



NATIONAL DEFENSE RESEARCH INSTITUTE

THE ARTS
CHILD POLICY
CIVIL JUSTICE
EDUCATION
ENERGY AND ENVIRONMENT
HEALTH AND HEALTH CARE
INTERNATIONAL AFFAIRS
NATIONAL SECURITY
POPULATION AND AGING
PUBLIC SAFETY
SCIENCE AND TECHNOLOGY
SUBSTANCE ABUSE
TERRORISM AND
HOMELAND SECURITY
TRANSPORTATION AND
INFRASTRUCTURE
WORKFORCE AND WORKPLACE

This PDF document was made available from www.rand.org as a public service of the RAND Corporation.

[Jump down to document](#) ▼

The RAND Corporation is a nonprofit research organization providing objective analysis and effective solutions that address the challenges facing the public and private sectors around the world.

Support RAND

[Purchase this document](#)

[Browse Books & Publications](#)

[Make a charitable contribution](#)

For More Information

Visit RAND at www.rand.org

Explore the [RAND National Defense Research Institute](#)

View [document details](#)

Limited Electronic Distribution Rights

This document and trademark(s) contained herein are protected by law as indicated in a notice appearing later in this work. This electronic representation of RAND intellectual property is provided for non-commercial use only. Unauthorized posting of RAND PDFs to a non-RAND Web site is prohibited. RAND PDFs are protected under copyright law. Permission is required from RAND to reproduce, or reuse in another form, any of our research documents for commercial use. For information on reprint and linking permissions, please see [RAND Permissions](#).

This product is part of the RAND Corporation technical report series. Reports may include research findings on a specific topic that is limited in scope; present discussions of the methodology employed in research; provide literature reviews, survey instruments, modeling exercises, guidelines for practitioners and research professionals, and supporting documentation; or deliver preliminary findings. All RAND reports undergo rigorous peer review to ensure that they meet high standards for research quality and objectivity.

Errata

To: Recipients of TR-756-OSD, RAND's Portfolio Analysis Tool (PAT): Theory, Methods, and Reference Manual

From: RAND Corporation Publications Department

Date: May 2010

Re: Corrections on pages 59, 60, 61, 62, 64, 67, 68, and 69

On pages 59, 60, and 61, in the first equation following subheads Weak Thresholds Method, Thresholds Method, and Weakest Link Method, $T_{j,k}$ in the numerator of the fraction should read $V_{j,k}^T$. In addition, in the first two conditions of the piecewise defined function, "if $V_{j,k}$ " should read "if $V_{i,j,k}$ ". The second condition should read, in full, "if $V_{i,j,k} \geq V_{j,k}^G$ " instead of "if $V_{j,k} \geq V_{j,k}^T$ ".

On page 62, in the first equation following the subhead Rankings Method, both the $W_{i,j}$ in the numerator and the $W_{i,k}$ in the denominator should read $W_{j,k}$.

On page 64, in the second paragraph, $S_{i,3}$ should read $M_{i,3}$.

On page 67, in the first paragraph, $S_{1,1}$ should read $M_{1,1}$. Table 5.8 contains revised data.

On page 68, Tables 5.9 and 5.10 contain revised data.

The authors are grateful to Callie Beagley of Systems Planning and Analysis, Inc., for noting some of these errors.

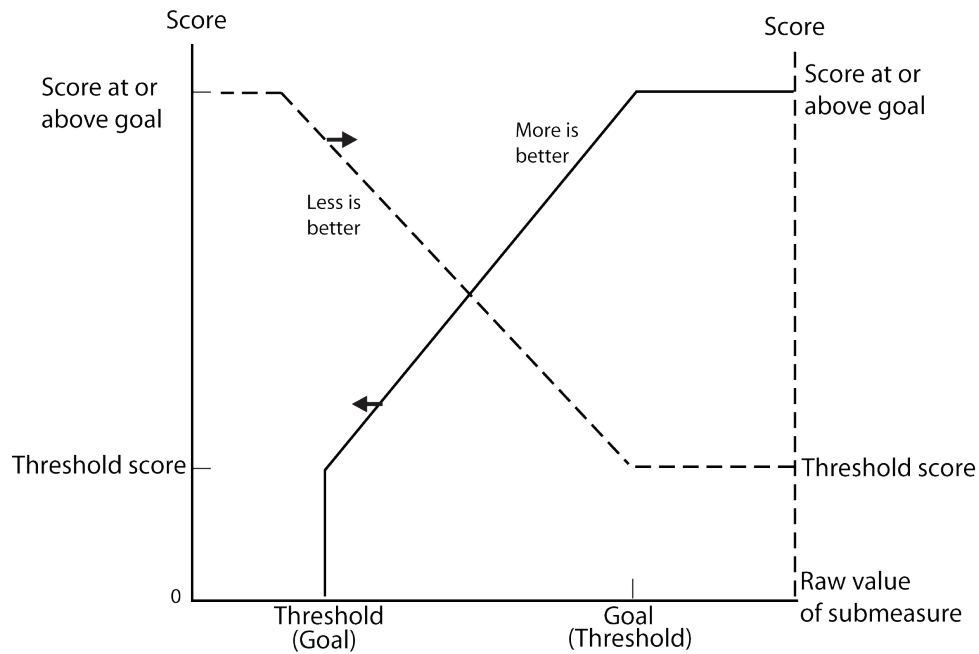
Weak Thresholds Method

Submeasure Scores with Weak Thresholds Method. With the Weak Thresholds method, each submeasure has a threshold and a goal. If the threshold is not reached, the submeasure score is 0. If the submeasure reaches or exceeds its goal, the submeasure score is its score for the goal value. In between, the score is determined by a linear relationship. That is, in cases where goodness increases with raw value:

$$\begin{aligned}
 S_{i,j,k} &= 0 && \text{if } V_{i,j,k} < V_{j,k}^T \\
 S_{i,j,k} &= G_{j,k} && \text{if } V_{i,j,k} \geq V_{j,k}^G \\
 S_{i,j,k} &= T_{j,k} + \frac{V_{i,j,k} - V_{j,k}^T}{V_{j,k}^G - V_{j,k}^T} (G_{j,k} - T_{j,k}) && \text{if } V_{j,k}^T \leq V_{i,j,k} \leq V_{j,k}^G
 \end{aligned}$$

If goodness decreases with raw value, the equations must be adjusted. Figure 5.2 shows graphically the resulting scoring method for cases in which more is better (solid line) and cases for which more is worse (dashed lines). The order of goal and threshold is reversed for the two cases—for the more-is-worse case (dashed lines), the threshold comes second and the goal first as one reads along the x-axis (see the parenthetical values).

Figure 5.2
Score Versus Raw Value for Goals and Thresholds Methods



Aggregation to Find Measure Scores with Weak Thresholds Method. The score of measure j is again given by a linear weighted sum, as in the Goals method, but the submeasure scores are different:

$$M_{i,j} = \frac{\sum_{k=1}^{n_j} W_{j,k} S_{i,j,k}}{\sum_{k=1}^{n_j} W_{j,k} G_{j,k}}$$

Aggregation to Find Overall Effectiveness with Weak Thresholds Method. Overall effectiveness with the Weak Thresholds method is given by a linear weighted sum, as previously, but the measure scores are different:

$$E_i = \frac{\sum_{j=1}^m C_j M_{i,j}}{\sum_{j=1}^m C_j}$$

Thresholds Method

Submeasure Scores with Thresholds Method. Submeasure scores are calculated with the same function as that used for the Weak Thresholds method:

$$\begin{aligned} S_{i,j,k} &= 0 && \text{if } V_{i,j,k} < V_{j,k}^T \\ S_{i,j,k} &= G_{j,k} && \text{if } V_{i,j,k} \geq V_{j,k}^G \\ S_{i,j,k} &= T_{j,k} + \frac{V_{i,j,k} - V_{j,k}^T}{V_{j,k}^G - V_{j,k}^T} (G_{j,k} - T_{j,k}) && \text{if } V_{j,k}^T \leq V_{i,j,k} \leq V_{j,k}^G \end{aligned}$$

Aggregation to Find Measure Scores with Thresholds Method. The difference in methods occurs here. If any submeasure fails to reach its threshold, the measure is assigned a score of 0:

$$\begin{aligned} M_{i,j} &= \frac{\sum_{k=1}^{n_j} W_{j,k} S_{i,j,k}}{\sum_{k=1}^{n_j} W_{j,k} G_{j,k}} && \text{if all submeasures reach their thresholds} \\ M_{i,j} &= 0 && \text{otherwise} \end{aligned}$$

Aggregation to Find Overall Effectiveness with Thresholds Method. Effectiveness is calculated precisely as before, but the measure scores are different:

$$E_i = \frac{\sum_{j=1}^m C_j M_{i,j}}{\sum_{j=1}^m C_j}$$

Weakest Link Method

The Weakest Link Method is even more stringent in enforcing the “requirements” represented by thresholds.

Submeasure Scores with Weakest Link Method. Submeasure scores are calculated with the same function as for the other threshold methods:

$$\begin{aligned} S_{i,j,k} &= 0 && \text{if } V_{i,j,k} < V_{j,k}^T \\ S_{i,j,k} &= G_{j,k} && \text{if } V_{i,j,k} \geq V_{j,k}^G \\ S_{i,j,k} &= T_{j,k} + \frac{V_{i,j,k} - V_{j,k}^T}{V_{j,k}^G - V_{j,k}^T} (G_{j,k} - T_{j,k}) && \text{if } V_{j,k}^T \leq V_{i,j,k} \leq V_{j,k}^G \end{aligned}$$

Aggregation to Find Measure Scores with Weakest Link Method. With the Weakest Link Method, the measure score is the minimum of the submeasure scores. If any submeasure fails to reach its threshold, the measure score will be 0, as in the Thresholds method, but if all submeasures reach their thresholds, the score will be different from that in the Thresholds method and typically smaller:

$$M_{i,j} = \min_k \{ S_{i,j,k} \}$$

Aggregation to Find Overall Effectiveness with the Weakest Link Method. Aggregation with the Weakest Link method is performed simply by taking the minimum measure score (which is identical to the minimum submeasure score of all submeasures). If any submeasure in any of the measures fails to reach its threshold, effectiveness will be 0. Even if that does not happen, effectiveness will typically be smaller in this method:

$$E_i = \min_j \{ M_{i,j} \}$$

Rankings Method

The Rankings method does not use goal or threshold values. Instead, for each submeasure, the investment options are simply ranked from best to worst, without regard to absolute performance. Instead of submeasure and measure scores, we refer to submeasure and measure ranks for this method.

Submeasure Ranks. Let $R_{i,j,k}$ be the rank of investment option i for measure j and for submeasure k . We define it as one more than the number of investment options that perform

strictly better than investment option i on submeasure k of measure j . Thus, if two investment options have the same raw value, they will have the same rank.

Aggregation to Find Measure Ranks with Rankings Method. The aggregation from submeasure rankings to measure rankings is again a linear weighted sum. We calculate the average rank $\bar{R}_{i,j}$ within measure j as

$$\bar{R}_{i,j} = \frac{\sum_{k=1}^{n_j} W_{j,k} R_{i,j,k}}{\sum_{k=1}^{n_j} W_{j,k}}$$

The value $\bar{R}_{i,j}$ is used to set the color (one of five shades of blue, as shown in Table 5.4 on p. 64) in the measure-summary table, depending on the quintile in which the average ranking resides.

Aggregation to Find Overall Effectiveness with Rankings Method. This aggregation function is best explained with words and examples. It is entirely different from the functions used in the other methods. For this effectiveness aggregation, each measure is assigned a score based on rankings.

Let us assume that K investment options are under consideration and that each submeasure has $K(K-1)/2$ points to distribute among the investment options. If there are no ties for the values in submeasure j , the submeasure score is given by

$$R'_{i,j,k} = K - R_{i,j,k}$$

If there are ties, the points that would have gone to the investment options in those positions are combined and then equally distributed among the investment options. For example, with ten investment options, the second- and third-place investment options would receive 8 and 7 points, respectively. If two investment options are tied for second place, they would receive $(7 + 8)/2 = 7.5$ points each. This scoring method is used instead of a linear transformation of the rankings to avoid producing artificially large numbers in the case of ties. To illustrate, if each of ten investment options had the same value for a submeasure, they would all tie for first. If the ranking effectiveness score did not take ties into account, each investment option would receive 9 points, meaning that the total (unweighted) contribution to the final score (summed across investment options) by that submeasure would be 90 points. By comparison, for a submeasure where every investment option had a different value, the total (unweighted) contribution to the final score by that submeasure would be 45 points (i.e., $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$).

As with the other scoring methods, each submeasure $S_{j,k}$ has a weight $W_{j,k}$ in the aggregation to the measure score. Similarly, the weights C_i determine the relative contribution of each measure to the effectiveness score for each investment option. The measure score for an investment option i for measure j is

Table 5.3
Mapping Measure Scores into Colors

Range of Scores	Color
$0 \leq \text{score} < 0.2$	Red
$0.2 \leq \text{score} < 0.4$	Orange
$0.4 \leq \text{score} < 0.6$	Yellow
$0.6 \leq \text{score} < 0.8$	Light green
$0.8 \leq \text{score} \leq 1$	Dark green

As shown in Table 5.4, the colors go from light blue to dark blue, denoting the quintile in which the weighted average rank of the investment option lies. For example, if there were ten investment options, an investment option would receive the lightest blue color if the weighted average rank was two or lower and would receive the darkest blue color if it was eight or greater. The same coloring method, based on quintiles, is used on the *Drilldown* sheets, where the rankings in each submeasure set the color for the cell.

Table 5.5 summarizes the various methods concisely.

Examples of Scoring and Aggregation Using Different Methods

To provide examples for each scoring method, we look at two measures, M_1 and M_2 , each consisting of three submeasures. All submeasures take values between 0 and 10, inclusive, with 2 being the threshold value and 7 being the goal value in each case. For both measures, submeasure $M_{i,3}$ will have weights twice those of the other two submeasures; all submeasures receive a score of 0 for not reaching the threshold value. By setting the weights of the three submeasures to 0.25, 0.25, and 0.5, we get a sum of 1, so there is no need to divide the sum of the submeasure scores by the sum of the weights. We also assume the weights of the measures are equal. We consider three investment options, which we call Investment Options A, B, and C. The raw values for each investment option for each submeasure are given in Table 5.6. (This and subsequent tables in this chapter are not PAT displays, but rather were constructed for the discussion.)

Table 5.4
Color-Coding in the Rankings Method

Meaning	Color
(Average) rank in first quintile	Lightest blue
(Average) rank in second quintile	Light blue
(Average) rank in third quintile	Medium blue
(Average) rank in fourth quintile	Dark blue
(Average) rank in fifth quintile	Darkest blue

Table 5.8
Illustrative Results for the Thresholds Method

Measure	M1	M1	M1		M2	M2	M2		
Level 2 measure	M1.1	M1.2	M1.3		M2.1	M2.2	M2.3		
Weight	0.25	0.25	0.5		0.25	0.25	0.5		
High or Low values desired?	High	High	High		High	High	High		
Threshold value	2	2	2		2	2	2		
Goal value	7	7	7		7	7	7		
Level 2 value for Threshold	0	0	0		0	0	0		
Level 2 value for Goal	1	1	1		1	1	1		
				M1 score				M2 score	Effectiveness
Option A	10	10	5		5	5	5		
Weighted score	.25	.25	.3	.8	.15	.15	.3	.6	.7
Option B	7	5	6		8	1	10		
Weighted score	.25	.15	.4	.8	.25	xxx	.5	0	.4
Option C	1	5	10		10	10	10		
Weighted score	xxx	.15	.5	0	.25	.25	.5	1	.5

Weak Thresholds Method

For the Weak Thresholds method, values that meet or exceed the goal value are highlighted in green; those that meet or exceed the threshold value but not the goal value are highlighted in yellow; and those that fail to meet the threshold value are highlighted in red (Table 5.9). We also show the measure score and effectiveness score and color the cells appropriately. We show the effectiveness score (assuming the measures have equal weight) for both measures in the rightmost column. Even though Investment Option C fails for submeasure $M_{1,1}$, reaching the goal value on four of six submeasures pushes its effectiveness above that of the other two options.

Weakest Link Method

For the Weakest Link, values that meet or exceed the goal value are highlighted in green; those that meet or exceed the threshold value but not the goal value are highlighted in yellow; and those that fail to meet the threshold value are highlighted in red (Table 5.10). We also show the (unweighted) submeasure score, the measure score (which is the minimum of the submeasure scores for each measure), and the effectiveness score (which is the minimum of the measure scores) for each investment option. The weights of the submeasures do not apply here. We color the effectiveness score cell for each measure to correspond to the color scheme used in PAT for this method. Because Investment Option A is the only option that did not fail on any submeasure, it has the highest effectiveness score under the Weakest Link method.

Table 5.9
Illustrative Results for the Weak Thresholds Method

Measure	M1	M1	M1		M2	M2	M2		
Level 2 measure	M1.1	M1.2	M1.3		M2.1	M2.2	M2.3		
Weight	0.25	0.25	0.5		0.25	0.25	0.5		
High or Low values desired?	High	High	High		High	High	High		
Threshold value	2	2	2		2	2	2		
Goal value	7	7	7		7	7	7		
Level 2 value for Threshold	0	0	0		0	0	0		
Level 2 value for Goal	1	1	1		1	1	1		
				M1 score				M2 score	Effect-iveness
Option A	10	10	5		5	5	5		
Weighted score	.25	.25	.3	.8	.15	.15	.3	.6	.7
Option B	7	5	6		8	1	10		
Weighted score	.25	.15	.4	.8	.25	0	.5	.75	.78
Option C	1	5	10		10	10	10		
Weighted score	0	.15	.5	.65	.25	.25	.5	1	.83

Table 5.10
Illustrative Results for the Weakest Link Method

Measure	M1	M1	M1		M2	M2	M2		
Level 2 Measure	M1.1	M1.2	M1.3		M2.1	M2.2	M2.3		
Weight	0.25	0.25	0.5		0.25	0.25	0.5		
High or Low values desired?	High	High	High		High	High	High		
Threshold value	2	2	2		2	2	2		
Goal value	7	7	7		7	7	7		
Level 2 value for Threshold	0	0	0		0	0	0		
Level 2 value for Goal	1	1	1		1	1	1		
				M1 score				M2 score	Effect-iveness
Option A	10	10	5		5	5	5		
Weighted score	1	1	.6	.6	.6	.6	.6	.6	.6
Option B	7	5	6		8	1	10		
Weighted score	1	.6	.8	.6	1	0	1	0	0
Option C	1	5	10		10	10	10		
Weighted score	0	.6	1	0	1	1	1	1	0

Rankings Method

In our example, the summary and effectiveness weights are identical (with the third submeasures of each measure having twice the weight of the other two), as shown in Table 5.11.

The average rank- and measure-score calculations are compressed slightly, and all values are rounded to two decimal places. For the measure score, the sum of the submeasure scores is divided by the product of the maximum submeasure score per measure (2) and the sum of the weights (1). The effectiveness value is just the average of the measure scores. Because Investment Option C has the best value for four of six submeasures, it is clearly the best option under the Rankings method.

**Table 5.11
Illustrative Results for the Rankings Method**

Measure	M1	M1	M1		M2	M2	M2		
Level 2 measure	M1.1	M1.2	M1.3		M2.1	M2.2	M2.3		
Weight	0.25	0.25	0.5		0.25	0.25	0.5		
High or Low values desired?	High	High	High		High	High	High		
Threshold value	2	2	2		2	2	2		
Goal value	7	7	7		7	7	7		
Level 2 value for Threshold	0	0	0		0	0	0		
Level 2 value for Goal	1	1	1		1	1	1		
				Avg. rank				Avg. rank	Effectiveness
Option A	10	10	5		5	5	5		
(rank)x(weight)	.25	.25	1.5	2	.75	.5	1.5	2.75	
(score)x(weight)	.5	.5	0	.5	0	.25	0	.13	.32
Option B	7	5	6		8	1	10		
(rank)x(weight)	.5	.5	1	2	.5	.75	.5	1.75	
(score)x(weight)	1	.125	.5	.44	.25	0	1	.63	.54
Option C	1	5	10		10	10	10		
(rank)x(weight)	.75	.5	.5	1.75	.25	.25	.5	1	
(score)x(weight)	.25	.125	1	.69	.5	.5	1	1	.85