

Nutritious or Nuisance on Net?

Values of Natural Capital in a Competitive Terrestrial Rangeland Ecosystem

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Abstract

Rangelands across the western United States provide considerable benefits. However, with nitrogen deposition, climate change and invasive species, the composition of these grassland ecosystems have experienced changes in the flow of services that they provide and, in turn, in the value of the natural capital that characterized the ecosystem. Expanding on the methods developed in Fenichel and Abbott (2014) in conjunction with a General Equilibrium Ecosystem Model (GEEM), we illustrate how the value of these grassland ecosystems may change as invasive species alter the landscape. In particular, by modeling three grass species as opposed to a single grass stock, we are able to identify two major sources of value to the system. First, grasses that provide nutritional value to cattle stocked on the rangeland provide a final ecosystem service value. Second, different populations of grass species competing for energy against other species can both provide both positive and negative values from these indirect, supporting ecosystem services. Our results show that even with positive nutrition values, competition effects can render the total marginal value of some grasses negative depending on the state of the ecosystem. Finally, we provide a validation check to the method by noting that the value of the land estimated by the extension of the Fenichel and Abbott method for a relatively intact native grass ecosystem corresponds almost exactly to land prices observed for agricultural rangeland in Wyoming.

Table of Contents

Abstract.....	iii
Figures.....	v
Tables.....	vi
Acknowledgments.....	vii
Abbreviations.....	viii
1. Introduction.....	1
2. Rangeland Ecosystem Model.....	4
2.1 Individual Vegetation Model and Per-Period Equilibria.....	4
2.2 Plant Population Dynamics.....	5
2.3 Cattle and Grazing.....	5
3. Numerical Approximation of the Value Function.....	8
3.1 Recurrence of Chebyshev Polynomials and Their Derivatives.....	9
3.2 Using the Matrix Method to Approximate the Value Function.....	11
4. Choice of Nodes and Evaluation of Ecosystem Model.....	13
5. Results.....	14
5.1 Overall Value of Rangeland.....	14
5.2 Accounting Prices of Each Grass Species.....	15
6. Discussion and Conclusion.....	19
References.....	21

Figures

Figure 1: Estimated Ecosystem Value as a Function of Cheatgrass Stocks (dollars/hectare)	15
Figure 2: Prices of Native Grass ($\$/10^7$ stems/hectare) with Moderate Leafy Spurge Populations (10^6 stems/hectare).....	17
Figure 3: Price of Cheatgrass ($\$/10^7$ stems/hectare) with Moderate Native Grass Populations (10^6 stems/hectare)	18

Tables

Table 1. Values Used in the Stocked Grassland Ecosystem Model	7
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Abbreviations

GEEM General Equilibrium Ecosystem Model

1. Introduction

Human well-being depends heavily on ecosystem services that are derived from natural capital. Although there has been considerable discussion and estimation of the value of ecosystem service flows, relatively little literature exists on how to value natural assets.¹ However, Fenichel and Abbott (2014) proposed a method to estimate the value of natural capital that is consistent with Jorgenson's (1963) theory of capital in private markets. In essence, the natural capital framework allows one to recover the net present value of a marginal change in natural capital stocks, which includes the value of all of the ecosystem services provided by those stocks. Importantly, this method does not rely on first-best optimal behavior by the economic agents in the system, but can accommodate behavioral adaptivity to changing states of the system by specifying the "economic program", or assumed human-system response (based, e.g., on empirical data).

Fenichel and Abbott have used their method to estimate the value of fisheries stocks in the Gulf of Mexico, while Fenichel et al. (2016) have used this framework to consider the value of groundwater in Kansas. Bond (2017) extended the method to consider multiple, linked natural capital stocks (fish biomass and wetlands) in the Gulf of Mexico, allowing for wetland stocks to provide both final ecosystem services (storm protection) and intermediate supporting services (fishery habitat).

In this paper, we continue this line of research into valuing natural capital. Rather than simply focusing on an aggregate of species as in Fenichel and Abbott (2014) and Bond (2017), this work extends the method to consider multiple species within a single terrestrial trophic layer as well as multiple trophic layers within an ecosystem. Specifically, we model a rangeland ecosystem with two primary trophic layers – cattle and vegetation – and consider three multiple competing species within the latter. We do so using a previously-published General Equilibrium Ecosystem Model (Finnoff, Strong, and Tschirhart, 2008) that allows for a much more complex representation of ecosystem dynamics in the context of natural capital valuation than in past literature, and provides the opportunity to explore how the role of supporting ecosystem services with complex interspecies relationships affects overall value. It also shows how relatively complex ecological models can work with natural capital valuation methods to capture values related to complex ecological interactions.

A secondary contribution of the paper is to show that the set of equations that govern the accounting prices of multiple natural capital stocks can be represented as a system of linear equations that can easily be solved through familiar matrix inversion or regression methods. Our hope is that by showing that the system is linear in estimation parameters even for complex,

¹ Notable exceptions include Barbier (2007) and Finnoff and Tschirhart (2003) and the citations therein.

multi-state systems, future researchers will face lower transaction costs when applying these methods.

The motivation behind this exercise is the livestock cattle industry in the western United States and Canada, which uses several natural capital stocks as inputs into the production process; namely, grasses. Until recently, much of the biomass production from native grass species such as blue grama (*Bouteloua gracilis*) and Buffalo Grass (*Bouteloua dactyloides*) was sufficient to support the demands for caloric intake necessary to remain profitably. Recently, however, climate change, nitrogen deposition, over grazing and other factors have resulted in cheatgrass (*Bromus tectorum*) gaining a competitive biological advantage (Strong and Oliver, 2014) and becoming a significant invasive species problem in Western states.

Although cattle are able to consume cheatgrass, the caloric content is roughly 75% that of native grasses (Huffaker and Cooper, 1995). At first glance, this would suggest that the value of cheatgrass on the rangeland would reduce the rangeland's capital value by roughly 25% compared to a native grass dominated landscape. However, as the current paper illustrates, this interpretation is based solely on the input-output relationship directly related to cattle production, and is not the complete story due to the dynamic competitive effects that take place within the ecosystem.

One of the key insights of this work is that even economically valuable species may have negative stock prices (accounting values) when there is competition within a trophic layer with a more valuable species.² In our case, two of the three plant species (native grass and cheatgrass) are consumed by cattle and the third is not, with cheatgrass having lower caloric content per unit land. In addition to positive production value, there is also an ecosystem competition effect that decreases the growth of the preferred species from a caloric content perspective. In some areas of the state space, the nutritional effect dominates (resulting in a positive accounting price), while in other areas, the competition effect dominates. Importantly, the value of land is derived from the sum of the net present values of the services provided by each grass stock jointly through the final ecosystem service of nutrition (as an input into the cattle production process) and the values related to supporting ecosystem function (the joint biological production of the three grass stocks). When one focuses solely on the ecosystem services related to cattle production and ignores the ecosystem function driven by the natural capital composition, these interactions become lost. Although these outcomes are presented in terms of ecosystems, these same properties may exist more broadly in natural systems including hydrologic systems where there are surface and groundwater interactions in terms of both water quantity and quality.

This paper is organized as follows. The next chapter summarizes the model in Finnoff, Strong, and Tschirhart (2008) for a grassland ecosystem. In Chapter 3, we extend the valuation model set up of Fenichel and Abbott (2014) and Bond (2017) to incorporate additional dimensionality of the grassland ecosystem. Chapter 4 details the estimation procedure that

² The accounting price is defined as the overall willingness to pay for a marginal unit of the capital stock.

combines these two models. Results are discussed in Chapter 5. Chapter 6 provides some concluding remarks.

2. Rangeland Ecosystem Model

In this section, we summarize the model of a western United States rangeland ecosystem affected by two invasive plant species, cheatgrass and leafy spurge, originally developed in Finnoff, Strong and Tschirhart (2008) and extended in Strong and Oliver (2014). This type of model is known as a General Equilibrium Ecosystem Model (GEEM), and has been applied in a variety of different ecosystems including: marine (Finnoff and Tschirhart, 2003 and 2008), rangeland (Hussain and Tschirhart, 2013), house mice (Kim, Tschirhart and Buskirk, 2007), plant competition (Finnoff and Tschirhart, 2005 and 2007), and the Great Salt Lake ecosystem (Finnoff and Caplan, 2004). In GEEMs, individual biological agents maximize net energy over a set of endogenously-determined energy prices to determine per-period equilibria. The dynamics of the system are determined by net energy of each species, with positive (negative) balances resulting in population increases (declines).

The three plant species considered are buffalo grass, a native perennial (g), cheatgrass, a non-native annual (c), and leafy spurge, a non-native perennial (s). The main distinction from a cattle grazing perspective of these three species is that cattle are able to consume both buffalo grass and cheatgrass, but cheatgrass has a lower nutritional content than buffalo grass. Leafy spurge is not palatable to cattle but does have ecosystem consequences in terms of biological competition with other plant species for space.

2.1 Individual Vegetation Model and Per-Period Equilibria

A representative plant from species $i = g, c, s$, is assumed to maximize fitness net energy for each period, $R_i(x_{it})$, through a choice of biomass x_{it} . The form of the net energy function is the same for all species but the physiological parameters differ across species. Individuals compete for access to sunlight both within species and across species within the same fixed area of the overall ecosystem. More specifically, the form of the net energy function is given by (1):

$$R(x_{it} | SEL_t) = (I_0 - SEL_t) x_{it} s_{it} - \alpha_i x_{it}^{q_{ii}} \left((N_t - N_i)^2 + 1 \right) - \beta_i \quad (1)$$

The first term on the right-hand-side of (1) is the total amount of energy produced by the plant through incoming radiation from the sun. I_0 is the incoming irradiation from the sun striking the plants photosynthetically active area (W / cm^2). SEL_t is the periodic “shading energy loss” incurred as other plants in the individual’s vicinity block out or diffuse a portion of the incoming light. SEL_t plays the role of a competitive “price” for access to sunlight or land. s_i converts biomass to energy.

The second term in (1) is variable respiration. It is assumed that there is an optimal level of an environmental stress N_i for each species, with stress coming from differences between the optimal and realized level (N_i). α_i converts biomass to respiration energy and is a species-specific constant calibrated by Finnoff, Strong and Tschirhart (2008). β_i is fixed respiration and does not change with changes in biomass size across periods.

SEL_t is determined as a “market clearing price” between the species when the area is completely filled with plants. When there is only a single species of plant on the landscape, SEL_t is determined by:

$$SEL_t = I_0 \left(1 - e^{-k_i L_{it}}\right), \quad (1)$$

where k_i is a light extinction coefficient and $L_{it} = \frac{n_{it} x_{it} s_i}{A}$ with n_{it} the number of individuals of species i and A is the area under consideration. When the landscape includes multiple species, this generalizes to

$$\sum_i \frac{n_{it} x_{it} s_i}{L_{it}} \leq A \quad (3)$$

An equilibrium within a period is defined when all species are maximizing net energy and the area is filled. If the area is not filled then $SEL_t = SEL^{NC}$, the minimum level of shading when plants only shade themselves.

2.2 Plant Population Dynamics

Unlike most ecological models used in economics, population growth is determined endogenously by the residual energy of the individuals within each species. If the net energy is positive (negative), population is assumed to increase (decrease). The discrete time population updating is given by:

$$n_{i,t+1} - n_{i,t} = \frac{n_{i,t}}{l_i} \left[\frac{R_{i,t} + f_{i,t}(x_{i,t}, N_t)}{f_{i,t}^s(x_i^s)} - 1 \right], \quad (4)$$

where l_i is the average lifespan of species i , x_i^s is the optimal biomass that occurs in steady state when only species i is present, $f_{i,t}(x_{i,t}, N_t)$ is the variable respiration and $f_{i,t}^s(x_i^s)$ is optimal respiration. In the steady state, $R_{i,t}$ is zero, $x_{i,t} = x_i^s$, and there is zero net population growth.

2.3 Cattle and Grazing

Cattle are added to the model to represent a “stocker” operation of the western mountain states. Cattle are stocked on the rangeland for the sole purpose of summer weight gain. Cattle function in a similar manner to plants in that they are maximizing fitness net energy while

preying on different species of plants. Strong and Oliver (2014) have considered alternatives to fitness maximization, but we adopt the simplest form here. The net energy function for cattle is given by:

$$R_{kt}(x_{kgt}, x_{kct}) = (e_g - e_{kgt})x_{kgt} + (e_c - e_{kct})x_{kct} - f_k(x_{kgt}, x_{kct}) - \beta_k. \quad (5)$$

The first two terms on the right-hand side are the net inflow of energy from consumption of the different grass species and these are the choice variables of the cattle. The parameters e_g and e_c are the embodied energy in each of the grass species and e_{kgt} and e_{kct} are the predation costs that cattle have to “pay” to prey on a particular species of grass. As a plant species become increasingly sparse (dense), the energy expended to forage increases (decreases). The third and fourth terms are respiration terms. For the third, we use a second order Taylor expansion in the biomasses of native grass and cheatgrass of the form:

$$f_k(x_{kgt}, x_{kct}) = \alpha_{kg}x_{kgt} + \alpha_{kc}x_{kct} + 0.5\alpha_k(x_{kgt}x_{kct} + x_{kgt}^2 + x_{kct}^2), \quad (6)$$

where the α 's are calibrated from the best available data as in Finnoff, Strong and Tschirhart (2008).

Given the introduction of cattle, the plants' net photosynthetic energy function must be modified to represent the outflow of energy from being foraged. In particular, the plants' energy function is appended by:

$$-e_i d_i x_{it}^{0.5}, \quad (7)$$

where $d_i x_{it}$ is per capita biomass supplied by species i to cattle and d_i is a calibrated parameter.

From the assumption that cattle are fitness net energy maximizers, two biomass demands or harvest functions arise from the first order conditions of (5). These are:

$$\begin{aligned} x_{kgt} &= \frac{4}{3} \left[\frac{e_g - e_{kgt} - \alpha_{kg}}{\alpha_k} - \frac{e_c - e_{kct} - \alpha_{kc}}{2\alpha_k} \right] \\ x_{kct} &= \frac{4}{3} \left[\frac{e_c - e_{kct} - \alpha_{kc}}{\alpha_k} - \frac{e_g - e_{kgt} - \alpha_{kg}}{2\alpha_k} \right]. \end{aligned} \quad (8)$$

In order to convert energy to weight gain, we follow Finnoff, Strong and Tschirhart (2008) and define weight gain as:

$$WG_t = \frac{182}{0.965} \left[\frac{R_{kt}(\cdot)}{182(1000)(0.635)(0.891(SBW))^{0.75}} \right]^{1.097} \quad (9)$$

Lastly, we relate weight gain to per period profits from the assumption of a stocker operation. Annual profit for the rancher is given by:

$$\Pi_t = p^f [WG_t + w_k] n_{kt} - p^s w_k n_{kt}, \quad (10)$$

where p^f is the final price of cattle per kg, p^s is the purchase price per kg of cattle, w_k is the initial weight of the purchased cattle assumed to be , and n_{kt} is the (exogenous and fixed) number

of cattle stocked.³ All of the models parameters are calibrated as in Finnoff, Strong and Tschirhart (2008), and represented in Table 1.

Table 1. Values Used in the Stocked Grassland Ecosystem Model

Variable	Native Grass	Cheatgrass	Leafy Spurge	Cattle
Mamimum Population (N/ha)	2.5×10^7	5×10^7	1.5×10^6	0.25
Biomass (X_{ij})	0.0002	0.00004	0.0035	1307-2818
Shaded leaf area (s_i)	10	10	11.429	--
Non-photosynthesizing respiration (r_i)	0.39	0.4	0.45	--
Extinction coefficient (k_i)	0.3	0.3	0.5	--
Variable respiration (α_i)	5.7670×10^5	3.773×10^6	24,015	$\alpha_{kg}, \alpha_{kc} = 1, \alpha_k = 1.139$
Fixed respiration (β_i)	1,799,313	483,495	26,476,080	1,250,554
Leaf area index (L_i)	1.5	0.6	1.8	--
Shaded energy loss in steady state (SEL_i)	1,310,989	59,596	214,696	--
Respiration power term (q_i)	2	2	2	--
Gross energy Content (e_i)	4,200	3,150	--	--
Average longevity (l_i)	5	2	10	--
Weight (w_i)	0.0002	0.00004	0.0035	273-589
Ideal nitrogen (N_i)	3	4	5	--
Herbivory(d_i)	1.257	1.054	--	--NA
Final price of Cattle (p^f)	--	--	--	\$1.50
Stocker price of Cattle (p^s)	--	--	--	\$1.98

³ Although we assume that stocking rates and ecosystem state are decoupled in this model in order to focus on the competitive ecosystem effects, the natural capital framework could accommodate endogenous stocking rules (either optimal or sub-optimal) that depend on the observable states of each of the three species.

3. Numerical Approximation of the Value Function

In this section, we provide the general outline for estimating the value of a vector of natural capital stocks in a manner similar to that of Fenichel and Abbott (2014). Consider a vector of natural capital stocks $n(t)$ that are related either directly in terms of predator-prey relationship or indirectly through trophic layer competition, as represented by a system of differential equations. Next, suppose that there exists a time autonomous behavioral response of the human system given by $h(n(t))$. In this case, the value function describing the system at time t is given by:

$$V(n(t)) = \int_t^{\infty} e^{-\delta(\tau-t)} \Pi(n(\tau), h(n(\tau))), \quad (11)$$

where Π is a real-valued instantaneous welfare function and $\delta > 0$ is the social discount rate. Given this formulation, the marginal values, or accounting prices, of each stock n_i are defined as $p_{n_i}(n) = \frac{\partial V(n)}{\partial n_i}$, $i = 1, 2, \dots, N$. These prices are, most generally, functions of each state variable represented in the ecosystem model.

Differentiating (11) with respect to t and solving for $V(n)$ yields:

$$V(n) = \frac{\Pi(n) + \sum_i p_{n_i}(n) \dot{n}_i}{\delta} \quad (12)$$

Equation (12) states that the value of being at state n is equal to the net present value of the instantaneous net benefits plus the value of the change in each of the capital stocks.

Differentiating Equation (12) with respect to each n_i yields:

$$\frac{\partial V}{\partial n_i} = p_{n_i} = \sum_j \frac{1}{\delta} \left(\Pi_{n_i} + \frac{\partial p_{n_i}}{\partial n_i} \dot{n}_i + p_{n_i} \frac{\partial \dot{n}_i}{\partial n_j} \right) = \sum_j \frac{1}{\delta} \left(\Pi_{n_i} + \dot{p}_{n_i} + p_{n_i} \frac{\partial \dot{n}_i}{\partial n_j} \right) \quad (13)$$

Mathematically, the accounting prices for each stock of natural capital are defined by a set of multi-dimensional first-order differential equations whose solution can be approximated using numerical methods by either a) approximating the value function directly by using (12) and then differentiating; or b) estimating the system implied by (13) by parameterizing each accounting

price function. Fenichel and Abbott (2014) and Bond (2017) used the former; here, we opt for the latter approach.⁴

3.1 Recurrence of Chebyshev Polynomials and Their Derivatives

We use the projection method detailed in Judd (1992) and expand the single-dimensional Chebyshev matrix method documented in Sezer and Kaynak (1996), Caporale and Cerrato (2010) and Mosino (2012) to solve for the value function that satisfies (12). Of particular interest is that this method does not require complex optimization routines, but rather simple matrix manipulations or equivalently, linear regression, that can quickly and easily approximate the multi-dimensional value function and the prices of fundamental interest. Our goal is to estimate the price of each stock as functions of the state variables by using Chebyshev polynomials.

The key insight of this method is that Chebyshev polynomials and their associated partial derivatives can be represented with a common set of basis functions and estimated coefficients, exploiting the recursive properties of the polynomials. In particular, for the single dimensional case, an approximation of the value function $V(n)$ using a Chebyshev series is:

$$\hat{V}(z) = \sum_{j=0}^J a_j \phi_j(z) \quad (14)$$

where $z \in [-1, 1]$, $\phi_j(z), j = 0, 1, \dots, J$ is the j th Chebyshev polynomial of the first kind, and a_j are coefficients to be estimated. The Chebyshev polynomials of the first kind are defined by the recurrence relation:

$$\begin{aligned} \phi_0(z) &= 1 \\ \phi_1(z) &= \cos(z) \\ \phi_{j+1}(z) &= 2 \cos(z) \phi_j(z) - \phi_{j-1}(z) \end{aligned} \quad (15)$$

By virtue of the nature of the Chebyshev formulation, Sezer and Kaynak (1996) and Mosino (2012) show that the n th derivative of $\hat{V}^n(z)$ with respect to z takes the form

$$\hat{V}^n(z) = 2^n \Phi(z) M^n a, \quad (16)$$

where $\Phi(z)$ is a $1 \times (J+1)$ vector such that $\Phi(z) = [\phi_0(z) \phi_1(z) \dots \phi_J(z)]$, a is the $(J+1) \times 1$ vector of coefficients $a = [a_0 \ a_1 \ \dots \ a_J]'$, and M is a $(J+1) \times (J+1)$ translation matrix as defined in Sezer and Kaynak (1996) that, when applied to a , creates the structure of effective coefficients on each polynomial for the derivative function in terms of the $J+1$ coefficients, taking advantage of the recursive structure of each term. More specifically, the matrix \mathbf{M} is formed using:

⁴ Parameterizing each accounting price function has the advantage of limiting variations in the second derivatives of the value function as a result of numerical approximation and highlighting the role of capital gains; however, it is computationally more complex than estimating the value function itself.

$$a_j^{n+1} = 2 \sum_{k=1}^{\infty} (j+2k+1) a_{j+2k+1}^n \quad (17)$$

where, in practice, it is assumed that $a_j^n = 0 \forall j > J$ (Sezer and Kaynak, 1996).

Using the current paper's notation, Sezer and Kaynak (1996) show that the matrix \mathbf{M} is defined using the recursion $a_j^{n+1} = 2 \sum_{k=1}^{\infty} (j+2k+1) a_{j+2k+1}^n$ assuming $a_j^n = 0 \forall j > J$:

$$\mathbf{M}_{(J+1) \times (J+1), J \text{ odd}} = \begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{3}{2} & 0 & \frac{5}{2} & \dots & \frac{J}{2} \\ 0 & 0 & 2 & 0 & 4 & 0 & \dots & 0 \\ 0 & 0 & 0 & 3 & 0 & 5 & \dots & J \\ \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & J \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

for odd J and

$$\mathbf{M}_{(J+1) \times (J+1), J \text{ even}} = \begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{3}{2} & 0 & \frac{5}{2} & \dots & 0 \\ 0 & 0 & 2 & 0 & 4 & 0 & \dots & J \\ 0 & 0 & 0 & 3 & 0 & 5 & \dots & 0 \\ \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & J \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

for even J .

The extension to multiple dimensions is straightforward. For tensor product approximation of degree J for several stocks, say (z_1, z_2, \dots, z_N) , the value function is parametrized by:

$$\hat{V}(\mathbf{z}) = \sum_{j_1=0}^J \sum_{j_2=0}^J \dots \sum_{j_N=0}^J a_{j_1 j_2 \dots j_N} \phi_{j_1}(z_1) \phi_{j_2}(z_2) \dots \phi_{j_N}(z_N). \quad (18)$$

For the partial derivatives of (18) with respect to some z_j , the recursion now takes the form:

$$a_{j_1 j_2 \dots j_N}^{n+1} = 2 \sum_{k=1}^{\infty} (j+2k+1) a_{j_1 j_2 \dots j+2k+1 \dots j_N}^n \quad (19)$$

which defines the terms of the N matrices $M_i, i = 1, 2, \dots, N$, corresponding to the derivative of each state. In other words, if $\Phi(z)$ is the $1 \times (J+1)^N$ tensor product associated with the terms in (18), then $\frac{\partial^n \hat{V}(z)}{\partial z_i^n} = 2^n \Phi(z) (\mathbf{M}_i)^n \mathbf{a}$.

However, as shown in Judd (1992) and Cai (2009), a complete polynomial specification in which the sum of the subscripts on the basis function are restricted to be less than or equal to J provides a more parsimonious approximation without much loss of accuracy. This is equivalent to assuming that $a_{j_1 j_2 \dots j_N} = 0 \forall (j_1 + j_2 + \dots + j_N > J)$ in both (18) and the derivative calculations.

The number of terms in the approximation is then reduced to $P = \frac{(N+J)!}{N!J!}$. We use a complete polynomial specification in this paper.

3.2 Using the Matrix Method to Approximate the Value Function

To recover the coefficients of the estimated value function, the appropriate approximation is substituted into (12) following the conversion of the state variable measurements to $[-1, 1]$:

$$\hat{V}(z_1, z_2, \dots, z_N) = \frac{1}{\delta} \left[\Pi(\cdot) + \sum_i \frac{\partial \hat{V}(\cdot)}{\partial z_i} \dot{z}_i(\cdot) \right] \quad (20)$$

Following Mosino (2012), we write this as a system of linear equations in the unknown coefficients as follows. First, assume that the approximation will occur at some number $M^N > P$ of the evaluation points (possibly Chebyshev nodes). Define \mathbf{w} as the $M^N \times 1$ vector associated with each evaluation point of $\delta^{-1} \mathbf{w}(z_m)$ and $\phi(z_m)$ as the $M^N \times P$ matrix of complete polynomial terms, with each row corresponding to a point in the state space. Let \mathbf{c} be the $P \times 1$ matrix of coefficients to be estimated, and \tilde{M}_i be the $P \times P$ translation matrix associated with each of the N stocks. As such, the estimated value function at each evaluation point is:

$$\hat{V} = \phi(z_m) \mathbf{c},$$

and the estimated derivative (and thus, accounting price) associated with each stock at each evaluation point is

$$\hat{V}_i^{(1)} = 2 \phi(z_m) \tilde{M}_i \mathbf{c}.$$

Substituting these expression into (20) and rearranging yields the following system of linear equations:

$$\left[\phi(z_m) - \frac{2}{\delta} \sum_i \text{diag} \left(\dot{n}_i(z_m) \right) \phi(z_m) \tilde{M}_i \right] \mathbf{c} = \delta^{-1} \mathbf{w}(z_m), \quad (21)$$

where $\text{diag}\left(\dot{n}_i(z_m)\right)$ is the $M^N \times M^N$ diagonal vector of state transition values for state i at each evaluation point. The unknown coefficients \mathbf{c} can be recovered using standard matrix or linear regression techniques depending on the number of evaluation points.

4. Choice of Nodes and Evaluation of Ecosystem Model

In order to evaluate the per-period profit and transition dynamics at each of the nodes there are two additional pieces of information that we need for the ecosystem model. First, there is an environmental parameter within the ecosystem model that must be chosen. In Finnoff, Strong, and Tschirhart (2008), the interpretation of N_i is the level of soil nitrogen and acts as a differential stress on each of the species of plants. We have chosen this parameter to be fixed across time and set it to 3.6 which will place a stress on each of the species but the stress is smallest for the native grass. At this level of environmental stress, the native grass species will be the dominant species in steady state. Additionally, we need to specify a time-autonomous stocking rule. For simplicity, we have chosen a constant stocking rate of one cattle per four hectares. This corresponds to roughly what Finnoff, Strong, and Tschirhart (2008) define as light stocking and is roughly 25% below what the Natural Resource Conservation Service recommended stocking rate for mixed-grass prairies in southwest Wyoming (Andales *et al.*, 2008; Hart and Ashby, 2008) in order to not force a different equilibrium for the natural equilibrium via the economic program for the given parameterization. We assume a discount rate of 5%.

We specify the nodes of evaluation in each dimension through construction of the Chebyshev nodes for each dimension of the ecosystem model. The Chebyshev nodes are defined on $[-1,1]$ as:

$$z_k = \cos\left(\frac{2k-1}{2n}\pi\right), k = 1, 2, \dots, n, \quad (22)$$

which are then translated to a compact state space in the units of the states in the ecosystem model. In this application, we ultimately choose 13 nodes in each dimension after testing, which when combined with the same number of coefficients per dimension provided a reasonably smooth approximation of the value function with relatively small errors.

The full factorial ($13^3=2,197$) of these individual states were taken as the evaluation points, and the growth in stocks \dot{n}_i and instantaneous profits were calculated numerically at each point (see equation (12)). We specified a complete polynomial of degree 13 (resulting in more evaluation points than coefficients to be estimated) and used regression to solve for the coefficients of the value function using (21), thus approximating the 3-dimensional value function. The slope of this function in any dimension gives the approximate accounting price of each stock.

5. Results

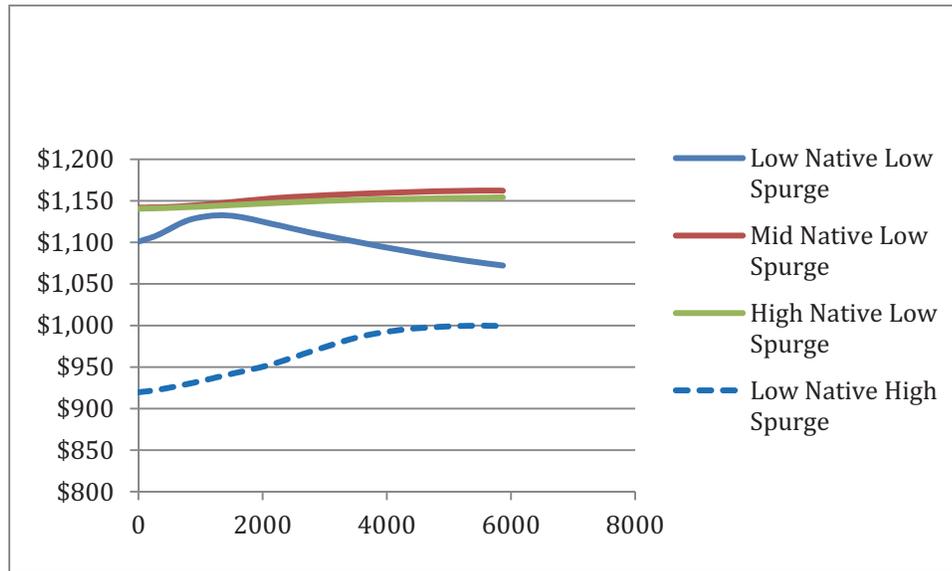
In order to characterize the ecosystem values, we solved for the intra-period equilibrium values from the grassland ecosystem model at each state node, and then used regression analysis to estimate approximate ecosystem values. There are two main (interdependent) sets of results that we highlight below: the overall value of rangeland by ecosystem state, and the accounting price of each ecosystem stock.

5.1 Overall Value of Rangeland

First, as in the semi-arid West, the only source of value in the terrestrial rangeland model is from the sale of cattle which grow (conditional on the stocking rate) in accordance with the state of the ecosystem. As such, the prices revealed through this method should roughly correspond to private land prices in the western United States where we expect landowners to maintain the ecosystem integrity rather than move the ecosystem to a cheatgrass dominated ecosystem with relatively high stocking rates. Figure 1 displays the estimated total value as a function of cheatgrass populations (stems per hectare) with various configurations of leafy spurge native populations. For ecosystems that have low leafy spurge and relatively high level of the native grass, the value of the ecosystem is roughly \$1,150 per hectare which corresponds to roughly \$465 per acre of land. In 2012, USDA estimated that the average price of land in Wyoming was approximately \$450 per acre⁵. Given that the ecosystem model is calibrated to the intermountain west with stocking rates that are roughly consistent with recommendations, this gives us confidence that the model is actually valuing the natural capital and the flow of benefits that accrue to ranchers.

⁵ http://trib.com/business/land-values-skyrocket-in-parts-of-wyoming/article_8ca06d9d-cd57-529c-abe2-e23be007b573.html

Figure 1: Estimated Ecosystem Value as a Function of Cheatgrass Stocks (dollars/hectare)



If, however, native grass stocks are relatively low, the overall value of the system tends to be lower (Figure 1), and this decline is more pronounced the higher is the invasive leafy spurge stock (dashed line). In other words, the marginal effect of an increase in leafy spurge stocks is unambiguously negative when native grass stocks are sufficiently low due to the competition effect and the complete lack of nutritional value of this species for cattle, seen here as the difference between the blue and dashed blue lines. When native grass stocks are sufficiently high, however, the structure of the ecosystem model and the natural limitations on cattle weight gain over time drive the competition effect from leafy spurge to zero, resulting in virtually identical values of land for mid to high-level native grass stocks (conditional on cheatgrass populations), seen here as the difference between the red and green lines.⁶

5.2 Accounting Prices of Each Grass Species

The second set of results focus on the prices of the different species stocks. Recall that the marginal value of the stock is equal to the net present value of the expected flows of services to which it contributes. As such, and because it has no nutritional value for cattle, the sign of the price of leafy spurge is always negative, as it competes with other species for energy and thus lowers the overall nutritional value of the ecosystem.

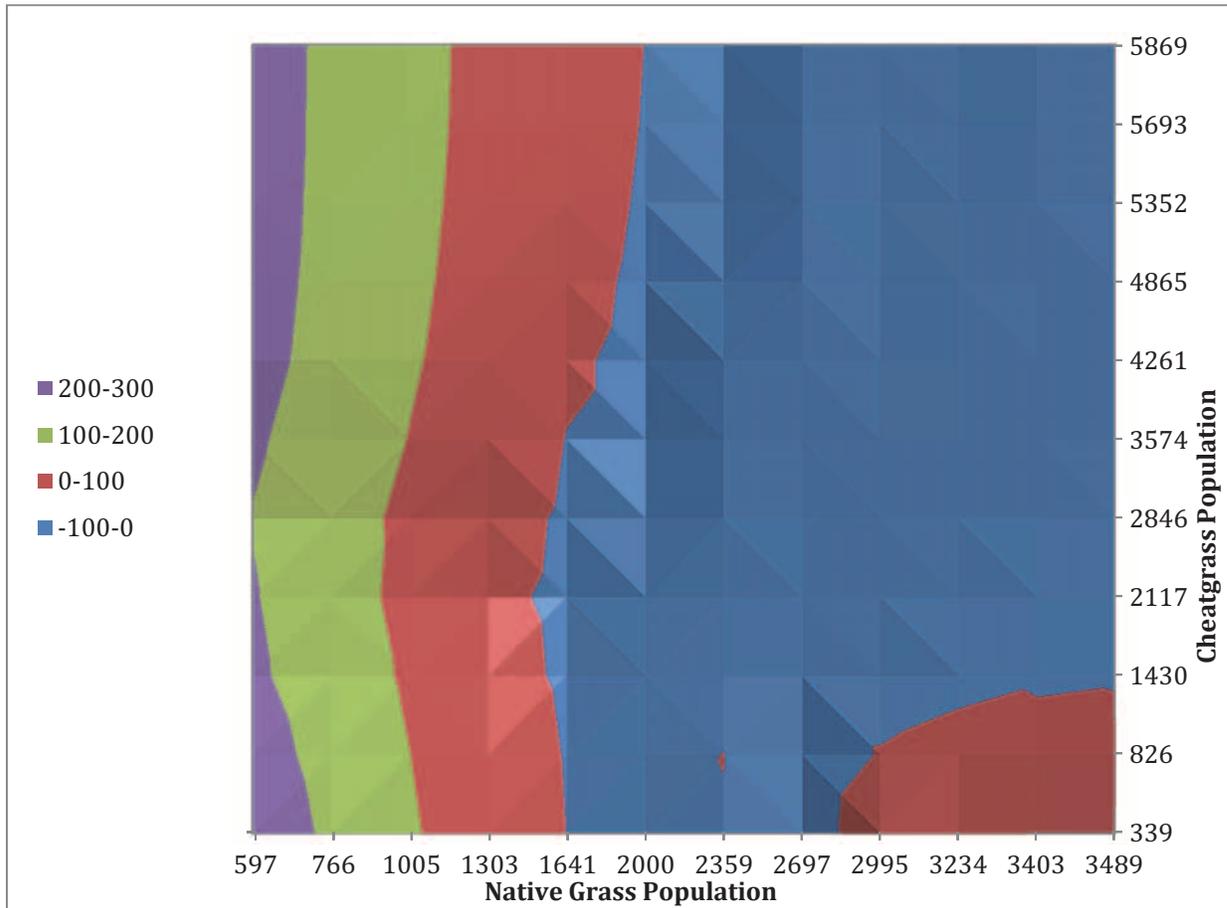
The other two species are more interesting, as values from nutrition interact with the value from competition effect for each plant species. Values from nutrition are relatively

⁶ Given the approximation methods used and the dimensionality of the problem, Figure 1 (and our other reported results) includes approximation error; we thus focus on insights to be gained from the overall quantitative and qualitative nature of the approximation.

straightforward, stemming from the positive effect on cattle weight gain and in turn on profits to the rancher. This suggests an unambiguously positive valuation. The competition effect, however, may be either positive or negative. For example, when the native grass competes with cheatgrass and leafy spurge, marginal values of native grass tend to be positive through the capitalization of increased available energy. However, at the landscape scale, native grass is also competing with itself, which tends to decrease marginal values at relatively high levels of abundance. As such, when the ecosystem has more plants (of any species) than can be supported by carrying capacity, valuations can turn negative as the competition effect outweighs the nutrition effect. Given ecosystem dynamics, then, even a grass species that is most nutritious for cattle, and thus appears to be a “good”, may be negatively valued due to the species’ effect on supporting ecosystem services. In other words, for some natural capital stocks, more is *not* always better.

Figure 2 provides an example by displaying the estimated price of native grass with a moderate level of leafy spurge population. We see that conditional on leafy spurge, native grass accounting prices are positive at lower population levels, but demand is downward-sloping as populations increase regardless of the level of cheatgrass populations. Ultimately, however, there is a threshold (determined by the dynamics of the ecosystem at each stock level) beyond which the marginal value of native grass turns negative due to the competition effect. Interestingly, at relative low levels of cheatgrass, the relative evolutionary strength of native grass at higher population levels of this species more aggressively competes with cheatgrass, reversing the negative valuation and showing the importance of expected system dynamics in the valuation of natural capital.

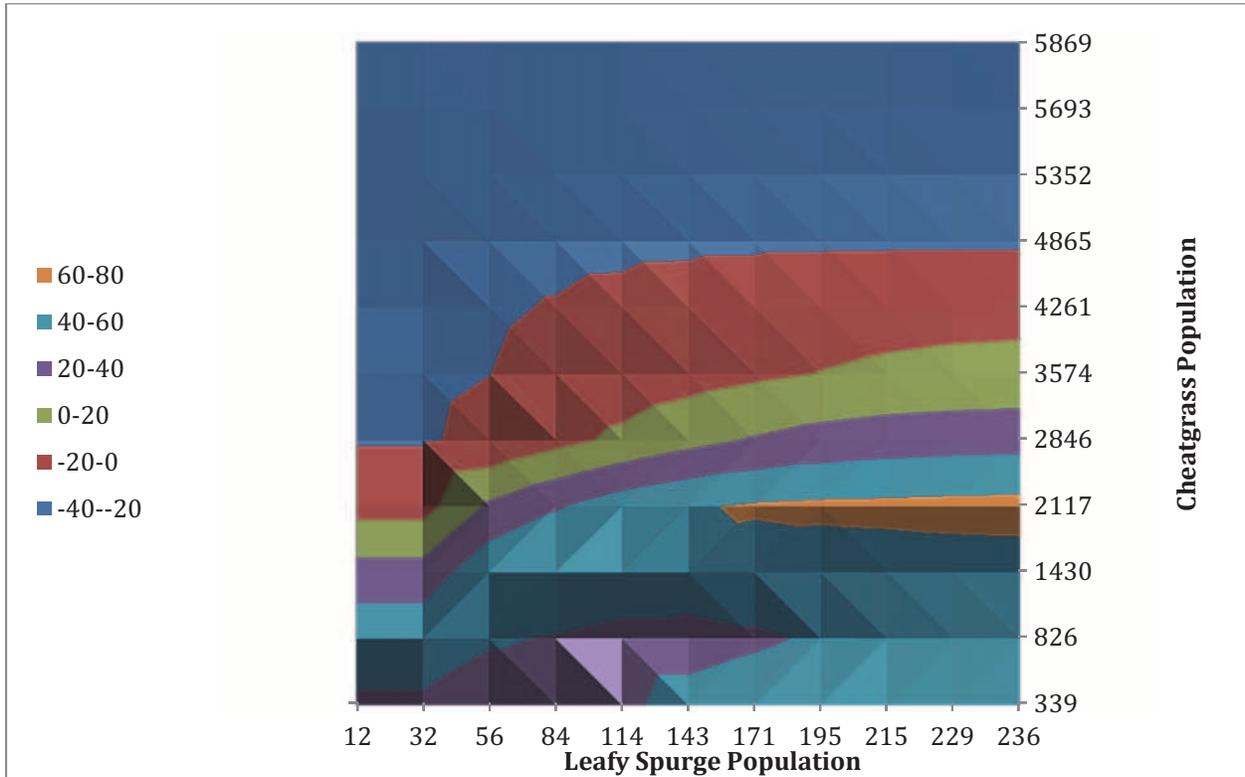
Figure 2: Prices of Native Grass ($\$/10^7$ stems/hectare) with Moderate Leafy Spurge Populations (10^6 stems/hectare)



Next, consider the effect of changes in the population of leafy spurge with moderate levels of the native grass present. Figure 3 displays these results. When the population of cheatgrass is relatively high, the price is negative due to the competition effect, even though cheatgrass can be consumed by cattle. The interesting case is when the cheatgrass population is moderate (around 2000 stems per hectare) and we increase the population of leafy spurge. With relatively high levels of leafy spurge, the price of cheatgrass is relatively high due to its nutritional value relative to leafy spurge and its ability to compete against the non-nutritional species. When the population of leafy spurge is relatively low, however, the competition effect (with native grass) dominates and the price of cheatgrass is negative. In other words, there is a positive effect on price of cheatgrass coming from the nutritional effect and the competition effect with leafy spurge, and a negative effect on price coming from the competition with the native grass. Which terms dominate, in terms of value, depends on the relative abundance of each species and the absolute abundance of all species in the ecosystem. This result shows that natural capital values are likely not only heterogeneous

across space, but also heterogeneous across system states due to the capitalization of the benefits (or costs) of ecosystem support services as the system evolves.

Figure 3: Price of Cheatgrass ($\$/10^7$ stems/hectare) with Moderate Native Grass Populations (10^6 stems/hectare)



6. Discussion and Conclusion

President Obama's October, 2015 memorandum requires Federal agencies to incorporate the value of ecosystem services into their decision-making processes wherever practical and applicable. Given the importance of ecosystem services in sustaining the welfare of the overall human system, this is a laudable objective; however, consistent methods of conceptualizing and estimating these values must be used in order to make informed decisions about the benefits and costs of proposed policy changes.

This paper extends the natural capital valuation framework originally proposed by Fenichel and Abbott (2014) to a multi-trophic, multi-species terrestrial rangeland ecosystem with three dynamically-changing stocks, including nutritional native grass and two invasive species, one of which provides direct production benefits for cattle (cheatgrass) and one which does not (leafy spurge). This example highlights the importance of understanding the overall biophysical linkages (i.e., the system of supporting functional ecosystem services) between natural capital stocks and incorporating those values into the overall price of an individual species (like a single grass species) or an aggregate resource (such as land) whose value depends on multiple interacting stocks.

In particular, for the rangeland values presented here, we use a GEEM to show that the interaction of two effects, nutrition and competition, interact across the three species to influence both the magnitude and the sign of the marginal value, or accounting price, of each individual grass stock, and thus the overall aggregate value of land. For the two stocks that both provide potential nutritional value and compete for energy given limited space (cheatgrass and native grass), the characterization of the stock as a "good" or a "bad" is state-dependent depending on which effect actually dominates. For the third (which offers nothing in the way of nutritional value for the only directly valuable species, cattle), the competition effects always dominate, and thus its marginal value is always negative.

There are several policy implications that result from these findings. First, analysis of invasive species management can be better accomplished by understanding the changes in the natural capital that arise with changes in composition. Eradication of invasive species is costly and the benefits, in terms of differences in natural capital values, may be small by comparison. Second, supporting ecosystem services or ecosystem functions have the potential to dominate final ecosystem services in terms of value. Not only do we care where the economy directly touches the ecosystem but indirectly through ecosystem function and this value may be greater than the use value.

Overall, this approach to natural capital valuation also provides additional insights into the economics of ecosystem function and the services that such systems provide. In particular, we typically think of ecosystem service values as positive; in our model, the natural example is

nutrition to cattle. But our results show that competition within a trophic layer, even by a beneficial species, can be harmful to the system as a whole, resulting in a negative marginal value for an individual species. This occurs because natural capital values and prices are dependent upon the dynamics of the system and not just the benefits that arise in any one period. Ignoring these complex dynamics can lead to very wrong conclusions about natural capital values, not only in magnitude but in sign as well. Just as with man-made capital, the value of the stock of natural capital is related to the flow of services that it provides; however, unlike man-made capital, the size and magnitude of appreciation or depreciation is endogenous to the system. Understanding the interconnections that drive this development using models such as GEEMs provide a promising way forward for the valuation of natural capital.

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