WORKING PAPER

Grossman's Health Threshold and Retirement

TITUS GALAMA
ARIE KAPTEYN
RAQUEL FONSECA
PIERRE-CARL MICHAUD

WR-658

December 2008

This paper series made possible by the NIA funded RAND Center for the Study of Aging (P30AG012815) and the NICHD funded RAND Population Research Center (R24HD050906).

RAND LABOR AND POPULATION

GROSSMAN'S HEALTH THRESHOLD AND RETIREMENT

Titus J. Galama* Arie Kapteyn* Raquel Fonseca* Pierre-Carl Michaud*

December 22, 2008

Abstract

We formulate a stylized structural model of health, wealth accumulation and retirement decisions building on the human capital framework of health provided by Grossman. We explicitly assume a functional form of the utility function and carefully account for initial conditions, which allow us to derive analytic solutions for the time paths of consumption, health, health investment, savings and retirement. We argue that the Grossman literature has been unnecessarily restrictive in assuming that health is always at Grossman's "optimal" health level. Exploring the properties of corner solutions we find that advances in population health (health capital) can explain the paradox that while population health and mortality have continued to improve in the developed world, retirement ages have continued to fall with retirees pointing to deteriorating health as an important reason for early retirement. We find that improvements in population health decrease the retirement age, while at the same time individuals retire when their health has deteriorated. In our model, workers with higher human capital (say white collar workers) invest more in health and because they stay healthier retire later than those with lower human capital (say blue collar workers) whose health deteriorates faster. Plausibly, most individuals are endowed with an initial stock of health that is substantially greater than the level required to be economically productive.

Keywords: health, demand for health, health capital, medical care, labor, retirement

JEL Codes : I10, I12, J00, J24, J26

---

*RAND Corporation; This research was supported by the National Institute on Aging, under grants R01AG030824, P30AG012815 and P01AG022481. We are grateful to Arthur van Soest, Erik Meijer, Jeffrey Tanner and Sarah Outcault for useful discussions and comments.

1 Introduction

Models of retirement need to be able to reconcile the counterintuitive observations that a) retirees mention deteriorating health as an important reason for early retirement, b) population health and mortality have continued to improve, but c) the age of retirement has declined for nearly a full century in the developed world (though the decline in retirement age has leveled off and reversed somewhat in the last decade; see, e.g., Blau and Goodstein, 2008). Some of this paradox could be explained by justification bias. Individuals may mention health as a reason to justify the fact that they are retired but in fact retire for other reasons, with health actually playing a minor role in the decision. For example, French (2005) estimates a life cycle model of labor supply, retirement, and savings behavior using the panel study of income dynamics (PSID). He finds that the structure of the Social Security system and of pensions are key determinants of the high observed job exit rates at ages 62 and 65 while Social Security benefit levels, health, and borrowing constraints are less important determinants of job exit at older ages. In line with this result Lazear (1986) finds that pensions are typically actuarially unfair and that sharp decreases in the actuarial value of retirement with continued work are used as a device by employers to induce earlier retirement of workers. Also Bazzoli (1985) finds that economic variables play a more important role than health in retirement decisions. On the other hand, Dwyer and Mitchell (1998) find the opposite: that health problems influence retirement plans more strongly than do economic variables. Specifically, Dwyer and Mitchell find that men in poor overall health retire between one and two years earlier than others. In other words, while there is agreement that health influences retirement there is disagreement about the importance of health in the retirement decision. Regardless of its current importance, the increased uptake of defined-contribution type pension vehicles such as 401(k)'s, which are actuarially fair, may reduce the importance of pension structure as a key determinant of retirement. This may warrant the inclusion of health as a more prominent determinant of future retirement.

While health may influence the decision to retire, it is unclear whether retirement in turn has an impact on health. Retirement may be a taxing event, resulting in the loss of friends and support networks, or retirement may be health preserving as it is work that is taxing, not retirement. Empirical evidence on the health effects of retirement is ambiguous (see for instance the literature review in Dave et al. 2006). Using the Health and Retirement Study (HRS), Dave et al. (2006) find that retirement has a detrimental effect on health. On the other hand, Coe and Zamarro (2007) in a cross country comparison find evidence that retirement may actually be health preserving. Lacking the possibility of a controlled experiment, establishing the direction of causality is wrought with difficulties. The decision to retire may be motivated by a desire to preserve health and/or by bad health hampering one's ability to be a productive member of the workforce.

With aging populations and trends towards earlier retirement despite significant improvements in the health of populations in the developed world, societies are increasingly burdened by the rising costs of a growing elderly economically inactive population that is supported by a relatively shrinking economically active group. Understanding what policy instruments can be used to reduce this burden is therefore essential and requires the inclusion of health in retirement models. Potential levers are: universal healthcare provision, subsidized healthcare for low income workers (weighing societal versus individual benefits from delayed retirement), promotion of healthy lifestyles etc.

The aim of this paper is to investigate the influence of various conditions, in particular that of an individual's health, on the decision to retire. In section 2, we formulate a stylized structural model of consumption, leisure, health, health investment, wealth accumulation and retirement decisions using the human capital framework of health provided by Grossman (1972). Health provides utility and healthy individuals have greater earnings causing individuals to invest in their health. Individuals can accumulate savings and/or borrow without restriction, and they are free to decide when to retire. In section 3 we first solve the optimal control problem conditional on retirement age. Specification of a functional form for the utility function allows us to derive analytical solutions for consumption, health, health investment and wealth, conditional on a given retirement age. In section 4 we discuss an extension of the model, and in section 5 we then maximize the implied indirect utility function with respect to the retirement age. In the model individuals find retirement increasingly attractive as they age as a result of three effects: (1) wage declines as a result of gradual health deterioration reducing income from work with age, (2) increased availability of leisure during retirement and (3) accumulation of pension wealth with years in the workforce. We provide simulations in section 6 and conclude in section 7. Detailed derivations are provided in section 8 (the appendix).

In a separate paper (Fonseca et al. 2008) we build and calibrate a health production model with endogenous retirement using a similar Grossman-type health-capital framework to investigate the effect of health insurance, pensions and health care systems on patterns of health and wealth accumulation over the life-cycle. An individual's health is not only subject to natural deterioration (aging) but also to sudden health shocks (e.g., stroke, cancer, etc) which have an increasing likelihood of occurrence with age. In Fonseca et al. (2008) we simulate health, consumption, health investment, retirement and wealth patterns and match the profiles to data from the Medical Expenditure Panel Survey (MEPS), PSID and the HRS. The two studies (the work presented here and the work presented in Fonseca et al. 2008) are complementary. The simplicity of the model presented here allows us to derive analytical solutions from which we can gain insight into the properties of the underlying model. The approach taken in Fonseca et al. (2008) allows us to introduce greater complexity and explore the role of uncertainty.

2 General framework: a health production model

A natural framework for our analysis is provided by Grossman (1972). For an excellent review of the basic concepts of this model see Muurinen and Le Grand (1985). Our formulation is most closely related to Case and Deaton (2003), Wagstaff (1986), Wolfe (1985) and Ehrlich and Chuma (1990).

Let us assume that a consumer is endowed with an intra-temporal utility function $U[L(t), C(t), H(t)]$ at age t, where leisure $L(t)$, consumption $C(t)$, and health $H(t)$ are all positive quantities. The utility function has diminishing marginal returns and is an increasing function in its arguments $L(t)$, $C(t)$ and $H(t)$. Let leisure during one's working life be equal to L_0 and during retirement equal to τL_0, with $\tau > 1$. Assuming separability of the utility function we can then write utility before retirement as $U_w[C(t), H(t)]$, and after retirement as $U_r[C(t), H(t)]$. Consumers maximize the life time utility function

$$\int_0^R U_w[C(t), H(t)]e^{-\beta t}dt + \int_R^T U_r[C(t), H(t)]e^{-\beta t}dt, \tag{1}$$

where T denotes total life time, R is the age of retirement and β is a subjective discount factor. Time t is measured from the time individuals begin employment. The objective function (1) is maximized subject to the following constraints:

$$\begin{aligned}
\dot{H}(t) &= \mu(t)m(t) - d(t)H(t) && 0 \le t \le T \\
\dot{A}(t) &= \delta A(t) + Y[H(t)] - C(t) - p(t)m(t) && 0 \le t \le T \\
Y[H(t)] &= \begin{cases} w_0(t) + \varphi(t)H(t) & 0 \le t \le R \\ b & R < t \le T \end{cases}
\end{aligned} \tag{2}$$

Furthermore we have initial and end conditions: $H(0)$, $A(0)$ and $A(T)$ are given.

$\dot{H}(t)$ and $\dot{A}(t)$ denote time derivatives of health $H(t)$ and assets $A(t)$. The first equation of (2) shows that an individual can invest in the stock of health $H(t)$ by investing $m(t)$ in medical care and/or other health promoting activities (e.g., exercise, diet, etc) with an efficiency $\mu(t)$ to improve health and counter the "natural" health deterioration rate $d(t)$. While Ehrlich and Chuma (1990) argue that medical technology should realistically exhibit diminishing returns to scale we use Grossman's (1972) original assumption of a medical technology that has constant returns to scale (as in the first equation of 2). As Grossman (2000) points out, diminishing returns to scale would greatly complicate the model, while the benefits (certainly for the purpose of our simplified analytical model) are limited. We further note that we impose diminishing returns of the utility of health, to ensure that infinite medical care is not demanded by consumers.

The second equation is simply the inter-temporal budget constraint, where δ is the interest rate, $Y[H(t)]$ is income, $C(t)$ is consumption and $p(t)$ is the price of health investment at time t. The product $p(t)m(t)$ is out-of-pocket medical expenditures. One way to interpret prices is by defining $m(t)$ as the "true" medical expenditures and $p(t)$ as the co-payment. In such a formulation "prices" vary dramatically depending on insurance status. For uninsured individuals in the U.S. the co-pay may effectively be 100%. The third equation in (2) shows how income $Y[H(t)]$ consists of earnings during working life and pension income during retirement. Earnings are a function of health, with $w_0(t)$ a base wage rate that is age dependent (but independent of health) and the marginal production benefit of health $\partial Y[H(t)]/\partial H(t) = \varphi(t) \geq 0$ determine the extent to which health increases one's wage. Retirement income b is independent of health. Note that the system dynamics change at the age of retirement R (where income, consumption, health investment and prices can be discontinuous and the dynamic equations change).

The essential features of the Grossman model are: 1) that the demand for medical care is a "derived" demand in that consumers demand good health, not medical care per se, 2) that health provides consumption benefits (utility is a function of health) and 3) that health provides production benefits (health increases earnings; see equation 2).

Integrating the second equation of (2) over the life time we obtain the life-time budget constraint

$$\begin{aligned} &\int_0^T C(t)e^{-\delta t}dt + \int_0^T p(t)m(t)e^{-\delta t}dt = \\ &A(0) - A(T)e^{-\delta T} + \int_0^R w_0(t)e^{-\delta t}dt + \tfrac{b}{\delta}(e^{-\delta R} - e^{-\delta T}) + \int_0^R \varphi(t)H(t)e^{-\delta t}dt. \end{aligned} \tag{3}$$

The left-hand side of (3) represents life-time consumption and life-time health investment, and the right-hand side represents life-time financial resources in terms of (from left to right): use of life-time assets, life-time income from wages and from benefits, and lastly, additional life-time earnings, resulting from good health and health investment.

Using the first equation of (2) we can write $H(t)$ as a function of health investment and initial health.

$$H(t) = H(0)e^{-\int_0^t d(s)ds} + \int_0^t \mu(x)m(x)e^{-\int_x^t d(s)ds}dx. \tag{4}$$

Individuals cannot "choose" health optimally. Instead they can invest in health $m(t)$ optimally. For one, we demand that health investment $m(t) \geq 0$, i.e., that individuals cannot "sell" their health through negative health investment $m(t)$. Health $H(t)$ at time t is path dependent; it is a function of the entire history $0 \leq t' < t$ of health investment $m(t')$ and of initial health $H(0)$. In the optimization problem we thus have to optimize with respect to the entire prior history of health investment $m(t')$.

Thus, we have the following optimal control problem: the objective function (1) is maximized with respect to the control functions $C(t)$ and $m(t)$ and subject to the constraints (2). The Lagrangean or generalized Hamiltonian (see, e.g., Seierstad and Sydsaeter 1987) of this problem is:

$$\Im = U[C(t), H(t)]e^{-\beta t}dt + p_A(t)\{\delta A(t) + Y[H(t)] - C(t) - p(t)m(t)\} + q(t)m(t), \tag{5}$$

where $U[C(t), H(t)] = U_w[C(t), H(t)]$ for $t \leq R$; $U[C(t), H(t)] = U_r[C(t), H(t)]$ for $t > R$; $p_A(t)$ is the adjoint variable associated with the differential equation (2) for assets $A(t)$ and $q(t)$ a multiplier associated with the condition that health investment $m(t) \geq 0$. The inclusion of the multiplier $q(t)$ is an essential difference between our formulation and prior formulations of the Grossman model. It allows us to explicitly impose the constraint that medical care is positive $m(t) \geq 0$ at all times. We discuss the implications of this choice and the arguments for making it in detail in section 3.

We have

$$\dot{p}_A(t) = -\tfrac{\partial \Im}{\partial A(t)} = -p_A(t)\delta, \tag{6}$$

the solution of which is

$$p_A(t) = p_A(0)e^{-\delta t}; \tag{7}$$

further $q(t) \geq 0$ and $q(t) = 0$ for $m(t) > 0$.

We proceed as follows. First we solve the optimal control problem conditional on retirement age R (i.e., for a fixed exogenous retirement age R) and specify a functional form for the utility function. For given exogenous time varying deterioration rate $d(t)$, prices $p(t)$, effciency $\mu(t)$, base wage rate $w_0(t)$, benefits b and production benefit $\varphi(t)$, we can then solve for the control variables $C(t)$ and $m(t)$ which in turn provides us with solutions for the state variables $H(t)$ and $A(t)$. We then maximize the resulting indirect utility function with respect to retirement age R. Health, savings and retirement thus are jointly determined in our model.

3 Exogenous retirement

The first order conditions for maximization of (1) subject to (2) are (for details see the Appendix):

$$\begin{aligned} \frac{\partial U_w(t)}{\partial C(t)} &= p_A(0)e^{(\beta-\delta)t} \quad (t \leq R) \\ \frac{\partial U_r(t)}{\partial C(t)} &= p_A(0)e^{(\beta-\delta)t} \quad (t > R), \end{aligned} \tag{8}$$

and

$$\begin{aligned} \frac{\partial U_w(t)}{\partial H(t)} &= p_A(0)\left[\pi_H(t) - \varphi(t)\right]e^{(\beta-\delta)t} + \frac{e^{\beta t}}{\mu(t)}\dot{q}(t) - \frac{e^{\beta t}}{\mu(t)}\left[\frac{\dot{\mu}(t)}{\mu(t)} + d(t)\right]q(t) \qquad (t \leq R) \\ \frac{\partial U_r(t)}{\partial H(t)} &= p_A(0)\pi_H(t)e^{(\beta-\delta)t} + \frac{e^{\beta t}}{\mu(t)}\dot{q}(t) - \frac{e^{\beta t}}{\mu(t)}\left[\frac{\dot{\mu}(t)}{\mu(t)} + d(t)\right]q(t) \qquad (t > R), \end{aligned} \tag{9}$$

where

$$\pi_H(t) \equiv \left[p(t)/\mu(t)\right]\left[d(t) + \delta - \dot{p}(t)/p(t) + \dot{\mu}(t)/\mu(t)\right] \tag{10}$$

is the the user cost of health capital at the margin (the interest rate δ represents an opportunity cost).

Equations (8) and (9) are similar to those by Case and Deaton (2005; their equations 5 and 6) for $q(t) = 0$, i.e., $m(t) > 0$. Equation (8) requires the marginal benefit of consumption to equal $p_A(0)$ (the shadow price of life-time wealth) times a time varying exponent that either grows or decays with time, depending on the sign of $\beta - \delta$ (the difference between the time preference rate β and the interest rate δ). The marginal benefit of health investment (equation 9) equals the product of the marginal benefit of consumption (equation 8) and the user cost of health capital at the margin $\pi_H(t)$ (equation 10) minus the marginal production benefits of health $\varphi(t)$ if the individual is working.[1]

We can make a number of observations with respect to the first order conditions for consumption and health investment (equations 8 and 9). For now we discuss the case where $q(t) = 0$, i.e., $m(t) > 0$. As we will discuss in more detail later, this represents a special case in which the evolution of an individual's health follows Grossman's solution for the "optimal" health stock. First, increasing lifetime resources will lower $p_A(0)$ and hence increase health investment and consequently health. Second, since health does not influence income after retirement (last equation of 2), retired individuals will reallocate away from health expenditures in the direction of more consumption. Third, a lower price of health investment increases health. This is pertinent in a cross-country comparison, but also when comparing across the life-cycle, for instance if health care is subsidized for certain age groups (like Medicare in the U.S.). Finally, more efficient health investment will lead to more health. Efficiency can explain variations within a country (if for instance individuals with a higher education level are more efficient in their health investment, Goldman and Smith, 2002) or across countries (if health care is more efficient in one country than in another).

In order to derive analytical solutions for consumption, health, health investment and wealth, we specify the following constant relative risk aversion (CRRA) form for the utility function (1):

$$U_w(C, H) = \frac{\left[C^{\zeta} H^{1-\zeta}\right]^{1-\rho}}{1-\rho}; \; U_r(C, H) = kU_w(C, H), \tag{11}$$

where ζ ($0 \leq \zeta \leq 1$) is the relative "share" of consumption $C(t)$ versus health $H(t)$ and ρ ($\rho > 0$) the coefficient of relative risk aversion.

The factor k is the ratio of utility when retired and when working. The factor k thus accounts for the utility derived from the greater availability of leisure during retirement. A simple way to motivate the introduction of the multiplicative factor k is to include leisure in the utility function as follows: $U(C, H, L) = \left[C^{\zeta} H^{1-\zeta} L^{\tau}\right]^{1-\rho}$, where L is leisure and where we have omitted the

[1] We impose that the user cost of health capital at the margin exceeds the marginal production benefit of health $\pi_H(t) \equiv [p(t)/\mu(t)]\,[d(t) + \delta - \dot{p}(t)/(p(t) + \dot{\mu}(t)/\mu(t)] > \varphi(t)$. Without this condition, the investment in health would finance itself by increasing wages by more than the user cost of health. As a result of this, consumers would choose infinite health investment paid for by infinite wage increases to reach infinite health.

multiplicative constant $1/(1-\rho)$. Assume that during the working years leisure is equal to L_0 while during retirement leisure is equal to $k_r L_0$ with $k_r > 1$. This implies that the ratio of utility before and after retirement is equal to $k \equiv k_r^{\tau(1-\rho)}$. This specification is consistent with the Stock and Wise (1990) specification in which the utility of consumption in retirement is a multiple of the utility of consumption when working. If $\rho < 1$ (i.e., utility is less concave than logarithmic) the ratio is greater than one. That is, at the same consumption level, utility is higher when retired. For $\rho > 1$ we have $k < 1$. In the latter case it is still the case that for a given consumption level, utility is higher in retirement, since utility is negative for $\rho > 1$.

This formulation can reproduce the drop in consumption observed at retirement (Banks, Blundell and Tanner 1998; Bernheim, Skinner and Weinberg 2001). For $k < 1$ and $\rho > 1$ (or for $k > 1$ and $\rho < 1$), and for a given consumption level the marginal utility of consumption is lower in retirement than while working and hence it is optimal to spend more money on consumption before retirement than after retirement.

Hurd and Rohwedder (2003, 2006) review some of the explanations put forward to explain the drop in consumption. The first of these is the occurrence of unanticipated shocks at the time of retirement, where, e.g., retirees are surprised to find that their economic resources are fewer than anticipated and adjust consumption accordingly. This would suggest that agents are insufficiently forward looking and would complicate employment of life-cycle models (used in this paper). However, Hurd and Rohwedder present evidence that the reductions are fully anticipated. In addition there are alternative explanations that are consistent with a life-cycle approach. For example, a second explanation involves uncertainty in the timing of retirement, where, e.g., workers retire because of a health event or unemployment resulting in a reduction in resources. Hurd and Rohwedder (2006) find that an unanticipated decline in lifetime resources caused by early retirement could explain a spending decline for part of the population. But the authors conclude that the empirical importance of health shocks is not great enough to explain fully the recollected declines in consumption. In line with this result, Blau (2008) suggests that a simple life cycle model in which individuals choose when to retire but are subject to shocks can account qualitatively for these stylized facts. However, Blau finds that the magnitude of the drop in consumption among households that experience a decline is too small in a calibrated model compared to the data. Blau concludes that other proposed explanations for the decline in consumption at retirement should continue to be explored in future research based on the life cycle framework. A third explanation is the increase in leisure at retirement, which would be consistent with $k < 1$. The increase in leisure can decrease consumption, e.g., because house keeping, home repairs etc. are performed by the consumer after retirement and no longer purchased. Hurd and Rohwedder (2006) report that a transition into retirement is associated with approximately a 5.5 hrs increase per week in time spent on home production. Hurd and Rohwedder (2006) conclude that this supports the view that the increased ability to engage in home production or more thrifty shopping during retirement is an important reason for the observed spending declines.

In our stylized formulation we employ a life-cycle model (e.g., we assume agents are to a large extent rational, rejecting the first explanation), specify a utility function that allows for a drop in consumption due to increased leisure at retirement (incorporating the third explanation), but do not incorporate uncertainty (i.e., we do not model the effect of the second explanation).

3.1 Model solutions: Grossman's "optimal" health stock

We begin analyzing the case where $q(t) = 0$, i.e. $m(t) > 0$. We will denote the solutions for consumption, health investment and health with $C_*(t)$, $m_*(t)$, and $H_*(t)$ for this special case (Grossman's "optimal" solutions) to distinguish from the more general solutions $C(t)$, $m(t)$, and $H(t)$. Solving the first order conditions (8) and (9) and using the Cobb-Douglas utility specification (11), we find the following solutions for the control functions $C_*(t)$ and $m_*(t)$ (for details see the Appendix):

$$C_*(t) = \zeta\Lambda\left[\pi_H(t) - \varphi(t)\right]^{1-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} \quad (t \leq R) \tag{12}$$

$$C_*(t) = k^{\frac{1}{\rho}}\zeta\Lambda\left[\pi_H(t)\right]^{1-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} \quad (t > R) \tag{13}$$

$$m_*(t) = \frac{1}{\mu(t)} e^{-\int_0^t d(s)ds} \frac{d}{dt}\left\{(1-\zeta)\Lambda\left[\pi_H(t) - \varphi(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} e^{\int_0^t d(s)ds}\right\} \quad (t \leq R) \tag{14}$$

$$m_*(t) = \frac{1}{\mu(t)} e^{-\int_0^t d(s)ds} \frac{d}{dt}\left\{k^{\frac{1}{\rho}}(1-\zeta)\Lambda\left[\pi_H(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} e^{\int_0^t d(s)ds}\right\} \quad (t > R), \tag{15}$$

where we have used the following definitions:

$$\chi \equiv \frac{1 + \rho\zeta - \zeta}{\rho}, \tag{16}$$

and

$$\Lambda \equiv \zeta^{\frac{1-\rho}{\rho}} \left(\frac{\zeta}{1-\zeta}\right)^{1-\chi} p_A(0)^{\frac{-1}{\rho}}. \tag{17}$$

The constant Λ [and hence $p_A(0)$] can be determined by substituting the solutions for health $H(t)$, consumption $C(t)$ and health investment $m(t)$ into the life-time budget constraint (3). The result can be written as a fraction $\Lambda \equiv \Lambda_n/\Lambda_d$, where the numerator Λ_n is similar to the expression for life-time resources (right hand side of 3). Hence increasing initial assets $A(0)$, base wages $w_0(t)$, retirement benefits b, production benefits of health $\varphi(t)$ or initial health $H(0)$ increases the constant Λ and thereby consumption $C(t)$, health investment $m(t)$, and health $H(t)$. The denominator Λ_d is a complicated function of the time paths of $d(t)$, $p(t)$, $\mu(t)$, $\varphi(t)$ and various model parameters:

$$\Lambda_d = \Lambda_d[d(t), p(t), \mu(t), \varphi(t), \delta, \beta, R, T, k, \rho, \zeta]. \tag{18}$$

The full solutions for Λ are provided in the Appendix for each of six scenarios (equations 73, 74, 75, 77, 78, 79, 91, 92, 93, 100, 101, and 102; for more details on the scenarios see section 3.2).

Consumption and health investment (equations 12 through 15) are functions of various combinations of the user cost of health capital at the margin $\pi_H(t)$ (see equation 10), minus the marginal production benefit of health $\varphi(t)$, to the power $1-\chi$ (consumption) or $-\chi$ (health investment).[2]

For constant time paths of $d(t) = d_0$, $p(t) = p_0$, $\mu(t) = \mu_0$, $\varphi(t) = \varphi_0$, consumption and health investment decrease (increase) exponentially with time if the time preference rate β is larger (smaller) than the interest rate δ. For $\beta = \delta$ we have constant time paths for consumption and for health investment except for jumps at retirement $t = R$ (due to the factor k associated with the increased availability of leisure during retirement).

Consumption increases with the user cost of health capital at the margin $\pi_H(t)$ and decreases with the marginal production benefit of health $\varphi(t)$ for $0 < \chi < 1$ (i.e. if $\rho > 1$ and $\zeta < 1$). The opposite pattern is found for $\chi > 1$ (i.e. if $0 < \rho < 1$ and $\zeta < 1$). For $\chi = 1$ (i.e. $\rho = 1$ or $\zeta = 1$) consumption is constant (for $\beta = \delta$), independent of the user cost of health capital at the margin and independent of the marginal production benefit of health.[3] Health investment shows a more complex dependence on the user cost of health capital at the margin and the marginal production benefit of health than consumption does (see equations 14 and 15).

For Grossman's "optimal" health stock $H_*(t)$ we find the following solutions:

$$H_*(t) = (1-\zeta)\Lambda\left[\pi_H(t) - \varphi(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} \quad (t \leq R) \tag{19}$$

$$H_*(t) = k^{\frac{1}{\rho}}(1-\zeta)\Lambda\left[\pi_H(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} \quad (t > R). \tag{20}$$

The trajectory described by equation (19) is the path that individuals would follow if initial health $H(0)$ would be exactly on this trajectory and is what is referred to in the literature as the "optimal" health stock (e.g., Grossman 1972, 2000). Similarly, equation (20) describes the trajectory that health would follow if health at retirement were exactly equal to $H_*(R_+)$.

As many authors have found (e.g., Case and Deaton 2005), Grossman's solution for the "optimal" health stock $H_*(t)$ is constant for constant time paths of $d(t) = d_0$, $p(t) = p_0$, $\mu(t) = \mu_0$, $\varphi(t) = \varphi_0$ (i.e., for a constant user cost of health capital) and for $\beta = \delta$, and decreases for an increasing deterioration rate with age $\dot{d}(t) > 0$.

At the age of retirement the Grossman solutions ($q(t) = 0$) for the "optimal" level of consumption (equations 12, 13), "optimal" level of health investment (equations 14, 15) and

[2]Notice that $\min\{1, 1/\rho\} \leq \chi \leq \max\{1, 1/\rho\}$, given that $\rho > 0,\ 0 \leq \varsigma \leq 1$.

[3]This can be understood as follows. The cost of holding the health stock increases with the user cost of health capital at the margin $\pi_H(t)$ and decreases with the marginal production benefit of health $\varphi(t)$ (see equation 9; $q(t) = 0$). Higher cost of holding the health stock would thus result in lower health levels $H_*(t)$. The marginal cost of consumption on the other hand does not change with changes in the user cost of health capital at the margin or with the marginal production benefit of health (see equation 8). In other words, the marginal benefit of consumption is also unchanged. The marginal benefit of consumption $\partial U(t)/\partial C(t) \propto H(t)^{\rho(\chi-1)} C(t)^{-\rho\chi}$ (where we have used 8 and 11 and $\beta = \delta$) increases with health for $\chi > 1$ and decreases with health for $\chi < 1$. In other words, higher costs of holding the health stock result in lower health levels and therefore lower (higher) consumption levels for $\chi < 1$ ($\chi > 1$). For $\chi = 1$ the marginal benefit of consumption is independent of health and hence there is no effect of health changes on the level of consumption.

"optimal" level of health (equations 19, 20) are discontinuous. These jumps represent the change in consumption and health investment as a result of utility derived from the increased availability of leisure time during retirement (depending on the value of k, leisure is a substitute or a complement of consumption and health) and because health has no effect on income after retirement.

3.2 Model solutions: general case

The Grossman literature generally assumes that individuals are capable of ensuring that their health is at the "optimal" level $H_*(t)$ (e.g., Grossman 1972a, 1972b, 2000; Case and Deaton 2005; Muurinen 1982; Wagstaff 1986; Zweifel and Breyer 1997, Ehrlich and Chuma 1990; Ried 1998). In other words, the literature assumes that either the initial health endowment $H(0)$ is at or very close to Grossman's "optimal" health stock $H_*(0)$ or that individuals find this health level desirable and are capable of rapidly dissipating or repairing any "excess" or "deficit" in health.

Unlike most discussion in the literature we argue instead that initial conditions are likely of importance and that health will in many circumstances not follow the Grossman solution for the "optimal" health stock. An essential characteristic of the Grossman model is that health cannot deteriorate faster than the natural deterioration rate $d(t)$. As equation (4) shows, any surplus in health above the equilibrium health path can at most dissipate at the natural rate of health deterioration $d(t)$ (this would correspond to individuals not investing in their health; $m(t) = 0$). As a result initial conditions cannot be dissipated rapidly (and what use would it be to shed any excess in health which provides utility and increases earnings?). Nor is there any reason to expect the endowment of health to exactly equal Grossman's "optimal" health stock (see also Wolfe 1985).

We allow health to have an initial value $H(0)$ that is different from Grossman's solution for the "optimal" health stock (see also Wolfe 1985). To take into account that any "excess" in health cannot dissipate faster than the natural deterioration rate $d(t)$ we explicitly demand that medical care is a positive quantity $m(t) \geq 0$ by introducing the multiplier $q(t)$ in the Lagrangean (equation 5). We thus allow for the existence of corner solutions where individuals do not invest in medical care $m(t) = 0$ for certain periods of time. As a result, given initial health $H(0)$, the Grossman "optimal" health stock is not the optimal solution. Any situation with "excessive" initial health (initial health $H(0)$ above $H_*(0)$) is preferable: individuals with excess initial health have higher levels of life-time health and consumption and therefore greater life-time utility. In other words, if individuals could choose they would always prefer "excessive" initial health $H(0)$ over Grossman's "optimal" health stock $H_*(0)$ (if $H(0) > H_*(0)$). In our formulation individuals use their "excess" health for the consumption and production benefit this "excess" in health provides.

The Grossman solution for the health stock $H_*(t)$ is instead the *minimum* level individuals "demand" for the productivity benefit and utility that good health provides. Individuals with health endowments $H(0)$ below Grossman's "optimal" health stock $H_*(0)$ will invest in medical care (an adjustment cost) to reach Grossman's "optimal" health level (see for details section 8.7 in the Appendix). For this reason we term the Grossman "optimal" health stock $H_*(t)$ the "minimally economically productive" or "minimally productive" health stock. Individuals only invest in health when they are "unhealthy" (health levels below or at the minimally productive level) and not when they are "healthy" (health levels above the minimally productive level). In other words, the minimally productive health level operates as a health threshold. In the following we will refer to what is traditionally called Grossman's "optimal" solution for health, as the "health threshold" or as the "minimally productive" level of health.

While the Grossman literature does not provide a convincing theoretical argument that health should be at or close to Grossman's "optimal" health stock $H_*(t)$, the ultimate test of our proposition that this assumption is invalid is to contrast its predictions with data. In separate work (Galama and Kapteyn, 2009) we propose structural and reduced form equations to test our proposition. We also contrast the predictions of our interpretation of the Grossman model (in which solutions where individuals do not invest in health $m(t) = 0$ for certain periods of time are allowed) with the "traditional" interpretation (in which health always follows Grossman's "optimal" health stock $H_*(t)$) and with the empirical literature. In a review of the empirical literature we find that the interpretation advocated here provides a better explanation for the observed evolution of health and of medical consumption. Importantly, our interpretation of the Grossman model can explain the observation that measures of medical care are negatively corelated with measures of health[4] while the traditional interpretation cannot (the Grossman model has received significant criticism regarding its inability to correctly predict this crucial relationship; see, e.g., Grossman 2000; Zweifel and Breyer, 1997). For more details see Galama and Kapteyn (2009).

We distinguish six scenarios as shown in figure 1. The health threshold $H_*(t)$ (dotted line) drops at the age of retirement R as a result of utility derived from the increased availability of leisure time during retirement (for our choice of parameters $k < 1$ leisure is a substitute of consumption and health) and because health has no effect on income after retirement. We show the simplest case in which the health threshold $H_*(t)$ is constant with time (e.g., for constant time paths of $d(t) = d_0$, $p(t) = p_0$, $\mu(t) = \mu_0$, $\varphi(t) = \varphi_0$ and for $\beta = \delta$) but the scenarios are valid for more general cases. Scenarios A, B, C and D begin with initial health $H(0)$ greater than the initial health threshold $H_*(0)$ and scenarios E and F begin with initial health $H(0)$ below the initial health threshold $H_*(0)$. In scenarios A and B health $H(t)$ reaches the health threshold $H_*(t)$ before the age of retirement R (at age t_1). In scenario A the health threshold $H_*(t)$ is once more reached at age t_2 before total life time T, but this is not the case in scenario B. In scenario C health $H(t)$ reaches the threshold $H_*(t)$ after the age of retirement R (at age t_2), and in scenario D health $H(t)$ never reaches the threshold $H_*(t)$ during the life of the individual. In scenarios E and F individuals begin working life with health levels $H(0)$ below the initial health threshold $H_*(0)$. Individuals will substitute initial assets $A(0)$ for improved initial health $H(0)$ such that initial health equals the initial health threshold $H(0) = H_*(0)$ (see section 8.7 in the Appendix for a more detailed discussion).

The detailed solutions for health $H(t)$, consumption $C(t)$ and health investment $m(t)$ for each

[4]Healthy individuals (above the threshold) do not invest in health while unhealthy individuals (at or below the threshold) do.

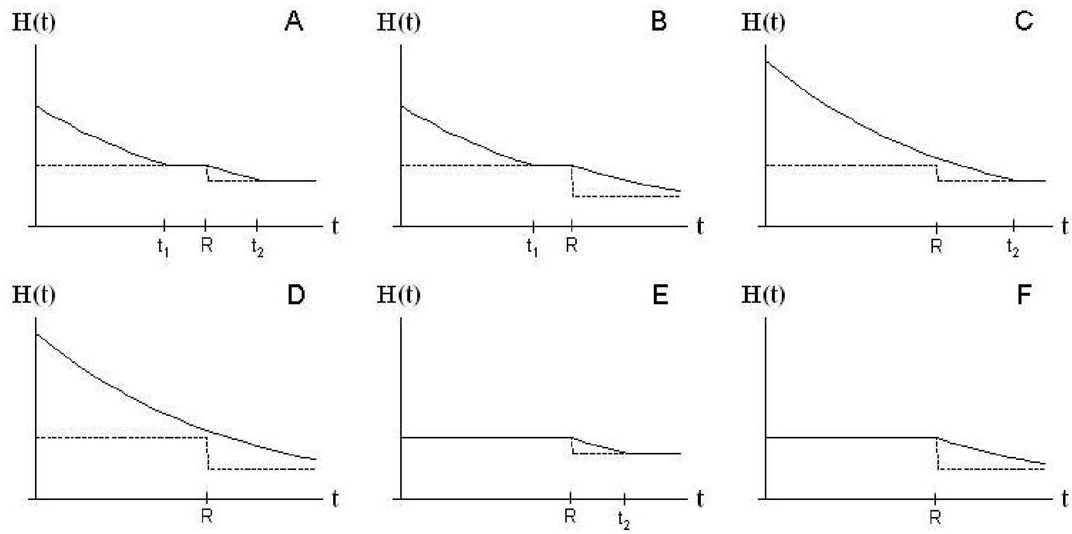

Figure 1: Six scenarios for the evolution of health. t_1 and t_2 denote the ages at which health (solid line) has evolved towards the health threshold (dotted line), and R denotes the age of retirement. The health threshold drops at the age of retirement R as a result of utility derived from the increased availability of leisure time during retirement (depending on the value of k, leisure is a substitute or a complement of consumption and health) and because health has no effect on income after retirement.

of the six scenarios are provided in the Appendix. Assets $A(t)$ can be derived by substituting the solutions for health $H(t)$, consumption $C(t)$ and health investment $m(t)$ as follows:

$$A(t) = \left\{ A(0) + \int_0^t \left[w_0(x) + \varphi(x)H(x) - C(x) - p(x)m(x) \right] e^{-\delta x} dx \right\} e^{\delta t}, \quad (t \leq R) \qquad (21)$$

$$A(t) = \left\{ A(R)e^{-\delta R} + \int_R^t \left[b - C(x) - p(x)m(x) \right] e^{-\delta x} dx \right\} e^{\delta t}, \quad (t > R) \qquad (22)$$

As a last note, each of the solutions are fully determined, that is by substituting the solutions for health $H(t)$, consumption $C(t)$, health investment $m(t)$ and assets $A(t)$ in the life-time budget constraint (equation 3) we can derive the constant Λ (or equivalently the constant $p_A(0)$). For more details see the Appendix.

4 Treatment of benefits

We introduce one further level of complexity to the model. In the set-up so far, benefits are independent of work history. Typically benefits are related to how long one has worked and the wages earned during working life. As a stylized representation of this we assume that a fraction of wages $\alpha w(t)$ are saved for retirement. Benefits accumulate with time and are invested with a return on investment of δ (the interest rate) as follows:

$$b(R) = b_0 + f(R)\alpha \int_0^R w(t)e^{\delta t}dt, \tag{23}$$

where the pension accumulation function $f(R)$ describes how benefits accumulate as a function of retirement age R and b_0 represents a base pension benefit.

The base pension benefit b_0 is provided regardless of years worked, e.g., it could represent a first-tier basic pension (OECD 2005) or a statutory poverty line. The remaining term in (23) represents the part of the pension that accumulates with years of work. This could represent a defined contribution (DC) plan or a defined benefit plan (DB) or it could represent an individual's portfolio of DB and DC plans. Pension wealth in retirement thus consists of a base pension b_0 (typically provided by the state), an individual private pension (either DB and/or DC) and accumulated assets $A(R)$ that can be drawn down during retirement.

A particular pension accumulation functional form of interest is $f(R) = \delta/[1-e^{-\delta(T-R)}]$, which is actuarially fair (accumulated pension wealth is paid out over the number of years in retirement $T-R$). Such a functional form is an approximation of a DC plan where the beneficiary can use his or her accumulated pension investment to purchase a life-time annuity.[5] The function $f(R)$ for a DB plan, on the other hand, would typically consist of an annual contribution rate per year worked and a conversion factor which would depend on R in a way which is not necessarily actuarially fair. As Lazear (1986) finds, the actuarial value of private pensions first rises but then declines as workers continue to work beyond a certain age. Lazear argues that sharp decreases in the actuarial value of retirement with continued work are used as a device by employers to induce earlier retirement of workers. Such a function could be represented by $f(R) = \delta/[1 - e^{-\delta(T-R)}]$ up to a retirement age R_* after which the function flattens to a constant or even slightly declining function of retirement age.

Replacing the assumed flat retirement benefits by (23) the previously derived equations remain valid with the following transformation

$$\begin{aligned} w_0(t) &\to (1-\alpha)w_0(t) \\ b &\to b_0 + \alpha f(R)\int_0^R w_0(t)e^{\delta t}dt \\ \varphi(t) &\to \varphi(t)\left[(1-\alpha) + f(R)\tfrac{\alpha}{\delta}\left(e^{-\delta R} - e^{-\delta T}\right)e^{2\delta t}\right] \end{aligned} \tag{24}$$

[5] In this example, the annuity is assumed to be actuarially fair. In a world with asymmetric information this assumption clearly needs to be modified.

Thus, even for constant time paths of $d(t) = d_0$, $p(t) = p_0$, $\mu(t) = \mu_0$, $\varphi(t) = \varphi_0$ and for $\beta = \delta$, consumption and health investment are not constant as the transformation for the marginal production benefit of health $\varphi(t)$ is a function of time. A derivation of transformation (24) is provided in the Appendix.

5 Endogenous retirement

Now let us finally return to the issue of the influence of health on the decision to retire. In our formulation, the decision to retire is determined by three factors. Individuals find retirement increasingly attractive as they age because of: (1) wage declines $w(t) = w_0(t) + \varphi(t)H(t)$ as a result of gradual health deterioration reducing income from work with age, (2) increased availability of leisure during retirement (factor k boost in utility) and (3) an increasing level of pension benefits $b(R)$ with years in the workforce.

Now consider the case where the age of retirement R can be chosen freely. The optimal R can be determined by inserting the solutions for $C(t)$, $H(t)$ into the "indirect utility function", $V(R)$, and differentiating $V(R)$ with respect to R.

$$V(R) \equiv \int_0^R U_w(t)e^{-\beta t}dt + \int_R^T U_r(t)e^{-\beta t}dt \tag{25}$$

Unfortunately the resulting expression for $V(R)$ turns out to be unwieldy for most of the scenarios A through F shown in Figure 1 (see the various solutions for $C(t)$ and $H(t)$ in the Appendix; note that we do not show the solution for $V(R)$ given its complexity). After differentiation of $V(R)$ with respect to R we do not find a simple solution for the optimal age of retirement R and therefore have to resort to numerically solving for the optimal retirement age R.

6 Simulations

In this section we begin by making some plausible assumptions about the model parameters and initial and terminal conditions. This will provide us with a starting point (our baseline model; section 6.1) from which we will subsequently deviate in order to investigate the impact of the various model levers on the decision to retire. For illustrative purposes we graph the solutions for consumption, health investment, health, assets etc and contrast the model with some stylized observations from the literature. We then briefly explore model simulations of health inequality (section 6.2) and the effect of health insurance on health and retirement (section 6.3). We discuss in detail the sensitivity of retirement age to model parameters (section 6.4) and discuss briefly the sensitivity of other model outcomes, such as, life-time consumption, life-time health investment, life-time health, and life-time assets, to changes in model parameters (section 6.5).

6.1 Calibration baseline model: white collar worker

Individuals begin work at age 20 (corresponding to $t = 0$), and, depending on the solution for the optimal retirement age, retire some 45 years later at an age of about 65 years (corresponding to $R \approx 45$). Individuals die with certainty at 85 years of age (corresponding to $T = 65$).

For simplicity we assume constant time paths of $d(t) = d_0$, $p(t) = p_0$, $\mu(t) = \mu_0$, $\varphi(t) = \varphi_0$, $w_0(t) = w_0$[6] and take $\beta = \delta$. We further assume an annual income of $w(t) \approx \$45{,}000$ for healthy "white collar" workers[7] and that healthy workers have a health stock of about 1.5 times that of unhealthy workers (we will discuss "blue collar" workers later). We can then obtain 25% higher earnings for healthy workers[8] for constant marginal production benefits of health $\varphi(t) = \varphi_0 \approx 1.5 w_0/H_H$ (where H_H is health for a healthy worker), and a constant base wage rate $w_0(t) = w_0 \approx \$20{,}000$ per year. A roughly 50% decline in wage between first employment ($t = 0$) and retirement ($t = R$)[6] requires that by the age of retirement health has fallen to one-fourth the level of health at first employment $H(0)$. We can simulate such results with an initial health $H(0)$ of \$30,000,[9] a constant health deterioration rate $d(t) = d_0$ of 5%, a contribution rate for retirement α of 15% of wages, zero basic benefits $b_0 = 0$, a coefficient of relative risk aversion $\rho = 1.32$, a constant health investment efficiency $\mu(t) = \mu_0 = 0.7\%$ and time preference rate and interest rate $\beta = \delta$ of 3%. We interpret prices $p(t)$ as the co-pay rates, which we take to be constant at $p(t) = p_0 = 20\%$, and $m(t)$ as the total annual medical expenditures – though this could include the cost of other health promoting activities such as exercise, diet, etc.

Hurd and Rohwedder (2003, 2006) find that "on average" consumption drops between 15 and 20% after retirement. We use this observation to determine the value for k by requiring that

---

[6] It is straightforward to use a more realistic wage profile, for example the commonly used earnings function by Mincer (1974) where the log of earnings is a quadratic function of age and linear in years of schooling. However, this would introduce additonal complexity into the model. The overal shape, i.e. height at peak, age at peak and curvature of the earnings function with age would influence the optimal age of retirement. In order to not complicate the interpretation of the effect on the retirement age of parameters that are of greater interest (than the parameters of the wage profile) we have chosen a simple constant base-wage rate $w_0(t) = w_0$.

[7] Median annual earnings for males were \$40,798 and for females \$31,223 in 2004, according to the US Census Bureau.

[8] French (2005) provides hourly wage and annual hours worked profiles for males by age and self-reported health status from the panel study of income dynamics (PSID). French finds that the effect of health on wages is relatively small: the hourly wage is about 10% higher and the annual hours worked are some 10% higher for healthy compared with unhealthy individuals. In our formulation we use annual wages, i.e. the product of hourly wages times the annual hours worked. Thus annual wages would be about 20-25% higher for healthy individuals. The hourly wage profiles show a wide hump (relatively flat between the ages of 40 and 60) for both healthy and unhealthy males with wages peaking near age 55 and a fairly rapid decline after age 60. The annual hours worked profiles show a relatively smooth decline with age, dropping by about 20% from age 30 to age 60 after which the decline accelerates and drops to 50% by age 70 (again compared with age 30).

[9] The dimension of health (dollars) can be understood as follows. Denoting the dimension of health by $[H]$ we have according to the first equation of 2 that $[\dot{H}] = [H]/[t] = [m][\mu]$ (where $[t]$ is the dimension of time [e.g., days, seconds etc], $[m]$ is the dimension of medical care [e.g., dollars per unit of time] and $[\mu]$ is the dimension of the efficiency of medical care $\mu(t)$). We then have $[H] = \$\,[\mu]$. For simplicity we assume the efficiency function is dimensionless and hence health is expressed in dollars.

consumption $C(t)$ drops at retirement to 85% of its value before retirement.[10] Hence we demand that (see for details the Appendix):

$$k^{\frac{1}{\rho\chi}} = 0.85. \tag{26}$$

For the values chosen, we have $k = 0.81$.

To ensure that health investment is not too far from the observed mean out-of-pocket medical expenditures (see Fonseca et al. 2008) of around \$3000 per year (corresponding to total medical expenditures of \$15,000) we assume $\varsigma = 0.85$, i.e. that an individual's preferences are significantly skewed towards consumption and away from health. We assume an actuarially fair benefits accumulation function $f(R) = \delta/[1 - e^{-\delta(T-R)}]$, i.e. as approximately in a DC plan. Lastly, we assume that individuals leave no bequests and receive no bequests, i.e. $A(0) = A(T) = 0$. There are likely many other plausible scenarios and parameter values. The current values are only for illustrative purposes.

For this set of parameters and assumptions (see Table 1 for a quick overview) we find ourselves in scenario A and determine an optimal age of retirement of 63.52 (corresponding to $R = 43.52$). Figures 2a-2e describe the evolution of income, consumption, assets, health and health investment for the optimal retirement age of 63.52 years.

As Figure 2a shows, earnings $Y[H(t)]$ during working life fall with declining health until the age of retirement when earnings are replaced by an annuity.[11] Consumption $C(t)$ (Figure 2b) is relatively constant over time as individuals smooth life-time consumption through the use of savings $A(t)$[12] (Figure 2c). Consumption shows a sudden drop at retirement to 85% of its level before retirement (this is the direct result of our choice for the value of leisure k; see Equation 26) as individuals substitute leisure for consumption. For the parameters chosen, individuals build up assets $A(R)$ of $\approx$ \$198,700 at the age of retirement (Figure 2c) and a pension b of \$18,800 per year (representing a present discounted value $(b/\delta)[1 - e^{-\delta(T-R)}]$ of \$222,800). Health $H(t)$ (the solid line in Figure 2d) declines fairly rapidly from a value of \$30,000 to about \$4,800 by age 56.6 ($t_1 = 36.6$) after which the individual starts investing in health (see Figure 2e). Health reaches \$5,800 by the age of retirement R and declines further to about \$2,000 by the end of life T. The dashed line in Figure 2d shows the health threshold. The health threshold increases over time up to the retirement age[13] after which it suddenly drops due to the substitution of health for leisure

[10] Hurd and Rohwedder (2006) argue that a number of explanations operate together to explain the magnitude of the observed drop in consumption at retirement. The substitution between leisure and consumption is only one such factor. In addition, there are individuals who do not experience a drop in consumption and there are those who experience more substantial drops in consumption. The assumed drop of magnitude 15% is for illustrative purposes only.

[11] As discussed earlier (see footnote 5) it is relatively easy to introduce more realistic wage age profiles. Because the shape of the wage age profile influences retirement and because we are primarily interested in the effect of health on the optimal retirement age we have chosen an simple wage profile where the base wage $w_0(t)$ is constant. Thus we can isolate the direct effect of parameter changes from any indirect effect that operates through the wage age profile.

[12] Note that individuals are also allowed to borrow at interest rate δ

[13] This is the result of the time dependence of the marginal benefit of health $\varphi(t)$ as a result of the benefit transformation (equation 24).

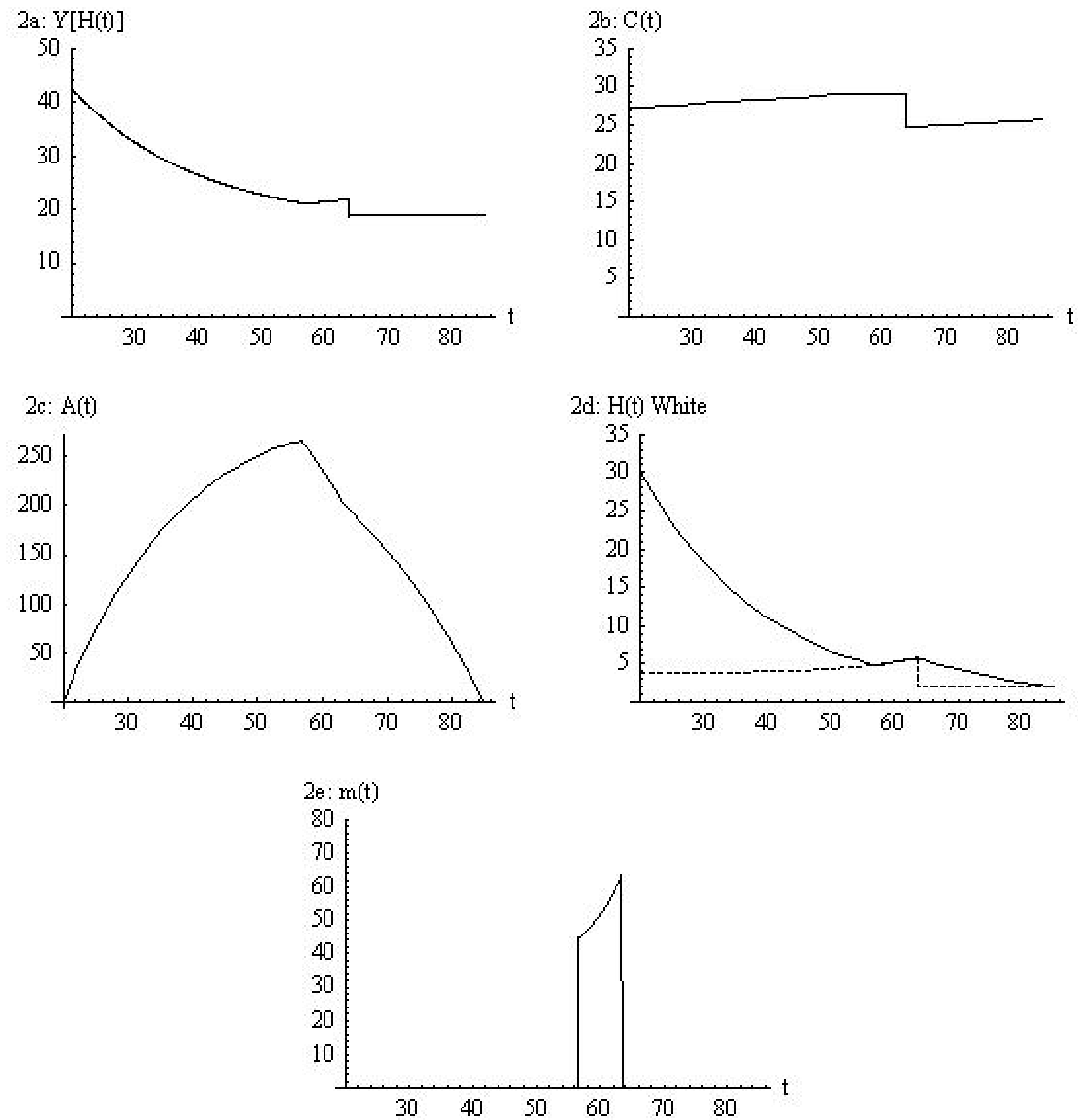

Figure 2: Income ($Y[H(t)]$; \$ thousands), consumption ($C(t)$; \$ thousands), assets ($A(t)$ \$ thousands), health ($H(t)$; \$ thousands; total health [solid line], health threshold [dashed line]) and health investment ($m(t)$; \$ thousands per year) versus age for a "white" collar worker.

and the disappearance of the effect φ on income during retirement. In our formulation and for the parameters chosen, the effect of retirement on an individual's health is negative – retirement is bad for health – as individuals lower their investment in health due to substitution of health for leisure and because health loses its relevance as a means to increase an individual's income.

Because the marginal production benefit of health $\varphi(t)$ is the only term in the transformation

(24) that is time dependent, and because the model solutions after retirement are not functions of $\varphi(t)$, we see that health threshold (Figure 2d) is constant over time during retirement (given our choice of constant health deterioration $d(t)$, prices $p(t)$, efficiency $\mu(t)$ and interest rate δ).

6.2 Health inequality

Case and Deaton (2003) show that "white collar" workers are in better health and have lower health deterioration rates than "blue collar" workers (based on self-reported health assessments). They, as well as Muurinen and Le Grand (1985), suggest this observation could be explained by the need for blue collar workers to perform more physically demanding work than non-manual occupations, which may not be open to lower educated workers. As a result blue collar workers "wear" their bodies out more quickly. An additional (or alternative) explanation could be that "blue collar" workers have lower health thresholds (lower levels of minimally productive health) $H_*(t)$ (but essentially the same "natural" health deterioration rate $d(t)$ as "white collar" workers) as a result of access to lower life-time resources. The lower value of $H_*(t)$ induces them to invest less in health.

Figure 3a shows the evolution of health for "blue collar" workers with a base wage rate of w_0=\$10,000 (half that of "white collar" workers; everything else held constant). The lower earnings of "blue collar" workers reduce their life-time income, their health threshold, and induce earlier retirement at age 53.16 (R = 33.16). As Figure 3b shows, health investment is lower over the life-time for "blue collar" workers. For these specific values workers do not invest in health during working life but only near retirement (scenario C). As a result health declines to about \$5,700 by the age of retirement 53.16 (R = 33.16) and to \$1,400 by age 81.62 when individuals start investing in health (t_2 = 81.62). Also, earlier retirement extends the retirement phase of life for "blue collar" workers which is characterized by a lower health threshold (lower level of minimally productive health) and therefore associated with lower levels of health investment and consequently lower health. As a result, at age 82 (t = 62), white collar workers are more than 40 percent healthier than blue collar workers.

6.3 Health insurrance

We now explore the role of health insurance on health, health investment and retirement. Figure 4 shows the impact of being uninsured. We use the same parameters as before for a white collar worker (our baseline model) but assume $p(t)$ = 1.0 (i.e., health investment is paid for one hundred percent out-of-pocket) before the age of Medicare eligibility. Afterwards $p(t)$ = 0.2 (i.e., we assume that after age 65 the uninsured are covered by a universal health insurance program, such as Medicare). Figures 4a, 4b and 4c show how uninsured individuals invest much less in health (health investment begins at age 73.88 [t_2 = 53.88]), therefore have higher effective health deterioration rates and are unhealthier (compare with Figure 2). Interestingly, consumption is not

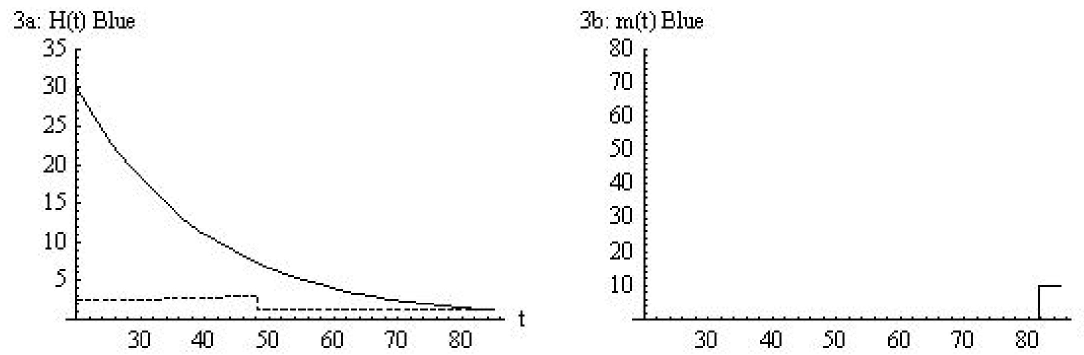

Figure 3: Blue collar health (3a left-hand side; health [solid line] and health threshold [dashed]; \$ thousands) and blue collar health investment (3b right-hand side; \$ thousands) versus age.

significantly affected while the age of retirement now coincides with the age of Medicare eligibility (age 65). Note the significantly lower level of the health threshold (the minimally productive health level) before the Medicare eligibility age of 65, during which health investment is paid one hundred percent out-of-pocket.

6.4 Retirement

We are further interested in the effect of assets, wages, benefits, health, health deterioration rates, and other variables and parameters on the decision to retire. Figures 5a through 5l show the effect of various model parameters on the decision to retire. The solid, dotted and dashed lines in each of the graphs show how, respectively, optimal retirement age R, t_1 and t_2 change in response to variation in a number of variables and parameters. As variables and parameters are varied, the solutions cycle through the scenarios A through F (see Figure 1). For example, figure 5b shows that as we increase the base wage rate w_0, we transition from scenario D ($t_1 > R$ and $t_2 > T$) for values of w_0 below ~ \$6,000 to scenario C ($t_1 > R$ and $t_2 < T$) for values of w_0 between ~ \$6,000 and ~ \$20,000. For values of w_0 between ~ \$14,000 and ~ \$20,000 the age of retirement R falls slightly as the optimal age of retirement tracks the evolution of t_1, i.e. the solution remains on the boundary between scenarios A and C ($t_1 = R$ and $t_2 < T$). Around $w_0 \sim$ \$20,000 we observe a jump in the age of retirement R as we move to scenario B for the remainder of the graph. Initially the solution remains on the boundary between scenarios B and D ($t_2 = T$) explaining the "flat" initial portion of the retirement graph for \$20,000 $< w_0 <$ \$24,000. For values $w_0 >$ \$24,000 we have $t_2 > T$ and retirement R continues its upward trend with increasing base wage rate w_0 (scenario B). Similar explanations hold for the other graphs in figure 5.

We now concentrate on the variation of the optimal retirement age R with the various variables and parameters (solid line in figures 5a through 5l). Figure 5a shows how greater initial assets $A(0)$ reduce the retirement age. Wealthy people have less incentive to work as they can fulfill all

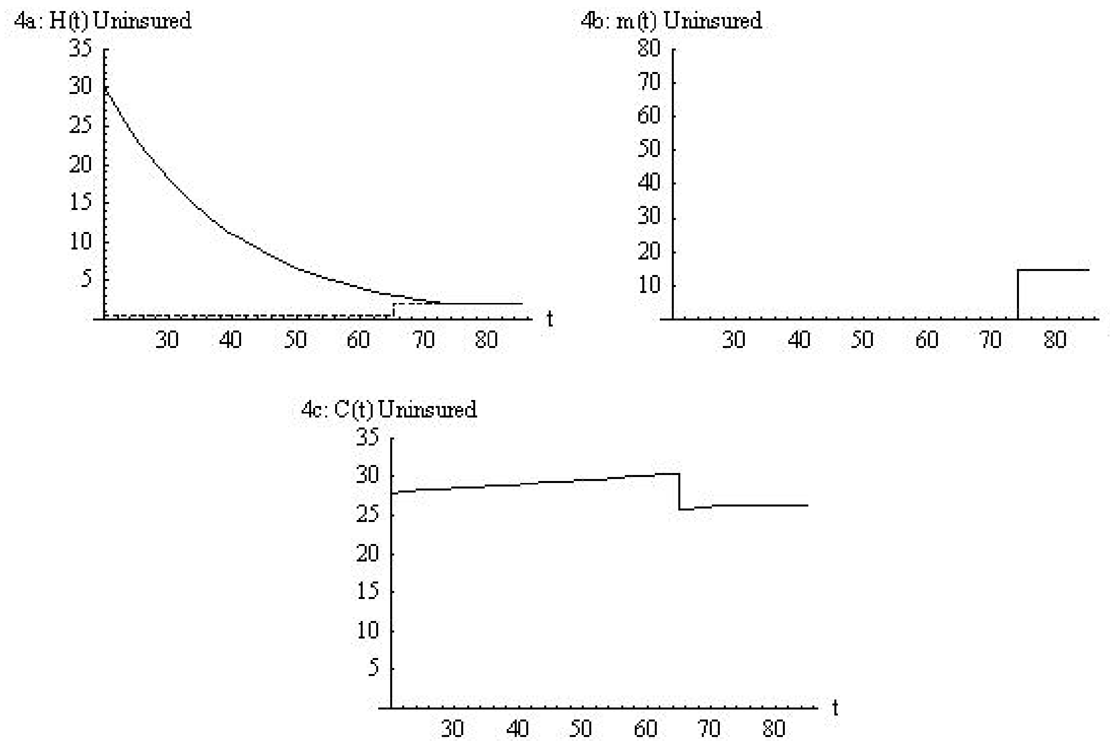

Figure 4: Health (4a left-hand side; $ thousands), health investment (4b center; $ thousands) and consumption (4c right-hand side; $ thousands) for the uninsured versus age.

or part of their consumption needs through inherited wealth. Figure 5b shows that higher wages w_0 increase the age at which individuals retire. Unlike a one-off contribution to life-time resources (such as initial assets $A(0)$), higher wages provide additional resources for as long as the individual works, thereby increasing the age of retirement. Indeed Mitchell and Fields (1984) find that higher earnings result in later retirement.

Figure 5c shows how increasing levels of basic benefits b_0 reduce the retirement age.[14] Indeed, we expect earlier retirement in countries with more generous benefits, as was shown in the cross-country comparison project of Gruber and Wise (1999, 2004, 2007).

Figure 5d shows that the higher the portion α of wages set aside for retirement the earlier an individual retires. Given that retirement in our formulation is completely the result of individual choice (benefits are approximately actuarially fair and the timing of retirement is not constrained) the role of pension wealth and that of regular savings is essentially the same. Lower pension savings will almost exactly be offset by larger accumulated savings (total life-time resources remain the same). In case retirement is not a choice variable (or at least restricted in various ways)

[14] Very early retirement in our model should probably be interpreted as the result of generous unemployment benefits rather than retirement benefits.

lower benefits will decrease life time resources, which will lower consumption and thereby also generate more asset accumulation. Indeed Kapteyn and Panis (2003) find a strong negative relation between wealth at retirement and replacement rates when comparing Italy, The Netherlands, and the U.S.

Figure 5e shows how increasing initial health $H(0)$ reduces the retirement age. Health provides "health capital" as can be seen from the equation for total life-time resources (right-hand side of 3). Initial health $H(0)$ thus operates qualitatively similar to assets and we observe a decrease in the age of retirement with increasing initial health.

The age of retirement increases with increasing rates of health deterioration $d(t) = d_0$ (Figure 5f). For one, higher health deterioration over one's life-time reduces the amount of additional life-time earnings resulting from an individual's inherited health $H(0)$ (see the last term in Equation 3). This would increase the retirement age as it reduces the "effective" initial health endowment. In addition, the user cost of health capital at the margin $\left[p_0/\mu_0\right]\left[d_0 + \delta\right] - \varphi_0$ is higher, which also leads to delayed retirement.

Similarly increasing prices of health care $p(t) = p_0$ (Figure 5g), decreasing health investment efficiency $\mu(t) = \mu_0$ (Figure 5h) and decreasing marginal productivity benefits of health $\varphi(t) = \varphi_0$ (Figure 5i) increase the cost of health capital at the margin and raise the retirement age.

The relationships between prices $p(t)$, health investment efficiency $\mu(t)$, the marginal production benefits of health $\varphi(t)$, the coefficient of relative risk aversion ρ (Figure 5k), and the factor k (Figure 5l; describing the increased utility from leisure during retirement) and retirement are particularly strong in that individuals never work ($R = 0$) or never retire ($R = T$) for certain parameter values. The relative utility weight ς given to consumption versus health has very little impact on the age of retirement (Figure 5j) except near the extreme of $\varsigma \approx 1$ (pure consumption model). Model simulations as well as observations of analytical solutions from simplified versions of our model (a $\varsigma \approx 1$ pure consumption model as in our simulation and $\delta = \beta = 0$) show that the parameters ρ (Figure 5k), k (Figure 5l) and retirement R are strongly related.

6.5 Sensitivity analysis

In addition to the effect of the various parameters on retirement it is of interest to understand more generally the sensitivity of the model to the model parameters. Table 1 displays the baseline model parameter values P_0 and the sensitivity to changes in each of the model parameters of life-time consumption, life-time health investment, life-time health, life-time assets and the age of retirement (endogenous in the model). The sensitivities were estimated by calculating the relative change in the quantity X of interest (e.g., life-time consumption) in response to a one percent change in model parameter values P_0(e.g., $\partial \ln X/\partial \ln P_0$). For example (see Table 1), a one percent increase in co-payment p_0 increases the age of retirement by 0.51 percent and life-time consumption by 0.28 percent. In other words, retirement and life-time consumption are not very sensitive to co-payment. On the other hand life-time health investment and life-time assets are more responsive to changes in co-payment, showing decreases by 1.12 percent and 1.44 percent,

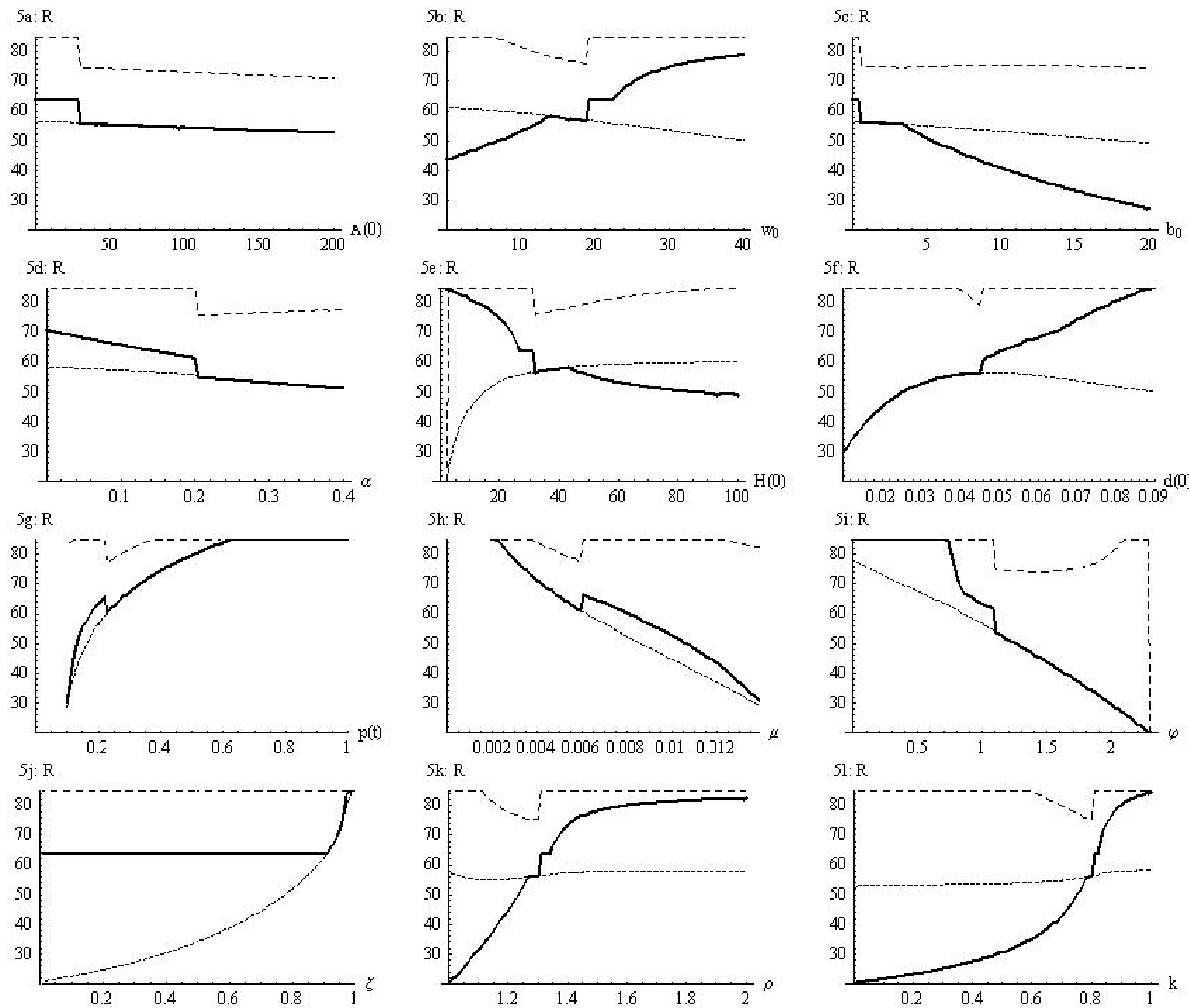

Figure 5: The effect of various variables and parameters on the decision to retire. Initial assets $A(0)$, base wage rate w_0, benefits b_0 are shown in \$ thousands, and initial health $H(0)$ in \$ thousands. Note: values for health deterioration $d(t) < 0.005$, prices $p(t) < 0.088$, health investment efficiency $\mu > 0.016$, and marginal production benefits of health $\varphi > 2.29$ are not shown as they correspond to a user cost of health capital at the margin $\left[p_0/\mu_0\right]\left[d_0 + \delta\right] - \varphi_0$ that is negative. Values of $\rho < 1$ and $k > 1$ are not shown as these require a change in specification; for $\rho = 1$ the utility function switches from being negative ($\rho > 1$ to positive ($\rho < 1$) values. For positive utility, values of $k < 1$ imply disutility from increased leisure, i.e. we need to also switch to values of $k > 1$.

respectively.

Table 1: Sensitivity (elasticities) of model outcomes to various variables and parameters.

P	P_0	$\frac{\partial \ln\left[\int_0^T C(t)e^{-\delta t}dt\right]}{\partial \ln P_0}$	$\frac{\partial \ln\left[\int_0^T m(t)e^{-\delta t}dt\right]}{\partial \ln P_0}$	$\frac{\partial \ln\left[\frac{\int_0^T H(t)e^{-\delta t}dt}{\int_0^T e^{-\delta t}dt}\right]}{\partial \ln P_0}$	$\frac{\partial \ln\left[\frac{\int_0^T A(t)e^{-\delta t}dt}{\int_0^T e^{-\delta t}dt}\right]}{\partial \ln P_0}$	$\frac{\partial \ln R}{\partial \ln P_0}$
p_0	0.20	+0.28	-1.12	+0.07	-1.44	+0.51
μ_0	0.7%	-0.25	+2.19	+0.10	+0.96	-0.52
φ	1.0	+0.20	+1.70	+0.06	+1.78	-0.52
d_0	5%	-0.05	+3.07	-0.54	-1.27	+0.68
β	3%	+0.04	-2.63	-0.01	-0.08	-0.17
δ	3%	+0.03	+0.01	+0.01	-0.16	+0.09
ρ	1.32	+0.01	-0.32	+0.01	+0.04	+0.06
ς	0.85	+0.42	-17.66	-0.42	-0.92	+0.00
k	0.81	+0.06	+0.81	+0.02	+0.23	+0.26
w_0	20k$	+0.55	+0.83	+0.04	+0.19	+0.00
α	15%	-0.10	-0.07	+0.01	-0.31	-0.15
$H(0)$	30k$ [μ]	+0.45	-1.27	+0.96	+0.81	+0.00

Elasticities greater than one indicate that the model is very sensitive to the particular parameter. Most noticeable is the parameter ς describing the relative "share" of consumption versus health in the utility function. A one percent change in ς decreases life-time health investment by nearly 18 percent. It should be noted though that the results in Table 1 are only valid for the particular parameter region close to the model calibration and that sensitivities will be different for different model calibrations.

7 Discussion

We have formulated a stylized structural model of consumption, leisure, health, health investment, wealth accumulation and retirement decisions using a Grossman type human capital framework of health. Specification of a functional form for the utility function and of initial conditions allows us to derive analytic solutions for consumption, health, health investment and wealth, conditional on a given retirement age.

We find that initial conditions are likely of importance and that health will under most circumstances not evolve as Grossman's "optimal" health stock $H_*(t)$. An essential characteristic of the Grossman model is that health cannot deteriorate faster than the natural deterioration rate $d(t)$. As a result initial health cannot dissipate rapidly, nor is there any reason to expect the endowment of health $H(0)$ to exactly equal Grossman's "optimal" health stock $H_*(0)$ (see also

Wolfe 1985). Wolfe (1985) assumes an initial surplus of health on the grounds that "… the human species, with its goal of self-preservation, confronts a different problem than the individual who seeks to maximize utility. The evolutionary solution to the former may entail an excessive health endowment in the sense that an individual might prefer to have less health and to be compensated with wealth in a more liquid form …" As Wolfe more or less suggests, humans may have been endowed with "excessive" health as a result of our evolutionary history which required good physical condition to hunt and gather food, defend ourselves, survive periods of hunger etc. Today's demands on human's physical condition are essentially based on the utility of good health and on economic productivity, which in an increasingly knowledge-intensive environment may be significantly smaller than in pre-historic times.

While Wolfe (1985) provides a convincing argument that high initial health endowments are plausible, we simply assume that initial health $H(0)$ can take any positive value (including values below Grossman's "optimal" health stock $H_*(0)$). Exploring corner solutions, in which individuals do not invest in medical care ($m(t) = 0$) for periods of time, we find that what is referred to in the literature as Grossman's "optimal" health stock (e.g., Grossman 1972, 2000) should, given initial condition $H(0)$, not be interpreted as an optimal solution but rather as a health threshold (given by the "minimally productive" level of health). Healthy individuals (whose health is above the threshold) do not invest in health, while unhealthy individuals (whose health is at or below the threshold) do. The threshold is the minimum health level individuals "demand" for the productivity benefits and utility that good health provides.[15]

In a review of the empirical literature Galama and Kapteyn (2009) find that the interpretation advocated here provides a better explanation for the observed evolution of health and of medical consumption. Importantly, our interpretation of the Grossman model can explain the observation that measures of medical care are negatively corelated with measures of health while the traditional interpretation cannot (see, e.g., Zweifel and Breyer, 1997, and references therein).

We employ the model to investigate the optimal age of retirement by maximizing the implied indirect utility function with respect to the retirement age. In the model individuals find retirement increasingly attractive as they age as a result of three effects: (1) wage declines as a result of gradual health deterioration reducing income from work with age, (2) increased availability of leisure during retirement and (3) accumulation of pension wealth (which can only be consumed after retirement) with years in the workforce.

The model can explain the paradox that the retirement age has continued to fall while retirees point to deteriorating health as an important reason for early retirement at the same time that population health and mortality have continued to improve in the developed world. If advances

[15]Wolfe (1985), to the best of our knowledge, is the only researcher who has attempted to explore the consequences of corner solutions in Grossman's model in some detail. His model and interpretation is however substantially different from ours. Wolfe employs a simplified Grossman model where health does not provide utility. Further, Wolfe interprets the onset of *" … a discontinuous mid-life increase in health investment … "* with retirement. We however do not associate the discontinous increase in health investment with retirement but with becoming unhealthy (health levels at or below the health threshold leading to health investment). Retirement in our model is the result of life-time utility maximization.

in population health are largely the result of better nutrition, preventative medicine (through, e.g., vaccination and other means), and better (less taxing) living, working and schooling environments then the overall health endowment $H(0)$ of the population increases and/or the health deterioration rate $d(t)$ decreases. Both effects result in earlier retirement.[16,17] Workers with higher earnings (say white collar workers) invest more in health and because they stay healthier retire later than those with lower earnings (say blue collar workers) whose health deteriorates faster. In other words, health is an important determinant of early retirement. Indeed Dwyer and Mitchell (1998) find that men in poor overall health are expected to retire one to two years earlier, an effect that persists after the authors correct for potential endogeneity of self-rated health problems.

Further, we can explain differences in the observed health deterioration rates between blue and white collar workers by differences in their health thresholds (their minimally productive level of health) and their resulting differences in health investment. We do not need to resort to physical effort or work-type related health effects (e.g., as in Case and Deaton 2007). Even though we do not find it unreasonable to assume that certain types of jobs result in higher health deterioration rates, we do offer that poorer individuals also invest less in health as their health thresholds (minimally productive levels of health) are lower than for richer individuals.

[16]If on the other hand advances in medical care or other advances increase the efficiency or lower the cost of health investment then retirement will be postponed.

[17]This prediction crucially depends on the assumption that a significant share of the population has health levels above the health threshold, i.e., that corner solutions are fairly common.

8 Appendix: derivations

8.1 First-order conditions

The objective function (1) is maximized subject to the constraints (2). Health can be solved as in (4). We now introduce Ł, the integral over time of the Lagrangian $\Im$ (equation 5).

$$\begin{aligned} Ł &= \int_0^T \Im dt = \int_0^R U_w[C(t), H(t)]e^{-\beta t}dt + \int_R^T U_r[C(t), H(t)]e^{-\beta t}dt \\ &+ p_A(0)\int_0^T \{\delta A(t) + Y[H(t)] - C(t) - p(t)m(t)\}e^{-\delta t}dt + \int_0^T q(t)m(t)dt. \end{aligned} \tag{27}$$

Maximizing Ł with respect to consumption $C(t')$ results in the following first order conditions:

$$\frac{\partial U_w(t')}{\partial C(t')} = p_A(0)e^{(\beta-\delta)t'} \quad t' \leq R \tag{28}$$

$$\frac{\partial U_r(t')}{\partial C(t')} = p_A(0)e^{(\beta-\delta)t'} \quad t' > R, \tag{29}$$

where we have used

$$\frac{\partial C(t)}{\partial C(t')} = \delta(t - t'), \tag{30}$$

where $\delta(t - t')$ is the Dirac delta function. The Dirac delta function is the continuous equivalent of the discrete Kronecker delta function. It has the property $\int_\Omega f(t)\delta(t - t')dt = f(t')\,(t' \in \Omega)$ and can informally be thought of as a function $\delta(x)$ that has the value of infinity for $x = 0$, the value zero elsewhere and has a area of 1 (normalized).

Using the functional form (11) of the utility function allows us to write the first order conditions with respect to consumption $C(t')$ (equations 28 and 29) as follows:

$$\frac{\partial U_w(t')}{\partial C(t')} = \varsigma C(t')^{\varsigma-\rho\varsigma-1}H(t')^{1-\varsigma-\rho+\rho\varsigma} = p_A(0)e^{(\beta-\delta)t'} \quad t' \leq R \tag{31}$$

$$\frac{\partial U_r(t')}{\partial C(t')} = k\varsigma C(t')^{\varsigma-\rho\varsigma-1}H(t')^{1-\varsigma-\rho+\rho\varsigma} = p_A(0)e^{(\beta-\delta)t'} \quad t' > R. \tag{32}$$

Before we continue with the first order conditions with respect to health investment $m(t')$, it is useful to look at the derivative of $H(t)$ (see equation 4) with respect to $m(t')$:

$$\frac{\partial H(t)}{\partial m(t')} = \mu(t')e^{-\int_{t'}^t d(s)ds} \quad t' \leq t \tag{33}$$

$$\frac{\partial H(t)}{\partial m(t')} = 0 \quad t' > t, \tag{34}$$

where once more we have used that

$$\frac{\partial m(t)}{\partial m(t')} = \delta(t - t'). \tag{35}$$

Maximizing Ł with respect to health investment $m(t')$ leads to

$$\begin{aligned}
\int_{t'}^{R} \frac{\partial U_w(t)}{\partial m(t')} e^{-\beta t} dt + \int_{R}^{T} \frac{\partial U_r(t)}{\partial m(t')} e^{-\beta t} dt &= \\
\int_{t'}^{R} \frac{\partial U_w(t)}{\partial H(t)} \mu(t') e^{-\int_{t'}^{t} d(s)ds} e^{-\beta t} dt + \int_{R}^{T} \frac{\partial U_r(t)}{\partial H(t)} \mu(t') e^{-\int_{t'}^{t} d(s)ds} e^{-\beta t} dt &= \\
p_A(0) p(t') e^{-\delta t'} - p_A(0) \int_{t'}^{T} \frac{\partial Y(t)}{\partial m(t')} e^{-\delta t} dt - q(t') & \qquad t' \leq R
\end{aligned} \tag{36}$$

$$\begin{aligned}
\int_{t'}^{T} \frac{\partial U_r(t)}{\partial m(t')} e^{-\beta t} dt &= \\
\int_{t'}^{T} \frac{\partial U_r(t)}{\partial H(t)} \mu(t') e^{-\int_{t'}^{t}} d(s)ds e^{-\beta t} dt &= \\
p_A(0) p(t') e^{-\delta t'} - p_A(0) \int_{t'}^{T} \frac{\partial Y(t)}{\partial m(t')} e^{-\delta t} dt - q(t') & \qquad t' > R,
\end{aligned} \tag{37}$$

where the lower integration limit t' reflects the fact that the stock of health (which utility and wages are functions of) is a function of past but not future health investment.

Using once more the functional form (11) of the utility function, using the Leibniz Integral Rule to differentiate equations (36) and (37) with respect to t' and substituting the result back into equations (36) and (37) we find:

$$\begin{aligned}
\frac{\partial U_w(t')}{\partial H(t')} &= (1 - \varsigma) C(t')^{\varsigma - \rho\varsigma} H(t')^{-\varsigma + \rho\varsigma - \rho} \\
&= p_A(0) \left[\pi_H(t') - \varphi(t') \right] e^{(\beta - \delta)t'} - \frac{e^{\beta t'}}{\mu(t')} \left[\frac{\dot{\mu}(t')}{\mu(t')} + d(t') \right] q(t') + \dot{q}(t') \frac{e^{\beta t'}}{\mu(t')} \\
&= \mathbb{A} p_A(0) e^{(\beta - \delta)t'} + \mathbb{B} \qquad\qquad (t' \leq R)
\end{aligned} \tag{38}$$

$$\begin{aligned}
\frac{\partial U_r(t')}{\partial H(t')} &= k(1 - \varsigma) C(t')^{\varsigma - \rho\varsigma} H(t')^{-\varsigma + \rho\varsigma - \rho} \\
&= p_A(0) \pi_H(t') e^{(\beta - \delta)t'} - \frac{e^{\beta t'}}{\mu(t')} \left[\frac{\dot{\mu}(t')}{\mu(t')} + d(t') \right] q(t') + \dot{q}(t') \frac{e^{\beta t'}}{\mu(t')} \\
&= \mathbb{A}' p_A(0) e^{(\beta - \delta)t'} + \mathbb{B} \qquad\qquad (t' > R),
\end{aligned} \tag{39}$$

where $\pi_H(t')$ is the user cost of health capital at the margin (equation 10) and the definitions for $\mathbb{A}$, $\mathbb{B}$, and $\mathbb{A}'$ follow directly from equations (38) and (39).

8.2 Solutions for health, consumption and health investment

Solving the first order conditions (equations 31, 32, 38 and 39) we find

$$C(t) = H(t)\left\{\frac{\zeta}{1-\zeta}\left[\mathbb{A} + \frac{\mathbb{B}e^{-(\beta-\delta)t}}{p_A(0)}\right]\right\} \qquad t \le R \tag{40}$$

$$C(t) = H(t)\left\{\frac{\zeta}{1-\zeta}\left[\mathbb{A}' + \frac{\mathbb{B}e^{-(\beta-\delta)t}}{p_A(0)}\right]\right\} \qquad t > R, \tag{41}$$

and the following solutions for $C(t)$ and $H(t)$:

$$C(t) = \zeta\Lambda\left[\mathbb{A} + \frac{\mathbb{B}}{p_A(0)}e^{-(\beta-\delta)t}\right]^{1-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} \qquad t \le R \tag{42}$$

$$C(t) = k^{1/\rho}\zeta\Lambda\left[\mathbb{A}' + \frac{\mathbb{B}}{p_A(0)}e^{-(\beta-\delta)t}\right]^{1-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} \qquad t > R \tag{43}$$

$$H(t) = (1-\zeta)\Lambda\left[\mathbb{A} + \frac{\mathbb{B}}{p_A(0)}e^{-(\beta-\delta)t}\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} \qquad t \le R \tag{44}$$

$$H(t) = k^{1/\rho}(1-\zeta)\Lambda\left[\mathbb{A}' + \frac{\mathbb{B}}{p_A(0)}e^{-(\beta-\delta)t}\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} \qquad t > R, \tag{45}$$

where once more we have used the definitions for χ (equation 16) and for Λ (equation 17).

Using equation (2) one can then solve for health investment $m(t)$:

$$m(t) = \frac{1}{\mu(t)}(1-\zeta)e^{-\int_0^t d(s)ds}\frac{\partial}{\partial t}\left\{\Lambda\left[\mathbb{A} + \frac{\mathbb{B}}{p_A(0)}e^{-(\beta-\delta)t}\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t}e^{\int_0^t d(s)ds}\right\} \qquad t \le R \tag{46}$$

$$m(t) = \frac{1}{\mu(t)}k^{1/\rho}(1-\zeta)e^{-\int_R^t d(s)ds}\frac{\partial}{\partial t}\left\{\Lambda\left[\mathbb{A}' + \frac{\mathbb{B}}{p_A(0)}e^{-(\beta-\delta)t}\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t}e^{\int_R^t d(s)ds}\right\} \qquad t > R. \tag{47}$$

With solutions for the control functions consumption $C(t)$ and health investment $m(t)$, and for the state variable health $H(t)$ we can find the solution for the state variable assets $A(t)$ using equations (21) and (22).

For positive health investment $m(t) > 0$ we have $q(t) = 0$ and $H(t) = H_*(t)$ and therefore $\mathbb{B} = 0$. These are the solutions for the health threshold (see equations 12, 13, 14, 15, 19 and 20). On the other hand, for initial conditions $H(0)$ and $H(R_+)$ that are above the health threshold (the minimally productive health level) $H_*(0)$ and $H_*(R_+)$ (see Figure 1 scenarios A through F) we have a situation of "excessive" initial health, i.e., the individual is endowed with an initial stock of health that is greater than the level required to be economically productive. In such cases individuals would want to "sell" their health, i.e., chose negative health investment $m(t) < 0$. Since this is not possible (health investment is a positive quantity) we have a corner solution where $m(t) = 0$. We can derive the solutions for consumption $C(t)$ and health $H(t)$ by imposing $m(t) = 0$. We then find a differential equation in $q(t)$ with the following solutions:

$$
\begin{aligned}
q(t) &= p_A(0)\int_0^t \mu(x)\left[H(0)e^{\left(\frac{\beta-\delta}{\rho}\right)x}e^{-\int_0^x d(s)ds}\frac{1}{\Lambda(1-\zeta)}\right]^{\frac{-1}{\chi}} e^{\int_x^t\left[\frac{\dot{\mu}(s)}{\mu(s)}+d(s)\right]ds}e^{-\delta x}dx \\
&- p_A(0)\int_0^t \mu(x)\left[\pi_H(x)-\varphi(x)\right]e^{\int_x^t\left[\frac{\dot{\mu}(s)}{\mu(s)}+d(s)\right]ds}e^{-\delta x}dx \\
&+ q(0)e^{\int_0^t\left[\frac{\dot{\mu}(s)}{\mu(s)}+d(s)\right]ds}, \qquad (t \leq R)
\end{aligned} \tag{48}
$$

$$
\begin{aligned}
q(t) &= p_A(0)\int_R^t \mu(x)\left[H(R)e^{\left(\frac{\beta-\delta}{\rho}\right)x}e^{-\int_R^x d(s)ds}\frac{1}{k^{1/\rho}\Lambda(1-\zeta)}\right]^{\frac{-1}{\chi}} e^{\int_x^t\left[\frac{\dot{\mu}(s)}{\mu(s)}+d(s)\right]ds}e^{-\delta x}dx \\
&- p_A(0)\int_R^t \mu(x)\pi_H(x)e^{\int_x^t\left[\frac{\dot{\mu}(s)}{\mu(s)}+d(s)\right]ds}e^{-\delta x}dx \\
&+ q(R)e^{\int_R^t\left[\frac{\dot{\mu}(s)}{\mu(s)}+d(s)\right]ds}. \qquad (t > R)
\end{aligned} \tag{49}
$$

Substituting the above solutions for $q(t)$ into those for consumption $C(t)$ (equations 42 and 43), health $H(t)$ (equations 44 and 45) and health investment $m(t)$ (equations 46 and 47), we find:

$$
\begin{aligned}
H(t) &= H(0)e^{-\int_0^t d(s)ds} \quad (t \leq R) && (50)\\
H(t) &= H(R)e^{-\int_R^t d(s)ds} \quad (t > R) && (51)\\
C(t) &= \zeta\Lambda^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}}H(0)^{-\left(\frac{1-\chi}{\chi}\right)}e^{\left(\frac{1-\chi}{\chi}\right)\int_0^t d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} \quad (t \leq R) && (52)\\
C(t) &= k^{1/\chi\rho}\zeta\Lambda^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}}H(R)^{-\left(\frac{1-\chi}{\chi}\right)}e^{\left(\frac{1-\chi}{\chi}\right)\int_R^t d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} \quad (t > R) && (53)\\
m(t) &= 0. && (54)
\end{aligned}
$$

A perhaps more intuitive way of arriving at the same result is by simply substituting $m(t) = \xi(t)^2$ and solving the optimization problem for the control variables $\xi(t)$ (instead of $m(t)$) and consumption $C(t)$ (i.e., one then does not have to resort to using the multiplier $q(t)$ associated with the condition that health investment $m(t) \geq 0$ in the Lagrangian 5). One then finds the same first order conditions for maximization with respect to consumption (equations 28 and 29). For the first order conditions for maximization with respect to $\xi(t)$ one finds that either $\xi(t) = 0$ (and hence $m(t) = 0$) or that the first order conditions conditions equations (36 and 37) are valid for $q(t) = 0$ ($\mathbb{B} = 0$).

We now have the material to solve the solutions for each of the scenarios A through F (see Figure 1) in detail.

8.3 Scenario A

8.3.1 Scenario A: $0 \le t \le t_1$

Figure 1 shows how in scenario A initial health $H(0)$ is above the initial health threshold $H_*(0)$ and individuals do not invest in health $m(t) = 0$. As a result health deteriorates with rate $d(t)$ untill age t_1 when health reaches the health threshold $H_*(t_1)$. We have the following condition $[H(t_1) = H_*(t_1)]$:

$$H(0)e^{-\int_0^{t_1} d(s)ds} = (1-\zeta)\Lambda_A \left[\pi_H(t_1) - \varphi(t_1)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t_1}, \tag{55}$$

and the following solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$:

$$\begin{aligned}
H(t) &= H(0)e^{-\int_0^t d(s)ds} && (56)\\
&= (1-\zeta)\Lambda_A \left[\pi_H(t_1) - \varphi(t_1)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t_1} e^{\int_t^{t_1} d(s)ds} && (57)\\
C(t) &= \zeta\Lambda_A^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}} H(0)^{-\left(\frac{1-\chi}{\chi}\right)} e^{\left(\frac{1-\chi}{\chi}\right)\int_0^t d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (58)\\
&= \zeta\Lambda_A \left[\pi_H(t_1) - \varphi(t_1)\right]^{1-\chi} e^{\left(\frac{1-\chi}{\chi}\right)\left(\frac{\beta-\delta}{\rho}\right)t_1} e^{-\left(\frac{1-\chi}{\chi}\right)\int_t^{t_1} d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (59)\\
m(t) &= 0. && (60)
\end{aligned}$$

8.3.2 Scenario A: $t_1 < t \le R$

Between the age t_1 and retirement R individuals invest in health $m(t) > 0$ and follow the health threshold (the minimally productive health path): $H_*(t)$, $C_*(t)$, and $m_*(t)$.

$$\begin{aligned}
H_*(t) &= (1-\zeta)\Lambda_A \left[\pi_H(t) - \varphi(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t}, && (61)\\
C_*(t) &= \zeta\Lambda_A \left[\pi_H(t) - \varphi(t)\right]^{1-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t}, && (62)\\
m_*(t) &= \frac{1}{\mu(t)} e^{-\int\limits_0^t d(s)ds} \frac{d}{dt}\left((1-\zeta)\Lambda_A \left[\pi_H(t) - \varphi(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} e^{\int\limits_0^t d(s)ds}\right). && (63)
\end{aligned}$$

8.3.3 Scenario A: $R < t \le t_2$

At retirement the health threshold drops to $H_*(R_+)$ and once more individuals do not invest in health ($m(t) = 0$) till age t_2 when health reaches the health threshold $H_*(t_2)$. We have the following condition $[H(t_2) = H_*(R\_)e^{-\int_R^{t_2} d(s)ds} = H_*(t_2)]$:

$$(1-\zeta)\Lambda_A \left[\pi_H(R) - \varphi(R)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)R} e^{-\int_R^{t_2} d(s)ds} = k^{1/\rho}(1-\zeta)\Lambda_A \left[\pi_H(t_2)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t_2}, \tag{64}$$

and the following solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$:

$$
\begin{aligned}
H(t) &= H_*(R\_)e^{-\int_R^t d(s)ds} && (65)\\
&= (1-\zeta)\Lambda_A\left[\pi_H(R)-\varphi(R)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)R}e^{-\int_R^t d(s)ds} && (66)\\
C(t) &= k^{1/\rho\chi}\zeta\Lambda_A^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}}H_*(R\_)^{-\left(\frac{1-\chi}{\chi}\right)}e^{\left(\frac{1-\chi}{\chi}\right)\int_R^t d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (67)\\
&= k^{1/\rho\chi}\zeta\Lambda_A\left[\pi_H(R)-\varphi(R)\right]^{1-\chi} e^{\left(\frac{1-\chi}{\chi}\right)\left(\frac{\beta-\delta}{\rho}\right)R}e^{\left(\frac{1-\chi}{\chi}\right)\int_R^t d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (68)\\
m(t) &= 0. && (69)
\end{aligned}
$$

8.3.4 Scenario A: $t_2 < t \leq T$

Between the age t_2 and the end of life T individuals invest once again in health ($m(t) > 0$) and follow the health threshold (the minimally productive health path): $H_*(t)$, $C_*(t)$, and $m_*(t)$.

$$
\begin{aligned}
H_*(t) &= k^{1/\rho}(1-\zeta)\Lambda_A\left[\pi_H(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t}, && (70)\\
C_*(t) &= k^{1/\rho}\zeta\Lambda_A\left[\pi_H(t)\right]^{1-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t}, && (71)\\
m_*(t) &= k^{1/\rho}\frac{1}{\mu(t)}e^{-\int_0^t d(s)ds}\frac{d}{dt}\left((1-\zeta)\Lambda_A\left[\pi_H(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t}e^{\int_0^t d(s)ds}\right). && (72)
\end{aligned}
$$

8.3.5 Scenario A: determination of Λ_A

Using the life-time budget constraint (3) and substituting the solutions for health $H(t)$, consumption $C(t)$ and health investment $m(t)$ we can determine the constant Λ_A. Define:

$$\Lambda_A \equiv \frac{\Lambda_{An}}{\Lambda_{Ad}}, \tag{73}$$

where Λ_{An} is the numerator and Λ_{Ad} is the denominator of Λ_A. We find:

$$
\begin{aligned}
\Lambda_{An} &= A(0) - A(T)e^{-\delta T} + \int_0^R w_0(x)e^{-\delta x}dx + \int_R^T b(x)e^{-\delta x}dx \\
&+ H(0)\int_0^{t_1} \varphi(x)e^{-\int_0^x d(s)ds}e^{-\delta x}dx \qquad (74) \\
\Lambda_{Ad} &= \int_{t_1}^{R} \left[\pi_H(x) - \varphi(x)\right]^{1-\chi} e^{-\kappa x}dx + k^{1/\rho}\int_{t_2}^{T} \left[\pi_H(x)\right]^{1-\chi} e^{-\kappa x}dx \\
&+ \zeta\left[\pi_H(t_1) - \varphi(t_1)\right]^{1-\chi} e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)t_1} \int_0^{t_1} e^{-\left(\frac{1-\chi}{\chi}\right)\int_x^{t_1} d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x}e^{-\delta x}dx \\
&+ \zeta k^{1/\rho\chi}\left[\pi_H(R) - \varphi(R)\right]^{1-\chi} e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)R} \int_R^{t_2} e^{\left(\frac{1-\chi}{\chi}\right)\int_R^{x} d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x}e^{-\delta x}dx \\
&+ (1-\zeta)\frac{p(R)}{\mu(R)}\left[\pi_H(R) - \varphi(R)\right]^{-\chi} e^{-\kappa R} - (1-\zeta)\frac{p(t_1)}{\mu(t_1)}\left[\pi_H(t_1) - \varphi(t_1)\right]^{-\chi} e^{-\kappa t_1} \\
&+ (1-\zeta)k^{1/\rho}\frac{p(T)}{\mu(T}\left[\pi_H(T)\right]^{-\chi} e^{-\kappa T} - (1-\zeta)k^{1/\rho}\frac{p(t_2)}{\mu(t_2)}\left[\pi_H(t_2)\right]^{-\chi} e^{-\kappa t_2}, \qquad (75)
\end{aligned}
$$

where we have used the following definition:

$$
\kappa \equiv \frac{\delta\rho + \beta - \delta}{\rho}. \qquad (76)
$$

8.4 Scenario B

8.4.1 Scenario B: $0 \leq t \leq t_1$

Figure 1 shows how similar to scenario A initial health $H(0)$ is above the health threshold $H_*(0)$ and individuals do not invest in health $m(t) = 0$. As a result health deteriorates with rate $d(t)$ untill age t_1 when health reaches the health threshold $H_*(t_1)$. The same condition $[H(t_1) = H_*(t_1)]$ holds as in scenario A (equation 55; replace Λ_A with Λ_B). Also the solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$ are the same as in scenario A (56, 57, 58, 59, and 60; replace Λ_A with Λ_B).

8.4.2 Scenario B: $t_1 < t \leq R$

As in scenario A, between the age t_1 and retirement R individuals invest in health $m(t) > 0$ and follow the health threshold (the minimally productive health path): $H_*(t)$, $C_*(t)$, and $m_*(t)$ (see equations 61, 62, and 63; replace Λ_A with Λ_B).

8.4.3 Scenario A: $R < t \leq T$

As in scenario A, at retirement the health threshold drops to $H_*(R_+)$ and once more individuals do not invest in health ($m(t) = 0$). In scenario B (unlike in scenario A) health, after the retirement age

R, does not deteriorate to the health threshold level $H_*(t)$ before the end of life T. The solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$ are given by equations 65, 66, 67, 68, and 69 (replace Λ_A with Λ_B) and are valid for $R < t \leq T$.

8.4.4 Scenario B: determination of Λ_B

Defining

$$\Lambda_B \equiv \frac{\Lambda_{Bn}}{\Lambda_{Bd}}, \tag{77}$$

where Λ_{Bn} is the numerator and Λ_{Bd} is the denominator of Λ_B, we find:

$$\begin{aligned}
\Lambda_{Bn} &= A(0) - A(T)e^{-\delta T} + \int_0^R w_0(x)e^{-\delta x}dx + \int_R^T b(x)e^{-\delta x}dx \\
&+ H(0)\int_0^{t_1} \varphi(x)e^{-\int_0^x d(s)ds}e^{-\delta x}dx \qquad (78)\\
\Lambda_{Bd} &= \int_{t_1}^R \left[\pi_H(x) - \varphi(x)\right]^{1-\chi} e^{-\kappa x}dx \\
&+ \zeta\left[\pi_H(t_1) - \varphi(t_1)\right]^{1-\chi} e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)t_1} \int_0^{t_1} e^{-\left(\frac{1-\chi}{\chi}\right)\int_x^{t_1} d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x} e^{-\delta x}dx \\
&+ \zeta k^{1/\rho\chi}\left[\pi_H(R) - \varphi(R)\right]^{1-\chi} e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)R} \int_R^T e^{\left(\frac{1-\chi}{\chi}\right)\int_R^x d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x} e^{-\delta x}dx \\
&+ (1-\zeta)\frac{p(R)}{\mu(R)}\left[\pi_H(R) - \varphi(R)\right]^{-\chi} e^{-\kappa R} - (1-\zeta)\frac{p(t_1)}{\mu(t_1)}\left[\pi_H(t_1) - \varphi(t_1)\right]^{-\chi} e^{-\kappa t_1}. \qquad (79)
\end{aligned}$$

8.5 Scenario C

8.5.1 Scenario C: $0 \leq t \leq R$

Figure 1 shows how similar to scenarios A and B initial health $H(0)$ is above the initial health threshold $H_*(0)$ and individuals do not invest in health $m(t) = 0$. But unlike scenarios A and B, health reaches the health threshold $H_*(t_2)$ only at age t_2, after the retirement age R. Individuals thus only invest in health during retirement and not during working life. A similar condition $[H(t_2) = H_*(t_2)]$ holds as in scenarios A and B (equation 55). We have:

$$H(0)e^{-\int_0^{t_2} d(s)ds} = k^{1/\rho}(1-\zeta)\Lambda_C\left[\pi_H(t_2)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t_2}, \tag{80}$$

and the following solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$:

$$
\begin{aligned}
H(t) &= H(0)e^{-\int_0^t d(s)ds} && (81)\\
&= k^{1/\rho}(1-\zeta)\Lambda_C\left[\pi_H(t_2)\right]^{-\chi}\, e^{-\left(\frac{\beta-\delta}{\rho}\right)t_2}e^{\int_t^{t_2} d(s)ds} && (82)\\
C(t) &= \zeta\Lambda_C^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}}H(0)^{-\left(\frac{1-\chi}{\chi}\right)}e^{\left(\frac{1-\chi}{\chi}\right)\int_0^t d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (83)\\
&= k^{-\left(\frac{1-\chi}{\rho\chi}\right)}\zeta\Lambda_C\left[\pi_H(t_2)\right]^{1-\chi}\, e^{\left(\frac{1-\chi}{\chi}\right)\left(\frac{\beta-\delta}{\rho}\right)t_2}e^{-\left(\frac{1-\chi}{\chi}\right)\int_t^{t_2} d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (84)\\
m(t) &= 0. && (85)
\end{aligned}
$$

8.5.2 Scenario C: $R < t \leq t_2$

The solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$ are:

$$
\begin{aligned}
H(t) &= H(0)e^{-\int_0^t d(s)ds} && (86)\\
&= k^{1/\rho}(1-\zeta)\Lambda_C\left[\pi_H(t_2)\right]^{-\chi}\, e^{-\left(\frac{\beta-\delta}{\rho}\right)t_2}e^{\int_t^{t_2} d(s)ds} && (87)\\
C(t) &= k^{1/\rho\chi}\zeta\Lambda_C^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}}H(0)^{-\left(\frac{1-\chi}{\chi}\right)}e^{\left(\frac{1-\chi}{\chi}\right)\int_0^t d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (88)\\
&= k^{1/\rho}\zeta\Lambda_C\left[\pi_H(t_2)\right]^{1-\chi}\, e^{\left(\frac{1-\chi}{\chi}\right)\left(\frac{\beta-\delta}{\rho}\right)t_2}e^{-\left(\frac{1-\chi}{\chi}\right)\int_t^{t_2} d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (89)\\
m(t) &= 0. && (90)
\end{aligned}
$$

8.5.3 Scenario C: $t_2 < t \leq T$

Between the age t_2 and the end of life T individuals invest once again in health ($m(t) > 0$) and follow the health threshold: $H_*(t)$, $C_*(t)$, and $m_*(t)$. The equations are the same as in scenario A (equations 70, 71, and 72; replace Λ_A with Λ_C).

8.5.4 Scenario C: determination of Λ_C

Defining

$$\Lambda_C \equiv \frac{\Lambda_{Cn}}{\Lambda_{Cd}}, \qquad (91)$$

where Λ_{Cn} is the numerator and Λ_{Cd} is the denominator of Λ_C, we find:

$$
\begin{aligned}
\Lambda_{Cn} &= A(0) - A(T)e^{-\delta T} + \int_0^R w_0(x)e^{-\delta x}dx + \int_R^T b(x)e^{-\delta x}dx \\
&+ H(0)\int_0^R \varphi(x)e^{-\int_0^x d(s)ds}e^{-\delta x}dx \qquad (92)\\
\Lambda_{Cd} &= k^{1/\rho}\int_{t_2}^T \left[\pi_H(x)\right]^{1-\chi} e^{-\kappa x}dx \\
&+ k^{-\left(\frac{1-\chi}{\rho\chi}\right)}\zeta\left[\pi_H(t_2)\right]^{1-\chi} e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)t_2}\int_0^R e^{-\left(\frac{1-\chi}{\chi}\right)\int_x^{t_2} d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x}e^{-\delta x}dx \\
&+ k^{1/\rho}\zeta\left[\pi_H(t_2)\right]^{1-\chi} e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)t_2}\int_R^{t_2} e^{-\left(\frac{1-\chi}{\chi}\right)\int_x^{t_2} d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x}e^{-\delta x}dx \\
&+ (1-\zeta)k^{1/\rho}\frac{p(T)}{\mu(T}\left[\pi_H(T)\right]^{-\chi} e^{-\kappa T} - (1-\zeta)k^{1/\rho}\frac{p(t_2)}{\mu(t_2)}\left[\pi_H(t_2)\right]^{-\chi} e^{-\kappa t_2}. \qquad (93)
\end{aligned}
$$

8.6 Scenario D

8.6.1 Scenario D: $0 \leq t \leq R$

Figure 1 shows how similar to scenarios A, B and C initial health $H(0)$ is above the initial health threshold $H_*(0)$ and individuals do not invest in health $m(t) = 0$. But unlike scenarios A, B and C health never reaches the health threshold $H_*(t)$ at any point during the individual's life time. Individuals are sufficiently endowed with initial health capital that they never need to invest in health during working life nor during retirement.

The solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$ are:

$$
\begin{aligned}
H(t) &= H(0)e^{-\int_0^t d(s)ds} \qquad (94)\\
C(t) &= \zeta\Lambda_D^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}}H(0)^{-\left(\frac{1-\chi}{\chi}\right)}e^{\left(\frac{1-\chi}{\chi}\right)\int_0^t d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} \qquad (95)\\
m(t) &= 0. \qquad (96)
\end{aligned}
$$

8.6.2 Scenario D: $R < t \leq T$

The solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$ are:

$$
\begin{aligned}
H(t) &= H(0)e^{-\int_0^t d(s)ds} \qquad (97)\\
C(t) &= k^{1/\rho\chi}\zeta\Lambda_D^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}}H(0)^{-\left(\frac{1-\chi}{\chi}\right)}e^{\left(\frac{1-\chi}{\chi}\right)\int_0^t d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} \qquad (98)\\
m(t) &= 0. \qquad (99)
\end{aligned}
$$

8.6.3 Scenario D: determination of Λ_D

Defining

$$\Lambda_D \equiv \frac{\Lambda_{Dn}}{\Lambda_{Dd}}, \tag{100}$$

where Λ_{Dn} is the numerator and Λ_{Dd} is the denominator of Λ_D, we find:

$$\begin{aligned}
\Lambda_{Dn}^{1/\chi} &= A(0) - A(T)e^{-\delta T} + \int_0^R w_0(x)e^{-\delta x}dx + \int_R^T b(x)e^{-\delta x}dx \\
&+ H(0)\int_0^R \varphi(x)e^{-\int_0^x d(s)ds}e^{-\delta x}dx \qquad (101)\\
\Lambda_{Dd}^{1/\chi} &= \zeta(1-\zeta)^{\frac{1-\chi}{\chi}} H(0)^{-\left(\frac{1-\chi}{\chi}\right)} \\
&\times \left[\int_0^R e^{\left(\frac{1-\chi}{\chi}\right)\int_0^x d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x}e^{-\delta x}dx + k^{1/\rho\chi}\int_R^T e^{\left(\frac{1-\chi}{\chi}\right)\int_0^x d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x}e^{-\delta x}dx\right]. \qquad (102)
\end{aligned}$$

8.7 Scenarios E and F

Figure 1 shows scenarios E and F. In these scenarios initial health $H(0)$ is below the initial health threshold $H_*(0)$. The simplified Grossman model that we employ here allows for complete repair. Case and Deaton (2003) point out that employing such technology is not realistic. Indeed wealthy individuals may have high health threshold levels and the ability to afford any kind of health investment, but they may not necessary be able to repair all types of poor health (e.g., cancer, aids, various disabilities such as blindness etc). Simply stated, not every illness has a cure. Further, while health in the Grossman model cannot deteriote faster than the deterioration rate $d(t)$ there is no intrinsic constraint on the rate at which health can be repaired. As such, in scenarios E and F individuals will seek to repair their health instantaneously when they enter the workforce at age 20 ($t = 0$), effectively substituting initial assets $A(0)$ for improved initial health $H(0)$ such that initial health equals the initial health threshold (the initial minimally productive level of health) $H(0) = H_*(0)$. An alternative interpretation is that individuals invest in health $m(t)$ well before they enter the workforce at age 20 ($t = 0$) to ensure their health is at the initial health threshold $H_*(0)$ at $t = 0$. Before they enter the workforce individuals don't consume yet (or at least consumption is paid for by their parents / caretakers) and have no assets $A(t)$ yet. In this case the end result is the same as if health investment were made in an infinitesimally small period of time at $t = 0$. We assume that individuals pay for the health investment themselves, i.e. they start with lower initial assets $A_*(0) = A(0) - p(0)m_*(0)$, where $m_*(0)$ is the quantity of health investment needed to arrive from initial health $H(0)$ to the initial health threshold $H_*(0)$. Approximating this initial health investment by a delta function, $m(t) = m_*(0)\delta(t - 0)$ (i.e., mathematically investment takes place at $t = 0$ during an infinitesimally small period of time) we find:

$$H_*(0) = H(0) + \mu(0)m_*(0), \tag{103}$$

and

$$\begin{aligned}
A_*(0) &= A(0) - p(0)m_*(0) \\
&= A(0) - \frac{p(0)}{\mu(0)}\left[H_*(0) - H(0)\right] \\
&= A(0) - \frac{p(0)}{\mu(0)}(1-\zeta)\Lambda_{E,F}\left[\pi_H(0) - \varphi(0)\right]^{-\chi} + \frac{p(0)}{\mu(0)}H(0), \qquad (104)
\end{aligned}$$

where $\Lambda_{E,F}$ denotes either Λ_E for scenario E or Λ_F for scenario F.

The solution for scenarios E and F can be derived from solutions A and B, respectively, by setting $t_1 = 0$ and replacing initial assets $A(0)$ and initial health $H(0)$ with the above expressions for $A_*(0)$ and $H_*(0)$. We leave this excercise to the reader.

8.8 Benefits transformation

Assuming that pension benefits accumulate over time as a fraction α of wages is invested with a return on investment of δ (the interest rate) as in equation (23) life-time income

$$\begin{aligned}
\int_0^T Y[H(t)]dt &= \int_0^R w(t)e^{-\delta t}dt + \int_R^T be^{-\delta t}dt \\
&= \int_0^R w_0(t)e^{-\delta t}dt + \frac{b}{\delta}(e^{-\delta R} - e^{-\delta T}) + \int_0^R \varphi(t)H(t)e^{-\delta t}dt, \qquad (105)
\end{aligned}$$

in the new formulation becomes

$$\begin{aligned}
\int_0^T Y[H(t)]dt &= (1-\alpha)\int_0^R w(t)e^{-\delta t}dt + \int_R^T be^{-\delta t}dt \\
&= (1-\alpha)\int_0^R w_0(t)e^{-\delta t}dt + \frac{1}{\delta}\left[b_0 + f(R)\alpha\int_0^R w_0(t)e^{\delta t}dt\right](e^{-\delta R} - e^{-\delta T}) \\
&+ \int_0^R \varphi(t)\left[(1-\alpha) + f(R)\frac{\alpha}{\delta}\left(e^{-\delta R} - e^{-\delta T}\right)e^{2\delta t}\right]H(t)e^{-\delta t}dt. \qquad (106)
\end{aligned}$$

Comparing (105) with (106) leads to the identifications made in (24). Note further that the transformations in (24) also preserve the form of the Lagrangean (5 and 27) and that the transformations are independent of the control variables $C(t)$ and $m(t)$. Thus the original solutions remain valid with the transformations as long as one includes the derivative of φ when calculating health investment (equations 14, 15, 46 and 47),

$$\dot{\varphi} = 2\varphi f(R)\alpha\left(e^{-\delta R} - e^{-\delta T}\right)e^{2\delta t}. \qquad (107)$$

9 References

Adams, P., D. McFadden, M.D. Hurd, A. Merril, T. Ribeiro (2003), "Tests for Direct Causal Paths between Health and Socioeconomic Status," Journal of Econometrics, 112 (1), pp. 3-56.

Banks, James, Richard Blundell and Sarah Tanner (1998), "Is There a Retirement-Savings Puzzle?", American Economic Review, 88 (4), 769-788.

Blau, David M. (2008), "Retirement and Consumption in a Life Cycle Model", Journal of Labor Economics, vol. 26, no. 1

Blau, David M. and Ryan Goodstein (2008), "Can Social Security Explain Trends in Labor Force Participation of Older Men in the United States?", in preparation.

Bazzoli, Gloria J. (1985), "The Early Retirement Decision: New Empirical Evidence on the Influence of Health", The Journal of Human Resources, Vol. 20, No. 2. (Spring, 1985), pp. 214-234.

Bernheim, B. Douglas, Jonathan Skinner and Steven Weinberg (2001), "What Accounts for the Variation in Retirement Wealth among U.S. Households?", American Economic Review, 91 (4) 832-857.

Case, A., and A. Deaton (2005), "Broken Down by Work and Sex: How our Health Declines", in: David A. Wise (ed.) (2005), Analyses in the Economics of Aging, The University of Chicago Press, Chicago, pp. 185-212.

Coe, Norma B. and Gema Zamarro, "Retirement Effects on Health in Europe", work in progress.

Dave, Dhaval, Inas Rashad, and Jasmina Spasojevic (2006), "The Effects of Retirement on Physical and Mental Health Outcomes", NBER Working Paper 12123.

Dwyer, Debra Sabatini and Olivia S. Mitchell, "Health problems as determinants of retirement: Are self-rated measures endogenous?," Journal of Health Economics, Volume 18, Issue 2, , April 1999, Pages 173-193.

Ehrlich, Isaac and Hiroyuki Chuma (1990), "A Model of the Demand for Longevity and the Value of Life Extension", The Journal of Political Economy, Vol. 98, No. 4, pp. 761-782

Galama, T.J. and Kapteyn, A. (2009), "Grossman's Health Threshold", Forthcoming.

Goldman, D.P. and J.P. Smith (2002), “Can Patient Self-Management Help Explain the SES Health Gradient?”, PNAS 99, 10929-10934.

Grossman, Michael. (1972a), “The Demand for Health-A theoretical and Empirical Investigation”. New York: National Bureau of Economic Research.

Grossman, Michael. (1972b), “On the Concept of Health Capital and the Demand for Health”, The Journal of Political Economy, Vol. 80, No. 2, pp. 223-255.

Grossman, Michael. (2000), “The Human Capital Model”, in *Handbook of Health Economics*, Volume 1, pp. 347-408, A.J. Culyer and J.P. Newhouse (Eds.), Elsevier Science.

Gruber, J., and D. Wise (eds) (1999), Social Security and Retirement Around the World, University of Chicago Press: Chicago.

Hurd, Michael and Susann Rohwedder (2003), ”The Retirement-Consumption Puzzle: Anticipated and Actual Declines in Spending at Retirement”, National Bureau of Economic Research Working Paper 9586, http://www.nber.org/papers/w9586

Hurd, Michael and Susann Rohwedder (2006), “Some Answers to the Retirement-Consumption Puzzle”, National Bureau of Economic Research Working Paper 12057, http://www.nber.org/papers/w12057

Kapteyn, A. and C. Panis (2005), “Institutions and Saving for Retirement: Comparing the United States, Italy, and the Netherlands” in: David A. Wise (ed.) (2005), Analyses in the Economics of Aging, The University of Chicago Press, Chicago, pp. 281-316.

Muurinen, Jaana-Marja (1982), “Demand for health: a generalized Grossman model”, Journal of Health Economics 1, p5-28.

Muurinen, Jaana-Marja, and Julian Le Grand (1985), “The Economic Analysis of Inequalities in Health”, Soc. Sci. Med, Vol. 20, No. 10, pp. 1029-1035

OECD (2005), “Pensions at a Glance: Public Policies Across OECD Countries”

Picone, G., M.Uribe and R.M. Wilson (1998), ”The effect of uncertainty on the demand for medical care, health capital and wealth” , Journal of Health Economics 17:171-186.

Ried, Walter (1998), “Comparative dynamic analysis of the full Grossman model”, Journal of Health Economics 17, p383425.

Seierstad, A. and K. Sydsaeter, ”Optimal Control Theory With Economic Applications”, in *Advanced Texbooks in Economics*, Volume 24, Series Editors: C.J. Bliss and M.D. Intriligator, Elsevier, North Holland, 1987

Stock, J.H. and D.A. Wise (1990), “Pensions, the Option Value of Work, and Retirement,” Econometrica 58, 1151-1180.

Stock, J.H. and D.A. Wise (1990), “The Pension Inducement to Retire: An Option Value Analysis,” in: D.A. Wise (ed.) Issues in the Economics of Aging, Chicago: University of Chicago Press, 1990, 205-30.

Wagstaff, A. (1986), The demand for health: Some new empirical evidence. , Journal of Health Economics, Vol. 5, pp. 195-233.

Wolfe, John R. (1985), “A Model of Declining Health and Retirement”, The Journal of Political Economy, Vol. 93, No. 6, pp. 1258-1267

Zweifel, P., Breyer, F., (1997), *Health Economics*, Oxford University Press, New York.