL \( j \) beyond the initial amount (\( IA_{ij} \)), this is a gain of inventory at FSL \( j \)

\( EI_{ij} \) inventory of commodity \( i \) (in tons) at FSL \( j \) that is reallocated to other FSLs, this is a loss of inventory at FSL \( j \)

\( AP_i \) the sum of the additional amount (in tons) of commodity \( i \) procured across the entire system, this is a gain of inventory across all FSLs

\( b_{ij} \) binary variable, \( = 1 \) if FSL \( j \) reallocates commodity \( i \) to other FSLs, \( = 0 \) otherwise

\( m \) a very large positive number to ensure that FSL \( j \) can reallocate commodity \( i \) to other FSLs or procure additional units of commodity \( i \) but not both

**Note:** There is an implicit assumption throughout the entire model that terms having an index value \( t \leq 0 \) are not considered.

**Objective Function**

\[
\min \sum_j \left( \Delta_j w_j + \Xi_{jnn} j + \Upsilon_{jww} j \right) + \sum_{ijkmt} \Omega_{ijkmt} x_{ijkmt} + \sum_{mh} \Theta_{mh} r_{mh} + \sum_i P_i AP_i \\
+ \sum_{ij} L_i EI_{ij} + \sum_{ikt} \Psi_{ikt} S_{ikt}
\]

(A.1)

The objective function minimizes the total cost, equal to the sum of the FSL opening cost, the cost for construction of new facilities, the facility operation cost, the transport cost, the cost of procuring new vehicles, the cost of procuring additional commodities, the cost for reallocating commodities between FSLs, and the shortfall cost for not satisfying demand requirements.

**Constraints**

\[
\sum_j q_{jmh} \leq (C_{mh} + r_{mh}) \quad \forall m, h
\]

(A.2)

This constraint limits the total number of available vehicles systemwide.
\[ \sum_{k \in \mu_k = h} y_{jkm} \leq v_{jm(t-1)h} \quad \forall j, m, h; t \geq 2 \quad (A.3) \]

This constraint limits the total number of vehicles that begin loading for transport at FSL \( j \) at time \( t \) to be no greater than the vehicles available there at the end of time \( t-1 \). Note that \( v_{jm} \geq 0 \) and constraint (B.21), taken together, eliminate the need for a version of this constraint at \( t=1 \).

\[ \sum_{k \in \mu_k = h} \sum_{m \in \text{AIR}(M)} \alpha_{m-1} \sum_{n=0}^{\alpha_{m-1}} [\rho_m(y_{jkm}(t-n))] \leq A_{rj} \quad \forall j, t, h \quad (A.4) \]

\[ \sum_{k \in \mu_k = h} \sum_{m \in \text{LAN}(M)} \alpha_{m-1} \sum_{n=0}^{\alpha_{m-1}} [\rho_m(y_{jkm}(t-n))] \leq A_{r2j} \quad \forall j, t, h \quad (A.5) \]

\[ \sum_{k \in \mu_k = h} \sum_{m \in \text{SEA}(M)} \alpha_{m-1} \sum_{n=0}^{\alpha_{m-1}} [\rho_m(y_{jkm}(t-n))] \leq A_{r3j} \quad \forall j, t, h \quad (A.6) \]

The previous three constraints are the Maximum on Ground constraints at the FSL. The constraints are defined in such a way as to account for both vehicle space on the ground and vehicle ground time.

\[ \sum_{j \in \text{AIR}(M)} \sum_{n=0}^{\beta_{m-1}} [\rho_m(y_{jkm}(t-\tau_{jkm} - \alpha_{m-n}))] \leq B_{r1k} \quad \forall k; \ z_k \leq t \leq \eta_k \quad (A.7) \]

\[ \sum_{j \in \text{LAN}(M)} \sum_{n=0}^{\beta_{m-1}} [\rho_m(y_{jkm}(t-\tau_{jkm} - \alpha_{m-n}))] \leq B_{r2k} \quad \forall k; \ z_k \leq t \leq \eta_k \quad (A.8) \]

\[ \sum_{j \in \text{SEA}(M)} \sum_{n=0}^{\beta_{m-1}} [\rho_m(y_{jkm}(t-\tau_{jkm} - \alpha_{m-n}))] \leq B_{r3k} \quad \forall k; \ z_k \leq t \leq \eta_k \quad (A.9) \]

The previous three FOL Maximum on Ground constraints similarly restrict the FOLs based on the unload space available at each FOL.

\[ \sum_{j, m} \sum_{n=1}^{t} x_{ijkm(n-\tau_{jkm} - \alpha_{m-\beta_{m-\pi_{2km}}})} \geq \left( \sum_{n=1}^{t} D_{ikn} \right) - s_{ikt} \quad \forall i, k; \ z_k \leq t \leq \eta_k \quad (A.10) \]

This cumulative demand constraint requires the cumulative arrivals by time \( t \) to be greater than or equal to the cumulative demand by time \( t \), with unmet demand recorded in the shortfall variable \( s \).

\[ \sum_{k \in \mu_k = h} \sum_{i, m, t} \phi_{i} x_{ijkm} \leq g\left(E_{j} w_{j} + n_{n_{j}}\right) \quad \forall j, h \quad (A.11) \]

\[ \sum_{k \in \mu_k = h} \sum_{i, m, t} \phi_{i} x_{ijkm} \leq g\left(w_{w_{j}}\right) \quad \forall j, h \quad (A.12) \]
\[ n_n j \leq (F_j - E_j) w_j \quad \forall j \] (A.13)

\[ w_j \leq w w_j \quad \forall j \] (A.14)

\[ w_j \leq E_j + n n_j \quad \forall j \] (A.15)

FSL storage is limited through the previous five constraints. This set of constraint types also controls the decision of whether to open an FSL at location \( j \).

\[
\sum_{k \in \mu_k = h} \sum_j \left( \sum_t \tau_{jkm}(y_{jkm}) + \left[ \|T\|^{-1} \sum_{t=1}^{\|T\|} \tau_{jkm} z_{jkm} \right] + \left[ \|T\|^{-1} \sum_{t=1}^{\|T\|} (\|T\|-t) z_{jkm} \right] \right) \leq \|T\| (C_m + r_m) \sigma_m \quad \forall m, h
\] (A.16)

This constraint limits the average fleetwide utilization over each phase to be less than the planning factor.

\[
\sum_i x_{ijkm} \leq \gamma_m y_{jkm} \quad \forall j, k, m
\] (A.17)

\[ \zeta_k + \pi_{1jm} \leq t \leq \eta_k - \tau_{jkm} - \alpha_m - \beta_m - \pi_{2km} \]

This constraint translates the tons of commodities transported via mode \( m \) into transport vehicles.

\[
\sum_i \phi_i x_{ijkm} \leq \phi_m y_{jkm} \quad \forall j, k, m
\] (A.18)

\[ \zeta_k + \pi_{1jm} \leq t \leq \eta_k - \tau_{jkm} - \alpha_m - \beta_m - \pi_{2km} \]

This constraint translates the square feet of commodities transported via mode \( m \) into transport vehicles.

\[
\sum_j z_{jkm} = \sum_j y_{jkm}(t - \tau_{jkm} - \alpha_m - \beta_m) \quad \forall k, m
\] (A.19)

\[ \zeta_k + \alpha_m + \beta_m \leq t \leq \eta_k - \pi_{2km} \]

After vehicles finish unloading at FOL \( k \) this constraint assigns those vehicles to return trips to FSLs.

\[ v_{jmh} = v_{jm(t-1)h} + \sum_{k \in \mu_k = h} [z_{jkm(t-\tau_{jkm})} - y_{jkm}] \quad \forall j, m, h; t \geq 2 \] (A.20)

\[ v_{jm''\gamma h} = q_{jmh} - \sum_{k \in \mu_k = h} y_{jkm''\gamma} \quad \forall j, m, h \] (A.21)
The previous two constraints are flow balance equations for the number of available vehicles, at
time periods \( t \geq 2 \) and \( t = 1 \), respectively. Vehicles available at FSL \( j \) at the end of time period \( t \) are
equal to the vehicles available at the end of time period \( t-1 \), less those that begin loading for
transport at the beginning of time period \( t \), plus those that return at the beginning of time period \( t \).

\[
\sum_{kmt} x_{ijkmt} + EI_{ij} \leq IA_{ij} + AU_{ij} \quad \forall i, j \quad (A.22)
\]

\[
\sum_{j} AU_{ij} \leq \sum_{j} EI_{ij} + AP_{i} \quad \forall i \quad (A.23)
\]

\[
EI_{ij} \leq b_{ij} \times m \quad \forall i, j \quad (A.24)
\]

\[
AU_{ij} \leq (1-b_{ij}) \times m \quad \forall i, j \quad (A.25)
\]

The previous four constraints all deal with reallocation and procurement of commodities. The
first two equations control how much of commodity \( i \) can be shipped out of each FSL and a
system wide equation to determine how much is realigned versus procured respectively. The
remaining two constraints ensure that amount of commodities reallocated from FSL \( j \) and the
amount of additional commodities needed at FSL \( j \) can both equal zero or one of them can be
greater than zero, but both of them cannot be greater than zero.

\[
\sum_{o} pp_{jho} \leq 1 \quad \forall h; j \in AFL(J) \quad (A.26)
\]

\[
\sum_{imt} \sum_{k \in FLSCEN} x_{ijkmt} \leq \left( \sum_{ikt} D_{ikt} \right) \sum_{h=SCENPHASE} pp_{jho} \quad \forall o; j \in AFL(J) \quad (A.27)
\]

These constraints allow an afloat FSL to support at most one scenario per phase.

The remaining constraints are necessary for mathematical “bookkeeping.”

\[
\sum_{kmt} x_{ijkmt} = xx_{ij} \quad \forall i, j \quad (A.28)
\]

\[
x_{ijkmt} = 0 \quad t < \zeta_{k} + \pi_{1jm}; \quad t > \eta_{k} - \tau_{jkm} - \alpha_{m} - \beta_{m} - \pi_{2km} \quad (A.29)
\]

\[
n_{ij}, q_{jm}, r_{mh}, s_{ikt}, v_{jmth}, x_{ijkmt}, xx_{ij}, y_{jkmt}, z_{jkm}, w_{ij}, AU_{ij}, EI_{ij}, AP_{i} \geq 0 \quad (A.30)
\]

\[
y_{jkmt}, z_{jkm} \quad \text{integer} \quad (A.31)
\]

\[
w_{ij}, pp_{jbo}, b \in \{0,1\} \quad (A.32)
\]

Several implicit assumptions are worth noting here: The forward support location
configuration will be determined and in place at the beginning of phase 1, and this FSL
configuration (i.e., the number, location, and size of FSL facilities) will then remain static across
all phases. The number of transport vehicles may vary across phases, but within any one phase
the number of transport vehicles may not vary. Idle vehicles at the FSL are assumed to be able to
“sit” somewhere and not consume MOG space at the FSL. Vehicles returning to an FSL do not
consume MOG space at that FSL, and vehicles departing an FOL for return to an FSL do not
consume MOG space at that FOL. The more complicated terms associated with $z$ in constraint (B.16) are necessary due to the fact that the travel time associated with a return trip to an FSL might occur over some period of time later than the maximum time period $||T||$, if a vehicle arrives at an FOL sufficiently close to this final time period. There is an implicit assumption that vehicles may not “sit” at an FOL; rather, immediately following unloading a vehicle, the vehicles must depart on a return trip to some FSL. Vehicles returning to an FSL at time $t$ are not available to begin loading for an FOL delivery until time $t + 1$. For all transport modes, no $j \rightarrow k$ route has transit time 0.

If an FOL is also a potential FSL, this collocation must be modeled such that no transportation or throughput resources are consumed to meet the FOL’s demand if the collocated FSL is opened. This can be accomplished through the use of a “dummy” vehicle. The dummy vehicle has transit time $\tau = 1$ from FSL $j$ to its collocated FOL, with $\tau = \infty$ over all other routes. For all other “nondummy” vehicles, the transit time $\tau = \infty$ is assumed from FSL $j$ to its collocated FOL. This dummy vehicle is assumed to consume no throughput, to incur no transport cost, and to have an infinite maximum load, with a utilization rate $\sigma = 1$. The fleetwide inventory of dummy vehicles can be set equal to the number of potential forward support locations.
Appendix B

General Algebraic Modeling System for the Non-Reliability Model

The following GAMS statements implement the optimization model described in this dissertation. (Some tables and parameters are abbreviated for space reasons)

Sets

I commodities /BOS, VEH, MIS, JDM/
J FSL index /Andersen,…, Tocumen/
SCEN list of scenarios /Ndjamena20_6,…, Payalebar11_3/
AFLOAT(J) APF FSLs /MUN2/
K FOL index /Alkharaj,…,Tuzla/
M mode of transport /C-17,HSS,Truck/
AIR(M) aircraft /C-17/
LAN(M) land vehicles /Truck/
SEA(M) sea vehicles /HSS/
H phase /H1*H7/
T time periods which divide up each phase /T1*T10/

ALIAS (T,N);

TABLE THETA(M,H) cost of obtaining an additional vehicle of mode M at the beginning of phase H

<table>
<thead>
<tr>
<th></th>
<th>H1</th>
<th>…</th>
<th>H7</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-17</td>
<td>99999999</td>
<td>…</td>
<td>77994138</td>
</tr>
<tr>
<td>HSS</td>
<td>99999999</td>
<td>…</td>
<td>77994138</td>
</tr>
</tbody>
</table>

TABLE PSI(I,K) shortfall cost per time unit per ton of commodity I not fulfilled at FOL K

<table>
<thead>
<tr>
<th></th>
<th>Alkharaj</th>
<th>…</th>
<th>Tuzla</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOS</td>
<td>0</td>
<td>…</td>
<td>0</td>
</tr>
<tr>
<td>JDM</td>
<td>0</td>
<td>…</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE OMEGA(I,J,K,M) cost per ton of commodity I transported from FSL J to FOL K via mode M

<table>
<thead>
<tr>
<th></th>
<th>Alkharaj.C-17</th>
<th>…</th>
<th>Tuzla.HSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOS.Anderson</td>
<td>11928.6384</td>
<td>…</td>
<td>0</td>
</tr>
<tr>
<td>JDM.Tocumen</td>
<td>15491.9648</td>
<td>…</td>
<td>0;</td>
</tr>
</tbody>
</table>
**TABLE PI1(J,M)** additional time needed before loading for commodities to leaving FSL J via mode M

<table>
<thead>
<tr>
<th></th>
<th>C-17</th>
<th>...</th>
<th>HSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anderson</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>Tocumen</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE PI2(K,M)** additional time needed following unloading for commodities to reach FOL K via mode M

<table>
<thead>
<tr>
<th></th>
<th>C-17</th>
<th>...</th>
<th>HSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alkharaj</td>
<td>0</td>
<td>...</td>
<td>1</td>
</tr>
<tr>
<td>Tuzla</td>
<td>0</td>
<td>...</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**TABLE TAU(J,K,M)** one-way transportation time from FSL J to FOL K via mode M

<table>
<thead>
<tr>
<th></th>
<th>Alkharaj.C-17</th>
<th>...</th>
<th>Tuzla.HSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anderson</td>
<td>0.7</td>
<td>...</td>
<td>13889</td>
</tr>
<tr>
<td>Tocumen</td>
<td>0.8</td>
<td>...</td>
<td>13889</td>
</tr>
</tbody>
</table>

**TABLE C(M,H)** planned systemwide inventory of mode M vehicles at the beginning of phase H

<table>
<thead>
<tr>
<th></th>
<th>H1</th>
<th>...</th>
<th>H7</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-17</td>
<td>50</td>
<td>...</td>
<td>50</td>
</tr>
<tr>
<td>HSS</td>
<td>4</td>
<td>...</td>
<td>4</td>
</tr>
</tbody>
</table>

**TABLE D(I,K,T)** cumulative demand in tons for commodity I at FOL K by time T

<table>
<thead>
<tr>
<th></th>
<th>T5</th>
<th>T10</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOS.Alkharaj</td>
<td>0</td>
<td>448.9</td>
</tr>
<tr>
<td>JDM.Tuzla</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE STEAMTIME(J,K)** APF FSLs

<table>
<thead>
<tr>
<th></th>
<th>GomezNinoApjay</th>
<th>...</th>
<th>PayaLebar</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUN2</td>
<td>2.5</td>
<td>...</td>
<td>0</td>
</tr>
</tbody>
</table>

**Parameters**

**L(I)** cost of reallocating tons of commodity I between FSLs
<table>
<thead>
<tr>
<th>Country</th>
<th>Code</th>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>/BOS</td>
<td>2000</td>
<td>P(I)</td>
<td>cost of procuring additional tons of commodity I</td>
<td>/BOS 10000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>JDM 10000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DELTA(J)</td>
<td>fixed cost incurred to open FSL J with E(J) square feet of storage space</td>
<td>/Anderson 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Tocumen 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>XI(J)</td>
<td>variable cost per square foot of storage space needed beyond E(J) for</td>
<td>/Anderson 2908800</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>commodities as FSL J</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Tocumen 172800/</td>
</tr>
<tr>
<td></td>
<td></td>
<td>UPSILON(J)</td>
<td>operating cost per warehouse at FSL J (total cost over the time horizon</td>
<td>/Anderson 3263491.442</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>discounted appropriately</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Tocumen 1938707.788/</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ALPHA(M)</td>
<td>number of time periods necessary to load a mode M vehicle</td>
<td>/C-17 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>BETA(M)</td>
<td>number of time periods necessary to unload a mode M vehicle</td>
<td>/C-17 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>GAMMA(M)</td>
<td>maximum load in tones per mode M vehicle</td>
<td>/C-17 45</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>HSS 400/</td>
</tr>
</tbody>
</table>
**Epsilon(M)** maximum square feet per mode M vehicle
/C-17 1080

/HSS 40000/

**Zeta(K)** contingency start date at FOL K
/Alkharaj 1

/Tuzla 1/

**Eta(K)** contingency finish date at FOL K
/Alkharaj 10

/Tuzla 10/

**Mu(K)** phase of contingency occurrence associated with FOL K
/Alkharaj 1

/Tuzla 5/

**Rho(M)** conversion factor for parking space for mode M
/C-17 1

/HSS 1/

**Sigma(M)** utilization rate expressed as the average flying hour goal per day divided by 24 hours
/C-17 0.6041

/HSS 1/

**Phi(I)** conversion factor for commodity I from tons to square feet of storage space
/BOS 21.6

/JDM 9.2 /

**A1(J)** max on ground in AIR equivalent vehicles at FSL J
A\textsubscript{2}(J) max on ground in LAN equivalent vehicles at FSL J

\begin{align*}
/\text{Anderson} & \quad 2 \\
/\text{Tocumen} & \quad 2/
\end{align*}

A\textsubscript{3}(J) max on ground in SEA equivalent vehicles at FSL J

\begin{align*}
/\text{Anderson} & \quad 6 \\
/\text{Tocumen} & \quad 6/
\end{align*}

B\textsubscript{1}(K) max on ground in AIR equivalent vehicles at FOL K

\begin{align*}
/\text{Alkharaj} & \quad 2 \\
/\text{Tuzla} & \quad 2/
\end{align*}

B\textsubscript{2}(K) max on ground in LAN equivalent vehicles at FOL K

\begin{align*}
/\text{Alkharaj} & \quad 6 \\
/\text{Tuzla} & \quad 6/
\end{align*}

B\textsubscript{3}(K) max on ground in SEA equivalent vehicles at FOL K

\begin{align*}
/\text{Alkharaj} & \quad 1 \\
/\text{Tuzla} & \quad 1/
\end{align*}

E(J) minimum square footage needed for an economically feasible FSL at location J for commodities

\begin{align*}
/\text{Anderson} & \quad 60.07 \\
/\text{Tocumen} & \quad 0/
\end{align*}

G maximum square feet per commodity I per warehouse

/20000/

F(J) maximum potential square feet of storage space at FSL J
FOLSCEN(K) scenario of FOL
/AlKharaj5_9  6
    ;
Tuzla1_7   11/

SCENPHASE(SCEN) phase of scenario
/1_7      5
    ;
7_10  7/;

Free Variables

OBJ objective;

Positive Variables

NN(J) additional number of notional igloos beyond E(J) needed at FSL J
WW(J) number of notional igloos utilized at FSL J
Q(J,M,H) number of mode M vehicles available at FSL J at the start of time T=1 during phase H
R(M,H) additional mode M vehicles obtained at the beginning of phase H
S(I,K,T) shortfall below demand in tons for commodity I at FOL K not fulfilled by time T
V(J,M,T,H) number of mode M vehicles available at FSL J at the end of time T during phase H
XX(I,J) total tons of commodity I sent out of FSL J
AU(I,J) additional tons of commodity I at FSL J beyond the initial amount
EI(I,J) reallocated tons of commodity I at FSL J
AP(I) additional tons of commodity I procured across the entire system
X(I,J,K,M,T) tons of commodity I sent from FSL J to FOL K via mode M beginning loading on time T;

Binary Variables

PP(J,H,SCEN) binary variable indicating if afloat prepositioned ship J supports scenario SCEN during phase H
b(I,J) binary variable to ensure that EI and AU are not both greater than zero
W(J) binary variable indicating status of FSL J;

Integer Variables

Y(J,K,M,T) number of mode M vehicles tasked to transport commodities and personnel from FSL J to FOL K beginning loading on time T;
Z(J,K,M,T) number of mode M vehicles tasked to make the return trip from FOL K to FSL J departing on time T;

Equations

OBJECTIVE objective function
TOTALNUMBERVEHICLES(M,H) constraint on the total number of mode M vehicles during phase H
FSLVEHAvAIL(J,M,T,H) constraint on mode M vehicle availability at FSL J during time T>1 for phase H
FSLMOGAIR(J,T,H) constraint on MOG of FSL J for air vehicles during time T for phase H
FSLMOGLAN(J,T,H) constraint on MOG of FSL J for land vehicles during time T for phase H
FSLMOGSEA(J,T,H) constraint on MOG of FSL J for sea vehicles during time T for phase H
FOLMOGAIR(K,T) constraint on MOG of FOL K for air vehicles during time T
FOLMOGLAN(K,T) constraint on MOG of FOL K for land vehicles during time T
FOLMOGSEA(K,T) constraint on MOG of FOL K for sea vehicles during time T
TOTSHIPTONS(I,J) constraint on the total tons of commodity I sent out of FSL J
DEMANDCONSTRAINT(I,K,T) constraint on meeting cumulative demand for commodity I at FOL K by time T
FSLSTORSF(J,H) constraint on storage space in square feet at FSL J during phase H
FSLSTORSF2(J,H) additional constraint on storage utilization in square feet at FSL J during phase H
FSLSTOR(J) additional constraint on storage space at FSL J
FSLUSAGE(J) constraint on minimal utilization at FSL J
FSLUSAGE2(J) additional constraint on minimal utilization at FSL J
UTERATE(M,H) constraint limiting the average fleetwide utilization of mode M vehicles over phase H
BOOKVEHTO(J,K,M,T) translates tons of commodities transported from FSL J to FOL K beginning loading on time T into mode M vehicles via tonnage
BOOKVEHSF(J,K,M,T) translates tons of commodities transported from FSL J to FOL K beginning loading on time T into mode M vehicles via storage in square feet
RETURNTRIPVEHICLES(K,M,T) assigns mode M vehicles to return trips following delivery and unloading to FOL K at time T
FLOWBALANCEVEH(J,M,T,H) tracks the number of mode M vehicles available at FSL J at the end of time T>1 for phase H
INITFLOWBALANCEVEH(J,M,H) tracks the number of mode M vehicles available at FSL J at the end of time T=1 for phase H
SHIPREAL(I,J) constraint on the amount of commodity I shipped out of FSL J and reallocated to other FSLs
SYSPROC(I) constraint on the additional amount of commodity I that is reallocated or procured across the entire system
REALL(I,J) constraint to ensure that if EI is greater than zero then AU must be equal to zero
ADDUNIT(I,J) constraint to ensure that if AU is greater than zero then EI must be equal to zero
AFLOATSCEN(J,H) afloat constraint
AFLOATSCEN2(J,SCEN) afloat constraint;
OBJECTIVE.. OBJ =E= 
SUM(J, DELTA(J)*W(J) + XI(J)*NN(J) + UPSILON(J)*WW(J)) + SUM(I, SUM(J, SUM(K, SUM(M, SUM(T, OMEGA(I,J,K,M)*X(I,J,K,M,T)))))) + SUM(M, SUM(H, THETA(M,H)*R(M,H))) + SUM(I, SUM(L(I)*EI(I,J))) + SUM(M, SUM(K, SUM(T, ORD(T)*PSI(I,K)*S(I,K,T))));
TOTALNUMBERVEHICLES(M,H).. SUM(J, Q(J,M,H)) =E= C(M,H) + R(M,H);
FSLVEHAVAIL(J,M,T,H)$(ORD(T)>1).. SUM(K$(MU(K)=ORD(H)),(Y(J,K,M,T))) =L= V(J,M,T-1,H);
FSLMOGAIR(J,T,H)..
SUM(K$(MU(K)=ORD(H)),SUM(M$AIR(M),SUM(N$(ORD(N)<=ALPHA(M)),RHO(M)*(Y(J,K,M,N+CEIL(ORD(T)-2*ORD(N)+1)))))) =L= A1(J);
FSLMOGLAN(J,T,H)..
SUM(K$(MU(K)=ORD(H)),SUM(M$LAN(M),SUM(N$(ORD(N)<=ALPHA(M)),RHO(M)*(Y(J,K,M,N+CEIL(ORD(T)-2*ORD(N)+1)))))) =L= A2(J);
FSLMOGSEA(J,T,H)..
SUM(K$(MU(K)=ORD(H)),SUM(M$SEA(M),SUM(N$(ORD(N)<=ALPHA(M)),RHO(M)*(Y(J,K,M,N+CEIL(ORD(T)-2*ORD(N)+1)))))) =L= A3(J);
FOLMOGAIR(K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K))).. 
SUM(J, SUM(M$AIR(M),SUM(N$(ORD(N)<=ORD(T)),X(I,J,K,M,N-CEIL(TAU(I,J,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))))) =G= 
SUM(N$(ORD(N)<=ORD(T)),D(I,K,N)-S(I,K,T));
FSLSTORSF(J,H).. 
SUM(K$(MU(K)=ORD(H)),SUM(I, SUM(M, SUM(T, PHI(I)*X(I,J,K,M,T))))) =L= 
G*(E(J)*W(J)+NN(J));
FSLSTORSF2(J,H)..
SUM(K$(MU(K)=ORD(H)),SUM(I, SUM(M, SUM(T, PHI(I)*X(I,J,K,M,T))))) =L= 
G*WW(J);
FSLSTOR(J)..< NN(J) =L= (F(J)-E(J))*W(J);
FSLUSAGE(J)..< W(J) =L= WW(J);
FSLUSAGE2(J)..< W(J) =L= E(J)+NN(J);
UTERATE(M,H)..< 
SUM(K$(MU(K)=ORD(H)),SUM(J, SUM(TAU(J,K,M)*(Y(J,K,M,T)))+SUM(TS(ORD(T)<= (CARD(T)-TAU(J,K,M))),TAU(J,K,M)*Z(J,K,M,T)+SUM(TS(ORD(T)>=(CARD(T)-TAU(J,K,M)+1)),(CARD(T)-ORD(T))*Z(J,K,M,T)))) =L= 
CARD(T)*(C(M,H)+R(M,H))*SIGMA(M);
BOOKVEHTO(J,K,M,T)$((ORD(T)>=ZETA(K)+PI1(J,M)) AND (ORD(T)<=ETA(K)-(TAU(J,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))).. SUM(I,X(I,J,K,M,T)) =L= GAMMA(M)*Y(J,K,M,T);
BOOKVEHSF(J,K,M,T)$((ORD(T)>=ZETA(K)+PI1(J,M)) AND (ORD(T)<=ETA(K)-(TAU(J,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))).. SUM(I,PHI(I)*X(I,J,K,M,T)) =L= EPSILON(M)*Y(J,K,M,T);
RETURNTRIPVEHICLES(K,M,T)$((ORD(T)>=ZETA(K)+ALPHA(M)+BETA(M)) AND (ORD(T)<=ETA(K)-PI2(K,M))).. SUM(J,Z(J,K,M,T)) =E= SUM(J,Y(J,K,M,T-CEIL(TAU(J,K,M)+ALPHA(M)+BETA(M))));
FLOWBALANCEVEH(J,M,T,H)$((ORD(T)>1).. V(J,M,T,H) =E= V(J,M,T-1,H)+SUM(K$(MU(K)=ORD(H)),Z(J,K,M,T-CEIL(TAU(J,K,M)))-Y(J,K,M,T));
INITFLOWBALANCEVEH(J,M,H). V(J,M,'T1',H) =E= Q(J,M,H)+SUM(K$(MU(K)=ORD(H)),Y(J,K,M,'T1'));
SHIPREAL(I,J). SUM(K,SUM(M,SUM(T,X(I,J,K,M,T))))+EI(I,J) =E= IA(I,J)+AU(I,J);
SYSPROC(I). SUM(J,AU(I,J)) =L= SUM(J,EI(I,J))+AP(I);
REALL(I,J). EI(I,J) =L= b(I,J)*100000;
ADDUNIT(I,J). AU(I,J) =L= (1-b(I,J))*100000;
FLOATSCEN(J,H)$$(AFLOAT(J)). SUM(SCEN,PP(J,H,SCEN)) =L= 1.0;
AFLOATSCEN2(J,SCEN)$$(AFLOAT(J)). SUM(1,SUM(K$(FOLSCEN(K)=ORD(SCEN)),SUM(M,SUM(T,X(I,J,K,M,T))))) =L= SUM(1,SUM(K,SUM(T,D(I,K,T))))*SUM(H$(ORD(H)=SCENPHASE(SCEN)),PP(J,H,SCEN)) ;

MODEL demo /ALL/;
demo.OPTFILE = 1;
OPTION ITERLIM = 500000000;
OPTION LIMROW = 10;
OPTION LIMCOL = 10;
OPTION SYSOUT = ON;
OPTION DMPSYM;
OPTION PROFILE = 3;
OPTION RESLIM=172800;
OPTION OPTCR=0.01;
OPTION MIP=CPLEX;
demo.PRIOROPT=1;
W.PRIOR(J)=1.0;
PP.PRIOR(J,H,SCEN)=1.0;
Y.PRIOR(J,K,M,T) = 2.0;
Z.PRIOR(J,K,M,T) = 3.0;
demo.HOLDFIXED=1;
SOLVE demo USING MIP MINIMIZING OBJ;

* separate the total cost into its facility construction, facility operation, transport, procurement, and reallocations components
PARAMETERS
FACCONCOST facility construction cost
FACOPERCOST facility operating cost
TRANCOST transport cost
PROCURECOST commodity procurement cost
REALLOCATECOST commodity reallocation cost
FROMTO(I,J,K,M) tons of commodity I shipped from FSL J to FOL K via mode M;
FACCONCOST = SUM(J,DELTA(J)*W.L(J)+XI(J)*NN.L(J));
FACOPERCOST = SUM(J,UPSILON(J)*WW.L(J));
TRANCOST = SUM(I,SUM(J,SUM(K,SUM(M,SUM(T,OMEGA(I,J,K,M)*X.L(I,J,K,M,T))))));
PROCURECOST = SUM(I,P(I)*AP.L(I));
REALLOCATECOST = SUM(I,SUM(J,L(I)*EI.L(I,J)));
FROMTO(I,J,K,M) = SUM(T,X.L(I,J,K,M,T));
Appendix C

FSL Site Selection and Transportation Reliability Model Formulation

In this appendix, we present the mathematical formulation of the reliability model. We will only give specific explanations of equations when they differ from what appears in the non-reliability model presented in Appendix B other than creating multiple versions for each subset of FSLs. For a further discussion, see Chapter 5. In this appendix, we will assume that set J contains 38 elements (in keeping with the example problem analyzed in this dissertation) to simplify notation.

Sets and Set Indices

\( i \in I \) commodities; \( I = \{ PAX, BOS, VEH, JDM, MIS, \ldots \} \)
\( j \in J \) FSL index; \( J = \{ FSL1, FSL2, \ldots, APF1, APF2, \ldots \} \)
\( J_1 (J) \) all but first FSL; \( J_1 \subseteq J; J_1 (J) = \{ ALDA, \ldots, WXWC \} \)
\( J_{38} (J) \) all but last FSL; \( J_{38} \subseteq J; J_{38} (J) = \{ AJJY, \ldots, VYHK \} \)
\( AFL(J) \) afloat FSLs; \( AFL(J) \subseteq J; AFL(J) = \{ APF1, APF2, \ldots \} \)
\( k \in K \) FOL index; \( K = \{ FOL1, FOL2, \ldots \} \)
\( m \in M \) modes of transport; \( M = \{ C-130, C-17, C-5, B747, TRUCK, HSS, \ldots \} \)
\( AIR(M) \) aircraft; \( AIR(M) \subseteq M; AIR(M) = \{ C-130, C-17, C-5, B747, \ldots \} \)
\( LAN(M) \) land vehicles; \( LAN(M) \subseteq M; LAN(M) = \{ TRUCK, \ldots \} \)
\( SEA(M) \) sea vehicles; \( SEA(M) \subseteq M; SEA(M) = \{ HSS, \ldots \} \)
\( h \in H \) phases; \( H = \{ 1, 2, \ldots \} \)
\( o \in SCN \) deployment scenarios; \( SCN = \{ 1, 2, \ldots \} \)
\( t \in T \) time periods which divide up each phase \( h \); \( T = \{ 1, 2, \ldots \} \)

Data Parameters:

\( \Delta_j \) fixed cost incurred to open FSL \( j \) with \( E_j \) square feet of storage space
\( \Theta_{mh} \) cost of obtaining an additional vehicle of mode \( m \) at the beginning of phase \( h \)
\( \Xi_j \) construction cost per unit of storage needed beyond \( E_j \) at FSL \( j \)
\( \Upsilon_j \) operating cost (discounted over the time horizon) per unit of storage at FSL \( j \)
\( \Psi_{ik} \) shortfall cost per time unit per ton of commodity \( i \) not fulfilled at FOL \( k \)
\( \Omega_{ijkm} \) cost per ton of commodity \( i \) transported from FSL \( j \) to FOL \( k \) via mode \( m \)
\( \alpha_m \) number of time periods necessary to load a mode \( m \) vehicle

\( \beta_m \) number of time periods necessary to unload a mode \( m \) vehicle

\( \gamma_m \) maximum load in tons per mode \( m \) vehicle

\( \varepsilon_m \) maximum load in square feet per mode \( m \) vehicle

\( \zeta_k \) contingency start period at FOL \( k \)

\( \eta_k \) contingency finish period at FOL \( k \)

\( \mu_k \) phase of scenario associated with FOL \( k \)

\( \pi_{1jm} \) additional time needed prior to loading for commodities departing FSL \( j \) via mode \( m \)

\( \pi_{2km} \) additional time needed following unloading for commodities to reach FOL \( k \) via mode \( m \)

\( \rho_m \) conversion factor for parking space for mode \( m \)

\( \sigma_m \) utilization rate, expressed (for airlift) as the average flying hour goal per day divided by 24 hours, for mode \( m \)

\( \tau_{jkm} \) one-way transportation time from FSL \( j \) to FOL \( k \) (or in opposite direction) via mode \( m \)

\( \phi_i \) conversion factor for commodity from tons to square feet of storage space

\( g \) maximum square feet per commodity \( i \) unit of storage

\( P_i \) cost of procuring additional tons of commodity \( i \)

\( I_{Aj} \) initial allocation (in tons) of commodity \( i \) at FSL \( j \)

\( L_i \) cost of reallocating tons of commodity \( i \) independent of FSL locations

\( STEAM_{jk} \) additional time needed for steaming to port before afloat FSL \( j \) can begin offloading at port associated with FOL \( k \)

\( FOLSCEN_k \) scenario associated with FOL \( k \)

\( SCENPHASE_o \) phase associated with scenario \( o \)

\( A_{Sj} \) max on ground, in class \( \{ \text{AIR}(M) = 1, \text{LAN}(M) = 2, \text{SEA}(M) = 3 \} \) equivalent vehicles, at FSL \( j \)

\( B_{Sk} \) max on ground, in class \( \{ \text{AIR}(M) = 1, \text{LAN}(M) = 2, \text{SEA}(M) = 3 \} \) equivalent vehicles, at FOL \( k \)

\( C_{mh} \) planned systemwide inventory of mode \( m \) vehicles at the beginning of phase \( h \)

\( D_{ikt} \) incremental demand, in tons, for commodity \( i \) at FOL \( k \) at time \( t \)

\( E_j \) minimum units of storage needed for an economically feasible FSL at location \( j \)

\( F_j \) maximum potential units of storage at FSL \( j \)
Variables

\(nn_j\) additional units of storage needed beyond \(E_j\) at FSL \(j\)

\(q_{jmh}\) number of mode \(m\) vehicles available at FSL \(j\) at the start of time period \(t=1\) during phase \(h\)

\(r_{mh}\) additional mode \(m\) vehicles obtained at the beginning of phase \(h\)

\(s_{ikt}\) shortfall below demand, in tons, for commodity \(i\) at FOL \(k\) not fulfilled by time \(t\)

\(s1_{ikt}\) shortfall below demand, in tons, for commodity \(i\) at FOL \(k\) not fulfilled by time \(t\) for subset \(J_f\)

\(S38_{ikt}\) shortfall below demand, in tons, for commodity \(i\) at FOL \(k\) not fulfilled by time \(t\) for subset \(J_{38}\)

\(v_{jmth}\) number of mode \(m\) vehicles available at FSL \(j\) at the end of time \(t\) during phase \(h\)

\(v1_{jmth}\) number of mode \(m\) vehicles available at FSL \(j\) (in subset \(J_1\)) at the end of time \(t\) during phase \(h\)

\(v38_{jmth}\) number of mode \(m\) vehicles available at FSL \(j\) (in subset \(J_{38}\)) at the end of time \(t\) during phase \(h\)

\(w_j\) binary variable indicating status of FSL \(j\), = 1 if opened, = 0 otherwise

\(x_{ijkmt}\) tons of commodity \(i\) sent from FSL \(j\) to FOL \(k\) via mode \(m\), beginning loading on time \(t\)

\(x1_{ijkmt}\) tons of commodity \(i\) sent from FSL \(j\) (in subset \(J_1\)) to FOL \(k\) via mode \(m\), beginning loading on time \(t\)

\(x38_{ijkmt}\) tons of commodity \(i\) sent from FSL \(j\) (in subset \(J_{38}\)) to FOL \(k\) via mode \(m\), beginning loading on time \(t\)

\(y_{jkmt}\) number of mode \(m\) vehicles tasked to transport commodities from FSL \(j\) to FOL \(k\), beginning loading on time \(t\). Integer

\(y1_{jkmt}\) number of mode \(m\) vehicles tasked to transport commodities from FSL \(j\) (in subset \(J_1\)) to FOL \(k\), beginning loading on time \(t\). Integer
number of mode $m$ vehicles tasked to transport commodities from FSL $j$ (in subset $J_{38}$) to FOL $k$, beginning loading on time $t$. Integer

Number of mode $m$ vehicles tasked to make the return trip from FOL $k$ to FSL $j$, departing on time $t$. Integer

Number of mode $m$ vehicles tasked to make the return trip from FOL $k$ to FSL $j$ (in subset $J_1$), departing on time $t$. Integer

Number of mode $m$ vehicles tasked to make the return trip from FOL $k$ to FSL $j$ (in subset $J_{38}$), departing on time $t$. Integer

Units of storage utilized at FSL $j$.

Maximum total tons of commodity $i$ sent out of FSL $j$ across the full set of FSLs and all 38 subsets of FSLs.

Binary variable indicating if afloat FSL $j$ supports scenario $o$ during phase $h$.

Additional units of commodity $i$ (in tons) that is procured at FSL $j$ beyond the initial amount ($IA_{ij}$), this is a gain of inventory at FSL $j$.

Inventory of commodity $i$ (in tons) at FSL $j$ that is reallocated to other FSLs, this is a loss of inventory at FSL $j$.

The sum of the additional amount (in tons) of commodity $i$ procured across the entire system, this is a gain of inventory across all FSLs.

Binary variable, = 1 if FSL $j$ reallocates commodity $i$ to other FSLs, = 0 otherwise.

A very large positive number to ensure that FSL $j$ can reallocate commodity $i$ to other FSLs or procure additional units of commodity $i$ but not both.

**Note:** There is an implicit assumption throughout the entire model that terms having an index value $t \leq 0$ are not considered.

**Objective Function**

$$\min \sum_j \left( \Delta_j w_j + \Xi_j n_{nj} + \Upsilon_j w_{wj} \right)$$

$$+ \sum_{ijkmt} \frac{\Omega_{ijkmt}}{39} x_{ijkmt} + \sum_{ikmt} \sum_{j \in J(J)} \frac{\Omega_{ijkmt}}{39} x_{ijkmt} + \ldots + \sum_{ikmt} \sum_{j \in J_{38}(J)} \frac{\Omega_{ijkmt}}{39} x_{38ijkmt}$$

$$+ \sum_{mh} \Theta_{mh} r_{mh} + \sum_i P_i AP_i + \sum_{ij} L_i EI_{ij} + \left[ \sum_{ikt} \frac{\Psi_{ik}}{39} s_{ikt} + \sum_{ikt} \frac{\Psi_{ik}}{39} s_{ikt} + \ldots + \sum_{ikt} \frac{\Psi_{ik}}{39} s_{38ikt} \right]$$

The objective function minimizes the average total cost, equal to the sum of the FSL opening cost, the cost for construction of new facilities, the facility operation cost, the average transport cost across all scenarios, the cost of procuring new vehicles, the cost for reallocating
commodities between FSLs, the cost of procuring additional commodities, and the average shortfall cost across all scenarios for not satisfying demand requirements.

Constraints

\[ \sum_{j} q_{jmh} \leq (C_{mh} + r_{mh}) \quad \forall m, h \]  
(C.2)

\[ \sum_{k \in \mu_k = h} y_{jkmt} \leq v_{jm(t-1)h} \quad \forall j, m, h; t \geq 2 \]  
(C.3)

\[ \sum_{k \in \mu_k = h} y_{1jkmt} \leq v_{1jm(t-1)h} \quad \forall m, h; j \in J_1(J), t \geq 2 \]  
(C.4)

\[ \sum_{k \in \mu_k = h} y_{38jkmt} \leq v_{38jm(t-1)h} \quad \forall m, h; j \in J_{38}(J), t \geq 2 \]  
(C.5)

\[ \sum_{k \in \mu_k = h} \sum_{m \in \text{AIR (M)}} \sum_{n=0}^{\alpha_{m-1}} [\rho_m(y_{jkmt-(n-1)})] \leq A_{1'j} \quad \forall j, t, h \]  
(C.6)

\[ \sum_{k \in \mu_k = h} \sum_{m \in \text{AIR (M)}} \sum_{n=0}^{\alpha_{m-1}} [\rho_m(y_{1jkmt-(n-1)})] \leq A_{1'j} \quad \forall t, h; j \in J_1(J) \]  
(C.7)

\[ \sum_{k \in \mu_k = h} \sum_{m \in \text{AIR (M)}} \sum_{n=0}^{\alpha_{m-1}} [\rho_m(y_{38jkmt-(n-1)})] \leq A_{1'j} \quad \forall t, h; j \in J_{38}(J) \]  
(C.8)

\[ \sum_{k \in \mu_k = h} \sum_{m \in \text{LAN (M)}} \sum_{n=0}^{\alpha_{m-1}} [\rho_m(y_{jkmt-(n-1)})] \leq A_{2'j} \quad \forall j, t, h \]  
(C.9)

\[ \sum_{k \in \mu_k = h} \sum_{m \in \text{LAN (M)}} \sum_{n=0}^{\alpha_{m-1}} [\rho_m(y_{1jkmt-(n-1)})] \leq A_{2'j} \quad \forall t, h; j \in J_1(J) \]  
(C.10)

\[ \sum_{k \in \mu_k = h} \sum_{m \in \text{LAN (M)}} \sum_{n=0}^{\alpha_{m-1}} [\rho_m(y_{38jkmt-(n-1)})] \leq A_{2'j} \quad \forall t, h; j \in J_{38}(J) \]  
(C.11)

\[ \sum_{k \in \mu_k = h} \sum_{m \in \text{SEA (M)}} \sum_{n=0}^{\alpha_{m-1}} [\rho_m(y_{jkmt-(n-1)})] \leq A_{3'j} \quad \forall j, t, h \]  
(C.12)
\[ \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}(\mathcal{C}) \cap \mathcal{M}(\mathcal{M})} \alpha^{-1}_{m} \sum_{n=0}^{m^2-1} [\rho_{m}(y_{1, jkm(t-n)})] \leq A_{3} \quad \forall t, h; j \in J_{1}(J) \]  
(C.13)

\[ \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}(\mathcal{C}) \cap \mathcal{M}(\mathcal{M})} \alpha^{-1}_{m} \sum_{n=0}^{m^2-1} [\rho_{m}(y_{38, jkm(t-n)})] \leq A_{3} \quad \forall t, h; j \in J_{38}(J) \]  
(C.14)

\[ \sum_{j \in J_{1}(J)} \sum_{m \in \mathcal{M}(\mathcal{C}) \cap \mathcal{M}(\mathcal{M})} \beta^{-1}_{m} \sum_{n=0}^{m^2-1} [\rho_{m}(y_{jkm(t-\tau_{jkm}(n))})] \leq B_{1} \quad \forall k; \zeta_{k} \leq t \leq \eta_{k} \]  
(C.15)

\[ \sum_{j \in J_{1}(J)} \sum_{m \in \mathcal{M}(\mathcal{C}) \cap \mathcal{M}(\mathcal{M})} \beta^{-1}_{m} \sum_{n=0}^{m^2-1} [\rho_{m}(y_{jkm(t-\tau_{jkm}(n))})] \leq B_{2} \quad \forall k; \zeta_{k} \leq t \leq \eta_{k} \]  
(C.16)

\[ \sum_{j \in J_{1}(J)} \sum_{m \in \mathcal{M}(\mathcal{C}) \cap \mathcal{M}(\mathcal{M})} \beta^{-1}_{m} \sum_{n=0}^{m^2-1} [\rho_{m}(y_{38, jkm(t-\tau_{jkm}(n))})] \leq B_{3} \quad \forall k; \zeta_{k} \leq t \leq \eta_{k} \]  
(C.17)

\[ \sum_{j \in J_{1}(J)} \sum_{m \in \mathcal{M}(\mathcal{C}) \cap \mathcal{M}(\mathcal{M})} \beta^{-1}_{m} \sum_{n=0}^{m^2-1} [\rho_{m}(y_{38, jkm(t-\tau_{jkm}(n))})] \leq B_{4} \quad \forall k; \zeta_{k} \leq t \leq \eta_{k} \]  
(C.18)

\[ \sum_{j \in J_{1}(J)} \sum_{m \in \mathcal{M}(\mathcal{C}) \cap \mathcal{M}(\mathcal{M})} \beta^{-1}_{m} \sum_{n=0}^{m^2-1} [\rho_{m}(y_{38, jkm(t-\tau_{jkm}(n))})] \leq B_{5} \quad \forall k; \zeta_{k} \leq t \leq \eta_{k} \]  
(C.19)

\[ \sum_{j \in J_{1}(J)} \sum_{m \in \mathcal{M}(\mathcal{C}) \cap \mathcal{M}(\mathcal{M})} \beta^{-1}_{m} \sum_{n=0}^{m^2-1} [\rho_{m}(y_{38, jkm(t-\tau_{jkm}(n))})] \leq B_{6} \quad \forall k; \zeta_{k} \leq t \leq \eta_{k} \]  
(C.20)

\[ \sum_{j \in J_{1}(J)} \sum_{m \in \mathcal{M}(\mathcal{C}) \cap \mathcal{M}(\mathcal{M})} \beta^{-1}_{m} \sum_{n=0}^{m^2-1} [\rho_{m}(y_{38, jkm(t-\tau_{jkm}(n))})] \leq B_{7} \quad \forall k; \zeta_{k} \leq t \leq \eta_{k} \]  
(C.21)

\[ \sum_{j \in J_{1}(J)} \sum_{m \in \mathcal{M}(\mathcal{C}) \cap \mathcal{M}(\mathcal{M})} \beta^{-1}_{m} \sum_{n=0}^{m^2-1} [\rho_{m}(y_{38, jkm(t-\tau_{jkm}(n))})] \leq B_{8} \quad \forall k; \zeta_{k} \leq t \leq \eta_{k} \]  
(C.22)

\[ \sum_{j \in J_{1}(J)} \sum_{m \in \mathcal{M}(\mathcal{C}) \cap \mathcal{M}(\mathcal{M})} \beta^{-1}_{m} \sum_{n=0}^{m^2-1} [\rho_{m}(y_{38, jkm(t-\tau_{jkm}(n))})] \leq B_{9} \quad \forall k; \zeta_{k} \leq t \leq \eta_{k} \]  
(C.23)
\[
\sum_{j,m} \sum_{n=1}^{t} x_{ijkm}(n-\tau_{jkm}-\alpha_m-\beta_m-\pi_{2km}) \geq \left( \sum_{n=1}^{t} D_{ikn} \right) - s_{ikt} \quad \forall i,k; \; \zeta_k \leq t \leq \eta_k \tag{C.24}
\]
\[
\sum_{m} \sum_{j \in J(J)} \sum_{n=1}^{t} x_{ijkm}(n-\tau_{jkm}-\alpha_m-\beta_m-\pi_{2km}) \geq \left( \sum_{n=1}^{t} D_{ikn} \right) - s_{ikt} \quad \forall i,k; \; \zeta_k \leq t \leq \eta_k \tag{C.25}
\]
\[
\sum_{m} \sum_{j \in J(J\text{38})} \sum_{n=1}^{t} x_{ijkm}(n-\tau_{jkm}-\alpha_m-\beta_m-\pi_{2km}) \geq \left( \sum_{n=1}^{t} D_{ikn} \right) - s_{38ikt} \quad \forall i,k; \; \zeta_k \leq t \leq \eta_k \tag{C.26}
\]
\[
\sum_{i} \phi_{ixx\bar{ij}} \leq g(E_j w_j + nn_j) \quad \forall j \tag{C.27}
\]

Constraint (C.27) restricts the amount of square footage of commodities of type \(i\) at FSL \(j\) that is shipped across all FOLs must be less than or equal to the maximum amount of square footage of commodities of type \(i\) that can be stored at FSL \(j\).

\[
\sum_{i} \phi_{ixx\bar{ij}} \leq g(w_j) \quad \forall j \tag{C.28}
\]

This constraint restricts the amount of square footage of commodities of type \(i\) at FSL \(j\) that is across each loss-of-access scenario must be less than or equal to the maximum amount of square footage of commodities of type \(i\) that is stored at FSL \(j\).

\[
nn_j \leq (F_j - E_j) w_j \quad \forall j \tag{C.29}
\]
\[
w_j \leq ww_j \quad \forall j \tag{C.30}
\]
\[
w_j \leq E_j + nn_j \quad \forall j \tag{C.31}
\]
\[
\sum_{k \neq k = h} \sum_{j} \left( \sum_{t} \tau_{jkm}(y_{jkm}) \right) + \left[ \sum_{t=1}^{\pi} \tau_{jkm} z_{jkm} \right] + \left[ \sum_{t=\pi-1}^{\pi+1} \tau_{jkm} z_{jkm} \right] \leq \tau_{(C_{mh} + r_{mh}) \sigma_m} \quad \forall m, h \tag{C.32}
\]
\[
\sum_{k \in \mathcal{K}_h} \sum_{j \in J_1(J)} \left( \sum_{t} \tau_{jkm} (y_{jkm}) + \left[ \left\| \mathbf{T} \right\|^{-\tau_{jkm}} \sum_{t=1}^{\left\| \mathbf{T} \right\|-1} (\| \mathbf{T} \| - t) z_{jkm} \right] \right) + \left[ \left\| \mathbf{T} \right\|^{-\tau_{jkm}} \sum_{t=1}^{\left\| \mathbf{T} \right\|-1} (\| \mathbf{T} \| - t) z_{jkm} \right] \leq \| \mathbf{T} \| (C_{mh} + r_{mh}) \sigma_m \quad \forall m, h \tag{C.33}
\]

\[
\sum_{k \in \mathcal{K}_h} \sum_{j \in J_{38}(J)} \left( \sum_{t} \tau_{jkm} (y_{38jkm}) + \left[ \left\| \mathbf{T} \right\|^{-\tau_{jkm}} \sum_{t=1}^{\left\| \mathbf{T} \right\|-1} (\| \mathbf{T} \| - t) z_{38jkm} \right] \right) + \left[ \left\| \mathbf{T} \right\|^{-\tau_{jkm}} \sum_{t=1}^{\left\| \mathbf{T} \right\|-1} (\| \mathbf{T} \| - t) z_{38jkm} \right] \leq \| \mathbf{T} \| (C_{mh} + r_{mh}) \sigma_m \quad \forall m, h \tag{C.34}
\]

\[
\sum_{i} x_{ijkmt} \leq y_{m, y_{jkm}} \quad \forall j, k, m \tag{C.35}
\]

\[
\zeta_k + \pi_{1jm} \leq t \leq \eta_k - \tau_{jkm} - \alpha_m - \beta_m - \pi_{2km}
\]

\[
\sum_{i} x_{1ijkmt} \leq y_{m, y_{jkm}} \quad \forall k, m; j \in J_1(J) \tag{C.36}
\]

\[
\zeta_k + \pi_{1jm} \leq t \leq \eta_k - \tau_{jkm} - \alpha_m - \beta_m - \pi_{2km}
\]

\[
\sum_{i} x_{38ijkmt} \leq y_{m, y_{38jkm}} \quad \forall k, m; j \in J_{38}(J) \tag{C.37}
\]

\[
\zeta_k + \pi_{1jm} \leq t \leq \eta_k - \tau_{jkm} - \alpha_m - \beta_m - \pi_{2km}
\]

\[
\sum_{i} \phi_{i} x_{ijkmt} \leq e_{m, y_{jkm}} \quad \forall j, k, m \tag{C.38}
\]

\[
\zeta_k + \pi_{1jm} \leq t \leq \eta_k - \tau_{jkm} - \alpha_m - \beta_m - \pi_{2km}
\]

\[
\sum_{i} \phi_{i} x_{1ijkmt} \leq e_{m, y_{1jkm}} \quad \forall k, m; j \in J_1(J) \tag{C.39}
\]

\[
\zeta_k + \pi_{1jm} \leq t \leq \eta_k - \tau_{jkm} - \alpha_m - \beta_m - \pi_{2km}
\]
\[\sum_{i} \phi_{i} x_{38_{jkm}} \leq \varepsilon_{m} y_{38_{jkm}} \quad \forall k, m, j \in J_{38}(J)\]  
(C.40)

\[\zeta_{k} + \pi_{1_{jm}} \leq t \leq \eta_{k} - \tau_{jk} - \alpha_{m} - \beta_{m} - \pi_{2_{km}}\]  
(C.41)

\[\sum_{j} z_{jkm} = \sum_{j} y_{jkm(t-\tau_{jk}-m-\beta_{m})} \quad \forall k, m\]  
(C.42)

\[\zeta_{k} + \alpha_{m} + \beta_{m} \leq t \leq \eta_{k} - \pi_{2_{km}}\]  
(C.43)

\[v_{jmth} = v_{jm(t-1)h} + \sum_{k \ni \mu_{k} = h} [z_{jkm(t-\tau_{jk})} - y_{jkm}] \quad \forall j, m, h, t \geq 2\]  
(C.44)

\[v_{1_{jmth}} = v_{1_{jm(t-1)h}} + \sum_{k \ni \mu_{k} = h} [z_{1_{jkm(t-\tau_{jk})}} - y_{1_{jkm}}] \quad \forall m, h, j \in J_{1}(J), t \geq 2\]  
(C.45)

\[\sum_{j \in J_{38}(J)} z_{38_{jkm}} = \sum_{j \in J_{38}(J)} [y_{38_{jkm(t-\tau_{jk}-m-\beta_{m})}}] \quad \forall k, m\]  
(C.46)

\[v_{jm_{1}h} = q_{jmh} - \sum_{k \ni \mu_{k} = h} y_{jkm_{1}h} \quad \forall j, m, h\]  
(C.47)

\[v_{1_{jm_{1}h}} = q_{jmh} + \sum_{k \ni \mu_{k} = h} [-y_{1_{jkm_{1}h}}] \quad \forall m, h, j \in J_{1}(J)\]  
(C.48)

\[\sum_{k \ni \mu_{k} = h} x_{38_{jkm}} \leq \varepsilon_{m} y_{38_{jkm}} \quad \forall k, m, j \in J_{38}(J)\]  
(C.49)

\[xx_{ij} + EI_{ij} \leq IA_{ij} + AU_{ij} \quad \forall i, j\]  
(C.50)

The maximum amount of commodity \(i\) shipped from FSL \(j\) across all FOLs across each loss-of-access scenario and the amount of commodity it reallocated to other FSLs must be less than or
equal to the sum of the initial allocation of commodity \(i\) and the additional tons of commodity \(i\) acquired at FSL \(j\).

\[
\sum_{j} A_{ij} \leq \sum_{j} E_{ij} + A_{P_i} \quad \forall i
\]  
(C.51)  
\[E_{ij} \leq b_{ij} \times m \quad \forall i, j
\]  
(C.52)  
\[A_{ij} \leq (1 - b_{ij}) \times m \quad \forall i, j
\]  
(C.53)  
\[
\sum_{o} p_{jho} \leq 1 \quad \forall h; j \in AFL(J)
\]  
(C.54)  
\[
\sum_{m} \sum_{k} \sum_{t} x_{ijkmt} \leq \left( \sum_{i} D_{ikt} \right) \sum_{h} p_{jho} \quad \forall o; j \in AFL(J)
\]  
(C.55)  
\[
\sum_{m} \sum_{k} \sum_{t} x_{1ijkmt} \leq \left( \sum_{i} D_{ikt} \right) \sum_{h} p_{jho} \quad \forall o; j \in AFL(J_1)
\]  
(C.56)  
\[
\sum_{m} \sum_{k} \sum_{t} x_{38ijkmt} \leq \left( \sum_{i} D_{ikt} \right) \sum_{h} p_{jho} \quad \forall o; j \in AFL(J_{38})
\]  
(C.57)  
\[
\sum_{k} x_{ijkmt} \leq x_{ij} \quad \forall i, j
\]  
(C.58)  
\[
\sum_{k} x_{1ijkmt} \leq x_{ij} \quad \forall i; j \in J_1(J)
\]  
(C.59)  
\[
\sum_{k} x_{38ijkmt} \leq x_{ij} \quad \forall i; j \in J_{38}(J)
\]  
(C.60)

Total tons of commodity \(i\) sent out from FSL \(j\) across all 39 scenarios is equal to the variable \(xx\).

\[
x_{ijkmt} = 0 \quad t < \zeta_k + \pi_{1jm}; \ t > \eta_k - \tau_{jkm} - \alpha_m - \beta_m - \pi_{2km}
\]  
(C.61)  
\[
x_{1ijkmt} = 0 \quad t < \zeta_k + \pi_{1jm}; \ t > \eta_k - \tau_{jkm} - \alpha_m - \beta_m - \pi_{2km}
\]  
(C.62)

\[
x_{38ijkmt} = 0 \quad t < \zeta_k + \pi_{1jm}; \ t > \eta_k - \tau_{jkm} - \alpha_m - \beta_m - \pi_{2km}
\]  
(C.63)
\(x_{ijkmt}, x_{ijkmt}^1, ..., x_{ijkmt}^38, x_{ij}^1, s_{ikt}, s_{ikt}^1, ..., s_{ikt}^38, s_{ikt}, s_{ikt}^1, ..., s_{ikt}^38, y_{jkm}, y_{jkm}^1, ..., y_{jkm}^38, y_{jkm}, y_{jkm}^1, ..., y_{jkm}^38, y_{jkm}\) 

\(z_{jkm}, z_{jkm}^1, ..., z_{jkm}^38, z_{jkm}^1, z_{jkm}^1, ..., z_{jkm}^38, z_{jkm}, v_{jmth}, v_{jmth}^1, ..., v_{jmth}^38, v_{jmth}, v_{jmth}^1, ..., v_{jmth}^38, v_{jmth}, v_{jmth}^1, ..., v_{jmth}^38, v_{jmth}\) 

\(w_j, p_{pp_{jho}}, b \in \{0, 1\}\) 

\((C.64)\)

\(\{0, 1\}\) 

\((C.65)\)

\((C.66)\)
Appendix D

General Algebraic Modeling System for the Reliability Model

The following GAMS statements implement the reliability optimization model described in Chapter 5 of this dissertation. (Some tables and parameters are abbreviated for space reasons)

Sets

\( I \) commodities /BOS, VEH, MIS, JDM/
\( J \) FSL index /Andersen,….Tocumen/
\( J_1(J) \) subset of all but first FSL /Al Udeid,….Tocumen/
\( J_2(J) \) subset of all but second FSL /Andersen,….Tocumen/
\( J_3(J) \) subset of all but last FSL /Andersen,….Spangdahlem/
\( SCEN \) list of scenarios /Ndjamena20_6,….Payalebar11_3/
\( AFLOAT(J) \) APF FSLs /MUN2/
\( K \) FOL index /Alkharaj,….Tuzla/
\( M \) mode of transport /C-17, HSS, Truck/
\( AIR(M) \) aircraft /C-17/
\( LAN(M) \) land vehicles /Truck/
\( SEA(M) \) sea vehicles /HSS/
\( H \) phase /H1*H7/
\( T \) time periods which divide up each phase /T1*T10/

ALIAS (T,N);

\textbf{TABLE THETA(M,H)} cost of obtaining an additional vehicle of mode \( M \) at the beginning of phase \( H \)

\begin{tabular}{lrr}
  & H1 & ... & H7 \\
\hline
C-17 & 99999999 & ... & 77994138 \\
: & : & : & : \\
HSS & 99999999 & ... & 77994138 \\
\end{tabular}

\textbf{TABLE PSI(I,K)} shortfall cost per time unit per ton of commodity \( I \) not fulfilled at FOL \( K \)

\begin{tabular}{lrr}
Alkharaj & ... & Tuzla \\
\hline
BOS & 0 & ... & 0 \\
: & : & : & : \\
JDM & 0 & ... & 0 \\
\end{tabular}
**TABLE OMEGA(I,J,K,M)** cost per ton of commodity I transported from FSL J to FOL K via mode M

<table>
<thead>
<tr>
<th></th>
<th>Alkharaj.C-17</th>
<th>Tuzla.HSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOS.Anderson</td>
<td>11928.6384</td>
<td>0</td>
</tr>
<tr>
<td>JDM.Tocumen</td>
<td>15491.9648</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE PI1(J,M)** additional time needed before loading for commodities to leaving FSL J via mode M

<table>
<thead>
<tr>
<th></th>
<th>C-17</th>
<th>HSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anderson</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Tocumen</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE PI2(K,M)** additional time needed following unloading for commodities to reach FOL K via mode M

<table>
<thead>
<tr>
<th></th>
<th>C-17</th>
<th>HSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alkharaj</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Tuzla</td>
<td>0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**TABLE TAU(J,K,M)** one-way transportation time from FSL J to FOL K via mode M

<table>
<thead>
<tr>
<th></th>
<th>Alkharaj.C-17</th>
<th>Tuzla.HSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anderson</td>
<td>0.7</td>
<td>13889</td>
</tr>
<tr>
<td>Tocumen</td>
<td>0.8</td>
<td>13889</td>
</tr>
</tbody>
</table>

**TABLE C(M,H)** planned systemwide inventory of mode M vehicles at the beginning of phase H

<table>
<thead>
<tr>
<th></th>
<th>H1</th>
<th>H7</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-17</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>HSS</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

**TABLE D(I,K,T)** cumulative demand in tons for commodity I at FOL K by time T

<table>
<thead>
<tr>
<th></th>
<th>T5</th>
<th>T10</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOS.Alkharaj</td>
<td>0</td>
<td>448.9</td>
</tr>
<tr>
<td>JDM.Tuzla</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
### TABLE STEAMTIME(J,K) APF FSLs

<table>
<thead>
<tr>
<th>GomezninoApjay</th>
<th>PayaLebar</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUN2</td>
<td>2.5</td>
</tr>
</tbody>
</table>

### Parameters

**L(I)** cost of reallocating tons of commodity I between FSLs

<table>
<thead>
<tr>
<th>/BOS</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>JDM</td>
<td>2000</td>
</tr>
</tbody>
</table>

**P(I)** cost of procuring additional tons of commodity I

<table>
<thead>
<tr>
<th>/BOS</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>JDM</td>
<td>10000</td>
</tr>
</tbody>
</table>

**DELTA(J)** fixed cost incurred to open FSL J with E(J) square feet of storage space

| /Anderson | 0       |
|           | :       |
| Tocumen   | 0/      |

**XI(J)** variable cost per square foot of storage space needed beyond E(J) for commodities as FSL J

| /Anderson | 2908800 |
|           | :       |
| Tocumen   | 1728000/ |

**UPSILON(J)** operating cost per warehouse at FSL J (total cost over the time horizon discounted appropriately)

| /Anderson | 3263491.442 |
|           | :           |
| Tocumen   | 1938707.788/ |

**ALPHA(M)** number of time periods necessary to load a mode M vehicle

<table>
<thead>
<tr>
<th>/C-17</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSS</td>
<td>0.2/</td>
</tr>
</tbody>
</table>
**BETA(M)** number of time periods necessary to unload a mode M vehicle
/C-17 0.2
: : 
HSS 0.2/

**GAMMA(M)** maximum load in tones per mode M vehicle
/C-17 45
: : 
HSS 400/

**EPSILON(M)** maximum square feet per mode M vehicle
/C-17 1080
: : 
HSS 40000/

**ZETA(K)** contingency start date at FOL K
/Alkharaj 1
: : 
Tuzla 1/

**ETA(K)** contingency finish date at FOL K
/Alkharaj 10
: : 
Tuzla 10/

**MU(K)** phase of contingency occurrence associated with FOL K
/Alkharaj 1
: : 
Tuzla 5/

**RHO(M)** conversion factor for parking space for mode M
/C-17 1
: : 
HSS 1/

**SIGMA(M)** utilization rate expressed as the average flying hour goal per day divided by 24 hours
/C-17 0.6041
: : 
HSS 1/
<table>
<thead>
<tr>
<th>PHI(I)</th>
<th>conversion factor for commodity I from tons to square feet of storage space</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>/BOS 21.6</td>
</tr>
<tr>
<td></td>
<td>:</td>
</tr>
<tr>
<td></td>
<td>JDM 9.2</td>
</tr>
<tr>
<td></td>
<td>:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A1(J)</th>
<th>max on ground in AIR equivalent vehicles at FSL J</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>/Anderson 2</td>
</tr>
<tr>
<td></td>
<td>:</td>
</tr>
<tr>
<td></td>
<td>Tocumen 2/</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A2(J)</th>
<th>max on ground in LAN equivalent vehicles at FSL J</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>/Anderson 6</td>
</tr>
<tr>
<td></td>
<td>:</td>
</tr>
<tr>
<td></td>
<td>Tocumen 6/</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A3(J)</th>
<th>max on ground in SEA equivalent vehicles at FSL J</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>/Anderson 1</td>
</tr>
<tr>
<td></td>
<td>:</td>
</tr>
<tr>
<td></td>
<td>Tocumen 1/</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B1(K)</th>
<th>max on ground in AIR equivalent vehicles at FOL K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>/Alkharaj 2</td>
</tr>
<tr>
<td></td>
<td>:</td>
</tr>
<tr>
<td></td>
<td>Tuzla 2/</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B2(K)</th>
<th>max on ground in LAN equivalent vehicles at FOL K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>/Alkharaj 6</td>
</tr>
<tr>
<td></td>
<td>:</td>
</tr>
<tr>
<td></td>
<td>Tuzla 6/</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B3(K)</th>
<th>max on ground in SEA equivalent vehicles at FOL K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>/Alkharaj 1</td>
</tr>
<tr>
<td></td>
<td>:</td>
</tr>
<tr>
<td></td>
<td>Tuzla 1/</td>
</tr>
</tbody>
</table>
**E(J)** minimum square footage needed for an economically feasible FSL at location J for commodities

/Anderson  60.07

:  

Tocumen  0/

**G** maximum square feet per commodity I per warehouse

/20000/

**F(J)** maximum potential square feet of storage space at FSL J

/Anderson  250

:  

Tocumen  250/

**FOLSCEN(K)** scenario of FOL

/AlKharaj5_9  6

:  

Tuzla1_7  11/

**SCENPHASE(SCEN)** phase of scenario

/1_7  5

:  

7_10  7/;

**Free Variables**

OBJ objective;

**Positive Variables**

NN(J) additional number of notional igloos beyond E(J) needed at FSL J

WW(J) number of notional igloos utilized at FSL J

Q(J,M,H) number of mode M vehicles available at FSL J at the start of time T=1 during phase H

R(M,H) additional mode M vehicles obtained at the beginning of phase H

S(I,K,T) shortfall below demand in tons for commodity I at FOL K not fulfilled by time T

S1(I,K,T) shortfall below demand in tons for commodity I at FOL K not fulfilled by time T for subset J1

:  

S38(I,K,T) shortfall below demand in tons for commodity I at FOL K not fulfilled by time T for subset J38

V(J,M,T,H) number of mode M vehicles available at FSL J at the end of time T during phase H
\( V_{1(J1,M,T,H)} \) number of mode M vehicles available at FSL J at the end of time T during phase H for Subset J1

\( V_{38(J38,M,T,H)} \) number of mode M vehicles available at FSL J at the end of time T during phase H for Subset J38

\( XX(I,J) \) maximum total tons of commodity I sent out of FSL J across the full set of FSLs and all 38 subsets of FSLs

\( AU(I,J) \) additional tons of commodity I at FSL J beyond the initial amount

\( EI(I,J) \) reallocated tons of commodity I at FSL J

\( AP(I) \) additional tons of commodity I procured across the entire system

\( X(I,J,K,M,T) \) tons of commodity I sent from FSL J to FOL K via mode M beginning loading on time T

\( X_{1(J1,K,M,T)} \) tons of commodity I sent from FSL J to FOL K via mode M beginning loading on time T for subset J1

\( X_{38(J38,K,M,T)} \) tons of commodity I sent from FSL J to FOL K via mode M beginning loading on time T for subset J38;

**Binary Variables**

\( PP(J,H,SCEN) \) binary variable indicating if afloat prepositioned ship J supports scenario SCEN during phase H

\( b(I,J) \) binary variable to ensure that \( EI \) and \( AU \) are not both greater than zero

\( W(J) \) binary variable indicating status of FSL J;

**Integer Variables**

\( Y(J,K,M,T) \) number of mode M vehicles tasked to transport commodities and personnel from FSL J to FOL K beginning loading on time T

\( Y_{1(J1,K,M,T)} \) number of mode M vehicles tasked to transport commodities and personnel from FSL J to FOL K beginning loading on time T for Subset J1

\( Y_{38(J38,K,M,T)} \) number of mode M vehicles tasked to transport commodities and personnel from FSL J to FOL K beginning loading on time T for Subset J38

\( Z(J,K,M,T) \) number of mode M vehicles tasked to make the return trip from FOL K to FSL J departing on time T

\( Z_{1(J1,K,M,T)} \) number of mode M vehicles tasked to make the return trip from FOL K to FSL J departing on time T for subset J1
Z38(J38,K,M,T) number of mode M vehicles tasked to make the return trip from FOL K to FSL J departing on time T for subset J38;

**Equations**

OBJECTIVE objective function
TOTALNUMBERVEHICLES(M,H) constraint on the total number of mode M vehicles during phase H
FSLVEHAVAIL(J,M,T,H) constraint on mode M vehicle availability at FSL J during time T>1 for phase H
FSLVEHAVAIL1(J1,M,T,H) constraint on mode M vehicle availability at FSL J during time T>1 for phase H for subset J1

FSLVEHAVAIL38(J38,M,T,H) constraint on mode M vehicle availability at FSL J during time T>1 for phase H for subset J38
FSLMOGAIR(J,T,H) constraint on MOG of FSL J for air vehicles during time T for phase H
FSLMOGAIR1(J1,T,H) constraint on MOG of FSL J for air vehicles during time T for phase H for subset J1

FSLMOGAIR38(J38,T,H) constraint on MOG of FSL J for air vehicles during time T for phase H for subset J38
FSLMOGLAN(J,T,H) constraint on MOG of FSL J for land vehicles during time T for phase H
FSLMOGLAN1(J1,T,H) constraint on MOG of FSL J for land vehicles during time T for phase H for subset J1

FSLMOGLAN38(J38,T,H) constraint on MOG of FSL J for land vehicles during time T for phase H for subset J38
FSLMOGSEA(J,T,H) constraint on MOG of FSL J for sea vehicles during time T for phase H
FSLMOGSEA1(J1,T,H) constraint on MOG of FSL J for sea vehicles during time T for phase H for subset J1

FSLMOGSEA38(J38,T,H) constraint on MOG of FSL J for sea vehicles during time T for phase H for subset J38
FOLMOGAIR(K,T) constraint on MOG of FOL K for air vehicles during time T
FOLMOGAIR1(K,T) constraint on MOG of FOL K for air vehicles during time T for subset J1
FOLMOGARI38(K,T) constraint on MOG of FOL K for air vehicles during time T for subset J38
FOLMOGLAN(K,T) constraint on MOG of FOL K for land vehicles during time T
FOLMOGLAN1(K,T) constraint on MOG of FOL K for land vehicles during time T for subset J1
FOLMOGLAN38(K,T) constraint on MOG of FOL K for land vehicles during time T for subset J38
FOLMOGSEA(K,T) constraint on MOG of FOL K for sea vehicles during time T
FOLMOGSEA1(K,T) constraint on MOG of FOL K for sea vehicles during time T for subset J1
FOLMOGSEA38(K,T) constraint on MOG of FOL K for sea vehicles during time T for subset J38
MAXSHIP(I,J) calculates the maximum amount of commodity I shipped from FSL J
MAXSHIP1(I,J) calculates the maximum amount of commodity I shipped from FSL J for subset J1
MAXSHIP38(I,J) calculates the maximum amount of commodity I shipped from FSL J for subset J38
DEMANDCONSTRAINT(I,K,T) constraint on meeting cumulative demand for commodity I at FOL K by time T
DEMANDCONSTRAINT1(I,K,T) constraint on meeting cumulative demand for commodity I at FOL K by time T for subset J1
DEMANDCONSTRAINT38(I,K,T) constraint on meeting cumulative demand for commodity I at FOL K by time T for subset J38
FSLSTORSF(J,H) constraint on storage space in square feet at FSL J during phase H
FSLSTORSF2(J,H) constraint on storage utilization in square feet at FSL J during phase H
FSLSTOR(J) additional constraint on storage space at FSL J
FSLUSAGE(J) constraint on minimal utilization at FSL J
FSLUSAGE2(J) additional constraint on minimal utilization at FSL J
UTERATE(M,H) constraint limiting the average fleetwide utilization of mode M vehicles over phase H
UTERATE1(M,H) constraint limiting the average fleetwide utilization of mode M vehicles over phase H for subset J1
UTERATE38(M,H) constraint limiting the average fleetwide utilization of mode M vehicles over phase H for subset J38
BOOKVEHTO(J,K,M,T) translates tons of commodities transported from FSL J to FOL K beginning loading on time T into mode M vehicles via tonnage
BOOKVEHTO1(J1,K,M,T) translates tons of commodities transported from FSL J to FOL K beginning loading on time T into mode M vehicles via tonnage for subset J1

BOOKVEHTO38(J38,K,M,T) translates tons of commodities transported from FSL J to FOL K beginning loading on time T into mode M vehicles via tonnage for subset J38
BOOKVEHSF(J,K,M,T) translates tons of commodities transported from FSL J to FOL K beginning loading on time T into mode M vehicles via storage in square feet
BOOKVEHSF1(J1,K,M,T) translates tons of commodities transported from FSL J to FOL K beginning loading on time T into mode M vehicles via storage in square feet for subset J1

BOOKVEHSF38(J38,K,M,T) translates tons of commodities transported from FSL J to FOL K beginning loading on time T into mode M vehicles via storage in square feet for subset J38
RETURNTRIPVEHICLES(K,M,T) assigns mode M vehicles to return trips following delivery and unloading to FOL K at time T
RETURNTRIPVEHICLES1(K,M,T) assigns mode M vehicles to return trips following delivery and unloading to FOL K at time T for subset J1

RETURNTRIPVEHICLES38(K,M,T) assigns mode M vehicles to return trips following delivery and unloading to FOL K at time T for subset J38
FLOWBALANCEVEH(J,M,T,H) tracks the number of mode M vehicles available at FSL J at the end of time T>1 for phase H
FLOWBALANCEVEH1(J1,M,T,H) tracks the number of mode M vehicles available at FSL J at the end of time T>1 for phase H for subset J1

FLOWBALANCEVEH38(J38,M,T,H) tracks the number of mode M vehicles available at FSL J at the end of time T>1 for phase H for subset J38
INITFLOWBALANCEVEH(J,M,H) tracks the number of mode M vehicles available at FSL J at the end of time T=1 for phase H
INITFLOWBALANCEVEH1(J1,M,H) tracks the number of mode M vehicles available at FSL J at the end of time T=1 for phase H for subset J1
INITFLOWBALANCEVEH38(J38,M,H) tracks the number of mode M vehicles available at FSL J at the end of time T=1 for phase H for subset J38

SHIPREAL(I,J) constraint on the amount of commodity I shipped out of FSL J and reallocated to other FSLs

SYSPROC(I) constraint on the additional amount of commodity I that is reallocated or procured across the entire system

REALL(I,J) constraint to ensure that if EI is greater than zero then AU must be equal to zero

ADDUNIT(I,J) constraint to ensure that if AU is greater than zero then EI must be equal to zero

AFLOATSCEN(J,H) afloat constraint

AFLOATSCEN2(J,SCEN) afloat constraint

AFLOATSCEN21(J1,SCEN) afloat constraint for subset J1

AFLOATSCEN238(J38,SCEN) afloat constraint for subset J38;

OBJECTIVE.. OBJ =E=
(SUM(I,SUM(J,SUM(K,SUM(M,SUM(T,(OMEGA(I,J,K,M)*X(I,J,K,M,T))/39)))))+SUM(I,SUM(J1,SUM(K,SUM(M,SUM(T,(OMEGA(I,J1,K,M)*X1(I,J1,K,M,T))/39)))))+SUM(I,SUM(J2,SUM(K,SUM(M,SUM(T,(OMEGA(I,J2,K,M)*X2(I,J2,K,M,T))/39)))))+SUM(I,SUM(J3,SUM(K,SUM(M,SUM(T,(OMEGA(I,J3,K,M)*X3(I,J3,K,M,T))/39)))))+SUM(I,SUM(J4,SUM(K,SUM(M,SUM(T,(OMEGA(I,J4,K,M)*X4(I,J4,K,M,T))/39)))))+SUM(I,SUM(J5,SUM(K,SUM(M,SUM(T,(OMEGA(I,J5,K,M)*X5(I,J5,K,M,T))/39)))))+SUM(I,SUM(J6,SUM(K,SUM(M,SUM(T,(OMEGA(I,J6,K,M)*X6(I,J6,K,M,T))/39)))))+SUM(I,SUM(J7,SUM(K,SUM(M,SUM(T,(OMEGA(I,J7,K,M)*X7(I,J7,K,M,T))/39)))))+SUM(I,SUM(J8,SUM(K,SUM(M,SUM(T,(OMEGA(I,J8,K,M)*X8(I,J8,K,M,T))/39))))+SUM(I,SUM(J9,SUM(K,SUM(M,SUM(T,(OMEGA(I,J9,K,M)*X9(I,J9,K,M,T))/39))))+SUM(I,SUM(J10,SUM(K,SUM(M,SUM(T,(OMEGA(I,J10,K,M)*X10(I,J10,K,M,T))/39))))+SUM(I,SUM(J11,SUM(K,SUM(M,SUM(T,(OMEGA(I,J11,K,M)*X11(I,J11,K,M,T))/39))))+SUM(I,SUM(J12,SUM(K,SUM(M,SUM(T,(OMEGA(I,J12,K,M)*X12(I,J12,K,M,T))/39))))+SUM(I,SUM(J13,SUM(K,SUM(M,SUM(T,(OMEGA(I,J13,K,M)*X13(I,J13,K,M,T))/39))))+SUM(I,SUM(J14,SUM(K,SUM(M,SUM(T,(OMEGA(I,J14,K,M)*X14(I,J14,K,M,T))/39))))+SUM(I,SUM(J15,SUM(K,SUM(M,SUM(T,(OMEGA(I,J15,K,M)*X15(I,J15,K,M,T))/39))))+SUM(I,SUM(J16,SUM(K,SUM(M,SUM(T,(OMEGA(I,J16,K,M)*X16(I,J16,K,M,T))/39))))+SUM(I,SUM(J17,SUM(K,SUM(M,SUM(T,(OMEGA(I,J17,K,M)*X17(I,J17,K,M,T))/39))))+SUM(I,SUM(J18,SUM(K,SUM(M,SUM(T,(OMEGA(I,J18,K,M)*X18(I,J18,K,M,T))/39))))+SUM(I,SUM(J19,SUM(K,SUM(M,SUM(T,(OMEGA(I,J19,K,M)*X19(I,J19,K,M,T))/39))))+SUM(I,SUM(J20,SUM(K,SUM(M,SUM(T,(OMEGA(I,J20,K,M)*X20(I,J20,K,M,T))/39))))+SUM(I,SUM(J21,SUM(K,SUM(M,SUM(T,(OMEGA(I,J21,K,M)*X21(I,J21,K,M,T))/39))))+SUM(I,SUM(J22,SUM(K,SUM(M,SUM(T,(OMEGA(I,J22,K,M)*X22(I,J22,K,M,T))/39))))+SUM(I,SUM(J23,SUM(K,SUM(M,SUM(T,(OMEGA(I,J23,K,M)*X23(I,J23,K,M,T))/39))))+SUM(I,SUM(J24,SUM(K,SUM(M,SUM(T,(OMEGA(I,J24,K,M)*X24(I,J24,K,M,T))/39))))+SUM(I,SUM(J25,SUM(K,SUM(M,SUM(T,(OMEGA(I,J25,K,M)*X25(I,J25,K,M,T))/39))))+SUM(I,SUM(J26,SUM(K,SUM(M,SUM(T,(OMEGA(I,J26,K,M)*X26(I,J26,K,M,T))/39))))+SUM(I,SUM(J27,SUM(K,SUM(M,SUM(T,(OMEGA(I,J27,K,M)*X27(I,J27,K,M,T))/39))))+SUM(I,SUM(J28,SUM(K,SUM(M,SUM(T,(OMEGA(I,J28,K,M)*X28(I,J28,K,M,T))/39))))+SUM(I,SU
TOTALNUMBERVEHICLES(M, H) ..
SUM(J, Q(J, M, H)) =E= C(M, H) + R(M, H);
FSLVEHAVAIL(J, M, T, H)$((ORD(T) > 1)) ..
SUM(K$((MU(K) = ORD(H))), (Y(J, K, M, T)) ) =L= V(J, M, T-1, H),
FSLVEHAVAIL1(J1, M, T, H)$((ORD(T) > 1)) ..
SUM(K$((MU(K) = ORD(H))), (Y1(J1, K, M, T)) ) =L= V1(J1, M, T-1, H),
FSLMOGAIR(J1, T, H) ..
SUM(K$((MU(K) = ORD(H))), SUM(M$AIR(M), SUM(N$((ORD(N) <= ALPHA(M))), RHO(M) * (Y(J, K, M, N + CEIL(ORD(T) - 2*ORD(N) + 1)))))) =L= A1(J1);, FSLMOGAIR38(J38, T, H) ..
SUM(K$((MU(K) = ORD(H))), SUM(M$AIR(M), SUM(N$((ORD(N) <= ALPHA(M))), RHO(M) * (Y38(J38, K, M, N + CEIL(ORD(T) - 2*ORD(N) + 1)))))) =L= A1(J38);
FSLMOGLAN(J,T,H)..
SUM(K$(MU(K)=ORD(H)),SUM(M$SLAN(M),SUM(NS(ORD(N)<=ALPHA(M)),RHO(M)*(Y(J,K,M,N+CEIL(ORD(T)-2*ORD(N)+1))))) =L= A2(J);
FSLMOGLAN1(J1,T,H)..
SUM(K$(MU(K)=ORD(H)),SUM(M$SLAN(M),SUM(NS(ORD(N)<=ALPHA(M)),RHO(M)*(Y1(J1,K,M,N+CEIL(ORD(T)-2*ORD(N)+1))))) =L= A2(J1);
.
.
FSLMOGLAN38(J38,T,H)..
SUM(K$(MU(K)=ORD(H)),SUM(M$SLAN(M),SUM(NS(ORD(N)<=ALPHA(M)),RHO(M)*(Y38(J38,K,M,N+CEIL(ORD(T)-2*ORD(N)+1))))) =L= A2(J38);
FSLMOGSEA(J,T,H)..
SUM(K$(MU(K)=ORD(H)),SUM(M$SEA(M),SUM(NS(ORD(N)<=ALPHA(M)),RHO(M)*(Y(J,K,M,N+CEIL(ORD(T)-2*ORD(N)+1))))) =L= A3(J);
FSLMOGSEA1(J1,T,H)..
SUM(K$(MU(K)=ORD(H)),SUM(M$SEA(M),SUM(NS(ORD(N)<=ALPHA(M)),RHO(M)*(Y1(J1,K,M,N+CEIL(ORD(T)-2*ORD(N)+1))))) =L= A3(J1);
.
.
FSLMOGSEA38(J38,T,H)..
SUM(K$(MU(K)=ORD(H)),SUM(M$SEA(M),SUM(NS(ORD(N)<=ALPHA(M)),RHO(M)*(Y38(J38,K,M,N+CEIL(ORD(T)-2*ORD(N)+1))))) =L= A3(J38);
FOLMOGAIR(K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K)))..
SUM(J,SUM(M$AIR(M),SUM(NS(ORD(N)<=BETA(M)),RHO(M)*(Y(J,K,M,N+CEIL(ORD(T)-TAU(J,K,M)-ALPHA(M)-2*ORD(N)+1))))) =L= B1(K);
FOLMOGAIR1(K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K)))..
SUM(J1,SUM(M$AIR(M),SUM(NS(ORD(N)<=BETA(M)),RHO(M)*(Y1(J1,K,M,N+CEIL(ORD(T)-TAU(J1,K,M)-ALPHA(M)-2*ORD(N)+1))))) =L= B1(K);
.
.
FOLMOGAIR38(K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K)))..
SUM(J38,SUM(M$AIR(M),SUM(NS(ORD(N)<=BETA(M)),RHO(M)*(Y38(J38,K,M,N+CEIL(ORD(T)-TAU(J38,K,M)-ALPHA(M)-2*ORD(N)+1))))) =L= B1(K);
FOLMOGLAN(K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K)))..
SUM(J,SUM(M$LAN(M),SUM(NS(ORD(N)<=BETA(M)),RHO(M)*(Y(J,K,M,N+CEIL(ORD(T)-TAU(J,K,M)-ALPHA(M)-2*ORD(N)+1))))) =L= B2(K);
FOLMOGLAN1(K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K)))..
SUM(J1,SUM(M$LAN(M),SUM(NS(ORD(N)<=BETA(M)),RHO(M)*(Y1(J1,K,M,N+CEIL(ORD(T)-TAU(J1,K,M)-ALPHA(M)-2*ORD(N)+1))))) =L= B2(K);
.
.

FOLMOGLAN38(K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K))).. SUM(J38, SUM(M$LAN(M), SUM(N$(ORD(N)<=BETA(M)), RHO(M)*(Y38(J38,K,M,N+CEIL(ORD(T)-TAU(J38,K,M)-ALPHA(M)-2*ORD(N)+1))))) = L = B2(K);
FOLMOGSEA(K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K))).. SUM(J, SUM(M$SEA(M), SUM(N$(ORD(N)<=BETA(M)), RHO(M)*(Y(J,K,M,N+CEIL(ORD(T)-TAU(J,K,M)-ALPHA(M)-2*ORD(N)+1))))) = L = B3(K);
FOLMOGSEA1(K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K))).. SUM(J1, SUM(M$SEA(M), SUM(N$(ORD(N)<=BETA(M)), RHO(M)*(Y1(J1,K,M,N+CEIL(ORD(T)-TAU(J1,K,M)-ALPHA(M)-2*ORD(N)+1))))) = L = B3(K);
FOLMOGSEA38(K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K))).. SUM(J38, SUM(M$SEA(M), SUM(N$(ORD(N)<=BETA(M)), RHO(M)*(Y38(J38,K,M,N+CEIL(ORD(T)-TAU(J38,K,M)-ALPHA(M)-2*ORD(N)+1))))) = L = B3(K);
MAXSHIP(I,J).. SUM(K, SUM(M, SUM(T, X(I,J,K,M,T)))) = L = XX(I,J);
MAXSHIP1(I,J1).. SUM(K, SUM(M, SUM(T, X1(I,J1,K,M,T)))) = L = XX(I,J1);

MAXSHIP38(I,J38).. SUM(K, SUM(M, SUM(T, X38(I,J38,K,M,T)))) = L = XX(I,J38);
DEMANDCONSTRAINT(I,K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K))).. SUM(J, SUM(M, SUM(N$(ORD(N)<=ORD(T)), X(I,J,K,M,N-CEIL(TAU(J,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))))) = G = SUM(N$(ORD(N)<=ORD(T)), D(I,K,N))-S(I,K,T);
DEMANDCONSTRAINT1(I,K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K))).. SUM(J1, SUM(M, SUM(N$(ORD(N)<=ORD(T)), X1(I,J1,K,M,N-CEIL(TAU(J1,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))))) = G = SUM(N$(ORD(N)<=ORD(T)), D(I,K,N))-S1(I,K,T);
DEMANDCONSTRAINT38(I,K,T)$((ORD(T)>=ZETA(K)) AND (ORD(T)<=ETA(K))).. SUM(J38, SUM(M, SUM(N$(ORD(N)<=ORD(T)), X38(I,J38,K,M,N-CEIL(TAU(J38,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))))) = G = SUM(N$(ORD(N)<=ORD(T)), D(I,K,N))-S38(I,K,T);
FSLSTORSF(J).. SUM(I, PHI(I)*XX(I,J)) = L = G*(E(J)*W(J)+NN(J));
FSLSTORSF2(J).. SUM(I, PHI(I)*XX(I,J)) = L = G*WW(J);
FSLSTORE(J).. NN(J) = L = (F(J)-E(J))*W(J);
FSLUSAGE(J).. W(J) = L = WW(J);
FSLUSAGE2(J).. W(J) = L = E(J)+NN(J);
UTERATE(M,H).. SUM(K$((MU(K)=ORD(H)), SUM(T, TAU(J,K,M)*(Y(J,K,M,T)))+SUM(T$ORD(T)<= (CARD(T)-TAU(J,K,M))), TAU(J,K,M)*Z(J,K,M,T))+SUM(T$ORD(T)>(CARD(T)-TAU(J,K,M)+1)), (CARD(T)-ORD(T))*Z(J,K,M,T))) = L = CARD(T)*(C(M,H)+R(M,H))*SIGMA(M);
UTERATE1(M,H)..
SUM(K$(MU(K)=ORD(H)),SUM(J1,SUM(T,TAU(J1,K,M)*(Y1(J1,K,M,T)))+SUM(T$(ORD(T)<=(CARD(T)-TAU(J1,K,M)),TAU(J1,K,M)*Z1(J1,K,M,T)))+SUM(T$(ORD(T)>(CARD(T)-TAU(J1,K,M)+1)),(CARD(T)-ORD(T))*Z1(J1,K,M,T)))) =L= CARD(T)*(C(M,H)+R(M,H))*SIGMA(M);

UTERATE38(M,H)..
SUM(K$(MU(K)=ORD(H)),SUM(J38,SUM(T,TAU(J38,K,M)*(Y38(J38,K,M,T)))+SUM(T$(ORD(T)<=(CARD(T)-TAU(J38,K,M)),TAU(J38,K,M)*Z38(J38,K,M,T)))+SUM(T$(ORD(T)>(CARD(T)-TAU(J38,K,M)+1)),(CARD(T)-ORD(T))*Z38(J38,K,M,T)))) =L= CARD(T)*(C(M,H)+R(M,H))*SIGMA(M);

BOOKVEHTO(J,K,M,T)$((ORD(T)>=ZETA(K)+PI1(J,M)) AND (ORD(T)<=ETA(K)-(TAU(J,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))).. SUM(I,X(I,J,K,M,T)) =L= GAMMA(M)*Y(J,K,M,T);

BOOKVEHTO1(J1,K,M,T)$((ORD(T)>=ZETA(K)+PI1(J1,M)) AND (ORD(T)<=ETA(K)-(TAU(J1,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))).. SUM(I1,X1(I1,J1,K,M,T)) =L= GAMMA(M)*Y1(J1,K,M,T);

BOOKVEHTO38(J38,K,M,T)$((ORD(T)>=ZETA(K)+PI1(J38,M)) AND (ORD(T)<=ETA(K)-(TAU(J38,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))).. SUM(I38,X38(I38,J38,K,M,T)) =L= GAMMA(M)*Y38(J38,K,M,T);

BOOKVEHSF(J,K,M,T)$((ORD(T)>=ZETA(K)+PI1(J,M)) AND (ORD(T)<=ETA(K)-(TAU(J,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))).. SUM(I,PHI(I)*X(I,J,K,M,T)) =L= EPSILON(M)*Y(J,K,M,T);

BOOKVEHSF1(J1,K,M,T)$((ORD(T)>=ZETA(K)+PI1(J1,M)) AND (ORD(T)<=ETA(K)-(TAU(J1,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))).. SUM(I1,PHI(I)*X1(I,J1,K,M,T)) =L= EPSILON(M)*Y1(J1,K,M,T);

BOOKVEHSF38(J38,K,M,T)$((ORD(T)>=ZETA(K)+PI1(J38,M)) AND (ORD(T)<=ETA(K)-(TAU(J38,K,M)+ALPHA(M)+BETA(M)+PI2(K,M)))).. SUM(I38,PHI(I)*X38(I,J38,K,M,T)) =L= EPSILON(M)*Y38(J38,K,M,T);

RETURNTRIPVEHICLES(K,M,T)$((ORD(T)>=ZETA(K)+ALPHA(M)+BETA(M)) AND (ORD(T)<=ETA(K)-PI2(K,M))).. SUM(J,Z(J,K,M,T)) =E= SUM(J,Y(J,K,M,T)-CEIL(TAU(J,K,M)+ALPHA(M)+BETA(M)));
RETURNTRIPVEHICLES38(K,M,T)\$(\text{ORD}(T) \geq \text{ZETA}(K) + \text{ALPHA}(M) + \text{BETA}(M)) \text{ AND} \ (\text{ORD}(T) \leq \text{ETA}(K) - \Pi_2(K,M)).. \ \text{SUM}(J38,Z38(J38,K,M,T)) = \text{E}= \text{SUM}(J38,Y38(J38,K,M,T-\text{CEIL}(\text{T}(J38,K,M) + \text{ALPHA}(M) + \text{BETA}(M))));

\text{FLOWBALANCEVEH}(J,M,T,H)\$(\text{ORD}(T)>1)\ldots \text{V}(J,M,T,H) = \text{E}= \text{V}(J,M,T-1,H) + \text{SUM}(K$(\text{MU}(K) = \text{ORD}(H)),Z(J,K,M,T-\text{CEIL}(\text{T}(J,K,M)))) - \text{Y}(J,K,M,T));

\text{FLOWBALANCEVEH1}(J1,M,T,H)\$(\text{ORD}(T)>1)\ldots \text{V1}(J1,M,T,H) = \text{E}= \text{V1}(J1,M,T-1,H) + \text{SUM}(K$(\text{MU}(K) = \text{ORD}(H)),Z1(J1,K,M,T-\text{CEIL}(\text{T}(J1,K,M)))) - \text{Y1}(J1,K,M,T));

\text{FLOWBALANCEVEH38}(J38,M,T,H)\$(\text{ORD}(T)>1)\ldots \text{V38}(J38,M,T,H) = \text{E}= \text{V38}(J38,M,T-1,H) + \text{SUM}(K$(\text{MU}(K) = \text{ORD}(H)),Z38(J38,K,M,T-\text{CEIL}(\text{T}(J38,K,M)))) - \text{Y38}(J38,K,M,T));

\text{INITFLOWBALANCEVEH}(J,M,H)\ldots \text{V}(J,M,'T1',H) = \text{E}= \text{Q}(J,M,H) + \text{SUM}(K$(\text{MU}(K) = \text{ORD}(H)), -\text{Y}(J,K,M,'T1'));

\text{SHIPLEAL}(I,J)\ldots \text{XX}(I,J) + \text{EI}(I,J) = \text{E}= \text{1A}(I,J) + \text{AU}(I,J);

\text{SYSPROC}(I)\ldots \text{SUM}(J,\text{AU}(I,J)) = \text{L}= \text{SUM}(J,\text{EI}(I,J)) + \text{AP}(I);

\text{REALL}(I,J)\ldots \text{EI}(I,J) = \text{L}= \text{b}(I,J) * 100000;

\text{ADDUNIT}(I,J)\ldots \text{AU}(I,J) = \text{L}= (1-\text{b}(I,J)) * 100000;

\text{AFLOATSCEN}(J,H)$\$(\text{AFLOAT}(J))\ldots \text{SUM}(\text{SCEN},\text{PP}(J,H,\text{SCEN})) = \text{L}= 1.0;

\text{AFLOATSCEN2}(J,SCEN)$\$(\text{AFLOAT}(J))\ldots \text{SUM}(I,\text{SUM}(K$(\text{FOLSCEN}(K) = \text{ORD}(SCEN)) , \text{SUM}(M, \text{SUM}(T, \text{X}(I,J,K,M,T)))) = \text{L}= \text{SUM}(I,\text{SUM}(K,\text{SUM}(T,D(I,K,T)))) * \text{SUM}(H$(\text{ORD}(H) = \text{SCENPHASE}(SCEN)), \text{PP}(J,H,\text{SCEN})) ;

\text{AFLOATSCEN21}(J1,SCEN)$\$(\text{AFLOAT}(J))\ldots \text{SUM}(I,\text{SUM}(K$(\text{FOLSCEN}(K) = \text{ORD}(SCEN)) , \text{SUM}(M, \text{SUM}(T, \text{X1}(I,J1,K,M,T)))) = \text{L}= \text{SUM}(I,\text{SUM}(K,\text{SUM}(T,D(I,K,T)))) * \text{SUM}(H$(\text{ORD}(H) = \text{SCENPHASE}(SCEN)), \text{PP}(J,H,\text{SCEN})) ;

\text{AFLOATSCEN238}(J38,SCEN)$\$(\text{AFLOAT}(J))\ldots \text{SUM}(I,\text{SUM}(K$(\text{FOLSCEN}(K) = \text{ORD}(SCEN)) , \text{SUM}(M, \text{SUM}(T, X38(I,J38,K,M,T)))) = \text{L}= \text{SUM}(I,\text{SUM}(K,\text{SUM}(T,D(I,K,T)))) * \text{SUM}(H$(\text{ORD}(H) = \text{SCENPHASE}(SCEN)), \text{PP}(J,H,\text{SCEN})) ;

\text{MODEL demo /ALL/;
demo.OPTFILE = 1;
OPTION ITERLIM = 500000000;
OPTION LIMROW = 10;
OPTION LIMCOL = 10;
OPTION SYSOUT = ON;
OPTION DMPSYM;
OPTION PROFILE = 3;
OPTION RESLIM=345600;
OPTION OPTCR=0.01;
OPTION MIP=CPLEX;
demo.PRIOROPT=1;
W.PRIOR(J)=1.0;
PP.PRIOR(J,H,SCEN)=1.0;
Y.PRIOR(J,K,M,T) = 2.0;
Z.PRIOR(J,K,M,T) = 3.0;
demo.HOLDFIXED=1;
SOLVE demo USING MIP MINIMIZING OBJ;

* separate the total cost into its facility construction, facility operation, transport, procurement,
and reallocations components
PARAMETERS
FACCONCOST facility construction cost
FACOPERCOST facility operating cost
TRANCOST transport cost
PROCURECOST commodity procurement cost
REALLOCATECOST commodity reallocation cost
FROMTO(I,J,K,M) tons of commodity I shipped from FSL J to FOL K via mode M;
FACCONCOST = SUM(J,DELTA(J)*W.L(J)+XI(J)*NN.L(J));
FACOPERCOST = SUM(J,UPSILON(J)*WW.L(J));
TRANCOST =
SUM(I,SUM(J,SUM(K,SUM(M,SUM(T,OMEGA(I,J,K,M)*X.L(I,J,K,M,T))))));
PROCURECOST = SUM(I,P(I)*AP.L(I));
REALLOCATECOST = SUM(I,SUM(J,L(I)*EL.L(I,J)));
FROMTO(I,J,K,M) = SUM(T,X.L(I,J,K,M,T));


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