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Errata

To: Recipients of *A Computational Model of Public Support for Insurgency and Terrorism*, by Paul K. Davis and Angela O'Mahony, TR-1220-OSD, 2013

From: The RAND Corporation

Date: June 2015

Re: The following revisions were made to the report:

- Page 15: Added the following sentence to the description of Table 2.3: "Table 2.3 shows (see last row) the original algorithm and a stronger revised version (see also Appendix D). The illustrative calculations in Chapter Four use the original algorithm."
- Page 15: Revised the equations in the last row of Table 2.3 and their description.
- Page 75: Revised Equation D.4 and the sentence that introduces it.
- Page 80: Revised Equation D.12.

The thresholds for the various factors can be different, and can be either small or large, providing a great deal of flexibility. For example, by setting the thresholds to Very Low levels, the formulation reduces to linear weighted sums. Also, if there is skepticism about the absolute necessity of a given factor (recall the "~" in the "~ands" of Figure 2.1), then its threshold may be set lower than those of the others.* Table 2.3 shows (see last row) the original algorithm and a stronger revised version (see also Appendix D). The illustrative calculations in Chapter Four use the original algorithm .

Table 2.3
Mathematical Expression of "And" Relationships

Expression	Explanation
$\mathbf{F}^0 = \{F_1^0, \ldots F_n^0\}$; $\mathbf{W}^0 = \{W_1^0, \ldots W_n^0\}$	Vectors of original factors and weights
$\mathbf{F} = \{F_1, \ldots F_n\}$; $\mathbf{W} = \{W_1, \ldots W_n\}$	Thresholded vectors and renormalized weights
$F_i = \begin{cases} F_i^0 \text{ if } F_i^0 \geq T_i \\ 0 \text{ otherwise} \end{cases}$	A given component of the thresholded vector is 0 if the original component failed to reach its threshold, but equals the original value otherwise.
$\mathbf{W}^0 \bullet \mathbf{F} = \{W_1^0 F_1, \ldots W_n F_n\}$	The vector of weighted scores: products of the original weights and the thresholded vector components
$\mathbf{W} = \dfrac{\mathbf{W^0} \bullet \mathbf{F}}{\sum_{j=1}^{n} W_n^0 F_n \text{ for j} \in \text{set of nonzero } \mathbf{F} \text{ components}}$	The vector of renormalized weights has components that are 0 or, for components with nonzero *F* values, are the original weights divided by the sum of original weights.
Original algorithm (weaker form) $N = \mathbf{W} \bullet \mathbf{F} = \sum_{i=1}^{n} W_i F_i$ Revised algorithm (see also Appendix D) $N = \left\{ \begin{matrix} 0 \text{ if } F_i < \mathrm{T}_i \text{ for any component } i \\ \mathbf{W^0} \bullet \mathbf{F^0} = \sum_{i=1}^{n} W_i^0 F_i^0 \text{ otherwise} \end{matrix} \right\}$	In the original algorithm, the node's value (i.e., the effects of the contributing factors) is the scalar product of the thresholded vector and renormalized weights. With the stronger version, the node's value is 0, if any of the factors is below its threshold, or the original scalar product.

* Another way to reflect the approximate nature of the relationship would be to add a random-error term to equations. Instead, we assess the "confidence" in results estimated with the basic equations. That is discussed in a later subsection.

This type of combining rule is often appropriate. Any system with two critical factors will, by definition, fail unless both are adequately effective. A military fighter aircraft is useless if it has everything else but not weapons. An effective organization must have adequate resources, not just leadership.

Generalization

A, B, and *E* may be negative, in which case a more general expression is

$$\begin{array}{l} \text{For } AA_0 \geq 0 \text{ and } BB_0 \geq 0 \\ \text{If } |A| < |A_0| \text{ or } |B| < |B_0| \\ \text{Then } 0 \\ \text{Else } E = E(A,B). \end{array} \qquad \text{Eq. D.3}$$

Also, if instead of two scalar variables A and B, we had a vector **F**, the condition would be*

$$\begin{array}{l} \text{For } \mathbf{FF}_0 \geq 0 \text{ (all components, to assure consistency of signs)} \\ \text{If } |\mathbf{F}| \geq |\mathbf{F}^0| \text{ for all components of } \mathbf{F} \\ \text{Then } E = E(\mathbf{F}) \\ \text{Else } E = 0. \end{array} \qquad \text{Eq. D.4}$$

Suppose next that both A and B (or, more generally, all elements of **F**) are at or above threshold values. What next? How do they combine?

Does the Effect Depend More on Largest Factors or on Averages?

In looking at factor trees for a number of applications, we concluded that it is necessary to distinguish among cases where A and B can counter, reinforce, or dilute each other’s effects. Factors of different sign naturally counter each other, but reinforcement versus dilution is less clear. For example, if A and B are 5 and 4, should E be more like 9, or at least something greater than 5 (reinforcement), or should it instead be 4.5, the average? If A and B are 8 and 2, should E be more like 8 or more like 5?

Both relationships exist in reality. For example, we may be effectively motivated by the strongest of the possible contributing motivations, with perhaps some reinforcement by another. In contrast, organizational effectiveness is likely to be more of a weighted average over such contributors as leadership, management, ideological package, and resources. Our assertions could be empirically tested, but the reader will presumably agree with the examples. Such rough-cut distinctions leave much unspecified, which we discuss below more rigorously.

A Minimum Set of Simple Combining Rules

Based on the questions of the last section, at least three combining rules (methods) are needed. All can be nonlinear, but they break into piece-wise linear forms. We call the three meth-

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* The first line is implemented in Analytica as, e.g., If Min (**F***$\mathbf{F}^0$, index) < 0 Then “Sign Error” Else, which means that if any of the elements of the product vector are negative, it implies a sign error. Implementation would be slightly different in other languages.

before thresholding. Let **T** be the corresponding vector of threshold values and $\mathbf{W^0}$ be the vector of initial weights. If the thresholds were all 0, then the result of combining the factors would be *E*, given by standard linear weighted sums:

$$\begin{aligned}&\text{For } \mathbf{W}^0 \geq 0 \text{ and } \mathbf{FF}^0 \geq 0 \text{ (consistency of signs)},\\ &E = \mathbf{W}^0 \bullet \mathbf{F}^0 = \sum_{i=1}^{n} W_i^0 F_i^0\\ &\text{(constrained to } [-10,10]).\end{aligned} \qquad \text{Eq. D.10}$$

The concept of thresholded linear weighted sums is that we compute the result as a scalar (dot) product but replace sub-threshold values of $\mathbf{F^0}$. If we do so, however, two issues arise: With what do we replace them and do we still sum over all variables?

If the Factors Matter Only When Above Threshold. If a given variable (factor) matters only if it is at or above threshold (in absolute-value terms), then we discard the factor and perform the average with the remaining variables. To do so, however, requires renormalization of weights. To write the mathematics it is convenient to define a "delta function" **D**(**X**). **X** can be any expression that can be evaluated as either true or false, i.e., as 1 or 0. Thus, it can be a vector or a vector of true-false tests.

$$\begin{aligned}&\mathbf{D}(\mathbf{X}):\\ &\text{If } \mathbf{X} \geq 0 \text{ Then 1 Else 0.}\end{aligned} \qquad \text{Eq. D.11}$$

Using the notation that ***XY*** is a simple vector multiplication such that the *i*-th element of the resulting vector is $\mathbf{X}_i\mathbf{Y}_i$, we then have for the TLWS method:

$$\begin{aligned}&\text{TLWS Method:}\\ &\text{For } \mathbf{F}^0\mathbf{T} \geq 0 \text{ (all elements; consistency of signs)}\\ &\mathbf{F} \equiv \mathbf{F}^0\mathbf{D}(|\mathbf{F}| - |\mathbf{T}| \geq 0)\\ &\mathbf{W} \equiv \mathbf{W}^0\mathbf{D}(|\mathbf{F}^0| - |\mathbf{T}|)\\ &\mathbf{W}^r \equiv \frac{\mathbf{W}}{\mathbf{W} \bullet \mathbf{I}^T} = \frac{\mathbf{W}}{\sum_{i=1}^{n} W_i}\\ &E = \mathbf{W}^r \bullet \mathbf{F} = \sum_{i=1}^{n} \frac{W_j F_j}{\sum_{j=1}^{n} W_j}\\ &\text{(constrained to } [-10,10]).\end{aligned} \qquad \text{Eq. D.12}$$

The modified vector **F** is the original except that its below-threshold elements are set to 0. The modified weighting vector **W** replaces the original weights of below-threshold factors