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TECHNICAL
REPORT



An Economic Model to
Estimate the Profits
Resulting from the
Employment of
Illegal Aliens

Claude Berrebi, Stephen J. Carroll, Jeffrey Sullivan

Sponsored by the Treasury Executive Office for Asset Forfeiture



INSTITUTE FOR CIVIL JUSTICE and
INFRASTRUCTURE, SAFETY, AND ENVIRONMENT

This research was sponsored by the Treasury Executive Office for Asset Forfeiture (TEOAF) and was conducted under the auspices of the RAND Institute for Civil Justice and the Safety and Justice Program within RAND Infrastructure, Safety, and Environment (ISE).

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Published 2009 by the RAND Corporation
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Preface

The Treasury Executive Office for Asset Forfeiture (TEOAF) asked the RAND Corporation to develop an economic model that can be used by federal law enforcement officials as a tool for determining appropriate amounts of forfeiture, or payment in lieu of forfeiture, in certain cases where firms knowingly employ undocumented workers. This report presents the economic model and the analysis used to develop the model. It also documents the decision-support tool developed to implement the model.

This research was sponsored by TEOAF, whose mission is to affirmatively influence the consistent and strategic use of asset forfeiture by Treasury and Department of Homeland Security law enforcement agencies to disrupt and dismantle criminal enterprises. This report should be of interest to law enforcement agencies concerned with the employment of undocumented workers.

This research was conducted under the auspices of the RAND Institute for Civil Justice and RAND Infrastructure, Safety, and Environment. Questions or comments about this report should be sent to the project leader, Stephen Carroll (Stephen_Carroll@rand.org).

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Summary

The Treasury Executive Office for Asset Forfeiture asked the RAND Corporation to develop an economic model that can be used by federal law enforcement officials as a tool for determining appropriate amounts of forfeiture, or payment in lieu of forfeiture, in cases where firms knowingly employ undocumented workers. We used basic concepts from accepted economic theory to develop a conceptual model that can be used to estimate what profits a firm would have realized if it had not employed any undocumented workers. The estimate of the profits it would have realized without employing undocumented workers can then be subtracted from the profits the firm actually obtained when employing undocumented workers to estimate the amount of profit the employer realized because of the employment of undocumented workers.

This report presents the economic model and the analysis used to develop the model. It also documents the decision-support tool developed to implement the model.

We assume that an employer who has employed illegal workers has organized his productive activities in a way that generates the maximum possible profits, given his resources (plant, equipment, labor of various types, etc.). We use the employer's actual resources, costs, revenues, and profits to estimate what must have been the relationship between resources and outputs, termed the production function, such that the resources available to him were used to generate the outputs necessary to yield his observed profits. Given the estimated production function, we can then estimate the profit-maximizing set of resources that the employer would have employed if he had not employed undocumented workers and what his profits would have been in that case. The difference between the employer's actual observed profits and the estimate of what his maximum profits would have been (given his production function) had he not employed illegal workers is the estimate of the profit the employer realized because he employed undocumented workers.

We developed a computer program that incorporates the calculations implied by the conceptual model.¹ A user can enter appropriate data for an employer who employed undocumented workers, and the program will generate an estimate of the resulting profits. The decision-support tool automatically generates alternative estimates of the profits realized as a result of employing undocumented workers and provides explanations of the assumptions that underlie the differences among the alternatives.

¹ The program is available at http://www.rand.org/pubs/technical_reports/TR599.

Acknowledgments

We owe thanks to many people for an enormous amount of help. A number of people in the Treasury Executive Office for Asset Forfeiture and in Immigration and Customs Enforcement contributed to our understanding of the workings of asset forfeiture. We are particularly indebted to Nadir Isfahani, Department of the Treasury, for his assistance throughout the course of this study. We cannot name every person who spent time with us; the list would be very long, and several of our conversations were on a no-attribution basis.

We are also indebted to RAND colleagues Kathleen Mullen and Mitch Tuller, who reviewed drafts of this report and provided numerous helpful comments, and Nancy Good and Christopher Dirks, who helped in preparing this report.

Abbreviations

BLS	Bureau of Labor Statistics
FTE	Full-Time Equivalent
ICE	Immigration and Customs Enforcement
INS	Immigration and Naturalization Service
MSA	Metropolitan Statistical Area
MOG	Major Occupational Group
TEOAF	Treasury Executive Office for Asset Forfeiture

Introduction

Purpose of This Report

The employment of undocumented workers is a criminal offense in the United States.¹ Anecdotal evidence indicates that this criminal activity is widespread. There are no national databases that document the number of illegal immigrants in the United States, but informed analysts believe that the number of illegal immigrants is growing rapidly and that most of the illegal immigrants to this country eventually find work.²

The mission of the Treasury Executive Office for Asset Forfeiture (TEOAF) is to affirmatively influence the consistent and strategic use of asset forfeiture by Treasury and Homeland Security law enforcement bureaus to disrupt and dismantle criminal enterprises. Accordingly, TEOAF has increased its attention to asset forfeiture cases based on the employment of undocumented workers. Federal law provides for forfeiture of the proceeds of this offense. However, the statutory law does not explicitly specify how to calculate the proceeds in these cases. Thus, there is little specific guidance to prosecutors in determining what amount of money would be an appropriate settlement in any particular case.

TEOAF asked the RAND Corporation to develop an economic model that can be used by federal law enforcement officials in determining settlement amounts in cases where firms knowingly employ undocumented workers. The model presented in this report is designed to serve as an analytical tool that law enforcement agencies can use to estimate the profits resulting from the employment of undocumented workers.

This report presents the economic model and the analysis used to develop the model. It also documents the decision-support tool developed to implement the model in any particular situation.

Background

Asset forfeiture is the divestiture without compensation of property used in a manner contrary to law. In the federal law enforcement context, it is the transfer to the U.S. government of all rights, title, and interest of property because of the property's involvement in or in relation to a crime or some unlawful activity. As with most offenses, forfeitures in cases involving the

¹ See 8 USC §1324 (a)(3) (making it an offense to knowingly hire during any 12-month period at least 10 individuals with actual knowledge that they are illegal aliens) and 8 USC §1324 (a) (6) (making employment of unauthorized aliens unlawful).

² See Johnson (2006) or Wasem (2006).

employment of illegal aliens can be either civil or criminal.³ Civil asset forfeiture provisions allow for the forfeiture of property, real or personal, which constitutes or is traceable to the proceeds obtained, directly or indirectly, from the violation.⁴ Property subject to criminal forfeiture includes that which “constitutes, or is derived from or is traceable to the proceeds obtained directly or indirectly” from the crime or is “used to facilitate, or is intended to be used to facilitate” the offense of which the person is convicted.⁵

Federal agencies, prosecutors, and courts have to place a dollar figure on the economic consequences of the employment of illegal aliens. While the statutory law provides the authority for asset forfeiture and describes the type of property that can be forfeited, there is little explicit guidance concerning the means of determining the appropriate levels of asset forfeiture, leaving each court to develop its own methods given the circumstances of the particular illegal employment case before it.

The law requires that five factors be considered in determining the appropriate civil money penalty to be levied on an employer who knowingly employed illegal aliens.⁶ These factors are: (1) the size of the employer’s business, (2) the good faith of the employer, (3) the seriousness of the violation, (4) the involvement of unauthorized alien employees, and (5) the employer’s history of previous violations. However, the law does not provide guidance as to how these factors should be considered in determining the appropriate civil money penalty to be assessed against an employer who has employed undocumented workers.

An Immigration and Naturalization Service (INS) Deputy District Counsel reviewed both the statutes and the case law regarding asset forfeiture and concluded that there is no standard approach to determining appropriate civil money penalties.⁷ She observed that the officer bringing a case must be able to clearly articulate valid reasons for a penalty amount. The model developed here is intended to provide federal law enforcement officials with a basis for determining the appropriate level of assets to be forfeited by firms engaged in the employment of undocumented workers.

Interpreting Wages Paid to Legal and Illegal Workers

Employers of illegal workers occasionally claim that they hired illegal workers only because legal workers were not available and that they paid the illegal workers the same wages as they paid legal workers. Hence, they did not profit from the employment of illegal workers. Employers also occasionally claim that they hired illegal workers only because the illegal workers were hard-working, industrious, responsive to direction, etc., whereas the available legal workers were lazy, poorly motivated, unresponsive, etc. In these cases, too, the employers argue

³ In criminal forfeiture, the defendant must be found guilty of the crime before property can be seized and forfeited, as criminal forfeiture is part of the defendant’s sentence. Civil forfeiture is an in rem action against the property itself for its involvement in a violation of the law.

⁴ See 18 USC §981(A)(1)(c).

⁵ See 18 USC §982(a)(6)(A).

⁶ U.S. Department of Justice (1998).

⁷ Giambastiani (no date).

that they did not profit from the employment of illegal workers because they paid the illegal workers as much as they paid their legal workers.

Both of these, and similar, arguments miss the central point. The issue is not what employers paid their illegal workers compared with what they paid their legal workers. Rather, the issue is what they paid their illegal workers compared with what they would have had to pay legal workers to meet their employment needs had they not employed illegal workers. It is a criminal offense to knowingly employ illegal workers. Doing so entails the risk of being caught and incurring fines and/or asset forfeiture, adverse publicity, legal fees, and so on. Why run the risk? Because the employers believe they are better off (i.e., make greater profits) employing illegal workers than employing legal workers to meet their labor needs.

Consider the employer who says legal workers are not available in his area. There are literally millions of unemployed legal workers in the United States. They may not live in the vicinity of the employer in question. But, if that employer offers a sufficiently high wage, they will commute long distances to work for him, or even pick up and move into the employer's area. Similarly, there are undoubtedly legal workers currently employed by other firms in the employer's area. If offered a sufficiently high wage, they will leave their current job to take a job with the employer in question. In sum, it simply is not true that the employer cannot find legal workers. What is true is that the employer has emptied the pool of legal workers willing to work for low wages and, rather than offer a wage sufficiently high to attract more legal workers, has chosen to instead hire illegal workers who will accept a low wage offer.

To illustrate, suppose a firm pays its workers at, say, \$12 an hour. The firm has found some number of legal workers willing to work for it for \$12 an hour, but the number of legal workers in the area willing to work for that wage is limited. The firm knows it could make more profits if it could hire more workers at that wage; it could hire more legal workers if it paid more, but then it would make less profit. The firm chooses to meet its remaining workforce needs with illegal workers willing to work for \$12 an hour.

If the firm had offered say, \$18 an hour, it could have attracted some legal workers willing to commute longer distances, willing to move into the area, or willing to quit a job with another employer in the area. And, if \$18 an hour would not attract a sufficient number of legal workers, \$28 an hour, or \$38 an hour, probably would. Unquestionably, there is some wage offer sufficiently high that the employer could attract additional legal workers if it offered that wage.

In any case, in this hypothetical example, the employer profited from the employment of illegal workers to the tune of \$6 (or \$16 or \$26) an hour for each of the illegals it hired at \$12 an hour rather than pay the \$18 (or \$28 or \$38) an hour it would have cost to hire legal workers instead. The fact that the firm found some legal workers willing to work for \$12 an hour is irrelevant to the question of what the employer gained from the employment of illegal workers.

The same arguments apply to the firm that claims it hired illegal workers because the illegal workers were better employees than the available legal workers. Here, too, the firm could hire all the legal workers with desirable attitudes and characteristics it wanted if it was willing to offer a sufficiently high wage. A sufficiently high wage offer would have attracted desirable workers from other geographical areas and from other employers. The firm chose to hire illegal workers because attractive illegal workers could be hired at lower wages than comparably attractive legal works. The employer gained through the employment of illegal workers to the extent that it was able to meet its employment needs, defined in terms of both the numbers

of employees needed and their attitudes/attributes, at a lower wage than it would have needed to pay comparable legal workers. Here, too, the fact that the firm was able to find some legal workers with the desired attitudes willing to work for a lower wage is irrelevant to the question of what the employer would have had to pay to meet its labor needs had it not hired illegal workers.

In sum, the proceeds from employing illegal workers are not reflected in the difference between what a firm employing illegal workers pays its legal workers and what it pays its illegal workers. Rather, the proceeds from employing illegal workers reflect the difference between what a firm employing illegal workers pays its illegal workers and what it would have had to pay to attract sufficient legal workers to meet its labor needs.

Conceptual Model

We reviewed statutory and case law and legal commentary (such as law-review articles) concerning civil forfeiture for illegal employment and other crimes. Our review confirmed the INS Deputy District Counsel's findings. A variety of approaches have been used to determine the appropriate amount of assets to be forfeited in any particular instance. However, we did not find any model that systematically applied basic economic principles to assess the extent to which an employer profited from the employment of undocumented workers.

We discussed the objective of asset forfeiture with officials from TEOAF and Immigration and Customs Enforcement (ICE) and with Assistant U.S. Attorneys who have been involved in asset forfeiture cases. We also discussed recent ICE enforcement actions with them to identify the amounts of assets forfeited, or payments made in lieu of forfeiture, in recent cases and to identify the boundaries of asset forfeiture based on those actions. We concluded that a primary objective of asset forfeiture is to force a violating firm to disgorge the proceeds realized through the employment of undocumented workers. To meet this objective, the appropriate amount of forfeiture would be the increased profits resulting from the employment of undocumented workers.

The profits resulting from the employment of undocumented workers are likely to vary across industries, firms of different sizes, location, and other factors. Accordingly, we used basic concepts from economic theory to develop a conceptual model that can be used to estimate what profits a firm would have realized if it had not employed undocumented workers. The estimate of the profits it would have realized without employing undocumented workers can then be subtracted from the profits the firm actually obtained when employing undocumented workers to estimate the amount of profit the employer realized because of the employment of undocumented workers.

We assume that an employer who has employed illegal workers has organized his productive activities in a way that generates the maximum possible profits, given his resources (plant, equipment, labor of various types, etc.). We also assume that data on his workforce (numbers and types of legal and illegal workers), the salaries and benefits paid to the workers, other costs, revenues, and profits are available. We use the data on the employer's resources, costs, revenues, and profits to estimate what must have been the relationship between resources and outputs, termed the production function, such that the resources available to him were used to generate the outputs necessary to yield his observed profits. That is, we ask: What must have been the relationship between resources and outputs such that this employer, when faced with the wage

rates and other resource costs he paid, would have chosen to employ the numbers and kinds of workers he hired and would have consequently realized the profits he made?

Given the estimated production function, we can then estimate the profit-maximizing set of resources that the employer would have employed if he had not employed undocumented workers and what his profits would have been in that case. The difference between the employer's actual observed profits and the estimate of what his maximum profits would have been (given his production function) had he not employed illegal workers is the estimate of the profit the employer realized because he employed undocumented workers.

Decision-Support Tool

We developed a decision-support tool, a computer program that incorporates the calculations implied by the conceptual model.⁸ A user can enter appropriate data for an employer who employed undocumented workers, and the program will generate an estimate of the resulting profits.

The wage rates an employer actually paid undocumented workers will probably be lower than the wages the employer would have had to pay comparable legal workers. Similarly, the employer contributions toward fringe benefits and such government programs as Social Security and unemployment insurance may be lower for illegal workers than they would have been for legal workers. Accordingly, the decision-support tool allows for alternative estimates based on alternative assumptions regarding prevailing wage rates and other labor costs. In particular, the decision-support tool includes Bureau of Labor Statistics (BLS) estimates of the average wage paid to workers, by occupational group and metropolitan area or state, and BLS estimates of total benefits paid to employees as a percentage of their total compensation.⁹ The tool allows users the option of using either their own estimates or the BLS estimates in calculating what an employer would have to pay legal workers.

The decision-support tool is designed to be user-friendly. It prompts the user regarding the data that must be entered to support the calculations. It also automatically generates alternative estimates of the profits realized as a result of employing undocumented workers and provides explanations of the assumptions that underlie the differences among the alternatives.

We have inserted commentary into the decision-support tool's code identifying the specific components of the conceptual model that underlie each of the tool's calculations. The commentary also identifies the data embedded in the model. The commentary guides users who seek to modify some portion of the tool or update the data incorporated in the tool.

Scope of the Analysis

A firm that employs illegal workers has an incentive to conceal its employment of illegal workers and, consequently, may choose not to report its illegal employees' wages to the IRS, to withhold taxes on their wages, to pay the employer's Social Security and Medicare contributions on their behalf, and so on. While these kinds of activities are often associated with the

⁸ The program is available at http://www.rand.org/pubs/technical_reports/TR599.

⁹ <http://data.bls.gov/cgi-bin/surveymost>

employment of illegal workers, they are crimes in their own right. Moreover, these crimes are sometimes committed by employers of legal workers. This study focuses on developing tools to assess the profits an employer gains through the employment of illegal workers. We do not attempt to include the profits an employer makes through other crimes, such as failure to report illegal employees' incomes and to withhold taxes on their incomes, even though those crimes are frequently associated with the employment of illegal workers. Therefore, our model will assume the existing information available from the employer to be correct.

Organization of This Report

Chapter Two presents the conceptual model. Chapter Three presents the decision-support tool. Chapter Four summarizes the work reported here. Appendices A, B, and C provide the technical details of certain calculations.

The Conceptual Model

Assumptions

For the conceptual model to be computationally feasible, we needed to make several assumptions. The assumptions make the model simpler while keeping the main characteristics similar enough to mimic reality and provide a sound estimate.

The main assumption in our economic model is that firms behave as profit-maximizing entities, and therefore will employ illegal workers to the extent that it is profitable. For computational simplicity, we additionally assume homogeneity in the firms' owners/managers risk aversion and their beliefs about their probability of being caught, and we assume homogeneous expectations with respect to the consequences of employing illegal workers across all firms.

These assumptions allow us to treat the firm's expected profit from employment of illegal workers as if it were known with certainty by the firm's decisionmakers. Obviously, such an assumption would better fit reality when the offender is employing illegal workers for a long period, whereas firms new to the employment of illegal workers might argue imperfect knowledge about any of the potential uncertainties.

Another important assumption is of competitive markets. In other words, we assume that firms are price takers in the product market and that they cannot influence inputs' prices through their employment decisions. These assumptions are quite realistic if enough firms compete in selling the same products or in producing the same services (i.e., the firm is absent monopoly power) and the firm's inputs could be easily used by other firms (i.e., the firm does not have monopolistic power). Simply put, the decision of the firm about whether to produce more or less or close down production altogether will not change market prices, nor will its decision to employ more or less employees change market wages.

Selecting the Model

The first consideration in selecting the appropriate model concerns the interrelationship between illegal workers and other workers or inputs in the production function of the firm. The aim is to design a model flexible enough to allow both substitution and complementarities between the production inputs. In some cases, illegal workers are used as a substitute for other legal workers or for expensive machinery, while in other cases, they complement the work of these other inputs. Since it is impossible to know in advance whether the appropriate production function of a particular firm exhibits complementarities or substitution between any two

inputs and to what extent, our model was designed generally to allow for the entire range of possibilities.

In considering the potential substitution and complementarities between the production inputs, it was clear that neither perfect substitution nor perfect complementarities could be possible. The existence of both legal and illegal workers contradicts the possibility of perfect substitution under the above assumptions, as it would necessarily lead to the employment of only one kind of workers (either legal or illegal).¹ Similarly, perfect complementarity would mean that it is impossible to produce without a positive amount from each of the inputs (i.e., employing illegal workers would be a necessary condition in the productive process), which is assumed not to be the case.

After considering several families of production functions, we concluded that the most appropriate function that answers our above requisites is of the Cobb-Douglas form (Cobb and Douglas, 1928).² Since the time span (one year) on which we will examine the function for any particular firm is short in terms of the properties of physical and human capital involved in production, it is reasonable to assume that no structural changes in the relative coefficients between the different inputs will occur, which makes this functional form very appropriate.

Theoretically, the Cobb-Douglas functional form allows for either constant, increasing, or decreasing returns to scale. However, under the above assumptions, constant returns to scale are inconsistent with positive profits (otherwise firms would increase production infinitely in aim to increase profits infinitely). Similarly, increasing returns to scale are inconsistent with competitive markets. Accordingly, the only viable option left is the one of decreasing returns to scale.

An important consideration in any production function is whether capital is fixed or variable. It is often the case that some or all of a firm's capital is tied into long-term or indivisible commitments (such as rent contracts, machinery, etc.) that cannot be adjusted. Some of the capital, however, can be easily sold and adjusted appropriately to the other inputs' requirements. Obviously, raw material depends almost entirely on the quantities produced and, in most cases, can be quickly adjusted. For those reasons, we decided to separate raw material and consider it as an entirely independent input. Additionally, we will repeat our computational analysis twice for every scenario, once assuming the remaining capital is fixed and could not be easily adjusted and once assuming variable capital.

Estimating the Model's Parameters

Unlike traditional economic exercises, in which the production function parameters are typically known (or assumed) and the economist's role is to solve for the optimal quantities for each of the input factors in order to maximize profit, we will solve our problem backward. As noted above, we already assumed that the firm is maximizing profits and holds complete information with respect to its own business. Consequently, we will use the input quantities used by the firm, combined with data about the firms' profits, revenues, and raw material costs, as well

¹ A profit-maximizing firm would employ the cheapest per-unit-of-output-alternative.

² In economics, the Cobb-Douglas functional form of production functions is widely used to represent the relationship of an output to inputs. It was proposed by Knut Wicksell (1851–1926) and tested against statistical evidence by Paul Douglas and Charles Cobb in 1928.

as wage bill obtained from the firms' financial books, to estimate the appropriate production function parameters for this particular firm. Once we have these parameters, we are then able to compute what the firm's profits would have been had it not employed illegal workers.

The functional form we use is

$$Q = K^\alpha \rho^\varepsilon L_1^\beta L_2^\gamma L_3^\delta \quad (1)$$

where L_1 is the number of skilled legal workers, L_2 is the number of illegal workers, and L_3 is the number of unskilled legal workers. K represents the amount of capital (in \$1 units); similarly, ρ equals the amount of raw material (in \$1 units). Finally, Q stands for the quantity of output produced.

Even though illegal workers are assumed to be mainly, if not entirely, unskilled, they cannot be assumed to be a perfect substitute for the unskilled legal workers. This restriction makes perfect sense theoretically, since otherwise only one type would be employed by a profit-maximizing firm (the type who is cheaper per unit of productivity), but it also makes perfect sense empirically, since the set of skills held by illegal workers is typically different from the set of skills held by legal workers, whether skilled or unskilled. The functional form allows for some substitution between the different types of workers, the extent of which is dependent on the resulting coefficient parameters.

In practice, we need to set an appropriate wage rate for each type of worker. Clearly, firms might offer more than three different wages; moreover, one's salary often depends on many factors beyond productivity (such as tenure or legal constraints). However, theoretically, a profit-maximizing firm should ignore other considerations; for computational tractability, we collapse wages into three main categories.³

In order to satisfy our decreasing-returns-to-scale requirement, we constrain our solution to $\alpha + \varepsilon + \beta + \gamma + \delta < 1$, while maintaining all the production coefficients positive (i.e., $\alpha, \varepsilon, \beta, \gamma, \delta > 0$).

The profit function that each firm maximizes can be described in terms of

$$\text{Max } \Pi,$$

where Π stands for the profit value and equals total revenues minus total costs ($\Pi = R - C$).

Total revenue (R) is equal to the quantity of output produced times the market price of this output and is available to us through examination of the firms' financial books, while costs (C) consist of all the quantities of input used times their prices (in the case of labor, the input price consists of wages and associated benefits paid to the employees) and are available to us through the firms wage bill and expense reports.

Mathematically, since

$$R = P_q Q, \quad (2)$$

³ For computational purposes, the decision-support tool allows the user several options in selecting a wage for each category. Chapter Three describes the tool.

$$C = P_k K + P_\rho \rho + P_{L_1} L_1 + P_{L_2} L_2 + P_{L_3} L_3, \quad (3)$$

and

$$Q = K^\alpha \rho^\varepsilon L_1^\beta L_2^\gamma L_3^\delta, \quad (4)$$

profit can be rewritten as

$$\Pi = P_q K^\alpha \rho^\varepsilon L_1^\beta L_2^\gamma L_3^\delta - P_k K - P_\rho \rho - P_{L_1} L_1 - P_{L_2} L_2 - P_{L_3} L_3. \quad (5)$$

We normalize $P_k \equiv P_\rho \equiv 1$, since physical capital and raw material are measured in units of capital/raw material valued at \$1 each. Hence,

$$\Pi = P_q K^\alpha \rho^\varepsilon L_1^\beta L_2^\gamma L_3^\delta - K - \rho - P_{L_1} L_1 - P_{L_2} L_2 - P_{L_3} L_3. \quad (6)$$

From the firms' financial books and observation we know L_1 , L_2 , L_3 , P_{L_1} , P_{L_2} , P_{L_3} , R , Π , and ρ .

Our objective is to compute the appropriate parameters (α , ε , β , γ , δ) and unknown variables (P_q and K) for our particular firm.

The first-order conditions to the firm's maximization problem yield a series of equations:

$$\frac{\partial \Pi}{\partial L_1} = \beta P_q K^\alpha \rho^\varepsilon L_1^{\beta-1} L_2^\gamma L_3^\delta = P_{L_1} \quad (7)$$

$$\frac{\partial \Pi}{\partial L_2} = \gamma P_q K^\alpha \rho^\varepsilon L_1^\beta L_2^{\gamma-1} L_3^\delta = P_{L_2} \quad (8)$$

$$\frac{\partial \Pi}{\partial L_3} = \delta P_q K^\alpha \rho^\varepsilon L_1^\beta L_2^\gamma L_3^{\delta-1} = P_{L_3} \quad (9)$$

$$\frac{\partial \Pi}{\partial \rho} = \varepsilon P_q K^\alpha \rho^{\varepsilon-1} L_1^\beta L_2^\gamma L_3^\delta = 1 \quad (10)$$

$$\frac{\partial \Pi}{\partial K} = \alpha P_q K^{\alpha-1} \rho^\varepsilon L_1^\beta L_2^\gamma L_3^\delta = 1. \quad (11)$$

By definition,

$$\frac{R}{Q} \equiv P_q = \frac{R}{K^\alpha \rho^\varepsilon L_1^\beta L_2^\gamma L_3^\delta} \quad (12)$$

and

$$\Pi^* = R - K - \rho - P_{L_1} L_1 - P_{L_2} L_2 - P_{L_3} L_3, \quad (13)$$

where Π^* stands for the optimized solution of $\text{Max } \Pi$.

We then solve these seven equations to find the seven unknowns. See Appendix A for detailed, step-by-step solutions.

And, as a result,

$$\Rightarrow \Pi^* = R - (K + \rho + P_{L_1} L_1 + P_{L_2} L_2 + P_{L_3} L_3) \Rightarrow K \text{ is solved}; \quad (14)$$

$$\alpha = \frac{K}{R} \Rightarrow \alpha \text{ is solved}; \quad (15)$$

$$\beta = \frac{L_1 P_{L_1}}{R} \Rightarrow \beta \text{ is solved}; \quad (15)$$

$$\gamma = \frac{L_2 P_{L_2}}{R} \Rightarrow \gamma \text{ is solved}; \quad (16)$$

$$\delta = \frac{L_3 P_{L_3}}{R} \Rightarrow \delta \text{ is solved}; \quad (17)$$

$$\frac{K}{R} \rho = \varepsilon K \Rightarrow \frac{\rho}{R} = \varepsilon \Rightarrow \varepsilon \text{ is solved}; \text{ and} \quad (18)$$

$$P_q = \frac{R}{K^\alpha \rho^\varepsilon L_1^\beta L_2^\gamma L_3^\delta} \Rightarrow P_q \text{ is solved.} \quad (19)$$

Now that we have solved for all the unknowns, we have a full specification of the technology used by the firm.

We will next solve the profit-maximization problem of the firm absent illegal workers and introduce legal replacement workers instead, denoted by L_4 .

Subject to the constraint of no illegal workers, the firm would face the profit function:

$$\Pi = P_q K^\alpha \rho^\varepsilon L_1^\beta L_4^\gamma L_3^\delta - K - \rho - P_{L_1} L_1 - P_{L_4} L_4 - P_{L_3} L_3. \quad (20)$$

Profit Maximization Assuming Variable Capital

We will start analyzing the case of variable capital (K). Similar to above, the first-order conditions are

$$\frac{\partial \Pi}{\partial L_1} = \beta P_q K^\alpha \rho^\varepsilon L_1^{\beta-1} L_4^\gamma L_3^\delta = P_{L_1}, \quad (21)$$

$$\frac{\partial \Pi}{\partial L_4} = \gamma P_q K^\alpha \rho^\varepsilon L_1^\beta L_4^{\gamma-1} L_3^\delta = P_{L_4}, \quad (22)$$

$$\frac{\partial \Pi}{\partial L_3} = \delta P_q K^\alpha \rho^\varepsilon L_1^\beta L_4^\gamma L_3^{\delta-1} = P_{L_3}, \quad (23)$$

$$\frac{\partial \Pi}{\partial \rho} = \varepsilon P_q K^\alpha \rho^{\varepsilon-1} L_1^\beta L_4^\gamma L_3^\delta = 1, \text{ and} \quad (24)$$

$$\frac{\partial \Pi}{\partial K} = \alpha P_q K^{\alpha-1} \rho^\varepsilon L_1^\beta L_4^\gamma L_3^\delta = 1. \quad (25)$$

Note that this time we are solving for the profit-maximizing quantities of L_1 , L_4 , L_3 , ρ and K (L_2 is constrained to equal zero), while we know α , β , γ , δ , ε , P_q , P_k , and P_ρ , from above, and P_{L_1} , P_{L_3} , and P_{L_4} from BLS data. For detailed solution, see Appendix B.

Once we solve for the profit-maximizing quantities in terms of known parameters, introducing our solutions for L_3 , L_1 , L_4 , ρ , and K into the profit function (equation 20) yields the highest profits (Π^*) the firm could obtain without illegal workers, for the case of variable capital (K).⁴ Subtracting the computed profit from the profit found on the firm's financial books

⁴ Note that if the profit-maximizing quantity of a labor category is less than 1, the decision-support tool will increase it to equal 1. Chapter Three describes the tool.

yields the amount of profitable attributable to the employment of illegal workers, under the assumption that capital is totally adjustable.

Profit Maximization Assuming Fixed Capital

We will next solve for the case of fixed capital.

The profit function remains equation 20 above. However, with fixed capital, the first-order conditions are only equations 21 through 24, since K is no longer a choice variable.

This time we are solving for the profit-maximizing quantities of L_1 , L_4 , L_3 , and ρ (L_2 is constrained to equal zero), while we know α , β , γ , δ , ε , P_q , P_k , and P_ρ from above and P_{L_1} , P_{L_3} , and P_{L_4} from BLS data, and we use the previously solved K (from equation 14). For detailed solution, see Appendix C.

Introducing our solutions for L_3 , L_1 , L_4 , and ρ into the profit function (equation 20) yields the highest profits (Π^*) the firm could obtain without illegal workers, for the case of fixed capital (K). Subtracting the computed profit from the profit found on the firm's financial books yields the amount of profit attributable to the employment of illegal workers, under the assumption that capital is not adjustable.⁵

⁵ In this case, as well, for profit maximizing quantities of labor of less than 1, the decision support tool increases the value to 1.

Decision-Support Tool

The decision-support tool implements the model described above with a graphical user interface that allows an operator to perform the calculations automatically.¹ The user is queried for the production function inputs, either directly or calculated from raw data by a wizard, and the tool computes the optimized results for two cases (fixed capital and variable capital) as well as one worst-case scenario. This section describes this process in detail.

User Input

In addition to the standard production-function inputs (e.g., wages), the tool requires the BLS major occupational group (MOG) for the workers who will be hired to replace the illegal immigrants. Along with the location of the firm (in the form of a metropolitan statistical area (MSA)), this is used to look up the average prevailing wage for those replacement workers. The user is not bound to this wage, but, unless instructed otherwise, it is the default used in further calculations.

To describe the firm as it operated, the user has two available methods that can be used in combination to compute the number of full-time equivalent (FTE) workers in the three categories (legal skilled labor, legal unskilled labor, and illegal labor)² as well as the yearly expense associated with one FTE in each category. The first method is simple direct entry. The second method uses the Wage Wizard to calculate the group totals and averages, using as much raw data as are available.

The wizard could be used to enter the wage and working hours for each employee individually, but it can also accept groups of individuals who work approximately the same hours for the same pay and benefits. In addition to entering the wages and working schedules for each group of employees and the number of employees in the group, each employee must be assigned to one of the three worker categories (legal skilled, legal unskilled, or illegal workers).

Once data for every employee or group have been entered, the wizard will calculate the FTE and yearly expense per FTE required to operate the firm and insert that data in to the direct entry form mentioned above. The user can then tweak, round, or redo the numbers before proceeding.

¹ The program is available at http://www.rand.org/pubs/technical_reports/TR599.

² All illegal labor is assumed to be unskilled.

Three of the inputs do not have an associated wizard: revenues, raw material costs, and profits. These values must be entered directly in to the tool, and any associated calculations are the responsibility of the user. The cost of capital is imputed from the rest of the data.

Determining the Production Function

Once the capital costs have been calculated, the parameters of the production function can be computed, with the assumption that the firm was attempting to maximize its profit. The results of these calculations, presented in Chapter Two, are implemented directly in the tool, so the production function is known as soon as the inputs are determined to be valid.

The validation tests that all the production exponents (alpha, beta, etc.) are between zero and one. It then tests that the sum of all the exponents is also between zero and one. Satisfying these conditions implies satisfaction of other, more commonsense, conditions, such as increased inputs leading to increased production or expenses plus profits not exceeding revenues. If a collection of inputs satisfies the validation criteria, then the production function is immediately known and is of the form described in Chapter Two.

Computing the Solutions

Each set of results (worst case, fixed capital, and variable capital) uses the same production function but calculates the outcome inputs in a different way. The mathematical forms of these are described in Chapter Two. This section describes how those forms were implemented.

In all cases, the maximum of P_{L_1} and the alternate wage (default BLS wage or set by the user) is used as P_{L_4} . This model is solved for the maximizing quantities of L_1 , L_2 , L_3 , and L_4 . However, for the purpose of the support tool in cases where the maximizing quantity of L is less than 1, the tool constrains it to be 1 in order to assume that at least one worker is available for each labor category.³ To determine outcome revenues, the estimated production function is used to determine quantity of goods or services produced, and this is multiplied by the price (determined from the original inputs).

For the worst-case scenario, the model uses the quantities of inputs observed in reality, when illegal workers are employed. These inputs are run through the production function calculated previously to determine the firm's revenues. The operating costs are determined by re-computing the wage expenses for the replacement workers and keeping all other costs the same. Expenses associated with illegal workers are, of course, not included. The resulting profit (revenue minus expenses) is reported, along with all the inputs used to arrive at this solution.

In the case of fixed capital, the capital expenditures and labor expense (per FTE) remain the same, but the number of FTE workers in each category is allowed to vary along with the amount of raw material purchased. An optimum, in terms of maximum profit, is implemented directly in the code as a closed-form solution, so the results are exact.⁴ In addition to the new

³ It should be noted that in the cases in which the support tool increases the value of L to 1, the solution is no longer optimal but rather an approximation.

⁴ Here, too, in the cases in which the support tool increases the value of L to 1, the solution is no longer optimal but rather an approximation.

revenue and profit, the number of FTE workers in each category and raw material expense are also updated and reported.

A closed-form solution also exists when capital costs are allowed to vary, and so this final scenario is solved in the same way as the fixed capital, though with a different optimum. As before, all the computed parameters are reported along with the final results.

All the results are summarized for the user in an easy-to-read tabular format that compares the original inputs to each of the possible outcomes. In addition, the detailed intermediate calculations are available to further document and explain the results.

For either level of detail, the results can be saved to file for storage.

Concluding Summary

We used basic concepts from accepted economic theory to develop a conceptual model that can be used to estimate what profits a firm would have realized if it had not employed any undocumented workers. The estimate of the profits it would have realized without employing undocumented workers can then be subtracted from the profits the firm actually obtained when employing undocumented workers to estimate the amount of profit that is attributable to the employment of undocumented workers.

We assume that an employer who has employed illegal workers has organized his productive activities in a way that generates the maximum possible profits, given his resources (plant, equipment, labor of various types, etc.). We also assume that data on his workforce (numbers and types of legal and illegal workers), the salaries and benefits paid to the workers, other costs, revenues, and profits are available. We use the data on the employer's resources, costs, revenues, and profits to estimate what must have been the relationship between resources and outputs, termed the *production function*, such that the resources available to him were used to generate the outputs necessary to yield his observed profits. That is, we ask: What must have been the relationship between resources and outputs such that this employer, when faced with the wage rates and other resource costs he paid, would have chosen to employ the numbers and kinds of workers he hired and would have consequently realized the profits he made?

Given the estimated production function, we can then estimate the profit-maximizing set of resources that the employer would have employed if he had not employed undocumented workers and what his profits would have been in that case. The difference between the employer's actual observed profits and the estimate of what his maximum profits would have been (given his production function) had he not employed illegal workers is the estimate of the profit the employer realized because he employed undocumented workers.

We developed a computer program that incorporates the calculations implied by the conceptual model. A user can enter appropriate data for an employer who employed undocumented workers, and the program will generate an estimate of the resulting profits. The decision-support tool automatically generates alternative estimates of the profits realized as a result of employing undocumented workers and provides explanations of the assumptions that underlie the differences among the alternatives.

Detailed Steps Toward the Solution of the Production Function Parameters and Variables

$$\frac{(7)}{(8)} \Rightarrow \frac{P_{L_1}}{P_{L_2}} = \frac{L_2 \beta}{L_1 \gamma}$$

$$\frac{(7)}{(9)} \Rightarrow \frac{P_{L_1}}{P_{L_3}} = \frac{L_3 \beta}{L_1 \delta}$$

$$\left. \begin{array}{l} (7) \Rightarrow P_{L_1} = \frac{\rho \beta}{L_4 \varepsilon} \\ (10) \Rightarrow P_{L_1} = \frac{K \beta}{L_1 \alpha} \end{array} \right\} \frac{\rho}{\varepsilon} = \frac{K}{\alpha} \Rightarrow \alpha \rho = \varepsilon K$$

$$\left. \begin{array}{l} (11) \Rightarrow P_q = \frac{1}{\alpha K^{\alpha-1} \rho^\varepsilon L_1^\beta L_2^\gamma L_3^\delta} \\ (12) \Rightarrow P_q = \frac{R}{K^\alpha \rho^\varepsilon L_1^\beta L_2^\gamma L_3^\delta} \end{array} \right\} \frac{K}{\alpha} = R$$

Inserting R into equation 13 solves K .

Detailed Steps Toward the Solution of Profit Maximization Assuming Variable Capital

$$\begin{aligned} \frac{(21)}{(23)} &\Rightarrow \frac{P_{L_1}}{P_{L_3}} = \frac{L_3\beta}{L_1\delta} \\ \frac{(21)}{(24)} &\Rightarrow P_{L_1} = \frac{\rho\beta}{L_1\varepsilon} \Rightarrow \rho = P_{L_1} L_1 \frac{\varepsilon}{\beta} \\ \frac{(22)}{(24)} &\Rightarrow P_{L_4} = \frac{\rho\gamma}{L_4\varepsilon} \Rightarrow \rho = P_{L_4} L_4 \frac{\varepsilon}{\gamma} \\ \frac{(22)}{(25)} &\Rightarrow P_{L_4} = \frac{K\gamma}{L_4\alpha} \Rightarrow K = P_{L_4} L_4 \frac{\alpha}{\gamma} \end{aligned}$$

Define

$$\Omega \equiv \frac{P_{L_3}}{P_{L_1}} \frac{\beta}{\delta}. \quad (26)$$

From $\frac{(21)}{(23)}$ above we get $L_1 = \Omega L_3$.

Using this in $\frac{(21)}{(24)}$ above we get $\rho = P_{L_1} \Omega L_3 \frac{\varepsilon}{\beta}$.

Combining this result with $\frac{(22)}{(24)}$ above we get

$$L_4 = \frac{P_{L_1} P_{L_3} \beta}{P_{L_4} P_{L_1} \delta} L_3 \frac{\varepsilon \gamma}{\beta \varepsilon} \Rightarrow L_4 = \frac{P_{L_3}}{P_{L_4}} L_3 \frac{\gamma}{\delta},$$

$$L_1 = \frac{P_{L_3}}{P_{L_1}} L_3 \frac{\beta}{\delta},$$

$$L_1 = \frac{P_{L_4}}{P_{L_1}} L_4 \frac{\beta}{\gamma},$$

$$\rho = P_{L_4} L_4 \frac{\varepsilon}{\gamma} = P_{L_1} L_1 \frac{\varepsilon}{\beta} = P_{L_3} L_3 \frac{\varepsilon}{\delta}, \text{ and}$$

$$K = P_{L_4} L_4 \frac{\alpha}{\gamma} = P_{L_1} L_1 \frac{\alpha}{\beta} = P_{L_3} L_3 \frac{\alpha}{\delta}.$$

Putting everything in terms of L_3 gives us

$$L_3 \equiv L_3, \tag{27}$$

$$L_1 = \frac{P_{L_3}}{P_{L_1}} L_3 \frac{\beta}{\delta}, \tag{28}$$

$$L_4 = \frac{P_{L_3}}{P_{L_4}} L_3 \frac{\gamma}{\delta}, \tag{29}$$

$$\rho = P_{L_3} L_3 \frac{\varepsilon}{\delta}, \tag{30}$$

$$K = P_{L_3} L_3 \frac{\alpha}{\delta}. \tag{31}$$

We now have a functional solution for each of the maximizing quantities in terms of L_3 and known parameters; inserting these solutions in any of the first-order conditions will give us the final answers entirely in terms of known parameters.

For example, using $\frac{\partial \Pi}{\partial \rho}$ (the fourth of the first order conditions, equation 24) we get

$$\varepsilon P_q \left(P_{L_3} L_3 \frac{\alpha}{\delta} \right)^\alpha \left(P_{L_3} L_3 \frac{\varepsilon}{\delta} \right)^{\varepsilon-1} \left(\frac{P_{L_3}}{P_{L_1}} L_3 \frac{\beta}{\delta} \right)^\beta \left(\frac{P_{L_3}}{P_{L_4}} L_3 \frac{\gamma}{\delta} \right)^\gamma L_3^\delta = 1. \quad (32)$$

Define

$$\frac{P_{L_3} L_3}{\delta} \equiv \phi_3 \Rightarrow L_3 = \frac{\phi_3 \delta}{P_{L_3}}; \quad (33)$$

$$\begin{aligned} \Rightarrow \phi_3^{\alpha+\varepsilon-1+\beta+\gamma} \left(\frac{\phi_3 \delta}{P_{L_3}} \right)^\delta \varepsilon P_q \alpha^\alpha \varepsilon^{\varepsilon-1} \left(\frac{\beta}{P_{L_1}} \right)^\beta \left(\frac{\gamma}{P_{L_4}} \right)^\gamma &= 1, \\ \Rightarrow \phi_3^{\alpha+\beta+\gamma+\delta+\varepsilon-1} P_q \alpha^\alpha \left(\frac{\beta}{P_{L_1}} \right)^\beta \left(\frac{\gamma}{P_{L_4}} \right)^\gamma \left(\frac{\delta}{P_{L_3}} \right)^\delta \varepsilon^\varepsilon &= 1, \\ \Rightarrow \phi_3^* &= \left[\frac{P_{L_1}^\beta P_{L_4}^\gamma P_{L_3}^\delta}{P_q \alpha^\alpha \beta^\beta \gamma^\gamma \delta^\delta \varepsilon^\varepsilon} \right]^{\frac{1}{\alpha+\beta+\gamma+\delta+\varepsilon-1}}. \end{aligned} \quad (34)$$

$\Rightarrow \phi_3^*$ is solved in terms of known parameters.

\Rightarrow using the above definition (equation 33), L_3 is solved in terms of known parameters, and, therefore, L_1 , L_4 , ρ , and K are also solved in terms of known parameters (equations 28 through 31).

Detailed Steps Toward the Solution of Profit Maximization Assuming Fixed Capital

Solving, like above, and putting everything in terms of L_3 gives us equations 27 through 30. Using the fourth of the first-order conditions (equation 24) for the case of fixed capital, we get

$$\varepsilon P_q K^\alpha \left(P_{L_3} L_3 \frac{\varepsilon}{\delta} \right)^{\varepsilon-1} \left(\frac{P_{L_3}}{P_{L_1}} L_3 \frac{\beta}{\delta} \right)^\beta \left(\frac{P_{L_3}}{P_{L_4}} L_3 \frac{\gamma}{\delta} \right)^\gamma L_3^\delta = 1. \quad (35)$$

Using definition 33,

$$\begin{aligned} \Rightarrow \phi_3^{\varepsilon-1+\beta+\gamma} \left(\frac{\phi_3 \delta}{P_{L_3}} \right)^\delta \varepsilon P_q K^\alpha \varepsilon^{\varepsilon-1} \left(\frac{\beta}{P_{L_1}} \right)^\beta \left(\frac{\gamma}{P_{L_4}} \right)^\gamma &= 1 \\ \Rightarrow \phi_3^{\beta+\gamma+\delta+\varepsilon-1} P_q K^\alpha \varepsilon^\varepsilon \left(\frac{\beta}{P_{L_1}} \right)^\beta \left(\frac{\gamma}{P_{L_4}} \right)^\gamma \left(\frac{\delta}{P_{L_3}} \right)^\delta &= 1 \\ \Rightarrow \phi_3^* &= \left[\frac{P_{L_1}^\beta P_{L_4}^\gamma P_{L_3}^\delta}{P_q K^\alpha \beta^\beta \gamma^\gamma \delta^\delta \varepsilon^\varepsilon} \right]^{\frac{1}{\beta+\gamma+\delta+\varepsilon-1}} \end{aligned} \quad (36)$$

- $\Rightarrow \phi_3^*$ for fixed capital is solved in terms of known parameters.
- \Rightarrow using equation 33 solves L_3 in terms of known parameters.
- $\Rightarrow L_1, L_4,$ and P are also solved in terms of known parameters (equations 28 through 30).

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