Chapter 5 - A Model of Technology Diffusion, Growth and the Environment

1. Introduction

Chapter 3 was concerned with a theoretical review of the process of technology diffusion and its linkages with the concept of social capital. In this chapter I operationalize these theoretical constructs into an applied model of technology diffusion and growth. As described in Chapter 1, this model has two main components. A module that endogenizes the process of technology diffusion, and a standard one sector macro-econometric model for the developing world, that incorporates an environmental component. The model has been calibrated to address the question of how developing countries should allocate over time investments in produced capital, technology incentives and the "consumption" of carbon emissions. Nonetheless, the model could be used to address other questions regarding sustainable growth, such as the optimal consumption of given natural resource over time.

Before presenting the model I review the current state of the art in applied models of technology diffusion and growth. This way, it will be easier to emphasize the fundamental differences between the model that I develop in this chapter and other models currently available. The chapter is therefore organized into four sections. Section 2 reviews modern applied models of technology diffusion and discusses their virtues and limitations. Section 3 introduces my agent-based model of technology diffusion. Section 4 describes the macro-econometric one sector model. Finally, Section 5 summarizes estimates model parameters and presents general results regarding steady state dynamics.

2. Applied Models of Technological Change

The need to model the process of technological change within applied simulation models goes back to the early '70s when the energy crisis forced analysts and policymakers to design and evaluate strategies to reduce dependence on costly imported oil (see Messner, 1997). Two modeling schools
have emerged since then: bottom-up models and top-down models. Bottom-up models emphasize the micro aspects of technological change, in particular the potential for new technologies to reduce their operation costs. Usually, these models are developed within a systems engineering perspective and are rich in the type of micro data that delimit diffusion trajectories (e.g., assumptions regarding the future costs and performances of new technologies). On the other hand, top-down models come from the macroeconometric tradition and operate through aggregate production functions treating technological progress as an exogenously changing parameter within these functions (e.g., the IMF's "Multimod Mark III"; and Laxton et al., 1998) or resulting from exogenously defined changes in capital vintages (e.g., OECD's "General Equilibrium Model of Trade and The Environment"; and Beghin et al., 1996). Early versions of these two types of models include BESOM, the Brookhaven Energy Systems Optimization Model (see Cherniavsky, 1974) and ETA-MACRO (see Manne, 1979). As stated by Grübler and Gritsevskii, "common to both modeling traditions is that the only endogenous mechanism of technological change is that of progressive resources depletion and resulting cost increases" (see Grübler and Gritsevskii, 1998).

Recently, efforts have been made to provide a more adequate representation of technological change, in particular to formalize the role of uncertainty and learning. We have already mentioned in Chapter 4 that uncertainty plays a key role in delaying the adoption of new technologies. On the other hand, learning is responsible for reductions in operation and adoption costs, as well as reductions in uncertainty itself. A model that attempts to endogenize these two elements of technological change is presented in Grübler and Gritsevskii (1998). The authors explain changes in the operation and adoption costs of new technologies through a learning process that results from commercial investments, research and development (R&D), and demonstration projects in niche markets (the classical learning by doing process). Uncertainty enters the model in the form of randomness in some of the model coefficients (e.g., future demand, learning coefficients). The learning process is "endogenized" on the basis of Wattanabe's empirically derived relationship between investments in new technologies (including R&D and demonstration projects) and operation costs (see Wattanabe, 1995). The model has one decision-maker and three technologies: "existing", "incremental" and
"revolutionary". These technologies differ in their actual costs and the potential for cost reductions (the learning coefficient in Wattanabe's function is uncertain). The goal is to identify the sequence of investments in each of the three types of technologies that will minimize operation costs over a given period of time (set to 200 years). One of the author's contribution is the implementation of an optimization algorithm that solves the stochastic intertemporal optimization problem by sampling points from the ex-ante defined distribution of model parameters. Using this algorithm, the author shows that gradual investments in the revolutionary technology, assumed to be 40 times more expensive than the existing technology, are optimal from a social point of view. However, in its current state, the model faces two limitations. First, the problem is solved under the assumption of the existence of a centralized decision-maker who controls the pace of future costs reductions via investments today. More likely, in a real situation, a set of heterogeneous agents will be making technology decisions in a decentralized environment, and therefore cannot directly influence reductions in operations costs. The second problem is that the learning coefficients linking investments to cost reductions are defined exogenously. Hence, once the distribution has been defined, the diffusion of each type of technology becomes fully determined: uncertainty vanishes! The model can be used to compute a reference path that the incremental and revolutionary technologies should follow. This is the socially optimal path. Yet, from a policy perspective, it is often valuable to understand the factors that may cause the diffusion of new technologies to deviate from this socially optimal path. In Chapter 3, I proposed that these factors lie at the core of the learning mechanism, specially in the process through which agents generate and share information about new technologies and the macroeconomic environment. These are the processes that, ultimately, one wishes to endogenize.

In the macroeconomic tradition, a novel contribution to endogenize technological change is due to Goulder and Matai (1997). The authors develop a model for the United States where technological progress results from investments in R&D. The fundamental goal is to formalize the market failure discussed in Chapter 3, where the private sector underinvests in research and development. Goulder and Matai show that a combined strategy of taxes and subsidies is the optimal strategy against climate change when social
spillovers are high. Nonetheless, the model ignores the process of technology diffusion and the role of social interactions in affecting this process. Technological progress occurs almost deterministically as investments in R&D increase. Hence, if we know the return to investments in R&D, it is possible to compute which is a socially optimal level of R&D, and therefore the optimal subsidy.

Another model in the macroeconomic tradition is developed in Meijers (1994). Meijers attempts to model the process of technology diffusion within a vintage framework. Hence, he relaxes the pervasive assumption that firms invest only in the state of the art vintage/technology (i.e., diffusion is instantaneous). Nonetheless, Meijers' adopts an epidemic framework. Thus, he defines the share of new investment in each type of technology as a function of the stock of knowledge about every technology. This stock accumulates as a function of the share of the total stock of capital invested in the technology. However, there is no explicit formalization of the adoption decision by individual heterogeneous firms.

The model I develop in the next section follows a fundamentally different approach. Technology diffusion results from choices by decentralized, heterogeneous decision makers who learn in an interactive environment not only about the performance and potential cost reductions for new technologies, but also the dynamics of other macroeconomic variables such as wages and prices.

### 3. Modeling the Diffusion of New Technologies through Heterogeneous Interactive Agents

The goal is to model the behavior of firms in developing countries with respect to production technology choices. The reader should have in mind not exclusively "big" firms, but mostly small firms, some times family owned firms, that operate in urban or rural areas. Ghana's cocoa producers can be considered as the prototype of the agents/firms considered in the analysis. Agent's technology choices determine not only factors' productivity but also depletion rates for alternative natural resources. The focus here will be on
the "consumption" of carbon emissions (i.e., the carbon intensity of the economy).

In this model, firms or agents are characterized by their ownership of capital and their geographic location. For simplicity, I work within a one-sector economy, and I describe the model from this perspective. However, the algorithm used to solve the model is able to operate in an N-sectors economy. Of course, adding sectors increases simulation time and expands considerably the number of parameters needed to calibrate, without necessarily providing additional insights. For similar reasons, I only work with two types of technologies: an "existing" and a "revolutionary" technology characterized by higher productivity and low dependence on fossil fuels, such as Low-NOx combustion, furnace sorbent injection, or duct injection, wet and dry scrubbers (see, Tavoulareas and Charpentier, 1995 for an extensive review of the cost-effectiveness of these technologies).

It is important to stress that the focus of the model is on the process of diffusion. Several non-technology policies such as monetary, fiscal, and even trade policies, affect this process. In the case of the latter, tariffs and quotas affect the costs of new technologies that are usually imported, but also the costs of inputs that are associated with the use of these technologies. Tariffs and quotas may also protect inefficient technologies. The role of trade policy in promoting technology diffusion constitutes a research in itself. In what follows the relative costs of new technologies with respect to traditional ones will be taken as a random variable to reflect variability in the type of distortions that may exist in the market for production technologies. Yet, trade instruments will not be considered explicitly.

### 3.1 Choosing Among Technologies

Each technology is associated with a production function. To choose a technology, agents need to solve three interrelated problems. First, agents need to derive the supply function associated with the technology. Second,
agents need to establish where along that supply function it is optimal to produce, given expectations about prices and wages. Finally, agents need to compare inter-temporal profits under alternative technologies. The basic assumption is that these choices are undertaken to maximize profits and that firms have a relatively short planning horizon. There is no scientific support for this assumption, but it is a generally accepted phenomena in the finance/business literature, that even big firms evaluate marketing and financial strategies within relatively short horizons that range between 5 and 10 years (see Higgins, 1998). Another important assumption is that firms are technology-costs takers, meaning that they cannot individually influence the dynamics of technology costs.

Production Functions

To characterize the production function associated with each production technology, I use a combined Cobb-Douglas/Constant Elasticity of Substitution (CES) specification. Three types of inputs enter this production function: human capital, produced capital, and natural capital. These three aggregate factors parallel the three components of the wealth of nations (see Chapter 2) although in the case of natural resources I consider exclusively the "consumption" of carbon emissions associated with the use of fossil fuels (e.g., oil, carbon and natural gas). Hence, for an agent $i$ using technology $j$ at time $t$, the production function is given by:

$$q_{ij} = A_{pj} k_{ij}^{\alpha_j} \left[ \left( (1-a_j) n_{ij} \right)^{\rho_j} + \left( a_j n_{ij} \xi_{ij} \right)^{\rho_j} \right]^{\frac{1}{\rho_j}} \alpha_j,$$

(5.1)

This apparently complicated formulation hides enormous flexibility and versatility. We observe that a Cobb-Douglas production function captures the trade-offs between produced capital ($k$) and non-produced capital ($l$ and $n$). Hence, the elasticity of substitution between produced capital and non-produced capital is equal to one (see Nordhaus and Yohe, 1983; and Edwards, 1991, for similar functional specifications). However, in the short run, agents take the level of produced capital as given (see Section 4 for a discussion on the dynamics of the stock of capital). On the other hand, the
trade-offs between human, and natural capital, are represented by a CES production function. Hence, in the short run, agents can substitute natural capital (n) and human capital (l) with an elasticity of \( \rho_j < 1 \). The other parameters of this function are as follows. The first parameter \( A_j \), a scale factor, parallels the exogenous technological progress coefficient of the standard Cobb-Douglas function. As suggested by the time index, \( A_j \) is assumed to change over time as agents learn about the characteristics of the technologies. Hence, \( A_j \) allows us to model "learning by using" and will be the channel through which I formalize knowledge spillovers. The coefficient \( \alpha_j \) is technology specific and represents the capital elasticity of output. Finally, the coefficient \( \xi_j \) captures the natural resources intensity of the technology. The higher \( \xi_j \), the lower the quantity of natural resources needed to produce a given level of output. In this case, the lower the carbon intensity of the economy. The parameter \( \xi_j \) will be one of the important uncertainties considered in this model.

**Cost Functions**

To derive the cost function associated with the production function (5.1), I proceed as follows. First, because capital is fixed in the short run, the cost minimization problem in terms of \( l \) and \( n \) can be written as:

\[
\begin{align*}
\text{Min}_{l, n, \xi_j} & : \bar{w}_l \bar{l} + z_n n_j \\
\text{s.t.} & \quad ((1-a_j)\bar{l})^{\rho_j} + (a_j n_j \xi_j)^{\rho_j} = F^{\rho_j}
\end{align*}
\]

where \( F = \left[ \frac{q_{ij}}{A_j k_{ij}^{a_j}} \right]^{\frac{1}{1-a_j}} \), \( \bar{w}_j \) is the cost of a unit of combined, high, and low quality labor, and \( z \) is the cost of consuming one unit of natural resources with technology \( j \). This last cost includes the costs associated with government regulations, such as permits or taxes.

The first order conditions for the minimization problem are:
\[
\begin{align*}
\bar{w}_j - \lambda \rho_j \left( (1-a_j) h^j \right)^{\rho_j - 1} (1-a_j) &= 0, \\
z_{jt} - \lambda \rho_j (a_n n_j) a_j &= 0
\end{align*}
\]

(5.3)

From (5.3), we get:

\[
\begin{bmatrix}
\bar{w}_j \\
\lambda \rho_j (1-a_j)
\end{bmatrix}^{\rho_j - 1} \left( 1-a_j \right) = \left( 1-a_j \right) h^j
\]

, (5.4)

\[
\begin{bmatrix}
z_{jt} \\
\lambda \rho_j a_j
\end{bmatrix}^{\rho_j - 1} = \left( a_n n_j \right)^{\rho_j}
\]

By replacing (5.4) in the constraint in (5.2) we get:

\[
\left( \lambda \rho_j \right)^{-\rho_j} = F^{\rho_j} \left\{ \begin{bmatrix}
\bar{w}_j \\
\lambda \rho_j (1-a_j)
\end{bmatrix}^{\rho_j - 1} + \begin{bmatrix}
z_{jt} \\
\lambda \rho_j a_j
\end{bmatrix}^{\rho_j - 1}
\right\}^{-1}
\]

, (5.5)

Finally by replacing (5.5) in (5.4), we get the conditional demand factor functions:

\[
\begin{bmatrix}
\bar{w}_j \\
\lambda \rho_j (1-a_j)
\end{bmatrix}^{\rho_j - 1} + \begin{bmatrix}
z_{jt} \\
\lambda \rho_j a_j
\end{bmatrix}^{\rho_j - 1} = \frac{F^{\rho_j}}{\left( 1-a_j \right)^{\rho_j}}
\]

\[
\begin{bmatrix}
\bar{w}_j \\
\lambda \rho_j (1-a_j)
\end{bmatrix}^{\rho_j - 1} + \begin{bmatrix}
z_{jt} \\
\lambda \rho_j a_j
\end{bmatrix}^{\rho_j - 1} = \frac{1}{\left( 1-a_j \right)^{\rho_j}}
\]

\[
\begin{bmatrix}
\bar{w}_j \\
\lambda \rho_j (1-a_j)
\end{bmatrix}^{\rho_j - 1} + \begin{bmatrix}
z_{jt} \\
\lambda \rho_j a_j
\end{bmatrix}^{\rho_j - 1} = \frac{1}{\left( a_j \right)^{\rho_j}}
\]

, (5.6)

System (5.6) provides the optimal demand for human capital and natural resources required to produce \( q_{it} \) units of output given factor prices. In particular we observe that given prices, the demand for natural resources will decrease as the parameter \( \xi_j \) increases.
By placing the conditional demand functions into the objective function in (5.2), we get the cost function for technology j:

$$c_j(w_j, z_j, k_u, q_j) = \left[ \frac{q_j}{A_j k_u^{\alpha_j}} \right]^{1-\alpha_j} \left\{ \left[ \frac{w_j}{(1-a_j)} \right]^{\rho_j} + \left[ \frac{z_j}{a_j} \right]^{\rho_j} \right\}^{\rho_j-1} \frac{1}{\varepsilon_j} + r_j k_u, \quad (5.7)$$

To ease the notation, I will write:

$$c_j(w_j, z_j, k_u, q_j) = q_j^{1-\alpha_j} \Gamma_{ij} + r_j k_u, \quad (5.8)$$

with

$$\Gamma_{ij} = \left[ \frac{1}{A_j k_u^{\alpha_j}} \right]^{1-\alpha_j} \left\{ \left[ \frac{w_j}{(1-a_j)} \right]^{\rho_j} + \left[ \frac{z_j}{a_j} \right]^{\rho_j} \right\}^{\rho_j-1} \frac{1}{\varepsilon_j}.$$

### Profit Functions

Given our level of aggregation, it is reasonable to assume that agents act competitively within their economic sectors. Hence, they maximize profits by setting marginal costs equal to the market price. Under this assumption, maximum profits using technology j are given by:

$$\pi_{ij}^* = \left[ \frac{p_g^*(1-\alpha_j)}{\Gamma_{ij}} \right]^{1-\alpha_j} \left[ \frac{1}{\Gamma_{ij}} \right]^{\alpha_j} - \left[ \frac{p_g^*(1-\alpha_j)}{\Gamma_{ij}} \right]^{1-\alpha_j} \Gamma_{ij} - r_j k_u, \quad (5.9)$$

where $p_g^*$ is the equilibrium price in sector g.

We notice that the profit function defined by (5.9) is based on an equilibrium price that is unknown at the time producers undertake their technology decisions. This implies that technology choices depend on agents' expectations about this clearing price. Further, agents do not have perfect information about the cost of human and natural capital embedded in $\Gamma_{ij}$, or about the opportunity cost of capital $r_j$ (although, because we assume that the latter is the same for all technologies, it will drop out of the choice
problem). For now, I will assume that each agent has well-defined expectations about each of the components of the profit function (i.e., the agents know the mean vector of these random variables, as well as their variance covariance matrix), and therefore are able to compute for each technology the mean profit $E[\pi_{ij}]$ and its variance $V[\pi_{ij}]$. Sub-section 3.4 shows how these calculations take place.

**Choices: Should I Stay or Should I Go**

Within a dynamic framework, agents need not only to decide whether to switch to a new technology, but also when to switch to that technology. Jaffe and Stavins (1994 and 1995) analyze this issue in the context of pollution regulation. They ask when is the optimal time to switch from a high polluting technology to a low polluting technology, given pollution taxes, technology subsidies, or quotas. My framework differs from Jaffe and Stavins in that they do not consider uncertainty.

An agent using the "existing" technology needs to compare the present value of expected profits to the present value of expected profits of the new technology. Therefore, I assume that at the end of period $t$, each agent first needs to evaluate whether switching to the new technology at the beginning of period $t+1$ is profitable. Further, I assume that profits are received at the end of each time period. Under these assumptions the profitability condition implies:

$$\sum_{k=t+1}^{T} E[\pi_{j'k}] \theta^{k-t} - \Lambda_{j' t} \geq \sum_{k=t+1}^{T} E[\pi_{jk}] \theta^{k-t},$$

(5.10)

where $\Lambda_{j' t}$ is a fixed cost associated with the adoption of technology $j'$ (for a similar specification, see Durlauf, 1993). The profitability condition can be rewritten as:

$$\Lambda_{j' t} \leq \sum_{k=t+1}^{T} \{ E[\pi_{j'k}] - E[\pi_{jk}] \} \theta^{k-t},$$

(5.11)

which states that switching is profitable if the cost of adoption is lower than the present value of the expect gains of adoption.
If condition (5.11) holds, then the agent needs to assess whether waiting to adopt will be even more profitable. Indeed, an agent may expect, for example, that the adoption cost will be lower in the future. Formally, waiting to adopt the technology at the beginning of time $t+1$ will be optimal if the following condition, the arbitrage condition, holds:

$$E[\pi_{ij+t+1}]\theta + \sum_{k=t+2}^{T} E[\pi_{ij+k}]\theta^{k-t} - E[\Lambda_{ij+t+1}]\theta \geq \sum_{k=t+1}^{T} E[\pi_{ij+k}]\theta^{k-t} - \Lambda_{ij+t},$$  

(5.12)

This condition states that it is optimal to wait if the net profits of switching tomorrow while facing a switching cost $E[\Lambda_{ij+t+1}]$ are higher than the profits received by switching today with a cost $\Lambda_{ij+t}$. Notice that

$$\sum_{k=t+1}^{T} E[\pi_{ij+k}]\theta^{k-t}$$

in (5.12) can be rewritten as $E[\pi_{ij+t+1}]\theta + \sum_{k=t+2}^{T} E[\pi_{ij+k}]\theta^{k-t}$. Therefore condition (5.12) can be rewritten as:

$$\Lambda_{ij+t} - E[\Lambda_{ij+t+1}]\theta \geq \{E[\pi_{ij+t+1}] - E[\pi_{ij+t}]\}\theta,$$

(5.13)

This condition states that it is optimal to wait if the expected gains from waiting (the first part of the inequality) are greater than expected costs of waiting (the forgone profit given by the second part of the inequality).

In summary, an agent should switch to the new technology at time $t$ only if:

$$X_p \geq 0 \quad \text{and} \quad X_a \geq 0,$$

(5.14)

where $X_p = \sum_{k=t+1}^{T} \{E[\pi_{ij+k}] - E[\pi_{ijk}]\}\theta^{k-t} - \Lambda_{ij+t}$ and $X_a = \{E[\pi_{ij+t+1}] - E[\pi_{ij+t}] + E[\Lambda_{ij+t+1}]\}\theta - \Lambda_{ij+t}$

I will show in Sub-section 3.4 that all the expectations in (5.14) are asymptotically normally distributed given a range of values for the support of the expectations. This being the case, $X_p$ and $X_a$ are normally distributed as well. I add the assumption that $X_p$ and $X_a$ are independent. Therefore, agents can compute:
\[
\phi_{jj'} = \Pr[X_p \geq 0] \Pr[X_d \geq 0],
\]  
(5.15)

which is the probability that the decision of switching will be correct.

My final assumption is that agents will switch to the new technology on the basis of their "reservation level" \(0 \leq \lambda_i \leq 1\), which depends on their risk aversion. Hence, an agent \(i\) at time \(t\) will switch from technology \(j\) to technology \(j'\) if:

\[
\phi_{jj'} \geq \lambda_i + \epsilon,
\]  
(5.16)

where \(\epsilon\) is white noise. This noise is introduced to take into account that agents do not always do the right thing, or that their decisions are influenced by factors not taken into account by our models (see Young, 1998; and Radner, 1996).

It is trivial to show that, given a vector of prices, the probability \(\phi_{jj'}\) increases with the "size" of the agent (i.e., its ownership of capital). Indeed, from (5.11) we see that the derivative of expected profits with respect to the gamma function is negative. We also observe from the definition of the gamma function, that its derivative with respect to the level of produced capital is negative. Thus, other things being equal, expected profits will increase with the level of produced capital. The implication, is that, other things being equal, agents with a higher endowment of produced capital will be more likely to switch to a new technology than agents with lower endowments, due to economies of scale. Thus, the model reproduces a well-known finding in the literature on technology diffusion (see Sahal, 1981). The idea is illustrated in Figure 5.2. The figure was constructed by simulating technology choices for a sample of 100 agents, using mean values for the model parameters (see Section 5).
3.2 Social Interactions and Cooperative Behavior

Up to this point, I have been assuming that choices are made given expectations, independently of the choices of other agents. Yet, Chapter 3 provided several reasons why cooperative behavior is an important factor influencing technology choices. In this sub-section, I will focus on the effects of cooperation on adoption costs. Our example of the farmer in the Andes Mountains falls into this category. In general, if the adoption decision is undertaken simultaneously by a community of potential users, adoption costs tend to be lower due to economies of scale, or because as a group adopters may have access to better prices.

To formalize this phenomenon, I recall the definitions of networks from Chapter 3. I assume that agents/firms in a given developing economy operate in a graph $G(V,E)$. The two dimensions of the space $V$ may be given economic interpretations. The first dimension ($K$) can be viewed as a one-dimensional social space. Agents' location in this space depends on their ownership of capital. The second dimension ($C$) is simply a one-dimensional geographic space. Agents are randomly located in this dimension. Hence, a vertex of the
A graph is a vector \( i = (k_i, c_i); k_i \in K, c_i \in C, V = K \times C \) that characterizes an agent in terms of its ownership of capital and its location in the geographical space. As usual, I define the neighborhood of agent \( i \) by the set of other agents with whom agent \( i \) shares an edge (i.e., has a connection), \( v(i) = \{ j \in V; (i, j) \in E, i \neq j \} \). As suggested in Chapter 3, networks can be, in part, characterized by the statistical process that governs the emergence of connections. To define this statistical process, I assume that the probability that two agents establish a connection is related to their distance in social and geographic space. For example, in Ecuador, small producers of corn are more likely to interact with other small producers of corn, and less likely to interact with producers of bananas, which are usually owners of large plantations. Similarly, corn producers from the Andes region are less likely to interact with corn producers from the coast. Formally, the probability that agent \( i \) and agent \( i' \) will be connected is given by:

\[
\Pr(i \leftrightarrow i') = \frac{1}{1 + \exp \left[ -\frac{2}{\beta_1 \text{Var}(k)} (k_i - k_{i'})^2 + \frac{2}{\beta_1 \text{Var}(c)} (c_i - c_{i'})^2 \right]},
\]

(5.17)

where \( \beta_1 \) is a connectivity parameter. As this parameter increases, the probability of connection also increases. Two examples of networks with 100 agents, constructed with \( \beta_1 = 0.1 \) and \( \beta_1 = 0.5 \) are given in Figure 5.2.
We observe that the average number of connections per capita increases as the connectivity parameter increases. For example, in the network $\beta_1 = 0.1$, agents have on average a single connection. At the extreme, in the network $\beta_2 = 0.5$, each agent is on average connected to 6 other agents (see Figure 5.3).
The network is also characterized by the prevalence of cooperative behavior among the members of the network. Therefore, with some probability $\chi$ - that is an intrinsic characteristic of the country under analysis - cooperative behavior between an agent and its neighbors can emerge. In summary, the network class is characterized by the vector $(\beta, \chi)$.

What happens when cooperative behavior emerges? As discussed in Chapter 3, the main implication of cooperative behavior is that when adoption of a new technology is undertaken simultaneously by a group of agents, costs may be lower than those faced by a single agent. To formalize this idea, I assume that the cost of adoption for the group decreases with the number of agents in the groups. This idea can be formalized through the popular logistic form:

$$\Lambda_{v(i)} = \Lambda_i |v(i)|^{-\beta_i}$$

(5.18)

where as before $\Lambda$ is the cost of adoption, $v(i)$ is the set of neighbors of agent $i$, $| |$ represents "the number of elements" in the set, and $\beta_i$ is a parameter that captures the level of social spillovers, or the percentage of decrease in the adoption cost that results from a one percent increase in the number of members in agent $i$'s neighborhood.

With cooperative behavior, agents analyze jointly the profitability and arbitrage conditions. In other words, they do not focus on an individual's expected profits, but rather add individual costs and benefits to come up with costs and benefits for the group. By definition, under cooperative behavior, costs are lower for everybody. However, it may be the case that for some agents the adoption of the new technology is still not profitable. Yet, "winners" can compensate "losers" in exchange for the marginal contribution to the reduction in adoption costs.

Similarly to the individual case, the condition for adoption by members of a group, composed by agent $i$ and its neighbors, is given by:

$$\phi_{v(i)j} > \lambda_{v(i)}$$

(5.19)
In the last three expressions, i indexes the agent who is analyzed, l indexes the members of the group, and j indexes technologies. We observe that expectations about profits by each agent l, are indexed by \( \nu(i) \). This is due to the fact that agents compute expected costs and benefits on the basis of social expectations about prices, wages, and the cost of natural resources. In other words, they take into account not their individual expectations of prices and wages, and the cost of natural resources, but rather the average of the expectations of all members of i's neighborhood. We also observe that the coefficient of risk aversion is the average of individual risk aversion coefficients.

### 3.3 Social Interactions and Knowledge Spillovers

In my discussion on social capital (Chapter 3), I showed that one of the main channels through which social networks affect the dynamics of the economy are knowledge spillovers. In the case of Africa, for example, Collier and Gunning (1999) suggested that low levels of knowledge spillovers explain in part poor economic performance. Also, in the case of the diffusion of hybrid cocoa in Ghana, social networks were important sources of "knowledge". This process can be viewed as a "learning by using" process— a demand characteristic of the technology. "Learning by using" differs from the more popular "learning by doing" process, a supply characteristic of the technology, that refers to the well-known phenomena that during the process of technology diffusion, adoption costs tend to decrease as the number of users increase (see Subsection 4.5 for a discussion of this process and its effects on the dynamics of \( \Lambda_j \)).

Learning by doing refers to the process by which economic agents discover more efficient ways of using a given technology. It also refers to the process by
which agents, in their social interactions, gain information about the performance of alternative production technologies and improvements to their modus operandi. In research, for example, we constantly learn from our colleagues about new theories, new ideas and statistical tools, or simply new computer programs. What in research appears to be pervasive, is pervasive in other activities as well. Physicians learn about new medicines and medical interventions from their peers; and farmers learn about new seeds or fertilizers from other farmers in their community. As I mentioned earlier, I formalize this idea through the technology factor $A_{ijt}$.

At the beginning of each simulation, this factor is the same across all agents, and varies only across technologies. As agents gain experience with the technology, they discover ways of making things work more efficiently, and $A_{ij}$ increases. These improvements, that are independent from the actions of other agents, are assumed to occur by chance. That is, there exists a probability distribution governing the arrival of improvements. I assume the arrival distribution is Poisson. Hence, for agent $i$ using technology $j$, the dynamics of $A_{ijt}$, in the absence of interactions, is given by:

$$
A_{ijt} = A_{ijt-1}(1 + \psi_0); \text{ with probability } e^{-\psi_1} \frac{\psi_1^t}{t!};
$$

$$
A_{ijt} = A_{ijt}; \text{ with probability } 1 - e^{-\psi_1} \frac{\psi_1^t}{t!};
$$

This implies that at any time $t$, an innovation/improvement by agent $i$ can be observed with a probability given by the first expression in (5.20). This improvement will increase $A_{ij}$ by $\psi_0 / 100$ percent. Thus, innovations are governed by the two parameters: $\psi_0$, the marginal innovation rate, and $\psi_1$, the mean arrival rate. We notice that because the arrival of improvements is Poisson distributed, the probability of observing these improvements decreases with time. This is a standard assumption when modeling productivity growth (see for example Pizer, 1998). It captures the well-known phenomena that innovations to a given technology accelerate during the first stages of the diffusion process, and decay afterwards (see Sahal, 1981).
I mentioned that learning by using also involves learning about the innovations of other agents, or more precisely, about their techniques. Hence, when agent \( i \) and agent \( l \) interact, they compare their techniques. If the technique of one of them, say \( l \), is better than that of the other (i.e., the \( A \) factor is greater), the agent with the least efficient technique will learn from the agent with the most efficient technique (in this case \( l \)). Learning involves increasing factor \( A_{ij} \) by a given fraction of the difference in levels of efficiency \( (A_{lj} - A_{ij}) \). We have:

\[
\begin{cases}
A_{ij} = A_{ij} + \psi_2 (A_{lj} - A_{ij}); & l \in \mathcal{V}(i) \text{ if } A_{ij} < A_{lj} \\
A_{ij} = A_{ij}; & \text{otherwise}
\end{cases}
\] (5.21)

The "quantity" of learning from the interaction is therefore regulated by the parameter \( \psi_2 \). This parameter becomes a proxy for the "quality" of network connections. Presumably, as the level of education of a given population of agents increases, \( \psi_2 \) should increase as well.

The dynamics of \( A_{ij} \) will affect the process of technology diffusion as well as the growth rate of the economy. Its dynamics will in turn be related to the density of connections in the network. To illustrate this idea, I have performed the following computational experiment. For different network classes, I have computed the average share of the population (average taken over 100 Monte Carlo simulations) that after a fixed period of time (set arbitrarily to 5 years) know about a given innovation that occurred at time 0 (i.e., agents who have increased their original \( A \) in proportion to the size of the innovation). The results of this experiment are summarized in Figure 5.4. It is clear that the "speed" at which information diffuses through the network depends on the level of connectivity. Higher connectivity leads to a higher share of the population being aware of the invention. However, higher connectivity will not necessarily be associated with a faster diffusion of new technologies. Indeed, information flows are not reserved to "revolutionary" technologies. Hence, higher connectivity may favor old technologies intensive in natural resources, and act as a limiting factor for economic growth (see Sub-section 5.3).
In summary, the effect that networks have on the diffusion of new technologies and productivity growth is captured by the vector of parameters \( (\beta_1, \chi, \beta_2, \psi_0, \psi_1, \psi_2) \), respectively the degree of connectivity, the probability of emergence of cooperative behavior, the degree of social spillovers, the marginal innovation rate, the average rate of innovation, and the quality of network connections. These six parameters determine what I have called a network class (see Chapter 3).

### 3.4 Generating Expectations about the Dynamics of the Economy

Agents in this model need to generate expectations about four macroeconomic variables: output prices, cost of labor, cost of natural resources including environmental regulations, and cost of new technologies. The dynamics of these variables can be approximated by a function of the form:

\[
y_t = y_0 t^b, \quad (5.22)
\]
Hence, depending on the value of \( b \), time series can grow exponentially (\( b > 1 \)), decrease exponentially (\( b < -1 \)), converge to a plateau from below (\( 0 < b < 1 \)), or converge to plateau from above (\( 1 < b < 0 \)).

For any time series of interest \( \{y_t\} \), agents are assumed to act as econometricians and generate estimates of \( b \) on the basis of observed trends. Equation (5.22) implies:

\[
y_t = y_{t-1} \left( \frac{t}{t-1} \right)^{b_t}; \quad t > 1,
\]

(5.23)

Therefore, agents estimate the model:

\[
\log \left( \frac{y_t}{y_{t-1}} \right) = b_t \log \left( \frac{t}{t-1} \right),
\]

(5.24)

The general estimation method that agents use is Recursive Least Squares (see Sargent, 1992). Hence, if we call \( z \) the endogenous variable and \( x \) the vector of exogenous variables (possibly including a vector of ones), the expected value of the vector of coefficients \( \beta_t \) in the model \( z_t = x_t \beta_t \) is given by:

\[
\beta_{t+1} = \beta_t + RR(\Omega_t^{-1} x_t)(\beta_{t,obs}^{obs} - \beta_t)
\]

while the variance is given by

\[
\Omega_{t+1} = \Omega_t + RR(x_t x_t' - \Omega_t)
\]

where \( RR \) is an exogenously defined adjustment factor that determines the speed of convergence, and \( \beta_{t,obs} \) is the observation of the parameter \( \beta_t \) at time \( t \). In this case of model (5.24), this observation will be generated by dividing \( \log \left( \frac{y_t}{y_{t-1}} \right) \) by \( \log \left( \frac{t}{t-1} \right) \).

This specification of the learning model supports a wide variety of formulations for the model driving the dynamics of the macroeconomic variables of interest. For our application, the formulation used in (5.24) is sufficient.

For a time series \( \{y_t\} \), given agents' estimator of \( b_t \), they can generate forecasts with mean and variances approximated by:  

5-23
\[ E[y_{t+k}] = y_t \left( \frac{t+k}{t} \right)^{E[b_i]} \]
\[ V[y_{t+k}] = V[b_i] \left[ y_t \left( \frac{t+k}{t} \right)^{E[b_i]} \log E[b_i] \right]^2 \]

In order to choose among technologies agents must have an estimate of future profits. In particular, the profit function requires expectations about the price of output, the cost of labor and the cost of natural resources. Agents use (5.25) to generate expectations about these variables. Given these expectations the process to estimate expected profits and their variance is a little more cumbersome. Indeed, given that the profit function is a non-linear function of random variables, the agents need to use the theorem for the Asymptotic Distribution of Non-Linear Functions (see Green, 1997):

Call \( \hat{\theta} \) the vector of estimates \( E[y] \) of the random variables \( y \) included in the profit function \( \pi(\theta) \), such that \( \hat{\theta} \rightarrow N[\theta,(1/n)V] \) where \( V \) is the variance covariance matrix; then \( \pi(\hat{\theta}) \rightarrow N[\pi(\theta),(1/n)\Pi(\theta)V\Pi(\theta)] \), where \( \Pi(\theta) = \left[ \partial \pi / \partial \theta_1 \ldots \partial \pi / \partial \theta_k \right] \) is a row vector of partial derivatives of the profit function with respect to each of the random variables.

Once \( \Pi(\theta) \) has been computed, it is straightforward to compute expected profits and their variance. Notice, however, that this is a way to compute profits at the expected values of the random variables. Given that the profit function is convex (see Varian, 1992) this value is lower than the true expected profit (the mean of profits integrating along the different random variables). This bias however applies to all the technologies. Since we are interested in choosing across technologies, we will assume this bias largely cancels out. That is, we allow choices to be based on profits computed at the mean value of the random variables.

### 3.5 Outputs from the Model of Technology Diffusion
Before moving to the description of the macro-econometric model (Section 4), I summarize in this sub-section the outputs that can be derived from the model of technology diffusion (see Sub-sections 3.1 to 3.4).

Production and the demand for labor and carbon emissions

Given his/her choice of technology and expectations about prices, wages, and the cost of natural resources, agent i computes the optimal level of output, \( q_{it} \), to be produced at time t. This level of output is given by the first bracket in the profit function (equation 5.9). Agent i also computes the optimal demand for labor, \( l_{it} \), and carbon emissions, \( n_{it} \), given by equation (5.6). Therefore, from the model of technology diffusion, by adding across agents we derive the aggregate output, \( Q \), and the aggregate demand for labor, \( l \), and carbon emissions, \( n \). We have:

\[
Q = \left( \sum_i q_{it} \right) \left( 1 - d_0 \left( \frac{n_{it}}{n_0} \right)^{d_1} \right),
\]

\[
n_i = \sum_i n_{it},
\]

and
\[
\tilde{l}_{it} = \sum_i \tilde{l}_{it}.
\]

We notice that total output is affected by the damages resulting from environmental degradation. In this case these damages are given by the quantity of carbon emissions at time t, \( n_{it} \), the initial level of carbon emissions, \( n_0 \), and unknown parameters \( d_0 \) and \( d_1 \).

Aggregates computed in equations (5.26-5.28) will be passed to the macro-econometric model that will in turn compute changes in prices, changes in the stock of capital, environmental damages, and other macroeconomic aggregates. The model is described in the next section.

4. The Macro-econometric Model

For analytical purposes, the model of technology diffusion is coupled to the macroeconometric model for the developing world developed in Huaque et al.
This model implements policies and computes output prices, wages, the savings rate of the economy, the costs of natural resources and new technologies, changes in the stock of natural and produced capital, as well as environmental damages.

4.1 Basic Identities

The core of the macro model is presented in Table 5.1. Most of its structure and parameters will be taken as given, so my discussion of the table will be limited. Huaque et al. (1993) offers a more detailed description of the model.

The functions in the table are respectively the aggregate production of the economy (A1), the consumption function (A2), the investment function (A3), the exports function (A4), the imports function (A5), the real money demand (A6), the equation determining the nominal interest rate (A7) and the monetary/external (A8) and income/expenditures identities (A9). The parameters of the model have been estimated in Huaque et al. (1993).

The production function A1, has been replaced by the production function (5.26) in the model of technology diffusion. For the other functions, A2 to A7, the specification is standard. Real consumption (A2) is assumed to be a function of disposable income (Yd), the real interest rate (r) and lagged values. Investment (A3) is defined as a function of the interest rate, aggregate output (Y), and lagged values. Exports (A4) and imports (A5) respond to changes in the real exchange rate (rer), aggregate output (Y), as well as lagged values. In the case of imports, the availability of international monetary reserves (IMR) also has an effect. Equation (A6) is the real money demand function, assumed to depend on the level of aggregate activity (Y) and the nominal interest rate (i). Finally, the domestic nominal interest rate depends on the degree of capital mobility, captured by the parameter \( \phi \). When \( \phi \) equals 0 (the case with no mobility), the domestic nominal interest rate equals the shadow interest rate (the interest that would prevail in the absence of capital flows from the rest of the world). When \( \phi \) equals one (perfect mobility), the domestic nominal interest rate is determined by the international interest rate and the expected devaluation of the nominal interest rate (ner) of the economy. Identity (A8) states that the
supply of money is given by the level of the international monetary reserves (IMR) and the domestic credit (DC).

A1) \[ Y_t = K_t^{0.12} L_t^{0.68} e^{0.11 t} \]

A2) \[ \log C_t = cte - 0.076 r_t + 1.010 \log C_{t-1} + 0.143 \log Y_t - 0.149 \log Y_{t-1} \]

A3) \[ I_t = cte - 0.113 (r_t - r_{t-1}) + 0.196 (Y_t - Y_{t-1}) + 0.809 I_{t-1} \]

A4) \[ \log X_t = cte + 0.0495 \log rer_t + 0.084 \log Y_t^* + 0.925 \log X_{t-1} \]

A5) \[ \log Z_t = cte - 0.157 rer_t + 0.161 \log Y_t + 0.038 \log \frac{IMR_{t-1}}{P_t Z_{t-1}} + 0.834 \log Z_{t-1} \]

A6) \[ \log \frac{M_t}{P_t} = -0.146 - 0.038 i_t + 0.571 \log Y_t - 0.397 \log Y_{t-1} + 0.881 \log \frac{M_{t-1}}{P_{t-1}} \]

A7) \[ i_t = \phi \left( r_t^* + \frac{E_{rer_{t+1}} - rer_t}{ner_t} \right) + (1 - \phi) i_{t-1}; \quad \phi = 0.91 \]

A8) \[ M_t = ner_t IMR_t + DC_t \]

A9) \[ Y_t = C_t + G_t + I_t + X_t - Z_t \]

**Table 5.1: Macroeconometric Model for the Developing World.**

Source: Huaque et al. (1993).

Finally, (A9) states that real gross domestic product identically equals domestic absorption (C+I+G) plus the current account balance (X-Z). The parameter cte in all equations refers to a constant that is computed to calibrate the model to initial conditions.

### 4.2 Dynamics of the Stock of Produced and Natural Capital

The stock of produced capital follows the discrete formulation:

\[ K_t = K_{t-1} (1 - \delta_t) + I_{t-1}, \tag{5.29} \]

where \( \delta \) is the depreciation rate.

Similarly, the dynamics of the stock of natural resources is given by:
where $R$ is the replenishment rate and $n$ is the quantity of natural resources consumed. However, given that our concern is with the dynamics of carbon emissions, in this application, the stock of the natural resources under consideration (fossil fuels) will play no role in our discussions.

**4.3 Dynamics of Wages**

To model the labor market, I have adopted a disequilibrium approach (see Muet, 1993). The fundamental reason is that in most developing countries, the labor market is not competitive and wages do not tend to adjust "immediately". For instance, there is extensive evidence of nominal wage downward-rigidity (see Akerlof et al., 1996 for a review). I make two simplifying assumptions. First, the supply of total labor force, $L$, is given by the growth rate of the population. We have:

$$L_t = L_0 (1 + \phi)^t, \quad (5.31)$$

Second, given the demand for labor (equation 5.28), the dynamics of wages is characterized by:

$$w_{t+1} = w_t \left[1 - \omega \left(\frac{L_t}{\bar{L}_t} - 1\right)\right], \quad (5.32)$$

where $\omega$ is a parameter used to capture the degree of "stickiness" in the labor market, or alternatively, the speed of adjustment.

**4.4 Dynamics of the Cost of Fossil Fuels**

The market for natural resources is forced to clear in each time period. While in practice the markets of fossil fuels have been heavily regulated, mostly subsidized during the seventies and eighties, this assumption is made given that in this analysis we are taking a normative approach. Hence, we are interested in determining the optimal "consumption" of carbon emissions over time. This optimal consumption implies some dynamics for the price, that
then can be compared to observed dynamics. Therefore, the supply of natural resources, \( n_t \), is treated as a policy variable, and the price of natural resources, \( Z \), is adjusted to ensure that the demand resulting from equation (5.27) is equal to the targeted supply. Implicitly, the level of \( n_t \) can be associated with the level of permits for the consumption of carbon that the government is willing to distribute.

There is an important caveat. To estimate model parameters, we cannot treat \( n_t \) as a policy variable. Indeed, we are interested in reproducing observed dynamics of the oil and carbon intensities of developing economies (see next section). This implies that \( n_t \) needs to be treated as an endogenous variable. This also implies, however, that we need to determine somehow the dynamics of the observed price for oil and carbon. While we do observe this price at the international level (see Table 5.2), we do not have time series on a country by country basis.

In fact, while at the international level the prices of both oil and carbon have been falling during the last two decades, at the national level the prices have been increasing as subsidies have been eliminated (see Appendix 8.6 for country specific oil, gas and carbon intensities). Furthermore, different countries have implemented different types of regulatory policies for the price of oil and carbon.

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Petroleum</td>
<td>224.0</td>
<td>173.0</td>
<td>100.0</td>
<td>83.0</td>
<td>78.0</td>
<td>69.0</td>
<td>63.0</td>
<td>63.0</td>
<td>78.0</td>
<td>77.0</td>
<td>55.0</td>
</tr>
<tr>
<td>Coal, Australian ($)/mt</td>
<td>55.9</td>
<td>49.2</td>
<td>39.7</td>
<td>38.8</td>
<td>36.2</td>
<td>29.5</td>
<td>29.3</td>
<td>33.0</td>
<td>33.4</td>
<td>32.4</td>
<td>28.1</td>
</tr>
<tr>
<td>Coal, US ($)/mt</td>
<td>59.9</td>
<td>67.9</td>
<td>41.7</td>
<td>40.6</td>
<td>38.1</td>
<td>35.7</td>
<td>33.1</td>
<td>32.9</td>
<td>32.6</td>
<td>33.6</td>
<td>33.1</td>
</tr>
<tr>
<td>Natural gas, Europe ($)/mmbtu</td>
<td>4.7</td>
<td>5.4</td>
<td>2.6</td>
<td>3.0</td>
<td>2.4</td>
<td>2.5</td>
<td>2.2</td>
<td>2.3</td>
<td>2.5</td>
<td>2.5</td>
<td>2.3</td>
</tr>
<tr>
<td>Natural gas, US ($)/mmbtu</td>
<td>2.2</td>
<td>3.6</td>
<td>1.7</td>
<td>1.5</td>
<td>1.7</td>
<td>2.0</td>
<td>1.7</td>
<td>1.4</td>
<td>2.4</td>
<td>2.3</td>
<td>2.0</td>
</tr>
<tr>
<td>Petroleum ($) /bbl</td>
<td>51.2</td>
<td>39.6</td>
<td>22.9</td>
<td>19.0</td>
<td>17.8</td>
<td>15.8</td>
<td>14.4</td>
<td>14.4</td>
<td>17.9</td>
<td>17.7</td>
<td>12.6</td>
</tr>
</tbody>
</table>

Table 5.2: Prices of Oil, Gas, and Carbon.

As a consequence, I have defined a general function for the dynamics of prices, which parameters need also to be estimated. Basically, I will ask the question of what type of price dynamics are consistent with observed
depletion rates for fossil fuels. The function that is used to be able to reproduce a wide class of dynamics across the developing world is given by:

\[
\log P_{n_t} = \log P_{n_{t-1}} + (\gamma_1 - 1)t + \gamma_2 u_t, \quad u \sim N(0,1),
\]  

(5.33)

This specification follows Nelson and Plosser (1982), and states that the price of fossil fuels follows a random walk with a drift that can be positive or negative. Other things being equal, countries where the price of fossil fuels has been increasing are more likely to reduce the consumption of fossil fuels. However, it is not impossible that countries where prices for fossil fuels have been decreasing also reduce their consumption given important innovations in production technologies that do not require fossil fuels as inputs. I emphasize that function (5.33) is only used for estimation purposes. It will not have any role when the model is used from a prescriptive point of view.

4.5 Dynamics of the Cost of New Production Technologies

There is extensive evidence that the production and distribution costs of new technologies decrease with the aggregate number of users (see Grübler, 1998 for a review). Developing countries are usually price takers in the world technology market. Hence, reductions in technology prices result mostly from investments that take place outside the developing country under analysis. Therefore, there is a component in the cost function for the new technology that is not linked to domestic demand for that technology. Given these considerations, I postulate that the cost of the new technology for a single agent is given by:

\[
\Lambda_t = \exp(\Lambda_0 - b_1 \log D_t - b_2 t + \mu)(1 - S_t),
\]  

(5.34)

where \( \Lambda_0 \) is the cost of the "first unit", \( D \) is the number of domestic users of the technology, \( t \) is time, and \( \mu \sim N(0,\sigma_u) \) is white noise. The last two terms of (5.34) are meant to capture changes in prices that result from exogenous factors. The variable \( S_t \) is a policy variable that gives the level
of the technology subsidy that the government implements. In summary, the
cost of a new technology for a given agent is affected by three factors: world
demand for the technology, aggregate domestic demand, and as discussed
earlier, local domestic demand (i.e., reduction in costs resulting from
coordinated actions with his/her neighbors: see equation 5.18).

4.6 Policy Variables and Aggregate Savings

The vector of policy instruments is given by: \( I, n, S \), that represent
respectively, investments in produced capital, consumption of natural
resources, and technology subsidies. As discussed in Chapter 4, these
instruments should be employed to maximize intertemporal social welfare,
measured by the utility function:

\[
U(C_t) = L_t \left( \frac{C_t / L_t}{1 - \tau} \right)^{\tau},
\]

The policy instruments also need to satisfy the following constraints:

\[
\begin{align*}
(Y_t - C_t - (X_t - Z_t) - g) &= I_t + S_t, \\
n_t &\leq N_t \quad \text{(5.36)}
\end{align*}
\]

where \( g \) are government net savings excluding investments in produced capital
(i.e., \( g \) includes tax net of wages and interest payments). For simplicity, I
will assume that \( X \) and \( Z \) result from an exchange rate policy that keeps the
real exchange rate constant. I also assume that monetary policy is
implemented in order to target 0 inflation). Therefore, given the dynamics of
\( X-Z \) (the current account deficit) and assuming a fixed \( g \), I will look for: i)
the share \( s^* \) of \( Y_t - (X_t - M_t) - g \) that should be saved; ii) the optimal
allocation of these savings between \( I \) and \( S \); and iii) the optimal consumption
of carbon emissions \( n_t \). This optimization problem is described in detail in
the next chapter. I do not make any assumptions regarding the ability of
markets or governments to generate optimal schedules for savings, investments,
and the consumption of carbon. Rather, I am taking a normative approach, and
estimating directly these optimal schedules. Eventually, these could be compared to observed macro aggregates as a mean to evaluate the performance of a given economy.

5. Parameters and Model Dynamics

5.1 Estimating Model Parameters

The model described in the previous two sections has 38 parameters. These have been classified into six categories: a) parameters describing agents characteristics; b) parameters describing technology characteristics; c) parameters describing networks; d) macroeconomic parameters; e) environmental parameters; and f) initial conditions (see Table 5.2).

Before we use the model to estimate optimal consumption, savings and investments schedules, it is necessary to estimate the distribution of the these parameters. For some of these parameters, I use estimates from the literature (e.g., the output-capital elasticity in the production function). For parameters that do not have an empirical counterpart, and that are not important from a theoretical/policy point of view (e.g., the mean and variance of the distribution of agents in the one dimension geographic space), I have fixed arbitrary values (these parameters have the label "subjective" in the source column). I have estimated the other parameters through moments simulation (for other applications of this method, see Robalino and Lempert, 1999; Dowlatabi and Oravetz, 1997; Akerloff et al. 1996; and Meijers, 1994). The main idea is to search for a set of model parameters that generate model dynamics that satisfy basic data constraints. I have limited myself to set three types of constraints: i) the distribution of GDP growth rates during the period 1984-1994 (given our assumption that the growth rate of the labor force is given by the growth rate of the population this is equivalent at looking at the growth rate of labor productivity); ii) the distribution of the growth rate of the ratio between GDP and the consumption of oil and carbon during the same period of time; and iii) historical rates of technology diffusion.
<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Values or Prior Distribution</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agents</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance in the distribution of capital per capita (% of the mean capital per capita)</td>
<td>$V_k$</td>
<td>$\sim N(200,200)$</td>
<td>Deininger and Squire (1998).</td>
</tr>
<tr>
<td>Mean and variance of the distribution in the one-dimensional geographic space.</td>
<td>$c, V_c$</td>
<td>100,30</td>
<td>Subjective</td>
</tr>
<tr>
<td>Mean and variance of the distribution of risk aversion</td>
<td>$\lambda, \sigma_\lambda$</td>
<td>0.5,0.01</td>
<td>Robalino and Lempert, 1999</td>
</tr>
<tr>
<td>Adjustment factor in learning model</td>
<td>$R_R$</td>
<td>0.5</td>
<td>Subjective</td>
</tr>
<tr>
<td>Technologies</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output elasticity of capital</td>
<td>$\alpha$</td>
<td>0.4</td>
<td>Pizer (1998)</td>
</tr>
<tr>
<td>Elasticity of substitution for old technology</td>
<td>$\rho$</td>
<td>$\sim U[0.2,0.6]$</td>
<td>Dowlatabi (1997)</td>
</tr>
<tr>
<td>Relative elasticity of substitution of the new to the old technology</td>
<td>$\rho_{new} = \rho_{old} \ast (1 + u)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight in CES function</td>
<td>$a$</td>
<td>0.5</td>
<td>Edwards, (1991)</td>
</tr>
<tr>
<td>Depletion rates for old and new technologies (old=1)</td>
<td>$\xi$</td>
<td>$\sim U[0,5]$</td>
<td>On the basis of Manns and Ritchels, 1988</td>
</tr>
<tr>
<td>Cost of first unit expressed as % of GDP if all agents switch.</td>
<td>$\bar{\Lambda}$</td>
<td>$\sim U[1,120]$</td>
<td>Tavoulareas and Charpentier (1995)</td>
</tr>
<tr>
<td>Domestic learning coefficient</td>
<td>$b_1$</td>
<td>$\sim U[0,0.3]$</td>
<td>Christianson (1995)</td>
</tr>
<tr>
<td>International Learning Coefficient</td>
<td>$b_2$</td>
<td>$\sim U[0,0.03]$</td>
<td>On the basis of productivity growth estimates by Pizer (1998)</td>
</tr>
<tr>
<td>Variance of the random shock in learning function</td>
<td>$\sigma_u$</td>
<td>$\sim U[0,0.1]$</td>
<td>On the basis of productivity growth estimates by Pizer (1998)</td>
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<tr>
<td>Networks</td>
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</tr>
<tr>
<td>Connectivity</td>
<td>$\beta_1$</td>
<td>$\sim U[0,0.5]$</td>
<td>On the basis of connectivity and Ethno-linguistic fractionalization index.</td>
</tr>
</tbody>
</table>

**Table 5.3: Model Parameters.**
<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Values or Prior-Distribution</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Innovation size</td>
<td>$\psi_0$</td>
<td>$\sim U[0,0.1]$</td>
<td>On the basis of productivity growth estimates by Pizer (1998)</td>
</tr>
<tr>
<td>Innovation rate</td>
<td>$\psi_1$</td>
<td>0.01</td>
<td>On the basis of productivity growth estimates by Pizer (1998)</td>
</tr>
<tr>
<td>Quality of Connections</td>
<td>$\psi_2$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Innovation rate in old technology</td>
<td>$\psi_3$</td>
<td>0.3</td>
<td>Subjective</td>
</tr>
<tr>
<td>Cooperative behavior</td>
<td>$\chi$</td>
<td>$\sim U[0,0.9]$</td>
<td>Subjective</td>
</tr>
<tr>
<td>Spillovers</td>
<td>$\beta_2$</td>
<td>$\sim U[0,0.3]$</td>
<td>Christianson (1995)</td>
</tr>
<tr>
<td>Macroeconomic and Institutional Parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population growth</td>
<td>$\phi$</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Inertia in the labor market</td>
<td>$\omega$</td>
<td>0.9</td>
<td>Akerlof et al. (1996)</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$\theta$</td>
<td>1.025</td>
<td>Cline, (1993)</td>
</tr>
<tr>
<td>Depreciation of the stock of produced capital</td>
<td>$\delta_k$</td>
<td>5%</td>
<td>Pizer, (1998)</td>
</tr>
<tr>
<td>Environmental Parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum threshold</td>
<td>$\delta_1$</td>
<td>given by initial conditions</td>
<td>Subjective</td>
</tr>
<tr>
<td>Damages at the threshold (%GDP)</td>
<td>$\delta_0$</td>
<td>$\sim U[1,10]$</td>
<td>Taking as reference catastrophic estimates of damages from climate change (see Cline, 1992).</td>
</tr>
<tr>
<td>Growth rate of damages below the threshold</td>
<td>$\delta_2$</td>
<td>1.3</td>
<td>Cline, 1992</td>
</tr>
<tr>
<td>Regeneration of the stock of natural capital</td>
<td>$R$</td>
<td>$\sim U[0,0.1]$</td>
<td>Subjective</td>
</tr>
<tr>
<td>Initial Conditions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depletion rate for fossil fuels</td>
<td></td>
<td>$U\sim [0.05,0.6]$</td>
<td>On the basis of World Bank (1998b)</td>
</tr>
<tr>
<td>Capital per Capita</td>
<td></td>
<td>$U\sim [100,2000]$</td>
<td></td>
</tr>
<tr>
<td>ICOR (GDP/K)</td>
<td></td>
<td>$U\sim [0.1,1]$</td>
<td></td>
</tr>
<tr>
<td>Savings rate</td>
<td></td>
<td>$U\sim [0.10,0.35]$</td>
<td></td>
</tr>
<tr>
<td>Stock of oil and carbon per capita (USD (1987) per capita)</td>
<td></td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Exports (% GDP)</td>
<td></td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>Imports (% GDP)</td>
<td></td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>Non Education expenditures (% GDP)</td>
<td></td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td></td>
<td>$\sim U[-0.001,0.001]$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td></td>
<td>$\sim U[0,0.1]$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3: Model Parameters (Continuation).

Given that my objective is to calibrate the model to an average developing country, I have included as parameters initial conditions (e.g., initial GDP per capital, initial stock of produced capital per capita, initial depletion rate for fossil fuels). Hence, the joint distribution of model parameters also controls for different initial conditions.

To proceed with the estimation it is first necessary to define a prior-distribution for each model parameter to be estimated. I have defined these
distributions on the basis of evidence from the literature, or as in the case of the price for natural resources, on the basis of exploratory analysis of plausible ranges of variation for the model parameters. For instance, a value of $\gamma_1$ in equation (5.33) above 0.1 causes unrealistic fluctuations in the long run. In general, I have tried to keep the variance of the prior distributions as large as possible. Also, given little information about priors I have used uniform probability distributions.

The algorithm used in the search conducted 20 cycles of 3,000 iterations (i.e., combinations of model parameters). For each iteration, the mean and variance of the endogenous variables was computed through 200 Monte Carlo. The results of the estimation are summarized in Figure 5.5. The figure presents the distribution of the simulated and observed average growth rates for GDP and the fossil fuels depletion rate. We observe that the model does a good job in replicating observed distributions, although in the case of the growth rate for the depletion rate, it fails to generate very high or very low values.

In the next Chapter I will present a detailed analysis of the effects of model parameters and policy choices on model dynamics. In the next section I limit my self to discuss the effects of the level of networks connectivity
Figure 5.5: Predicted and observed distributions for the growth rates of fossil fuel depletion rates and GDP per capita (1984-1994).

5.2 Networks Connectivity and Steady States

The model developed in the previous sections shares many of the characteristics of the models within the Social Interactions approach that I described in Chapter 3. As in Young's model (1999), the benefits that agents derive from a particular technology depend on the choices of their neighbors. In the absence of coordination, these choices may fail to be socially efficient, given spillover effects. Yet with some probability, that I define here exogenously, cooperation emerges. More important, the model implicitly defines a set of transition probabilities between technology states that
parallels Durlauf (1993). Indeed, the productivity of the new technology for a given agent depends on the previous choices of its neighbors. If most of them are using the new technology, then it is likely that spillover effects will take place, and that for the agent in question, switching to the new technology will be profitable. Thus, as in Durlauf (1993), one can define

$$\mu(w_t = \prod_{t-1} w_{j-1}, \forall j \in v(i))$$

as the probability that an agent will adopt the new technology given the previous choices of its neighbors. Given the relative complexity of our model, starting with the fact that the networks are not regular and fully connected as in Durlauf (1993), I cannot show that his proofs apply. However, I use simulations to analyze the effects that connectivity has on model steady states. Intuitively, one should expect, as in Durlauf, that multiple steady states would emerge and that the model will display non-ergodic properties, in the sense that initial conditions do not fully characterize the steady state that is chosen.

This is indeed what happens when network connectivity increases. In Figure 5.6, I have graphed the range of variation of the steady state level of GDP as a function of the level of connectivity and the frequency and magnitude of technological innovation (parameters $\psi_0, \psi_1$ in equation 5.20). The dynamics of the model are driven by changes in the stock of produced capital, the supply of labor, the supply of natural resources, and technological progress. Therefore, a pure steady state where neither the economy nor the population are growing, implies a fixed stock of produced and human capital, a constant supply of natural resources, and no technological change. The line in the center can be interpreted as the average level of GDP in the steady state. The top line describes the maximum steady state level of GDP and the bottom line represents the minimum steady state level of GDP. It is clear that as connectivity increases multiple steady states become possible. Hence, the economy can reach high output equilibria (e.g., somewhere along the top line) or low output equilibria. Which equilibria is chosen depends on the series of technology shocks experienced by the economy.

The interpretation of this result is the following. As connectivity increases, information flows regarding technological innovations also increase. However, increased flows benefit both the old and the revolutionary technology. Therefore, a sequence of positive shocks for the old technology
will make it more difficult for the new technology to penetrate the market. Hence, while on average more connected economies have more growth potential (i.e., the average level of GDP of steady state GDP tends to be higher) they may also generate more inertia for the old technology.

The empirical implication of this result is that, other things being equal, countries with high connectivity will face more variability on their long-run levels of GDP. An empirical implication is that the variance of the error term in an econometric model of growth that does not control for connectivity, should increase with the level of connectivity. Therefore one can think about testing the empirical implication of this model, by looking at the relationship of the variance of the residual with the level of connectivity. Given time constraints I have not implemented this test but this is something to keep in mind for future research.
Figure 5.6: Connectivity and GDP Steady States.
6. Conclusion

This chapter has described the construction of an agent-based model of growth with endogenous technology diffusion that emphasize the role of social interactions in agents expectations about the macro-economic environment and the characteristics of new technologies. The chapter has also described the method used to estimate the model parameters. The results show that the model is able to generate consistent dynamics for macro variables of interest. Our analysis has drawn attention on the importance that network connectivity and technology characteristics have for the dynamics of variables such as aggregate output and depletion rates of the economy, and the non-linear character of these relationships. The chapter has also illustrated on the basis of simulations that the model displays non-ergodic dynamics when connectivity and the rate of innovation are high.

The next chapter will use the model to analyze how a benchmark economy facing an environmental constraint should choose savings rates, investments in produced capital, technology incentives, and the consumption of fossil fuels, in order to maximize intertemporal social welfare. The chapter will pay particular attention to the role of network structures in determining these policy choices.

1 Other models that have followed this vein of research are Mattson (1997) and Messner (1992).
2 These coefficients capture the effects of learning by doing and learning by using.
3 I will be using interchangeably the word “firm” and the word “agent”.
4 Physicists at the Santa Fe institute in New Mexico, have estimated that, on average, each individual is directly related to 300 other individuals (rates are of course higher among politicians or businessmen) [informal conversation with James Cruchtfield].
5 The use of a probability is of course a shortcut to keep the level of complexity of the model at a minimum. However, the reader is referred to Young (1999), Kranton and Minehart (1999), and Kranton (1996), for more complex formalizations of the process through which cooperation emerges.
6 This is the so-called “Delta Method” (see Green, 1997).
7 In the dynamic optimization problem of chapter 6, this saving rate is assumed to be a control variable.